TORSION AND THE GRAVITY DUAL OF PARITY SYMMETRY BREAKING IN ADS4/CFT3

Based on:
JHEP 0903:033 with N. N. Hoang, and R. G. Leigh
and
on work in progress with N. N. Hoang, R. G. Leigh and D. Minic

Tassos Petkou
UoC

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$$I_{EH} \equiv -16\pi G_4 S_{EH} = \int \left(R^{ab} \wedge e^c \wedge e^d - \frac{\Lambda}{6} e^a \wedge \cdots \wedge e^d \right) \epsilon_{abcd}$$

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WITH THE USUAL DEFINITIONS FOR THE VIELBEIN AND THE SPIN-CONNECTION

$$R^{ab} = d\omega^{ab} + \omega^a{}_c \wedge \omega^{cb}\,, \;\; T^a = de^a + \omega^a{}_b \wedge e^b \qquad (a, b = 0, 1, 2, 3)$$

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TO MAKE CONTACT WITH THE METRIC FORMALISM WE NOTE:

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THE 3+1 SPLIT IS A REFINED ADM FORMULATION FOR 4-D GRAVITY:

WE ASSUME A LOCAL 3-D SLICING AND SPLIT EVERYTHING ACCORDINGLY

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\begin{aligned}
e^a \quad &\rightarrow \quad e^0 = N dt \;\; e^i = \tilde{\epsilon}^i + N^i dt \\
\omega^{ab} \quad &\rightarrow \quad \omega^0{}_i = \sigma_\perp K_i + q^0{}_i dt\,, \;\; \omega^i{}_j = \sigma \epsilon^i{}_{jk}(B^k + Q^k dt)
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$$I_{GH} = 2\sigma_\perp \int_{\partial\mathcal{M}} \epsilon_{ijk}\, K^i \wedge \tilde{\epsilon}^j \wedge \tilde{\epsilon}^k$$

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“GAUSS LAW” $(\tilde{D}K)_i \equiv (\tilde{d}K_i + \epsilon_i{}^{jk} B_k \wedge K_j)$

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THE (SYMMETRIC) PART OF THE "MAGNETIC FIELD" GIVES A CONSTRAINT

$$\frac{\delta}{\delta B_i} I = 0 \Rightarrow \tilde{T}^i = \tilde{d}\tilde{\epsilon}^i - \sigma \epsilon^i_{\ jk} B^j \wedge \tilde{\epsilon}^k = 0 \qquad \left[\vec{B} = \vec{\nabla} \times \vec{A}\right]$$

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SPATIAL TORSION CARRIES THE NON-DYNAMICAL "GRAVITATIONAL MAGNETIC" D.O.F.

THE ZERO TORSION CONDITION RELATES THEM TO THE "DREIBEIN".

THE ANNOUNCED TALK WOULD HAVE CONTINUED AS...

Example 1: Electromagnetism

$$I = \int dt \wedge \left\{ \dot{A} \wedge *_3 E - \frac{1}{2}\left(E \wedge *_3 E + B \wedge * B\right) - A_0 \tilde{d} *_3 E \right\}, B = *_3 \tilde{d} A$$

$$E \mapsto -B\,,\ B = *_3 \tilde{d} A \mapsto *_3 E$$

$$I \mapsto \int dt \wedge \left\{ -\dot{A}_D \wedge *_3 B - \frac{1}{2}\left(E \wedge *_3 E + B \wedge * B\right) + A_0 \tilde{d} *_3 B \right\}, \tilde{d} A_D = *_3 E$$

The Gauss Law maps to the Bianchi identity. Then, we can write:

$$I \mapsto I_D = I - \int A_D \wedge \tilde{d} A$$

The boundary modification is a Chern-Simons term.

The q-constraints give:

$$q^{\alpha\beta} \Rightarrow K_{[\alpha,\beta]} = 0$$

and also:

$$\epsilon_{\alpha\beta\gamma}\tilde{T}^{\beta} \wedge \tilde{e}^{\gamma} = \epsilon_{\alpha\beta\gamma}\tilde{d}\tilde{e}^{\beta} \wedge \tilde{e}^{\gamma} - \sigma_{\perp} B_{\beta} \wedge \tilde{e}^{\beta} \wedge \tilde{e}_{\alpha} = 0$$

We require that the latter transforms like a vector under SO(3) rotations of the dreibein. The magnetic field term in an obstruction.

$$B_{[\alpha,\beta]} = \epsilon_{\alpha\beta}{}^{\gamma} V_{\gamma}\,, \quad V = V_{\alpha}\tilde{e}^{\alpha}\,, \quad \tilde{e}^{\alpha} \mapsto \Lambda^{\alpha}{}_{\beta}\tilde{e}^{\beta}$$

The choice:

$$\left(\Lambda^{-1}\right)^{\gamma}{}_{\beta}\,\tilde{d}\Lambda^{\alpha}{}_{\gamma} = \sigma_{\perp} V \delta^{\alpha}_{\beta} \qquad \Rightarrow \quad B_{[\alpha,\beta]} = 0$$

and shows that the antisymmetric part of the magnetic field is a gauge d.o.f.

This is equivalent to choosing the de-Donder gauge:

$$\epsilon_{\alpha\beta\gamma}\tilde{d}\tilde{e}^{\alpha} \wedge \tilde{e}^{\gamma} = 0$$

However, since the magnetic field does not appear in the kinetic term, its variation gives an algebraic equation; the zero-torsion equation.

$$\tilde{T}^{\alpha} = \tilde{d}\tilde{e}^{\alpha} + \epsilon^{\alpha\beta\gamma} B_{\beta} \wedge \tilde{e}_{\gamma} = 0$$

Only the symmetric part of the magnetic field contributes to that.

Next use the shifted electric field:

$$\hat{K}^{\alpha} = K^{\alpha} = \rho\tilde{e}^{\alpha} \ , \ \rho^2 = \sigma_{\perp}\Lambda$$

For, symmetric electric and magnetic fields and zero torsion we get:

$$I_{HP} = \int dt \wedge \Big\{ \dot{\tilde{e}}^{\alpha} \wedge \hat{\Pi}_{\alpha} - 4\sigma_{\perp} N^{\alpha} \epsilon_{\alpha\beta\gamma} (\tilde{D}\hat{K})^{\beta} \wedge \tilde{e}^{\gamma}$$
$$-2\sigma_{\perp} N \epsilon_{\alpha\beta\gamma} \Big(B^{\alpha} \wedge B^{\beta} + \hat{K}^{\alpha} \wedge \hat{K}^{\beta} + 2\rho\hat{K}^{\alpha} \wedge \tilde{e}^{\beta} \Big) \wedge \tilde{e}^{\gamma} \Big] \Big\}$$

Linearize as:

$$\tilde{e}^\alpha = \underline{\tilde{e}}^\alpha + E^\alpha \,, \;\; N = 1 + n \,, \;\; N^\alpha = n^\alpha$$

$$B^\alpha = \underline{B}^\alpha + b^\alpha \,, \;\; \hat{K}^\alpha = \underline{\hat{K}}^\alpha + k^\alpha$$

Make an educated guess for a nice background i.e. the vacuum:

$$\underline{B}^\alpha = 0 = \underline{\hat{K}}^\alpha$$

The action becomes:

$$I_{HP} = \int dt \wedge \Big\{ (\dot{E}^\alpha + \rho E^\alpha \wedge p_\alpha - 2\sigma_\perp \epsilon_{\alpha\beta\gamma}(b^\alpha \wedge b^\beta + k^\alpha \wedge k^\beta) \wedge \underline{\tilde{e}}^\gamma$$
$$-4\sigma_\perp \eta^\alpha \epsilon_{\alpha\beta\gamma} \tilde{d} k^\beta \wedge \underline{\tilde{e}}^\gamma + n(4\sigma_\perp \tilde{d} b_\gamma + \rho p_\gamma) \wedge \underline{\tilde{e}}^\gamma \Big\}$$

$$p_\alpha = -4\sigma_\perp \epsilon_{\alpha\beta\gamma} k^\beta \wedge \underline{\tilde{e}}^\gamma$$

The vanishing of the linear terms gives:

$$\dot{\underline{\tilde{e}}}^\alpha + \rho \underline{\tilde{e}}^\alpha = 0$$

This is solved by (A)dS4:

$$\underline{\tilde{e}}^0 = dt\,, \quad \underline{\tilde{e}}^\alpha = e^{-\rho t} dx^\alpha\,, \quad \underline{K}^\alpha = \rho \underline{\tilde{e}}^\alpha$$

$$\underline{Ric}_{ab} = -\frac{3\sigma_\perp}{L^2}\eta_{ab} \;\;, \;\; \underline{R} = -\frac{12\sigma_\perp}{L^2}$$

Finally - the duality map:

$$k^\alpha \mapsto -b^\alpha\,, \quad b^\alpha \mapsto k^\alpha \qquad\qquad E \mapsto \mathcal{E}\;,\; p \mapsto -p_D$$

$$\epsilon^{\alpha\beta\gamma} b_\beta \wedge \underline{\tilde{e}}_\gamma + \tilde{d}E^\alpha \mapsto \epsilon^{\alpha\beta\gamma} k_\beta \wedge \underline{\tilde{e}}_\gamma + \tilde{d}\mathcal{E}^\alpha = 0$$

The action dualizes to:

$$I \mapsto I_D = \int dt \wedge \Big\{ -\dot{\mathcal{E}}^\alpha \wedge p_{D,\alpha} - \rho \tilde{E}^\alpha \wedge p_{D,\alpha}$$
$$-2\sigma_\perp \epsilon_{\alpha\beta\gamma}(b^\alpha \wedge b^\beta + k^\alpha \wedge k^\beta) \wedge \underline{\tilde{e}}^\gamma$$
$$+4\sigma_\perp n^\alpha \epsilon_{\alpha\beta\gamma} \tilde{d} b^\beta \wedge \underline{\tilde{e}}^\gamma + n(4\sigma_\perp \tilde{d} k_\gamma + \rho p_{D,\alpha}) \wedge \underline{\tilde{e}}^\gamma \Big\}$$

This differs from the initial action by $\rho \mapsto -\rho$

Nevertheless, this does not affect the second order e.o.m.

The constraints also dualize to:

$$C_\alpha \equiv \epsilon_{\alpha\beta\gamma} \tilde{d} k^\beta \wedge \underline{e}^\gamma \mapsto -\epsilon_{\alpha\beta\gamma} \tilde{d} b^\beta \wedge \underline{\tilde{e}}^\gamma$$

$$C_0 \equiv -\sigma_\perp (\tilde{d} b_\gamma - \rho \epsilon_{\alpha\beta\gamma} k^\alpha \wedge \underline{\tilde{e}}^\beta) \wedge \underline{\tilde{e}}^\gamma \mapsto -\sigma_\perp (\tilde{d} k_\gamma + \rho \epsilon_{\alpha\beta\gamma} b^\alpha \wedge \underline{\tilde{e}}^\beta) \wedge \underline{\tilde{e}}^\gamma$$

Recall the linearized Bianchi identities:

$$B_T^\alpha = -\epsilon_{\alpha\beta\gamma}\tilde{d}b^\beta \wedge \underline{\tilde{e}}^\gamma + \cdots$$

$$B_T^0 = -\sigma_\perp(\tilde{d}k_\alpha + \rho\epsilon_{\alpha\beta\gamma}b^\beta \wedge \underline{\tilde{e}}^\gamma) \wedge \underline{\tilde{e}}^\alpha + \cdots = 0$$

The duality maps the constraints into the Bianchi identities.

$$C_\alpha \mapsto B_{T,\alpha}\,,\; C_0 \mapsto B_T^0 \qquad B_{T,\alpha} \mapsto -C_\alpha\,,\; B_T^0 \mapsto -C_0$$

Lastly, we notice that the modified duality transformations;

$$k^\alpha \mapsto -b^\alpha - 2\rho\mathcal{E}^\alpha\,,\;\; b^\alpha \mapsto k^\alpha$$

Leave the action invariant, up to additional terms in the constraints.

$$-8\rho n^\alpha k_\beta \wedge \underline{\tilde{e}}_\alpha \wedge \underline{\tilde{e}}^\beta \qquad\qquad 8n\Lambda\epsilon_{\alpha\beta\gamma}\mathcal{E}^\beta \wedge \underline{\tilde{e}}^\gamma \wedge \underline{\tilde{e}}^\alpha$$

Using the relationship between the dual dreibein and the electric field;

$$\mathcal{E}^\alpha_{\ \beta} = \frac{1}{\partial^2}\epsilon^\alpha_{\ \delta\gamma}\partial^\gamma k^\delta_{\ \beta}$$

we can show that the additional terms vanish. Hence, gravity with a c.c. requires a modified duality transformation.

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$$I = -c\int_\epsilon^\infty dr \int d^3\vec{x}\left[E_i\partial_r A_i - \frac{1}{2}(E_iE_i - B_iB_i)\right]\ , \quad B_i = \epsilon_{ijk}\partial_j A_k$$

$$\delta I_{on\ shell} = -c \int d^3\vec{x} E_i(r,\vec{x}) \delta A_i(r,\vec{x}) \Big|_{\epsilon}^{\infty} \ + \ \text{e.o.m} \ + \ \text{Dirichlet B.C.}$$

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Two bulk theories

(in terms of the two sets of variables)

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Two bulk theories (in terms of the two sets of variables) $\longrightarrow$ Two boundary 1-pt functions (responses)

$$\langle J_i(\vec{x}) \rangle_{A_i} = E_i(\vec{x}) \equiv \mathrm{i}\tilde{B}_i(\vec{x}) = \mathrm{i}\epsilon_{ijk}\partial_j \tilde{A}_k(\vec{x})$$

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generalizes to e.m. tensor and higher-spin currents [TCP,& Leigh (03-04)]

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$$T^{A} = 0 \Rightarrow de^{A} + \omega^{A}_{\ B} \wedge e^{B} = 0$$

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WE OBTAIN:

$$I_{EH} + I_{GH} + I_{NY} = \int dt \wedge \Big[\dot{\tilde{\epsilon}}^i \wedge \left(-4\sigma_\perp \epsilon_{ijk} \left[K^j - \theta B^j\right] \wedge \tilde{\epsilon}^k\right) + 2\sigma_\perp \theta \epsilon_{ijk} \dot{B}^i \wedge \tilde{\epsilon}^j \wedge \tilde{\epsilon}^k + \cdots \Big]$$

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IMPLICATIONS FOR 2+1D FLUID DYNAMICS (IN PROGRESS..)

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THE SIMPLEST CASE IS:

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FROM THE SECOND E.O.M. WE FIND

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NOTICE THE - SIGN, CONSISTENT

WITH A EUCLIDEAN PSEUDOSCALAR

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$$*_4 dF = \frac{2}{3} H$$

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USING THE E.O.M FOR THE TWO-FORM POTENTIAL (KALB-RAMOND)

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SPACETIME-DEPENDENT COUPLINGS FOR TOPOLOGICAL INVARIANTS BRING ON ADDITIONAL D.O.F. INTO THE GAME: THEY ENLARGE THE HOLOGRAPHIC APPLICATIONS.

(NO NEED FOR KINETIC TERMS - CONTRARY TO PECCEI-QUINN)

THE TORSION DOMAIN WALL

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LOOK FOR AN EXACT SOLUTION OF THE FORM ("DOMAIN WALL ANSATZ")

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INTEGRATION CONSTANT

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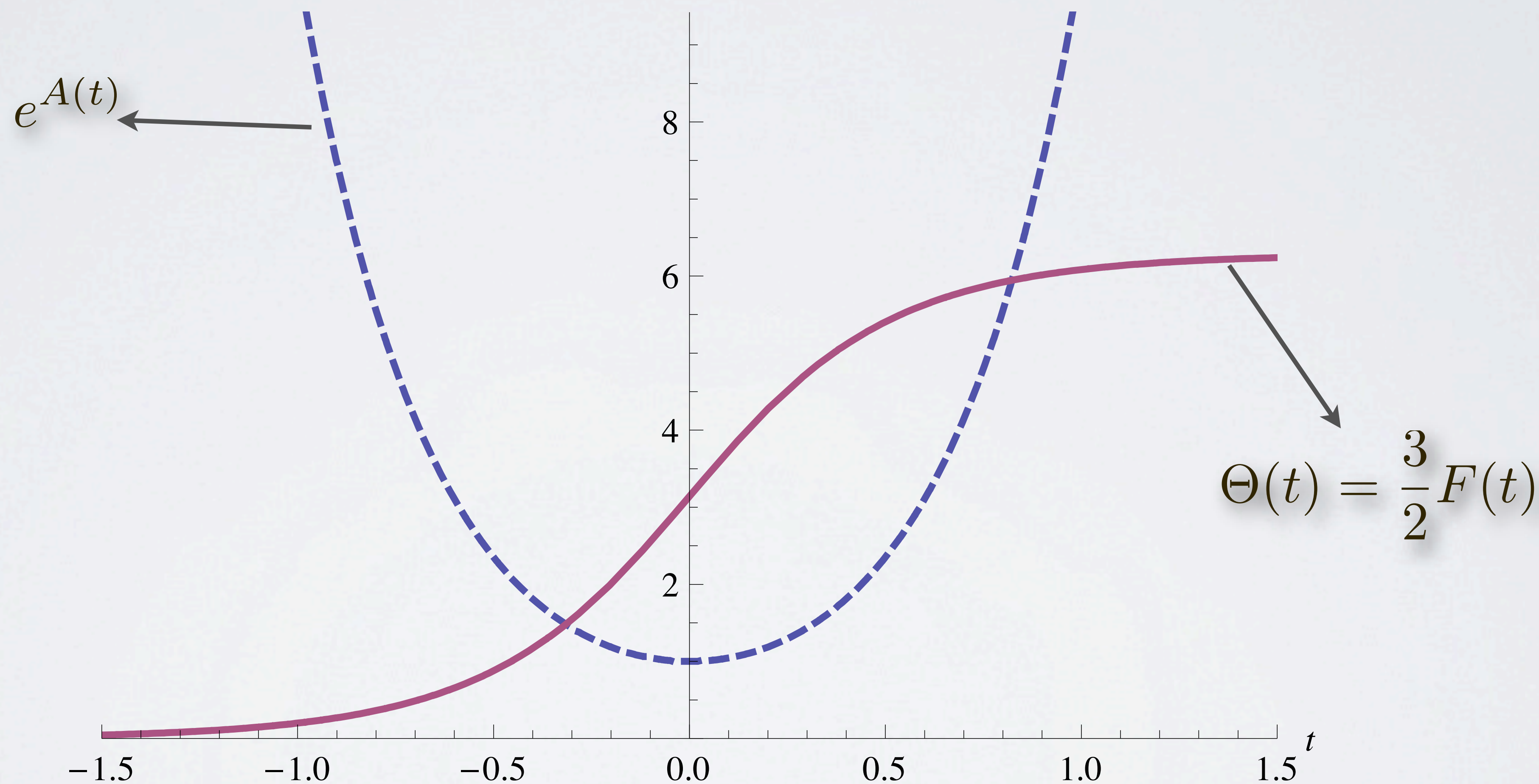

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ONE FOR EACH ASYMPTOTIC ADS4 REGION

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HOLOGRAPHY OF THE TORSION DOMAIN WALL

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VACUUM

HOLOGRAPHY OF THE TORSION DOMAIN WALL

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NO TUNNELING IN 3D AT INFINITY VOLUME

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EXPAND THE AUXILIARY FIELD AS $\sigma = \sigma_* + \frac{1}{\sqrt{N}}\lambda$ TO GET THE GAP EQUATION

$$\frac{1}{G} = \int^\Lambda \frac{d^3\vec{p}}{(2\pi)^3}\frac{2}{\vec{p}^2 + \sigma_*} = (Tr\mathbf{1})\left[\frac{\Lambda}{\pi^2} - \frac{|\sigma_*|}{\pi^2}\arctan\frac{\Lambda}{|\sigma_*|}\right]$$

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JUST ADD $S_{CS}^{def} = -iq \int A \wedge dA \, , \quad q = \frac{Ne^2}{4\pi}$ TO THE $\sigma_* = m$ VACUUM

AND THE FERMIONIC INTEGRATION WILL LEAD TO THE $\sigma_* = -m$ VACUUM.

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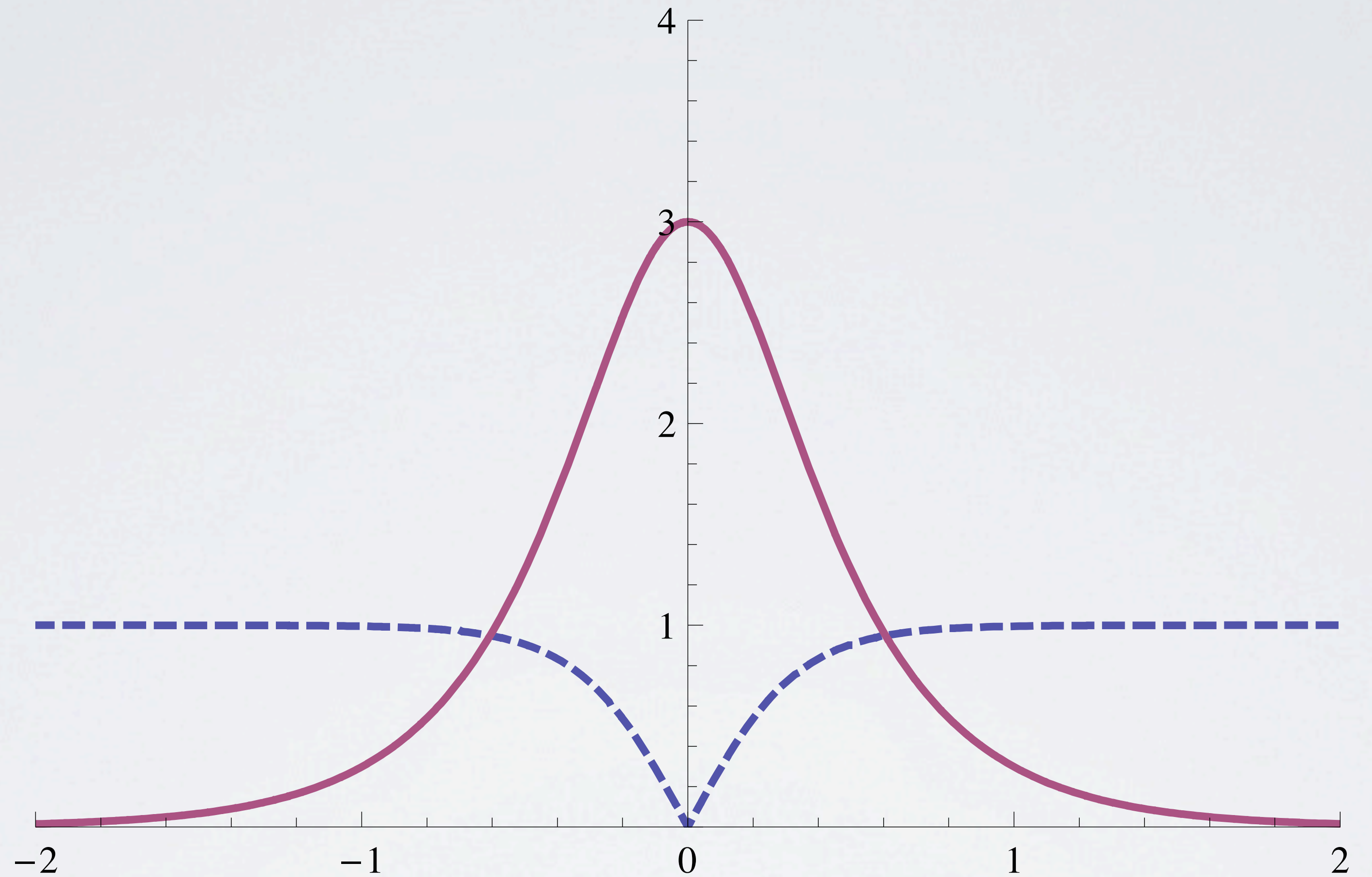

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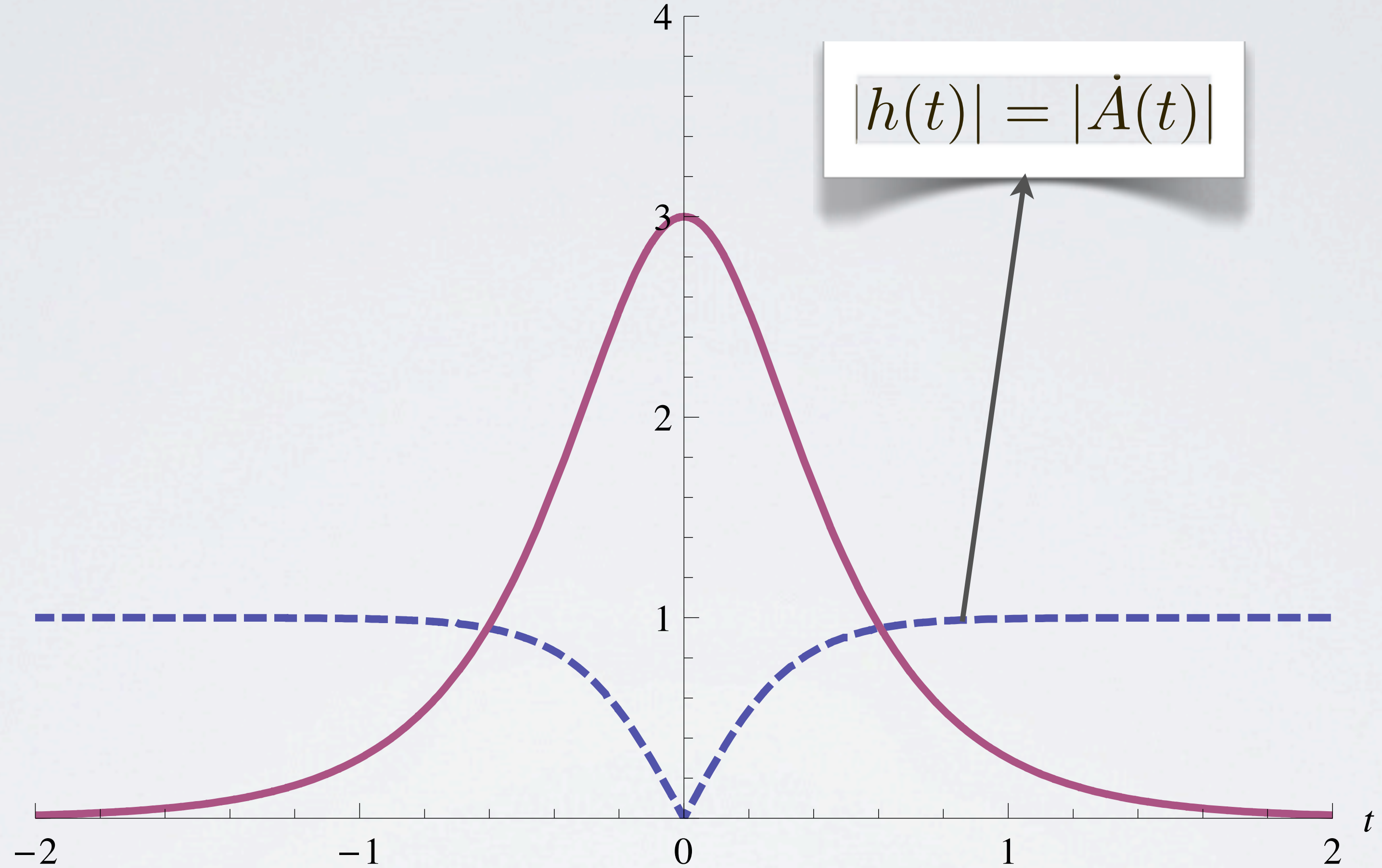

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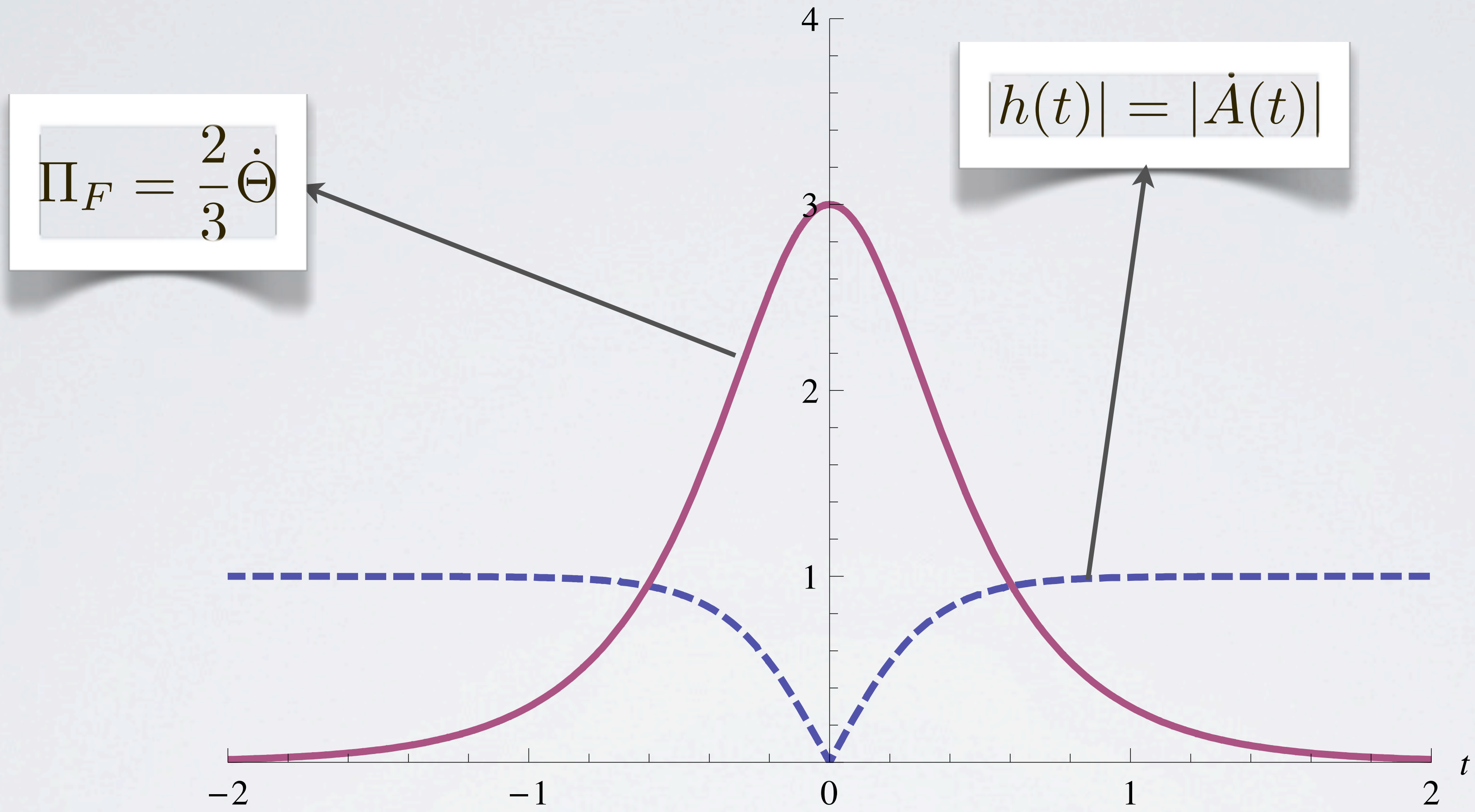

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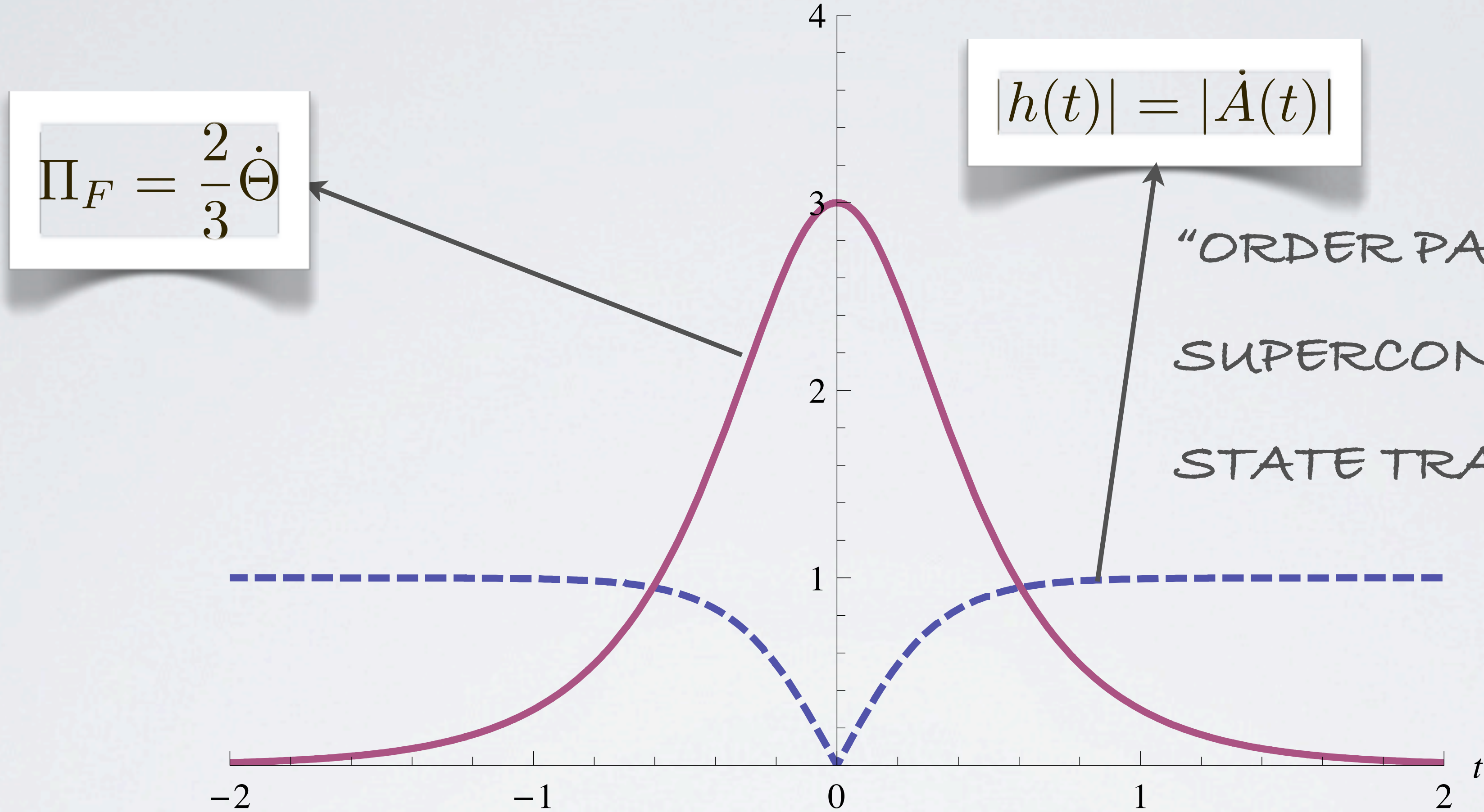

"ORDER PARAMETER" FOR THE SUPERCONDUCTING/NORMAL STATE TRANSITION

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$$|h(t)| = |\dot{A}(t)|$$

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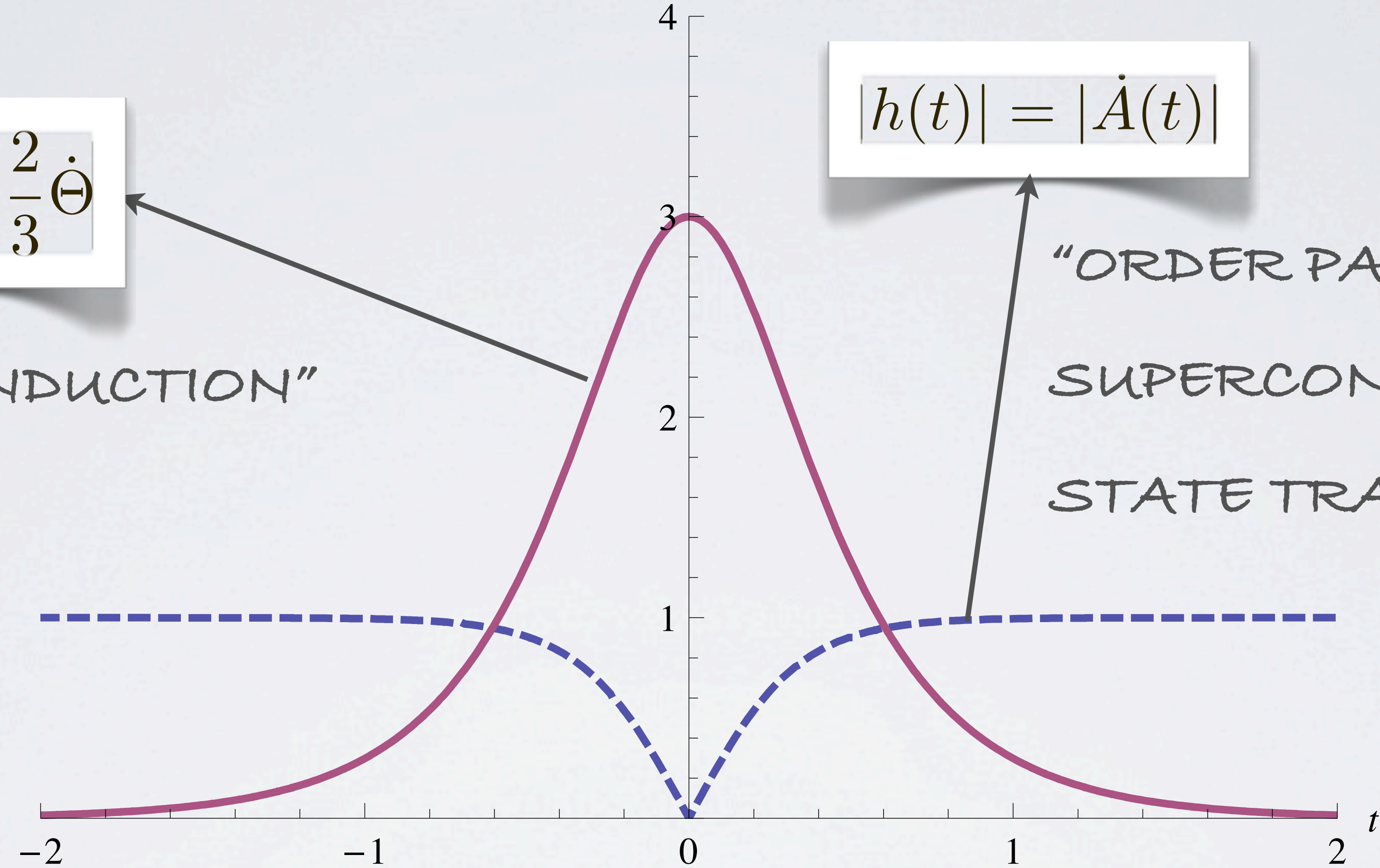

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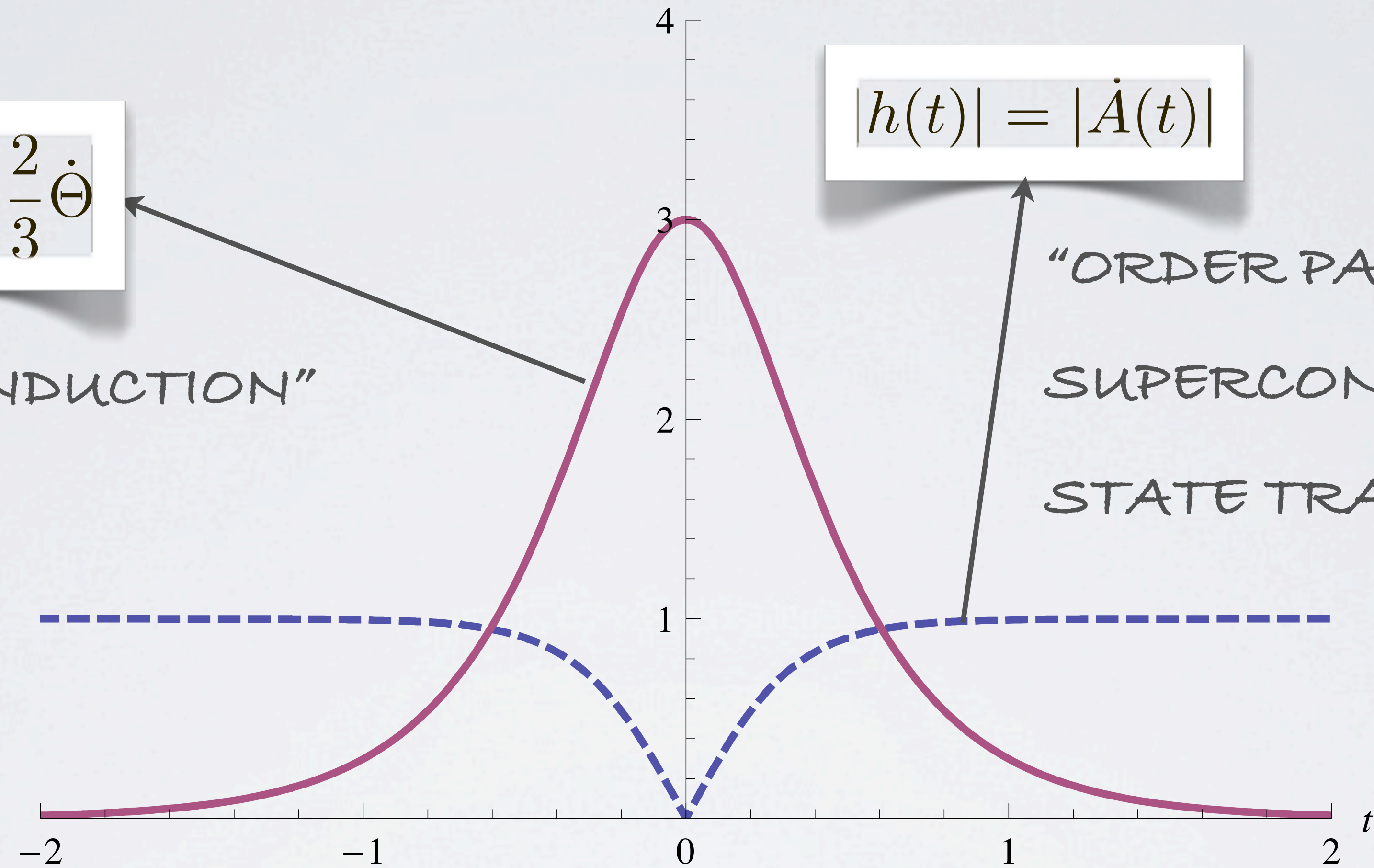

THE TORSION DW RESEMBLES TWO SUPERCONDUCTING REGIONS (ASYMPTOTICALLY ADS4 SPACES) JOINED BY A NORMAL-STATE REGION (FLAT SPACE). PERHAPS A JOSEPHSON JUNCTION?

THE SPACE AT $\underline{t=0}$ IS FLAT. I.E. WE THINK IN TERMS OF A PSEUDOSCALAR COUPLED TO GRAVITY

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FLAT SPATIAL SLICE

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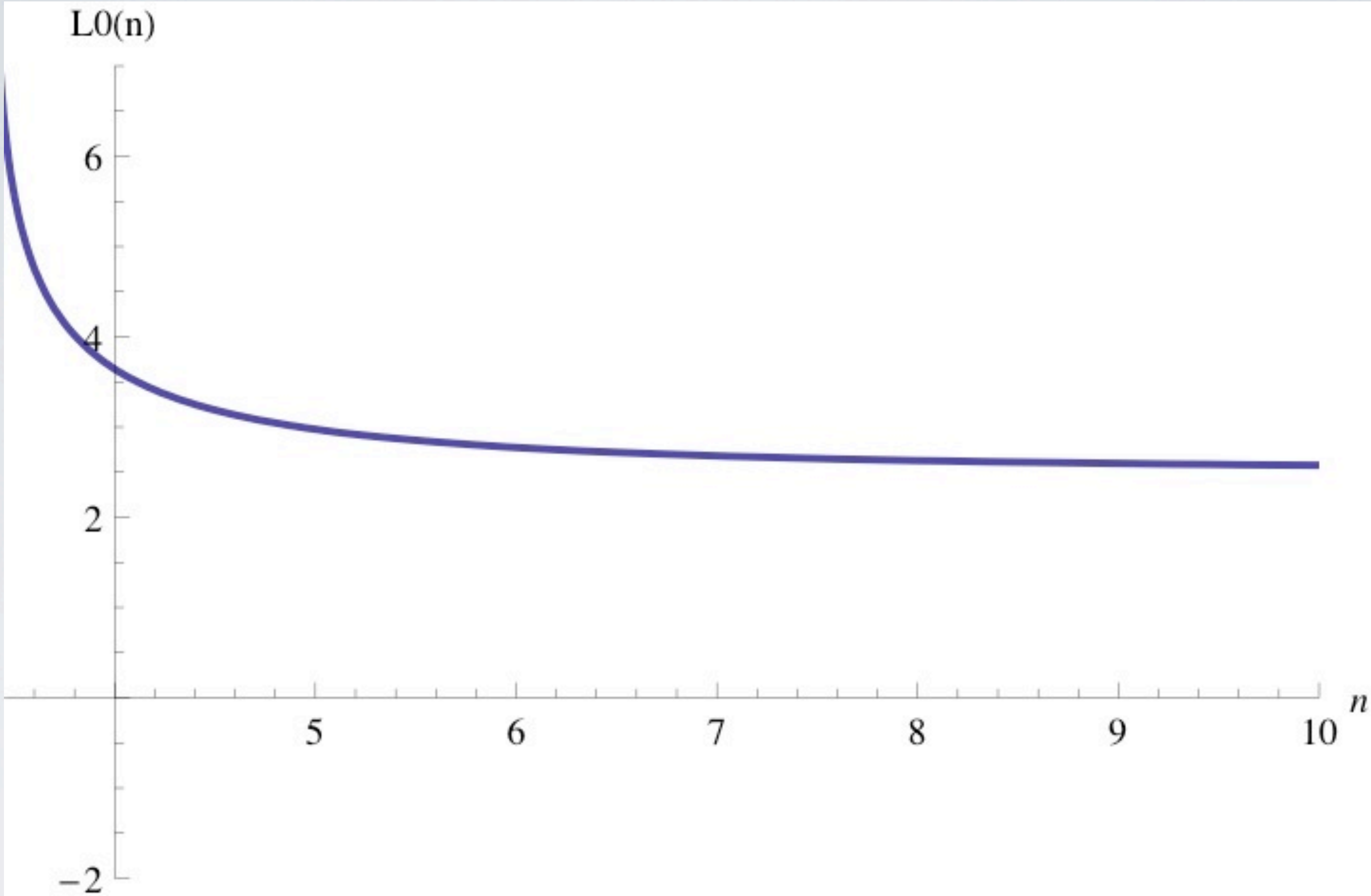
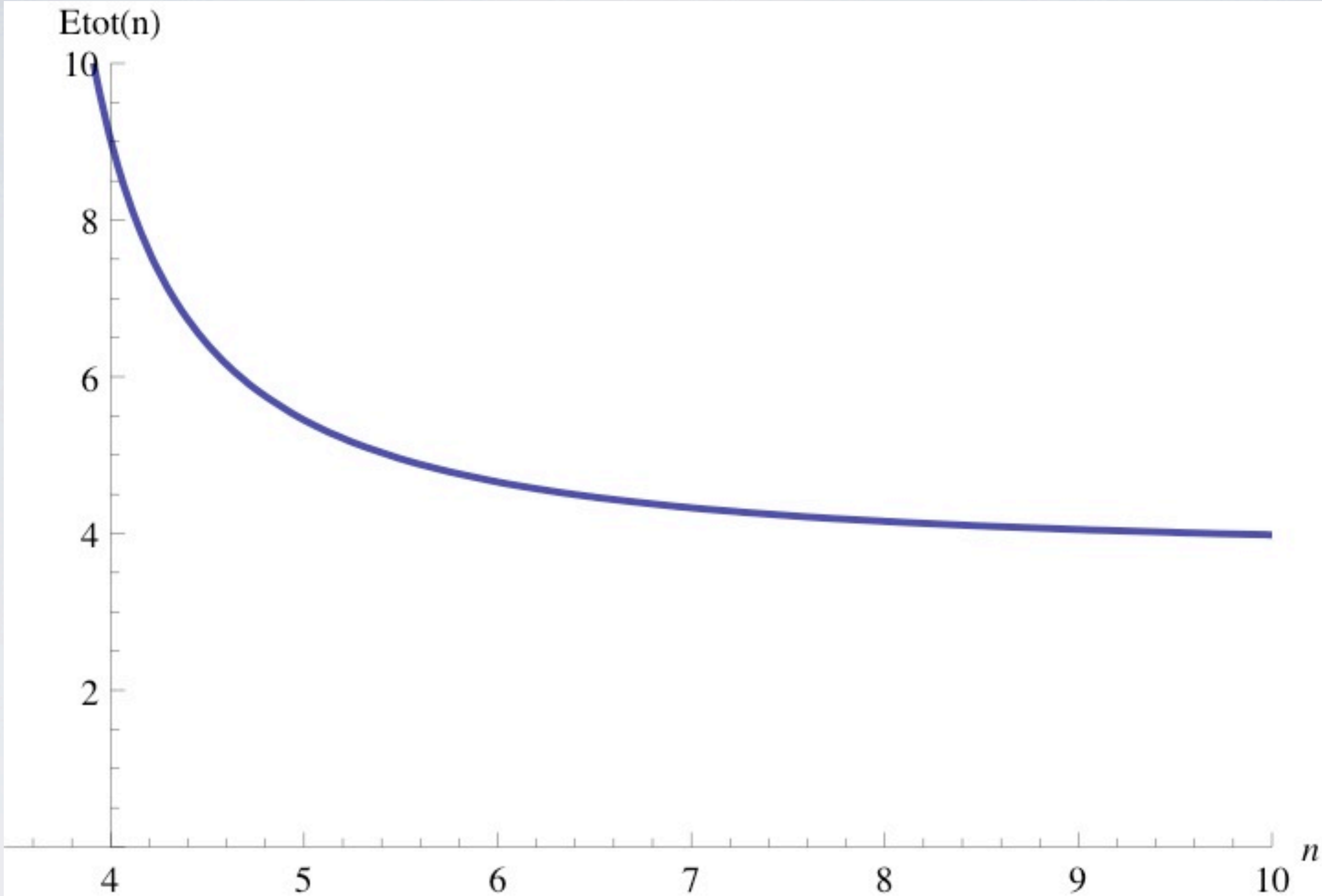
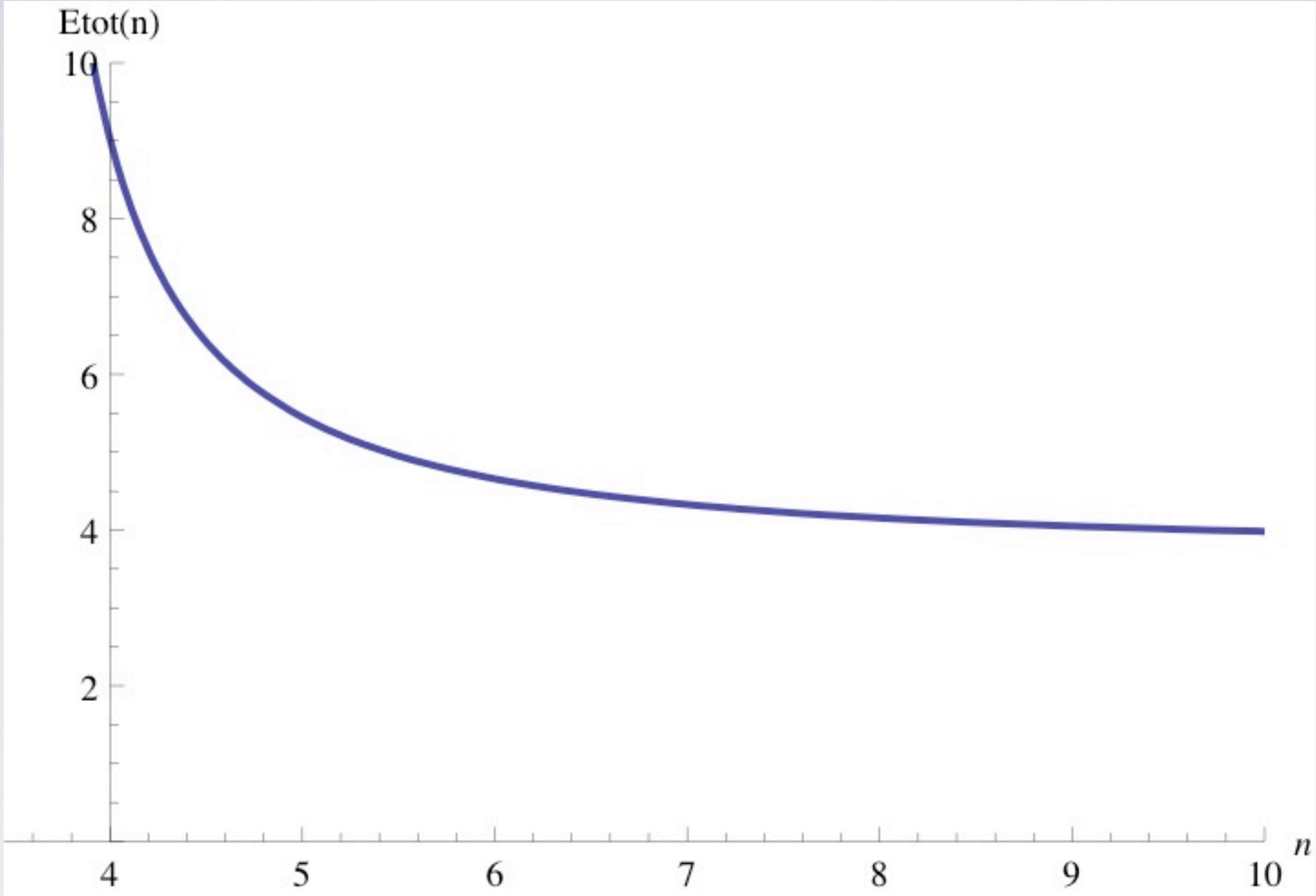
PROCEED AS ABOVE TO CALCULATE THE FLUX QUANTIZATION CONDITION. WE OBTAIN A RELATIONSHIP BETWEEN THE NUMBER OF DWS, THE MAGNETIC FIELD AND THE SIDE OF THE "DROPLET".



L0(n)
6
4
2
−2
5
6
7
8
9
10
n



Etot(n)
10
8
6
4
2
n
4
5
6
7
8
9
10



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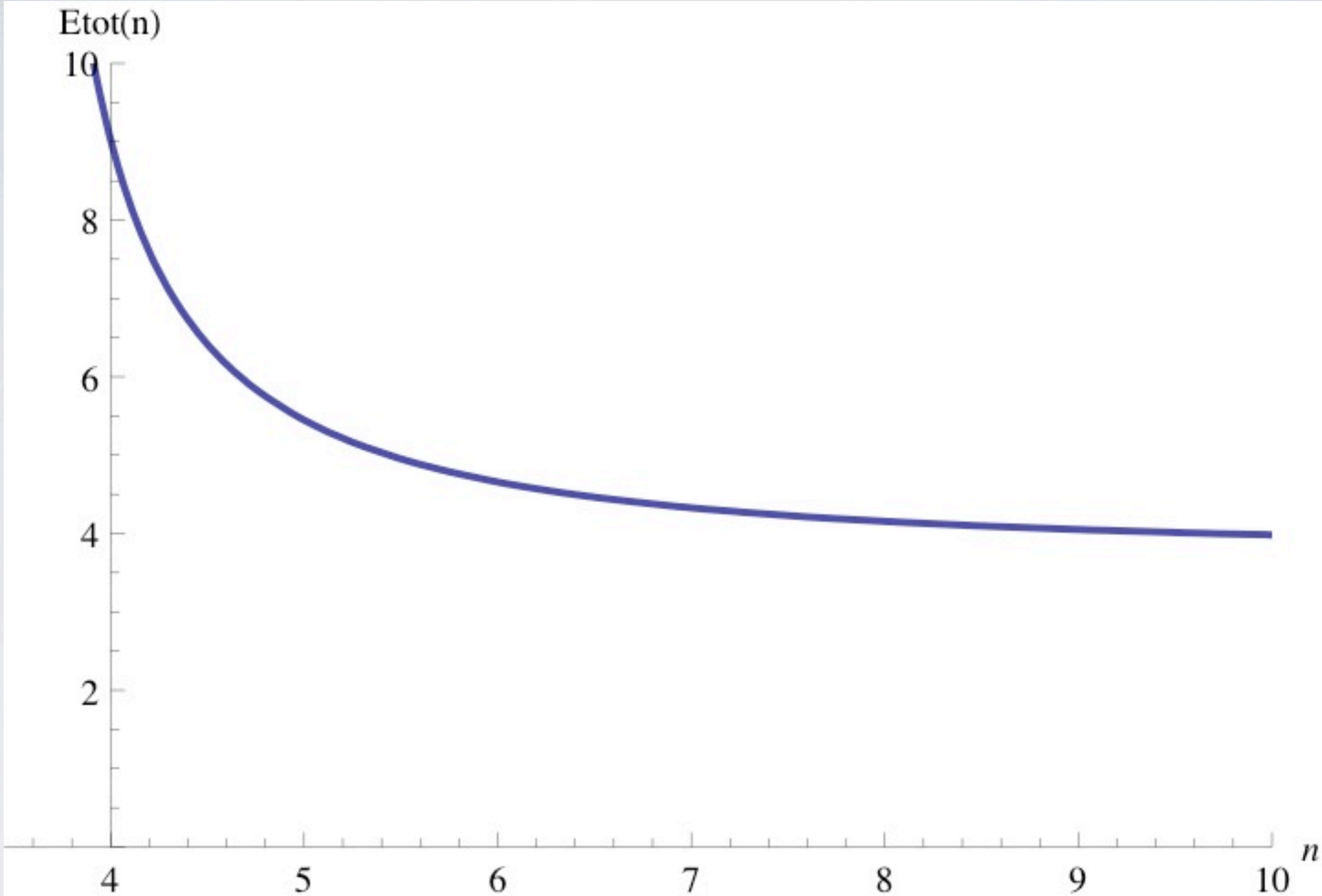

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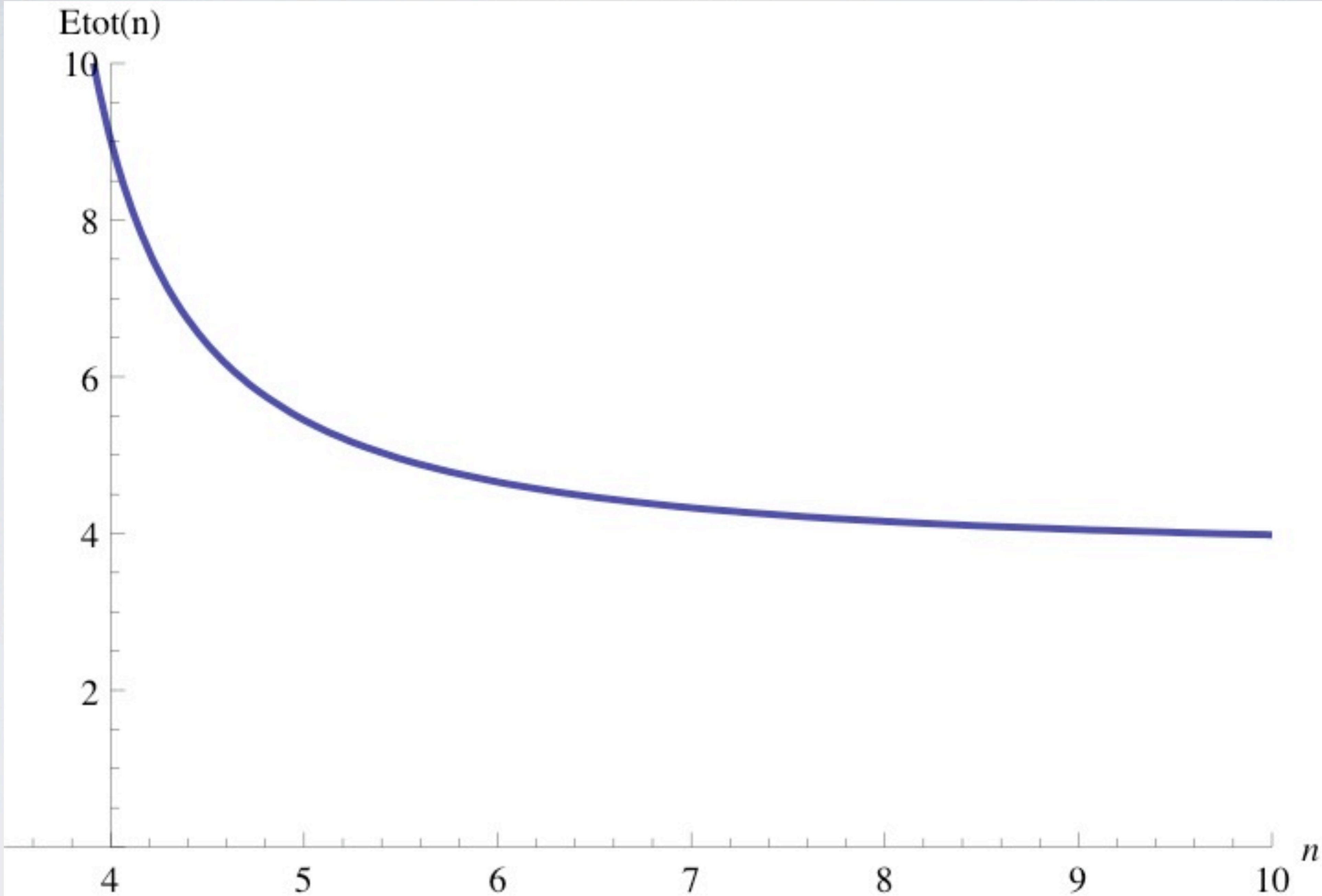

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in progress: with R. G. Leigh, N. Hoang and D. Minic

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THE GRAVITY SUPERCONDUCTIVITY PHASE DIAGRAM

(THE WORMHOLE/DW TRANSITION)

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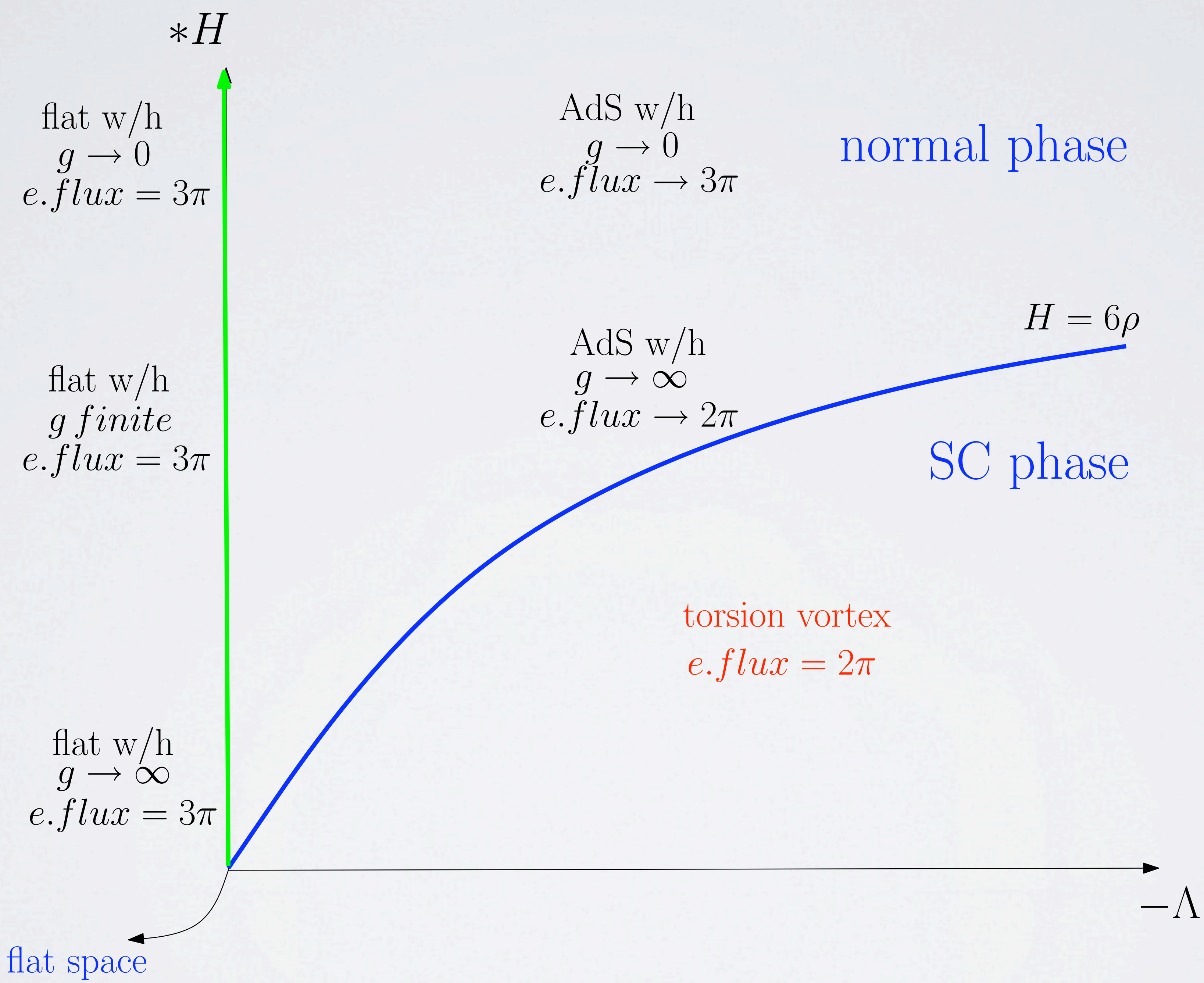

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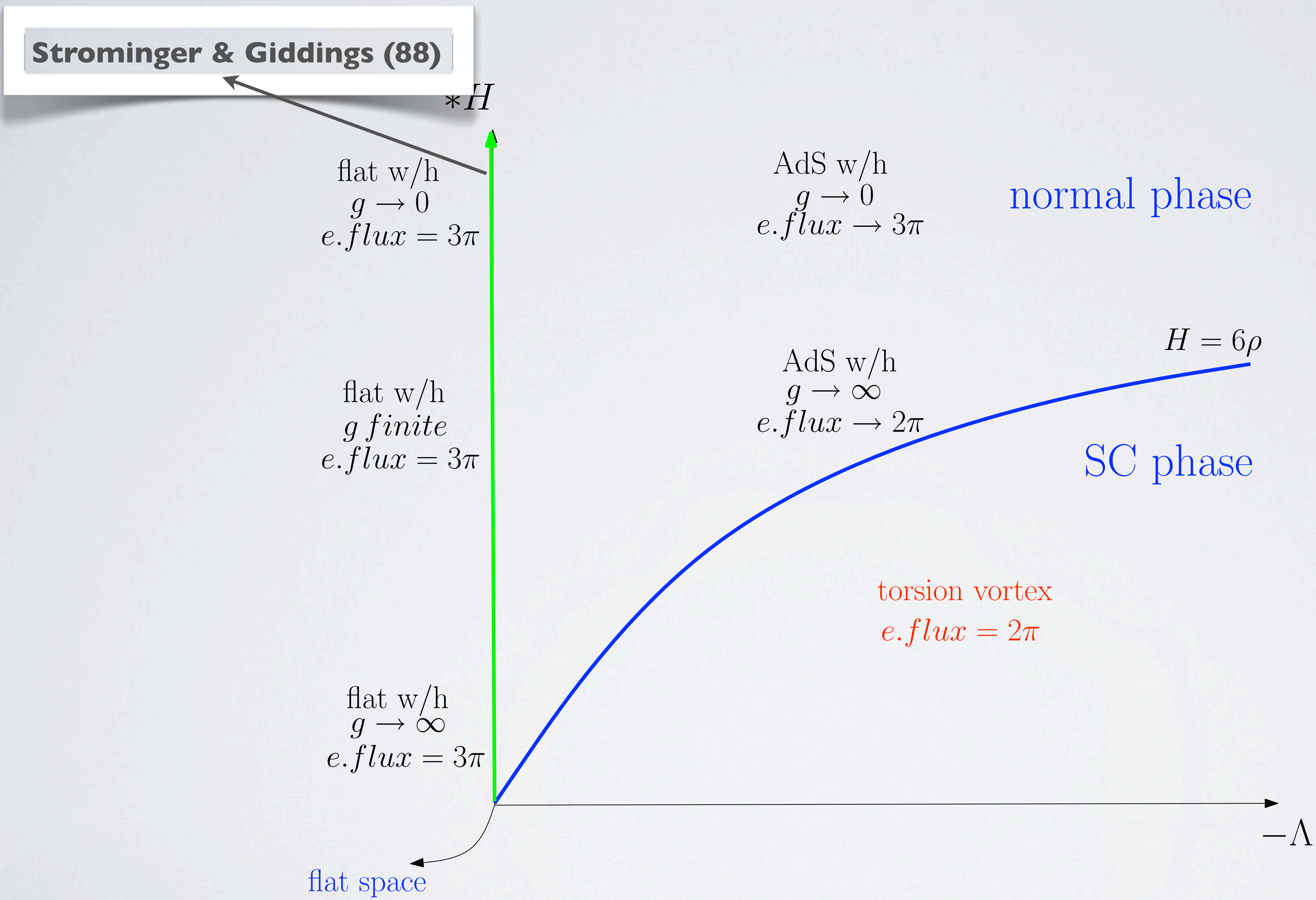

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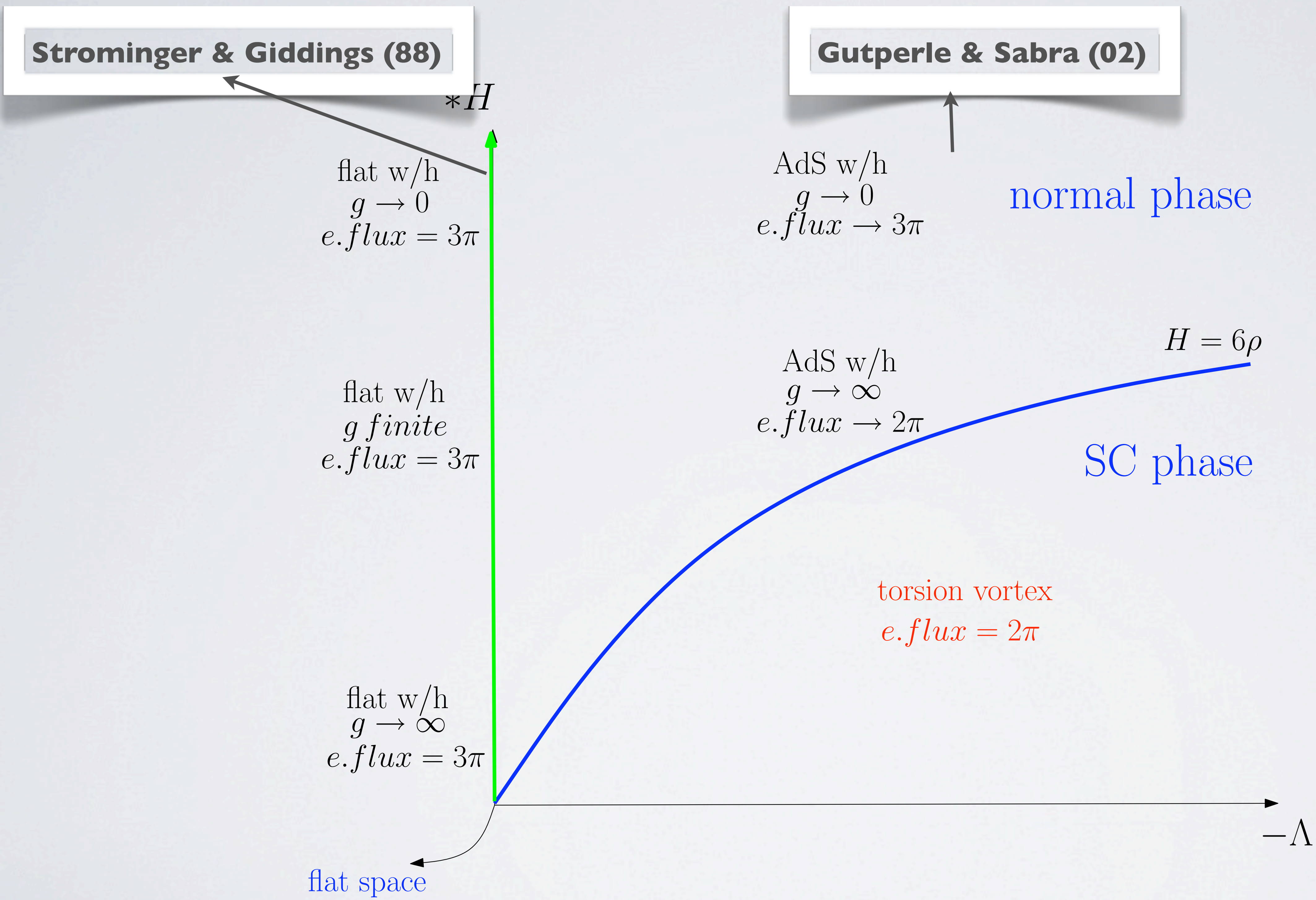

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