Holographic Spectral Functions in Metallic AdS/CFT

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Outline

A little light advertising

AdS/CFT and Novel Approaches to Hadron and Heavy Ion Physics (2010-10-11 To 2010-12-03)

Coordinators: Stanley Brodsky, Nick Evans, Hong Liu, Craig Roberts, Dam Son, Xin-Nian Wang, Urs Wiedemann, Jonathan Shock, Rong-gen Cai, Yu-xin Liu, En-ke Wang, Mei Huang, Peng-Fei Zhuang

Topics: AdS/CFT, Heavy ion physics, strongly coupled QCD

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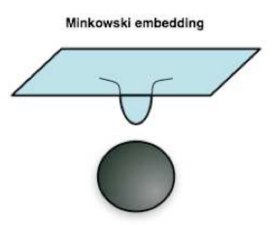
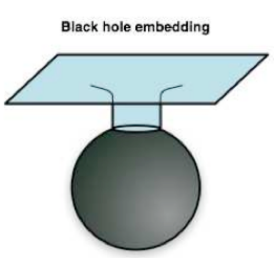
Motivation

- Studies of finite temperature theories from AdS/CFT have already given us some remarkable insight into the strongly coupled plasma
- Clearly there is a lot of phase space to study: baryon and isospin chemical potential, external EM fields, dispersive behaviour....
- Adding a finite electric field gives interesting behaviour at the classical embedding level
- We would like to understand more about the dynamics of such a system:
  - spectral functions
  - transport coefficients

Phase transitions in AdS/CFT

- Hawking Page (Confinement-Deconfinement) See talk by Charmousis
- Fundamental phase transition (discrete to continuous spectral function) See talk by Argyres and Benincasa





Figures from hep-th/0611099

- Magnetic field $\rightarrow$ chiral symmetry breaking phase transition.
- $B + T \rightarrow$ interesting phase structure $\supset$ second order phase transition.
- refs: (0701001, 0706.3811, 0709.1547, 0709.1551, 0709.1554, 0802.5345, 0903.5345) $\in$ {Albash, Bergman, Erdmenger, Filev, Johnson, Kundu, Lifshitz, Lippert, Meyer, Rashkov, JS, Viswanathan}

The setup

We have results for general Dp/Dq intersections but for illustration we will look at $D3/D7$:

Finite temperature AdS/CFT - $AdS_5 \times S^5$ with a non-extremal horizon:

$$ds_{10}^2 = r^2\left(-\left(1-\frac{r_h^4}{r^4}\right)dt^2 + dx_3^2\right) + \frac{dr^2}{r^2\left(1-\frac{r_h^4}{r^4}\right)} + \boxed{\frac{d\psi^2}{1-\psi^2}+\psi^2\,d\theta^2} + d\Omega_3^2$$

Non-dynamical flavour from a small number of D7 probe brane:

$$\mathcal{S}_{DBI} = -N_f\,T_{D7}\int_{D7} d^8\zeta\sqrt{-det(g_{ab}+2\pi\alpha' F_{ab})}$$

Finite baryon density+electric field+current (maybe)+excitations:

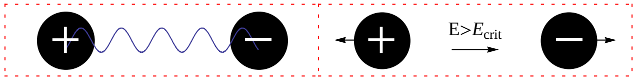
$$A = A_0(r)\,dx^0 + (E_z t + A_z(r))\,dx^z + \mathcal{A}_\perp(\mathbf{t},\mathbf{z},\mathbf{r})\mathbf{dx}_\perp$$

$$\frac{\delta S_{DBI}}{\delta A_0'} = \frac{n_q}{\Omega_n} \qquad \frac{\delta S_{DBI}}{\delta A_p'} = \frac{j_z}{\Omega_n}$$

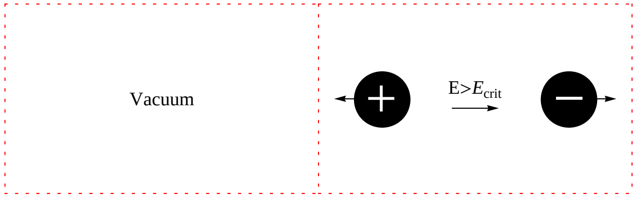
Predictions: Meson melting redux

- The meson modes on the D7-brane are made of quarks charged under a $U(1)_B$.
- Turn on an electric field: If the field is strong enough to pair create then the mesons will be ripped apart.
- Finite baryon number density $\rightarrow$ current for arbitrarily small fields
- With large enough electric field the current will be enhanced by pair dissociation/creation
- Once pair creation occurs and/or in the presence of finite baryon density the spectral functions should no longer be discrete

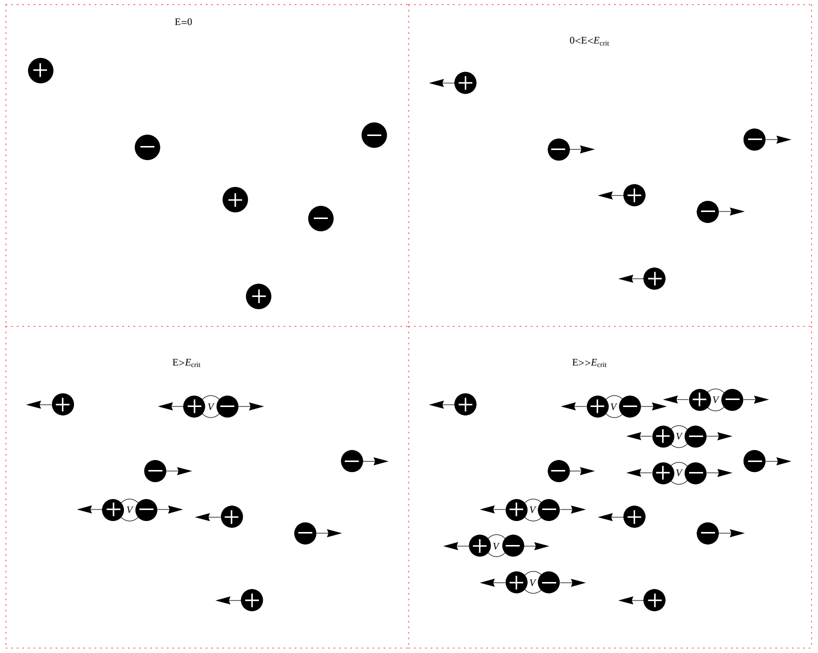
At zero baryon density we have bound states which are ripped apart when the field is large enough



With this electric field there can be pair production from the vacuum



At finite baryon density the story is a little more complicated



Example embeddings: Parameter space$(\tilde{E}, \tilde{n}_q, \tilde{m}_q)$

Classical embedding calculated from the following legendre transformed lagrangian:

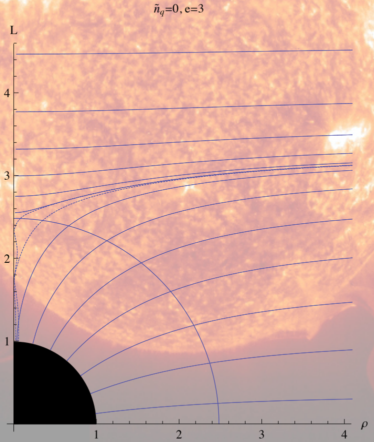
$$\tilde{\mathcal{L}} \sim \sqrt{-(g_{tt}g_{ii} + {E_z}^2)g_{ii}^3 g_{rr}}\sqrt{1 + \frac{g_{tt}{n_q}^2 + g_{ii}{j_z}^2}{g_{tt}g_{ii}^3}}$$

- factors including $g_{tt} \rightarrow$ zero at r_{ss} (singular shell) in the square roots
- Branes which don't have profiles which cut r_{ss} will not be effected
- Branes which do cut the singular shell need to have a tuned current: $j_z = j_z(E_z, m_q, n_q)$

Example embeddings: Parameter space $(\tilde{E}, \tilde{n}_q, \tilde{m}_q)$

Zero baryon density we have:

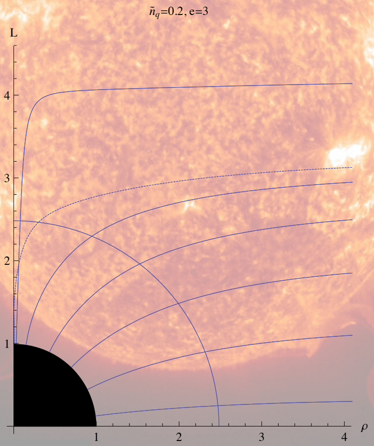
- Minkowski embeddings
- Singular shell embeddings:
  - conically singular embeddings
  - black hole embeddings



Example embeddings: Parameter space$(\tilde{E}, \tilde{n}_q, \tilde{m}_q)$

Finite baryon density we have:

- Only singular shell embeddings:
  - canonically singular embeddings
  - black hole embeddings



The conductivity

The current was constrained by regularising the singular shell. From this we can calculate the conductivity:

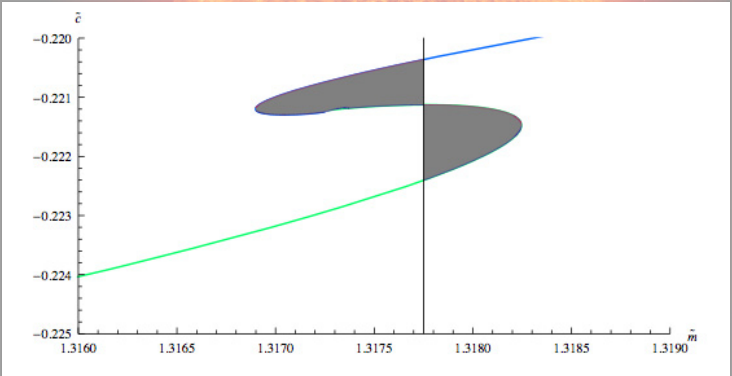
$$\sigma = \frac{j_z}{E_z} \sim \sqrt{\sqrt{1+{E_z}^2}(1-{\psi_{ss}}^2)^3 + \frac{{n_q}^2}{1+{E_z}^2}}$$

NB. Two terms coming from clear physics (Karch and O'Bannon 0705.3870)

We will use this result to check the microscopic calculation of the same quantity using the Kubo relation

Cross over versus first order phase transitions

For some regions of parameter space we find double valuedness in certain parameters near the black hole critical embedding.



Cross over versus first order phase transitions

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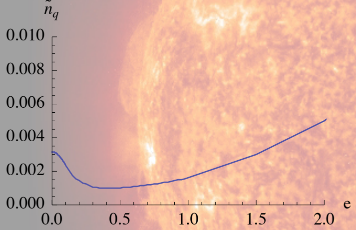
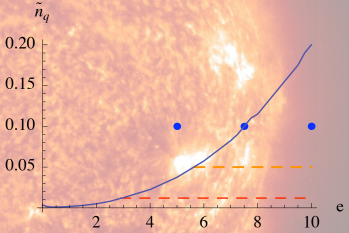
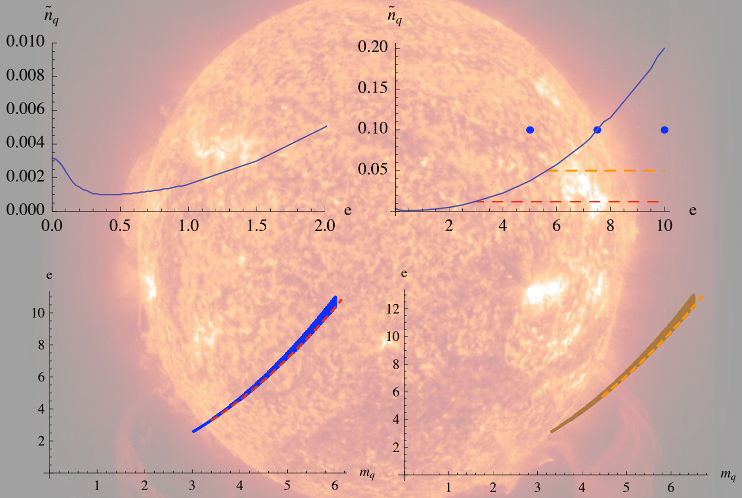




Above the line of second order phase transitions there are fast crossovers in behaviour. Below the line there are first order phase transitions

Cross over versus first order phase transitions



It appears that the conical embeddings are related to, but not equal to, a series of unstable embeddings.

Spectral functions from AdS/CFT

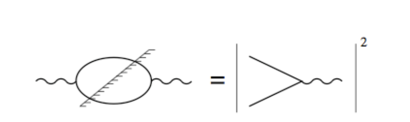
By looking at perturbations on top of the classical embedding we can calculate the spectral function of the different operators of the theory. From this we can calculate many quantities:

- Meson masses and decay widths when there is a quasiparticle interpretation
- Transport properties from Kubo relations: susceptibility, diffusion, conductivity, viscosity, etc.
- photoproduction when we treat the system as weakly coupled to a photon

Spectral functions from AdS/CFT

Not only can we look at the phase structure, but we can try to understand the dynamics and stability by looking at the spectral function.

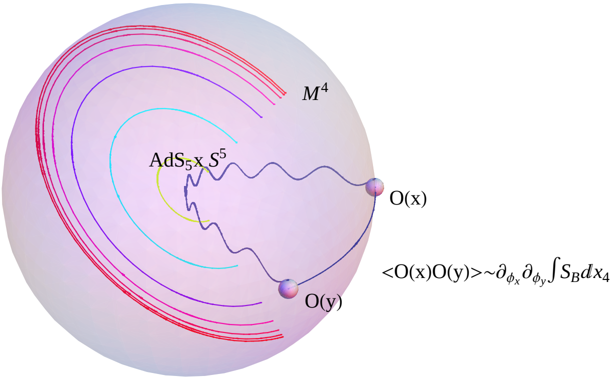
$$\chi^{\mu}_{\mu} \xrightarrow{\text{optical theorem}} -2ImG^{R}_{\mu\nu} :$$



$$-2ImG^{R}_{\mu\nu} \longrightarrow \Big\langle J_{\perp,||}(k) J_{\perp,||}(-k) \Big\rangle$$

$$\Big\langle J_{\perp,||}(k) J_{\perp,||}(-k) \Big\rangle \longrightarrow \boxed{\frac{\delta \int d^4\zeta \mathcal{L}_{D7}|_{\text{boundary action}}}{\delta A_{\perp,II}(k)_b \delta A_{\perp,ll}(-k)_b}}$$

Graphically speaking



Excitations

- We need to set up certain boundary conditions to look at the spectral function
- Set up the boundary conditions on the singular shell and on the horizon with a Frobenius expansion
- two possible indices at each singular point:

$$\eta^h_{\pm} = \frac{1 \pm 1}{2} - \frac{i\omega}{4\pi T} \qquad \eta^{ss} = (0, func(E, \omega, n_q, \psi_0))$$

- the last function is very complicated, but in the $E \to 0$ limit goes to $\frac{i\omega}{2\pi T}$.

Excitations

- We use two criteria to work out the correct combination of indices:

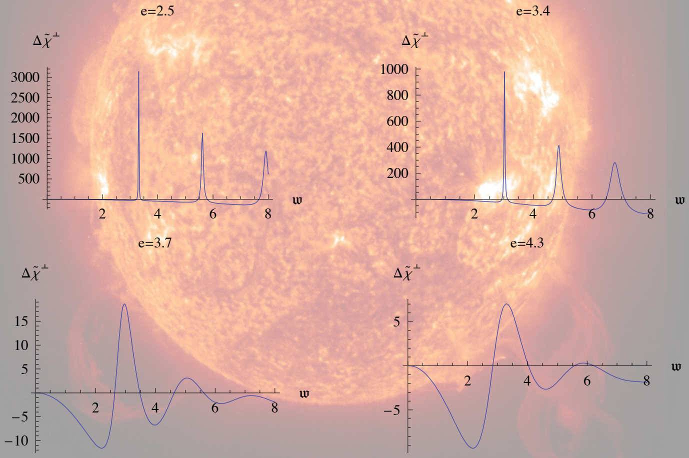
$\eta^{(lim)}$	η^h_+	η^h_-
η^{ss}_0	$1 - i\,\frac{\omega}{4\pi T}$	$-i\,\frac{\omega}{4\pi T}$
η^{ss}_r	$1 + i\,\frac{\omega}{4\pi T}$	$i\,\frac{\omega}{4\pi T}$

Table: *The four possibilities for the index on the horizon, $\eta^{(lim)}$, in the limit of vanishing electric field.*

- The correct limiting behaviour is given by $\eta^h_- \eta^{ss}_0$
- Use the conductivity calculation as a cross-check.
- This can be done analytically in the hydrodynamic approximation: **gives exactly the same answer as the Karch, O'Bannon macroscopic calculation**.

Spectral functions

- Calculate the spectral function and see what is happening across the region of conical embeddings/phase transition. Quasiparticle$\rightarrow$ non-quasipartile transition.



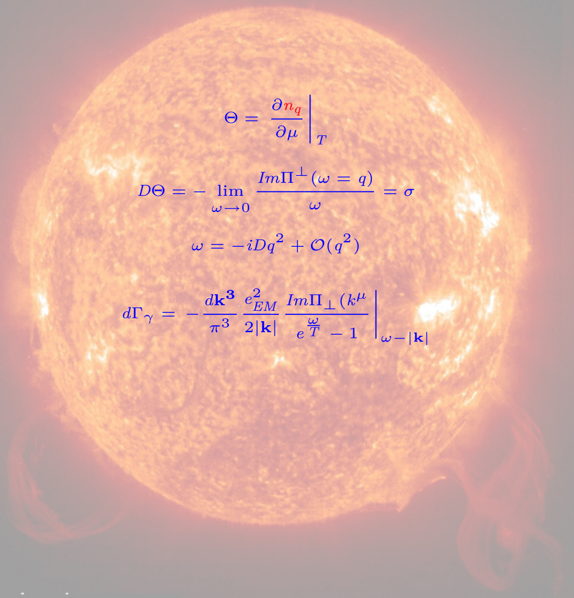
Spectral functions

- We can calculate the photoproduction rate (with a caveat) and see a large jump in the rate across the phase transition region.
- It is tempting to think that the width of spectral functions are directly related to the size of the induced horizon but this is not so...
- The induced horizon size does not increase monotonically with the induced singular shell area, which does seem to set the lifetime of the mesons.

Conclusions

- There is a region where the behaviour in the system changes rapidly
- This can be a first order, second order or cross over transition
- Closely associated with this region are a series of conical embeddings, possibly unstable
- A full thermodynamic study of the phase space has yet to be completed
- The singular shell has an attractor-like mechanism, meaning that what is happening inside and outside are completely independent. Any force applied to the brane inside the singular shell will not be felt outside - this may not be true in the unquenched approximation.

Thank You!



$$\Theta = \left.\frac{\partial n_q}{\partial \mu}\right|_T$$

$$D\Theta = -\lim_{\omega \to 0} \frac{Im\Pi^{\perp}(\omega = q)}{\omega} = \sigma$$

$$\omega = -iDq^2 + \mathcal{O}(q^2)$$

$$d\Gamma_\gamma = -\frac{d\mathbf{k}^3}{\pi^3}\frac{e_{EM}^2}{2|\mathbf{k}|}\left.\frac{Im\Pi_{\perp}(k^\mu}{e^{\frac{\omega}{T}} - 1}\right|_{\omega - |\mathbf{k}|}$$