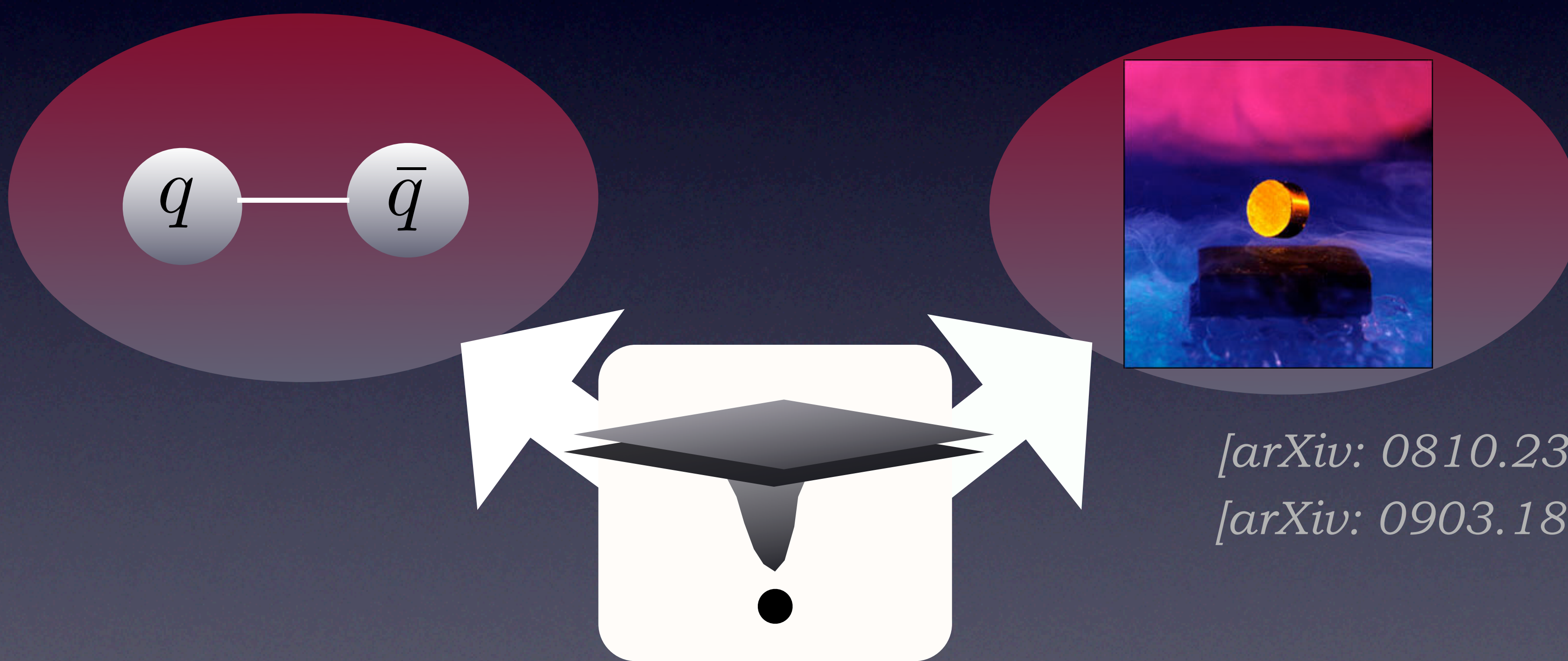
Flavor Superconductivity/ Superfluidity

Fifth Aegean Summer School, Adamas, Milos, September 2009



[arXiv: 0810.2316]
[arXiv: 0903.1864]

by Matthias Kaminski (IFT-UAM/CSIC Madrid)

in collaboration with M.Ammon, J.Erdmenger, P.Kerner (MPI Munich)

Outline

I. Invitation: Superconductivity & Holography

II. Review: Holographic Concepts

III. Details: Flavored Plasma (D3/D7)

IV. Results: Flavor Superconducting Phase (D3/D7)

V. Discussion

Remarks on references:
All references are hyperlinked in this document.
Publications quoted here are chosen because they review or explain certain aspects in a (pedagogical) way which is accessible to the unexperienced reader.

I. Invitation: Conventional Superconductors

Examples *[see lectures by Horowitz]*

- Color superconducting phase at high densities *[Alford, Rajagopal, Wilczek '97]*
- Higgs mechanism: Superconductivity of vacuum *[e.g.Weinberg]*

Weak coupling concepts

- charged condensate of Cooper-pairs
- (gauge) symmetry: electromagnetic $U(1)_{\rm em}$
- local symmetry spontaneously broken
- Goldstone bosons eaten

(photons in SC become massive → Meissner effect)

Theory: BCS (Bardeen-Cooper-Schrieffer) well established

Superfluidity: global symmetry spontaneously broken, Goldstone bosons survive (become hydro modes)

→ weakly gauge boundary theory

I. Invitation: Unconventional Superconductors

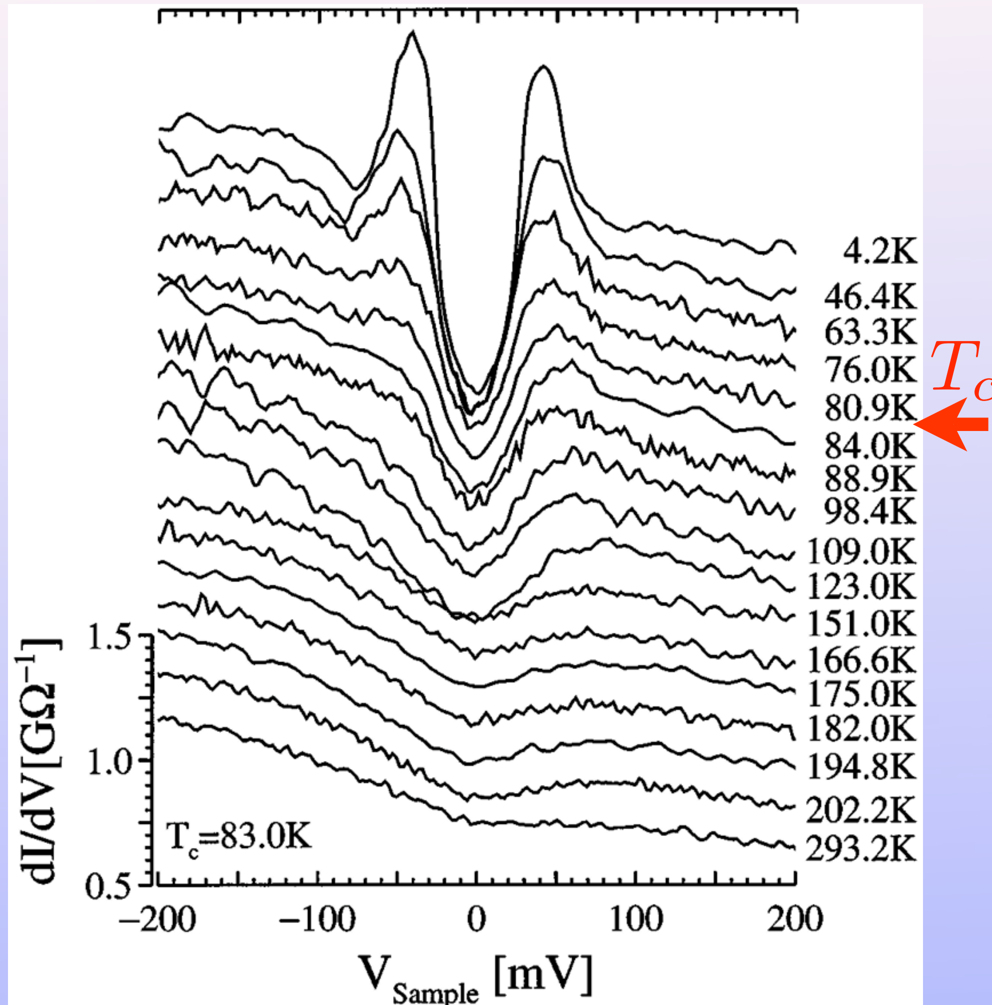
[see talk by Panagopoulos]

Typical signatures

- magnetic field expulsion (c)
- energy gap (peak at edge)
- pseudo gap
- underdoped: strong coupl.

Figure: Tunneling spectra measured in high temperature superconductor $Bi_2Sr_2CaCu_2O_{8+\delta}$.

[Renner et al., Phys. Rev. Lett. 80, 149 - 152 (1998)]

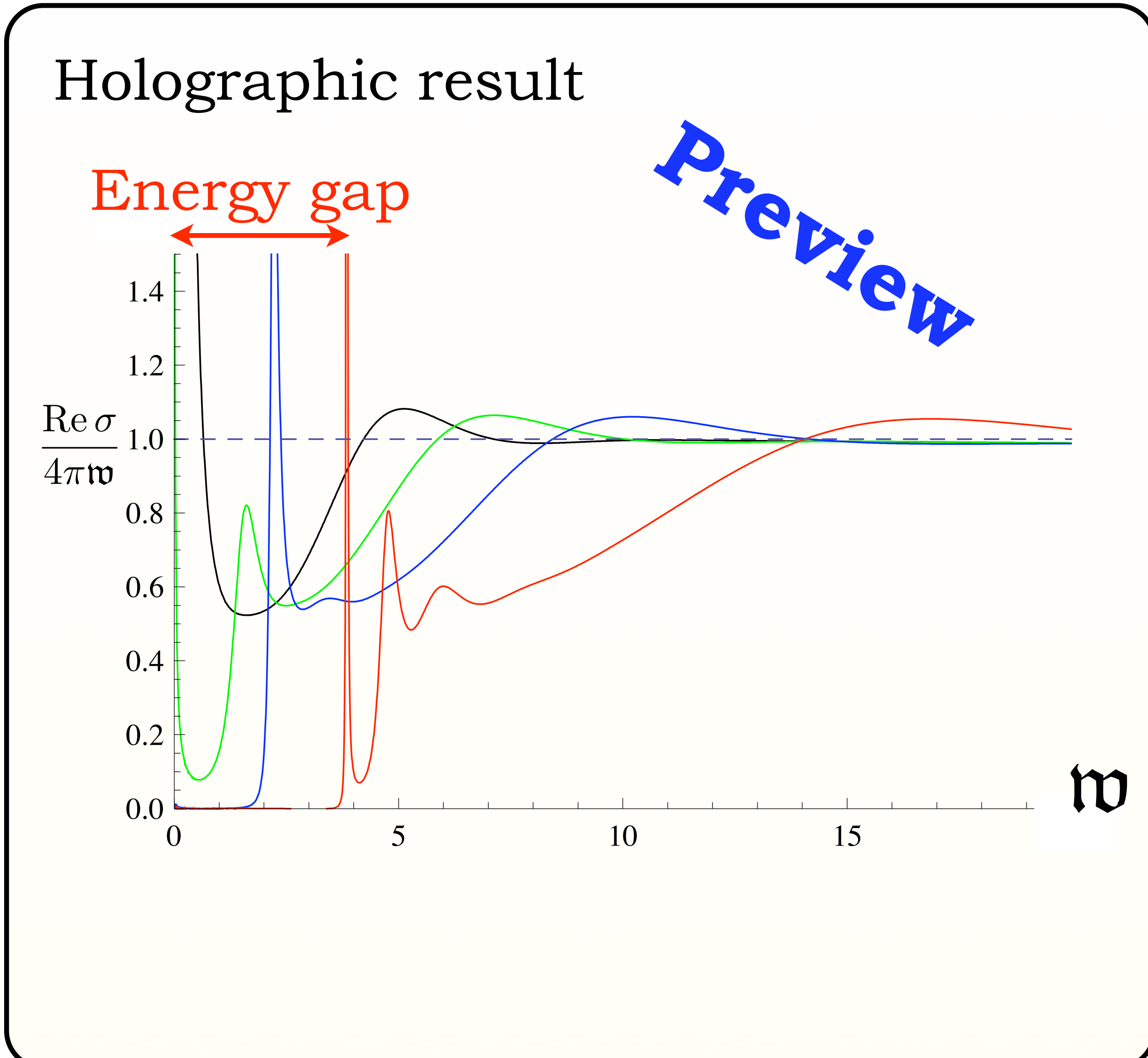


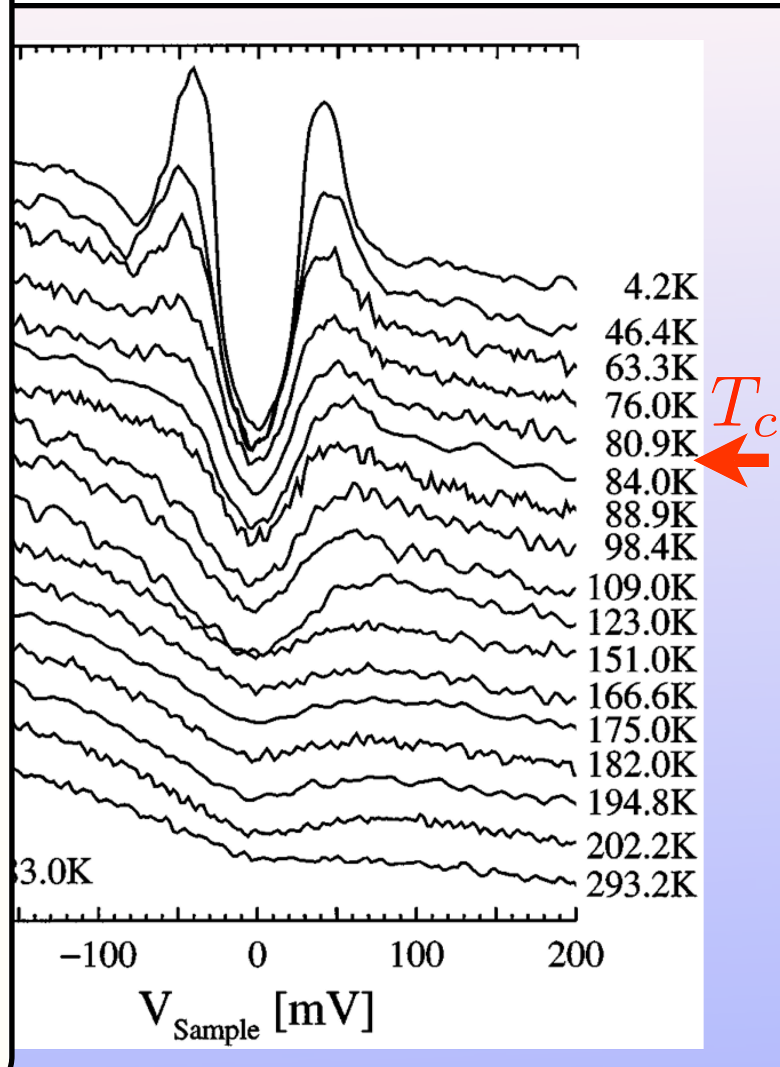
Theory? Pairing mechanism? Meissner effect?

[see lectures by Sadchdev]

I. Invitation: Unconventional Superconductors

[see talk by Panagopoulos]





Theory? Pairing mechanism? Meissner effect?

[see lectures by Sadchdev]

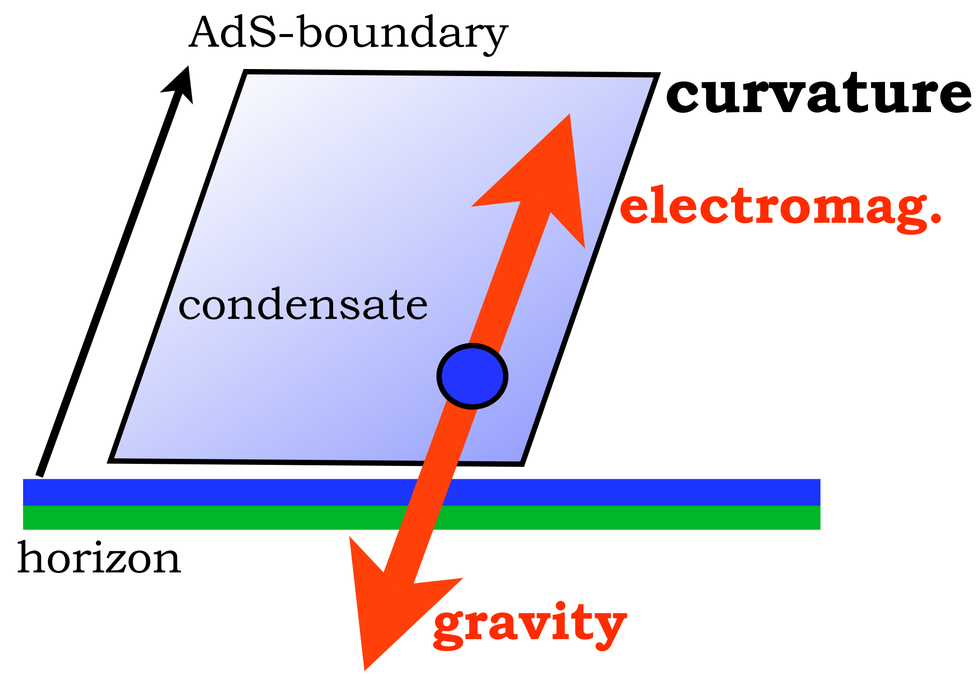
I. Invitation: Building a Holographic SC

Field Theory

What do we need?

- charged condensate (vev)
- no source
- condensate of charge carriers
- finite temperature

Gravity *[Gubser, Pufu 0805.2960]*



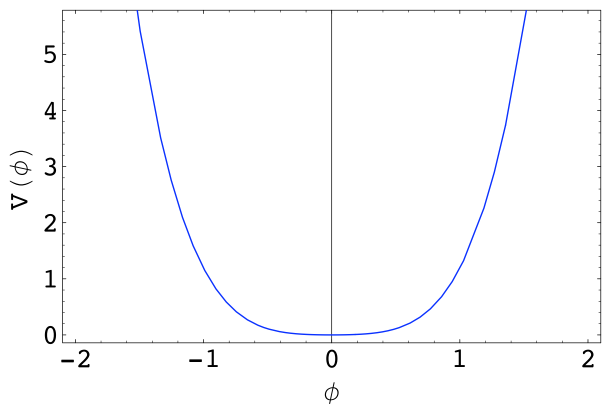
- introduce normalizble mode
- no non-normalizable mode
- condensate hovers over horizon
- black hole

Is this stable?

I. Invitation: Get some intuition

Field Theory

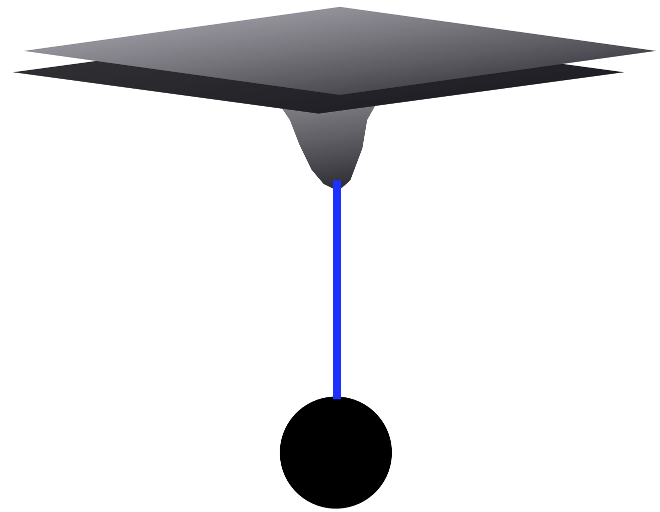
$$\mathcal{L} \sim D_\nu \phi D^\nu \phi \sim ({M_q}^2 - {\mu_{\text{isospin}}}^2)\phi^2$$



- charged particles condense at large enough chemical potential

$$\mu_{\text{isopin}} \sim M_q$$

Gravity



- strings (D3-D7) give FT charges
- cannot put infinitely many
- second brane is important

Why do we need a non-Abelian structure?

I. Invitation: Get some intuition

Field Theory

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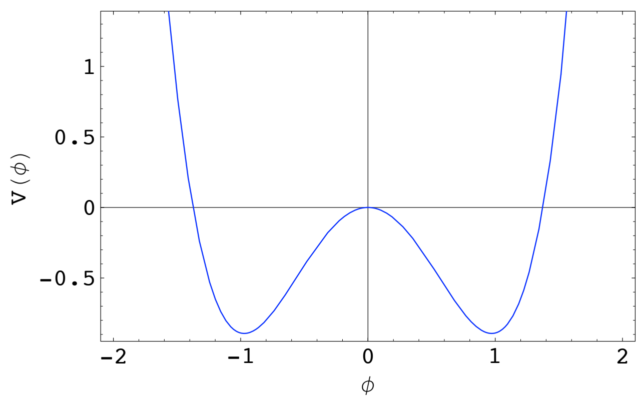

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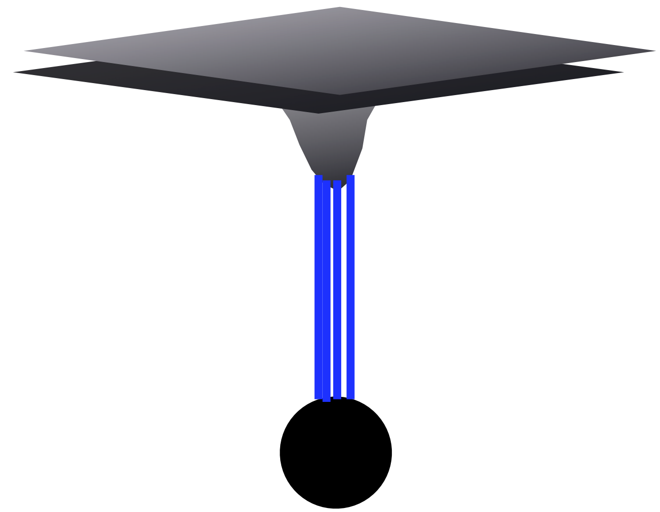

- strings (D3-D7) give FT charges
- cannot put infinitely many
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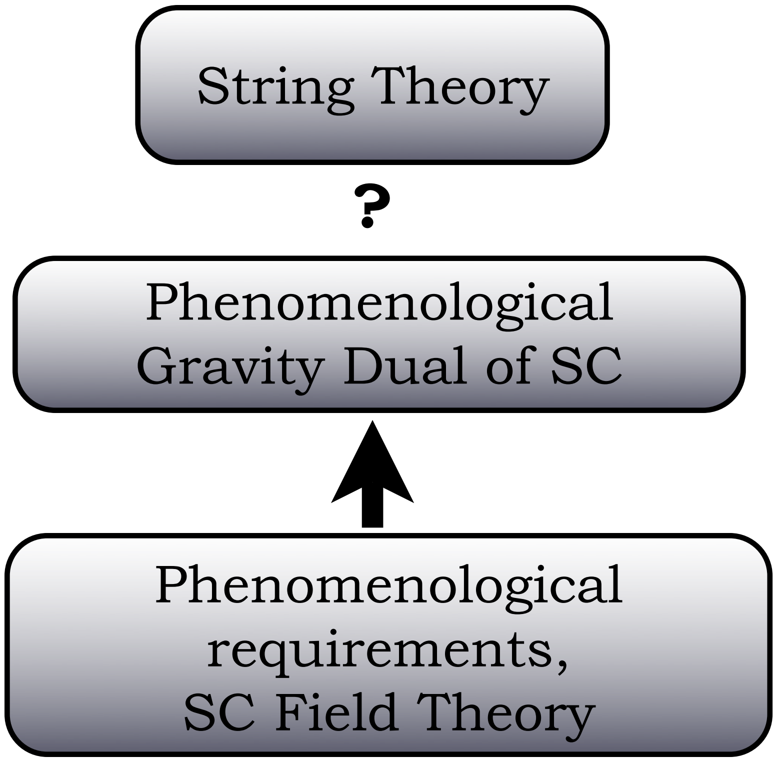
Why do we need a non-Abelian structure?

I. Invitation: Why so complicated?

Bottom-up

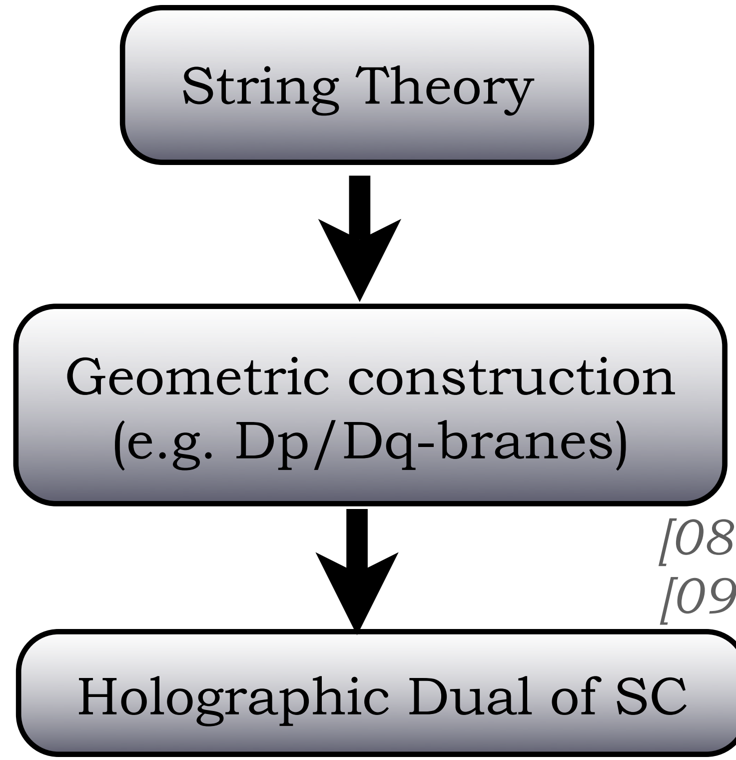


[Gubser, Pufu 0805.2960]
[Hartnoll, Herzog, Horowitz 0803.3295]

- study effects in clean setup
- seperate effects

[see lectures by Horowitz]

Top-down



[0810.2316]
[0903.1864]

- string theory derived
- identification of FT degrees o.f.
- ‘dirty’
- many effects at once

Pairing mechanism!

Navigator

✓ Invitation: Superconductivity & Holography

II. Review: Holographic Concepts

also reviewed in [M.K. 0808.1114]

III. Details: Flavored Plasma (D3/D7)

IV. Results: Flavor Superconducting Phase (D3/D7)

V. Discussion

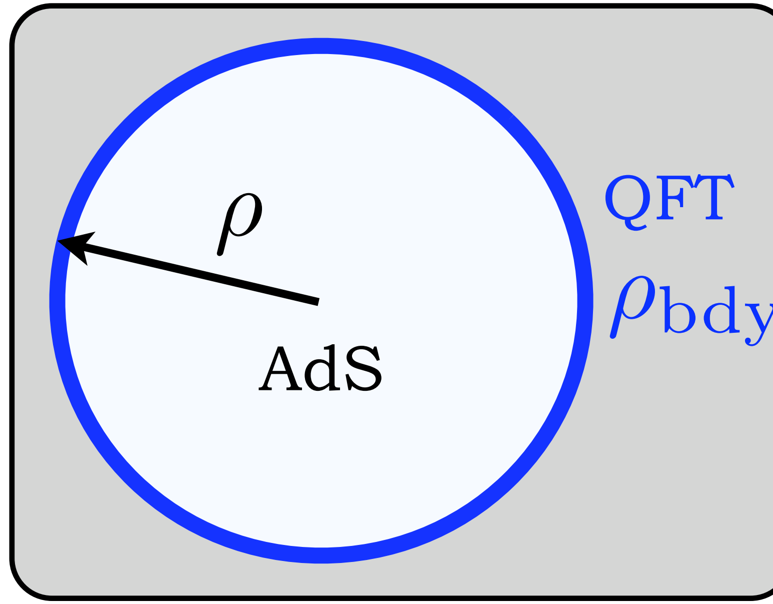
II. Review: Boundary Asymptotics

non-normalizable (source) — normalizable (vev)

$$A = A^{(0)} + \frac{A^{(2)}}{\rho^2} + \dots$$

$$(\rho \to \infty)$$

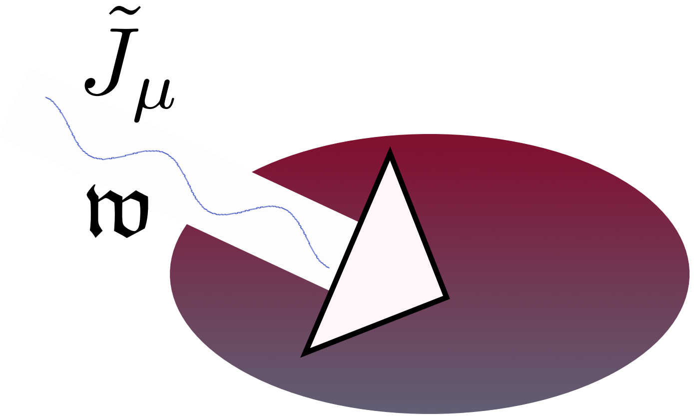
[see lectures by Argyres]



Dictionary

QFT FEATURE (energy scale)	$\longleftrightarrow$	GEOMETRY (radial coord. ρ)
operator J_μ	$\longleftrightarrow$	field A_μ (gauge)
vev	$\longleftrightarrow$	$A^{(2)}$ (charge)
source	$\longleftrightarrow$	$A^{(0)}$ (chem. pot.)

II. Review: Correlators & Spectral Functions



Gauge Theory Problem (strong): Find retarded two-point function of flavor current in YM-plasma. ➜ Gravity problem (weak): Find solution for equation of motion of vector field in SUGRA.

$$J_\mu \longleftrightarrow A_\mu$$

$$G^{\text{ret}}(\omega, q) = -\mathrm{i} \int \mathrm{d}^4x \, \mathrm{e}^{\mathrm{i}\vec{k}\vec{x}} \, \theta(x^0) \, \langle [J(\vec{x}), J(0)] \rangle \longleftrightarrow \frac{\delta^2}{\delta A_{\text{bdy}} \delta A_{\text{bdy}}} S_{\text{Sugra}}$$

Recipe:

$$S_{\text{Sugra}} \sim \int \partial_\rho A \, \partial_\rho A \;\rightarrow\; S_{\text{on-shell}} \sim \int A \, \partial_\rho A \;\rightarrow\; G^{\text{ret}} \sim \lim_{\rho\to\infty} \frac{A}{A} \frac{\partial_\rho A}{A}$$

Thermal spectral function: $\mathfrak{R}(\omega, \mathbf{q}) = -2 \, \mathrm{Im} G^{\text{ret}}(\omega, \mathbf{q})$

[Son,Starinets hep-th/0205051]

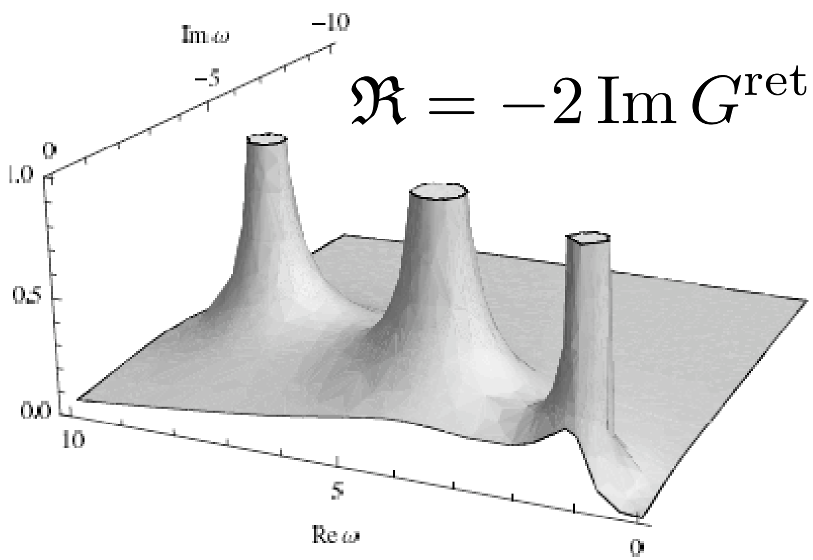
II. Review: Quasinormal modes *[Berti et al. 0905.2975]*

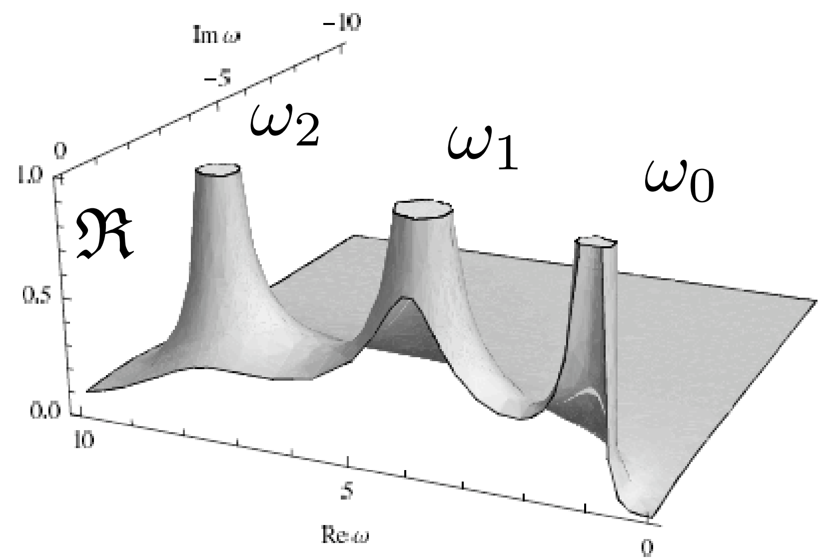
Special frequencies: $\omega_{\mathrm{n}} \in \mathbb{C}$; $\lim_{\rho \to \rho_{\mathrm{bdy}}} |\tilde{A}(\omega_{\mathrm{n}})|^2 = 0$
(quasinormal)

$$e^{-i\omega r} = e^{-i\mathrm{Re}\{\omega\} r} e^{\mathrm{Im}\{\omega\} r}$$

Example:

$$G^{\mathrm{ret}} = \frac{N_f N_c T^2}{8} \lim_{\rho \to \rho_{\mathrm{bdy}}} \left(\rho^3 \frac{\partial_\rho \tilde{A}(\rho)}{\tilde{A}(\rho)} \right)$$

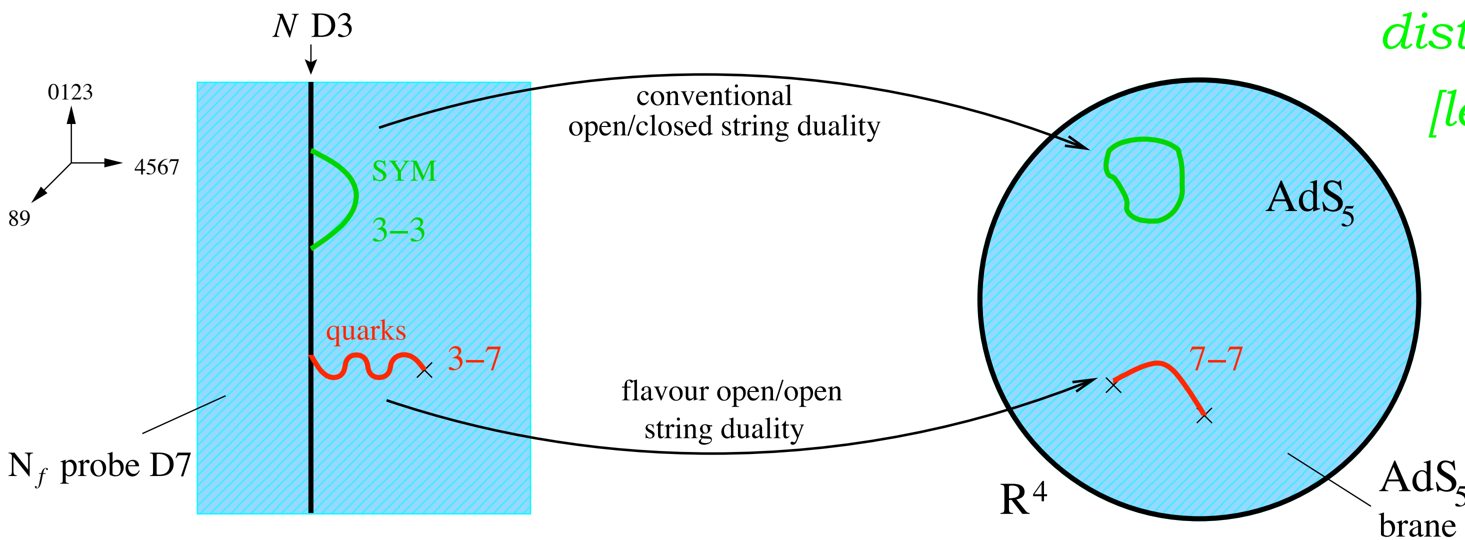




Gravity:
quasinormal frequencies ⟷ Gauge theory:
poles of correlator
(energy, damping, stability
of mesonic excitations)

II. Review: D3/D7-Brane Setup (1)

Flavor Probe Branes (D7)



distinct approaches:

[lectures by Kiritsis]

[talk by Cotrone]

[Karch, Katz hep-th/0205236]

Review: [Erdmenger et al. 0711.4467]

Dirac-Born-Infeld (DBI) action

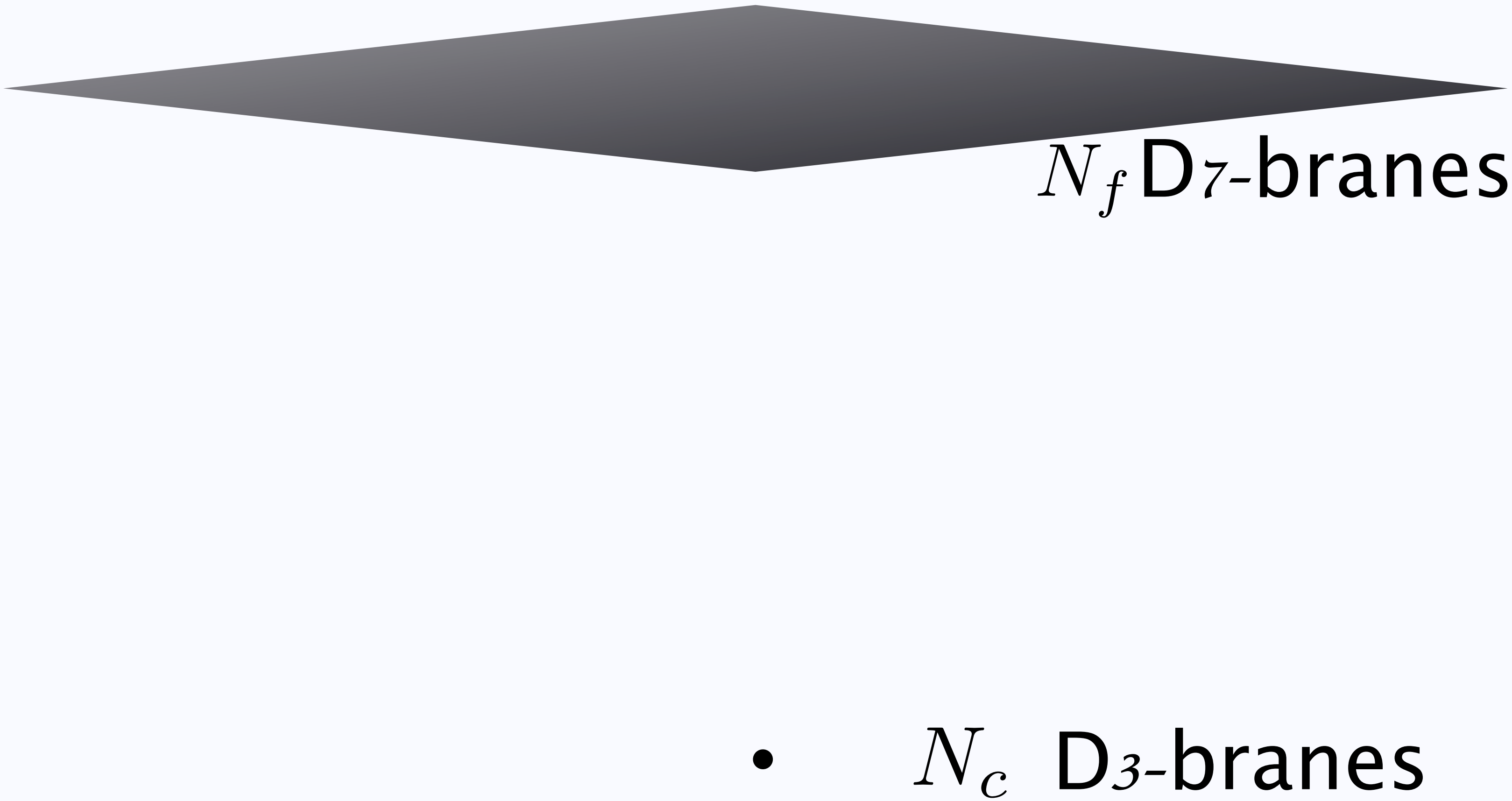
$$S_{\mathrm{DBI}} = -T_7 \int \mathrm{d}^8\sigma \left(\sqrt{-\det(P[G+B]_{\mu\nu} + (2\pi\alpha')^2)F_{\mu\nu}}\right)$$

$$B \equiv 0$$

II. Review: D3/D7-Brane Setup (2)

- N_c D3-branes

II. Review: D3/D7-Brane Setup (2)



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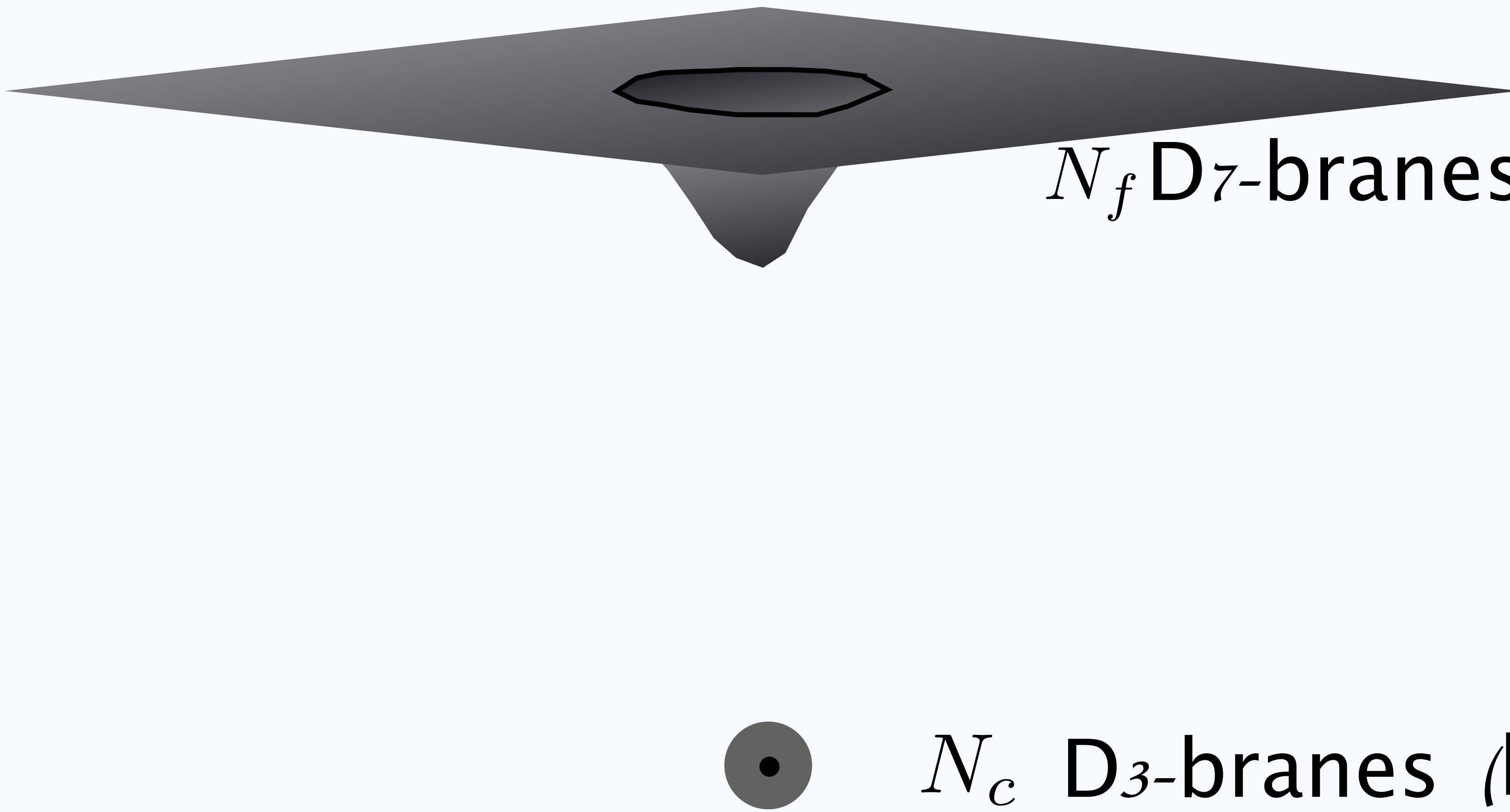

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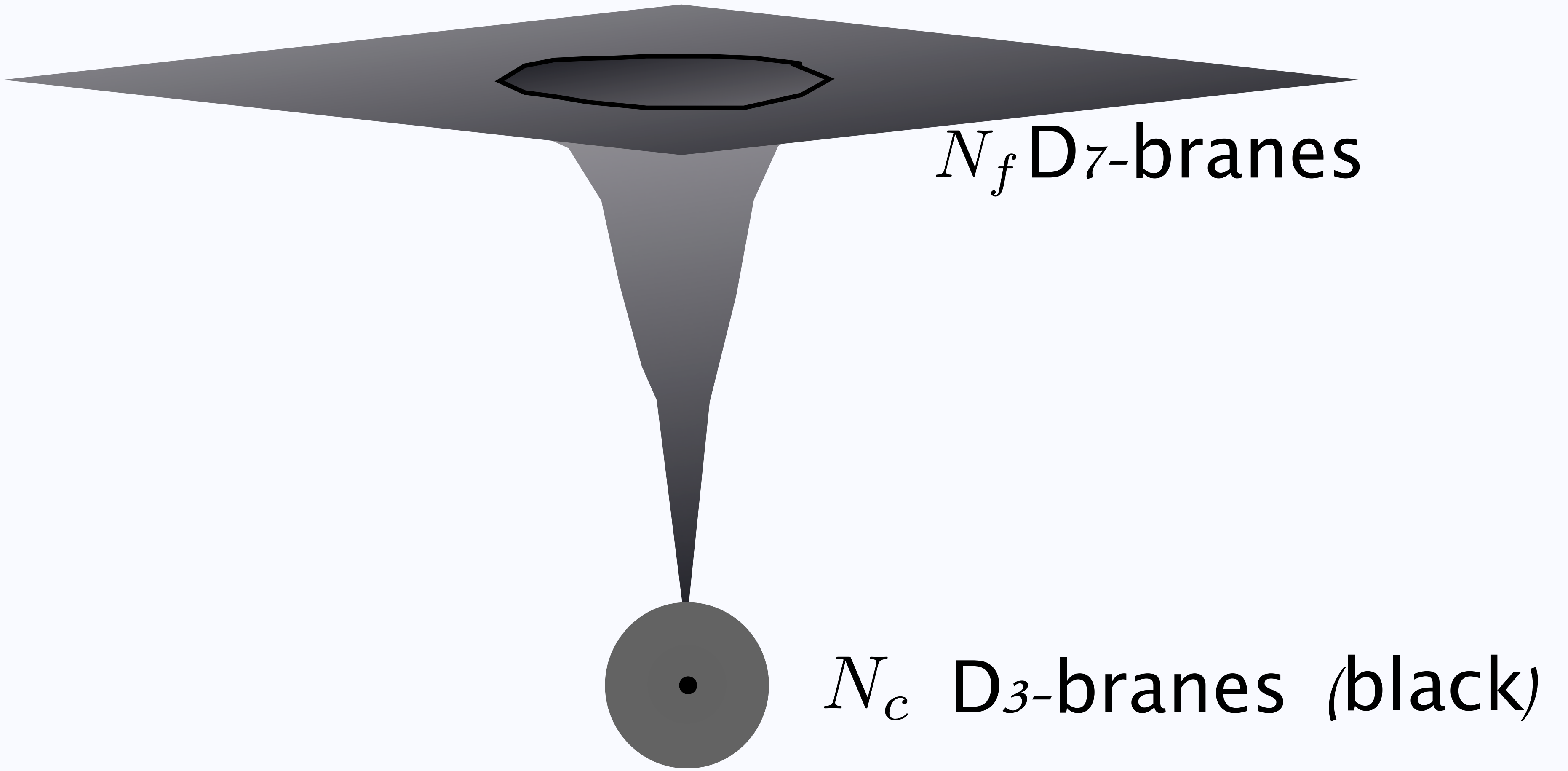

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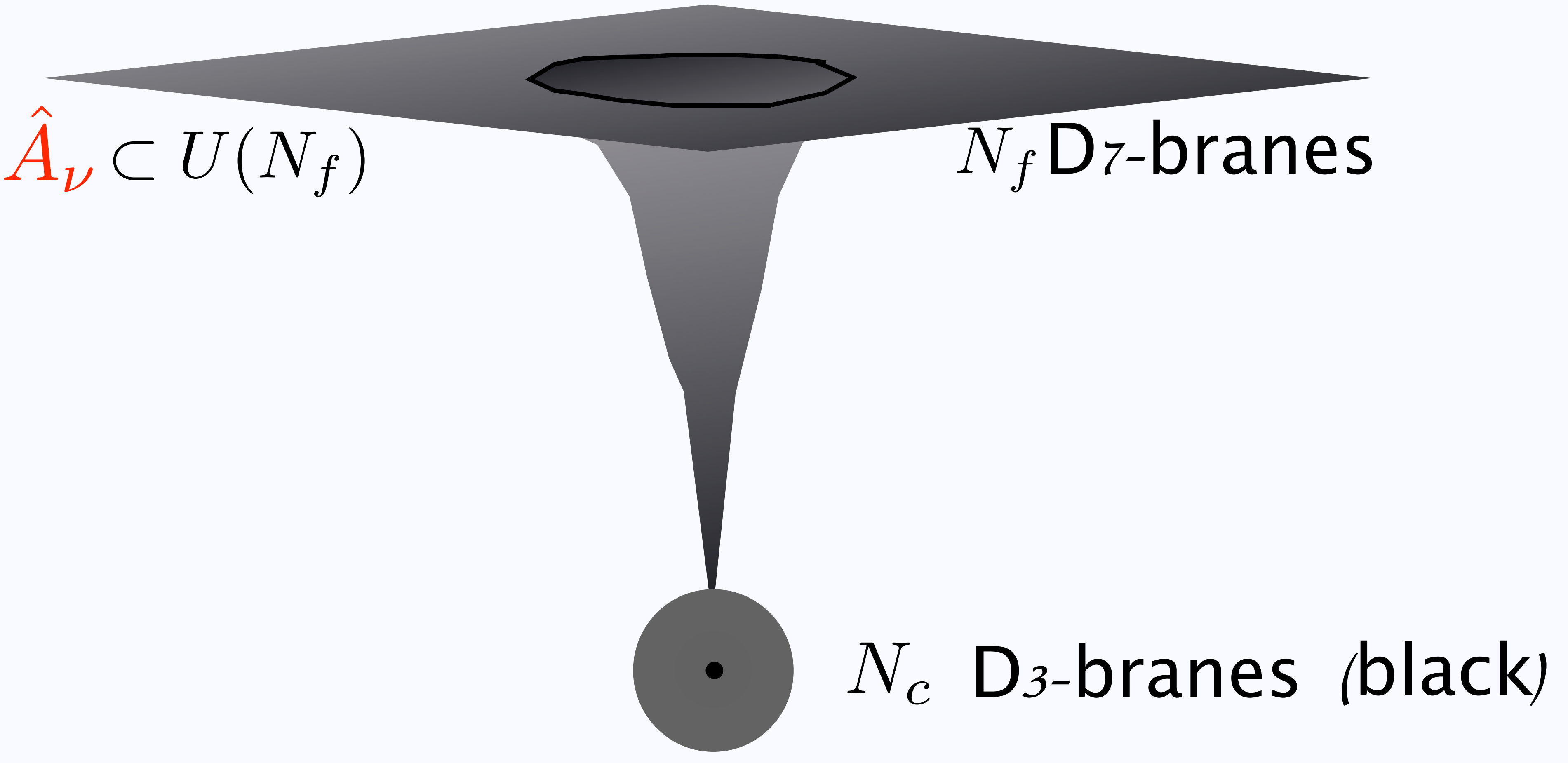

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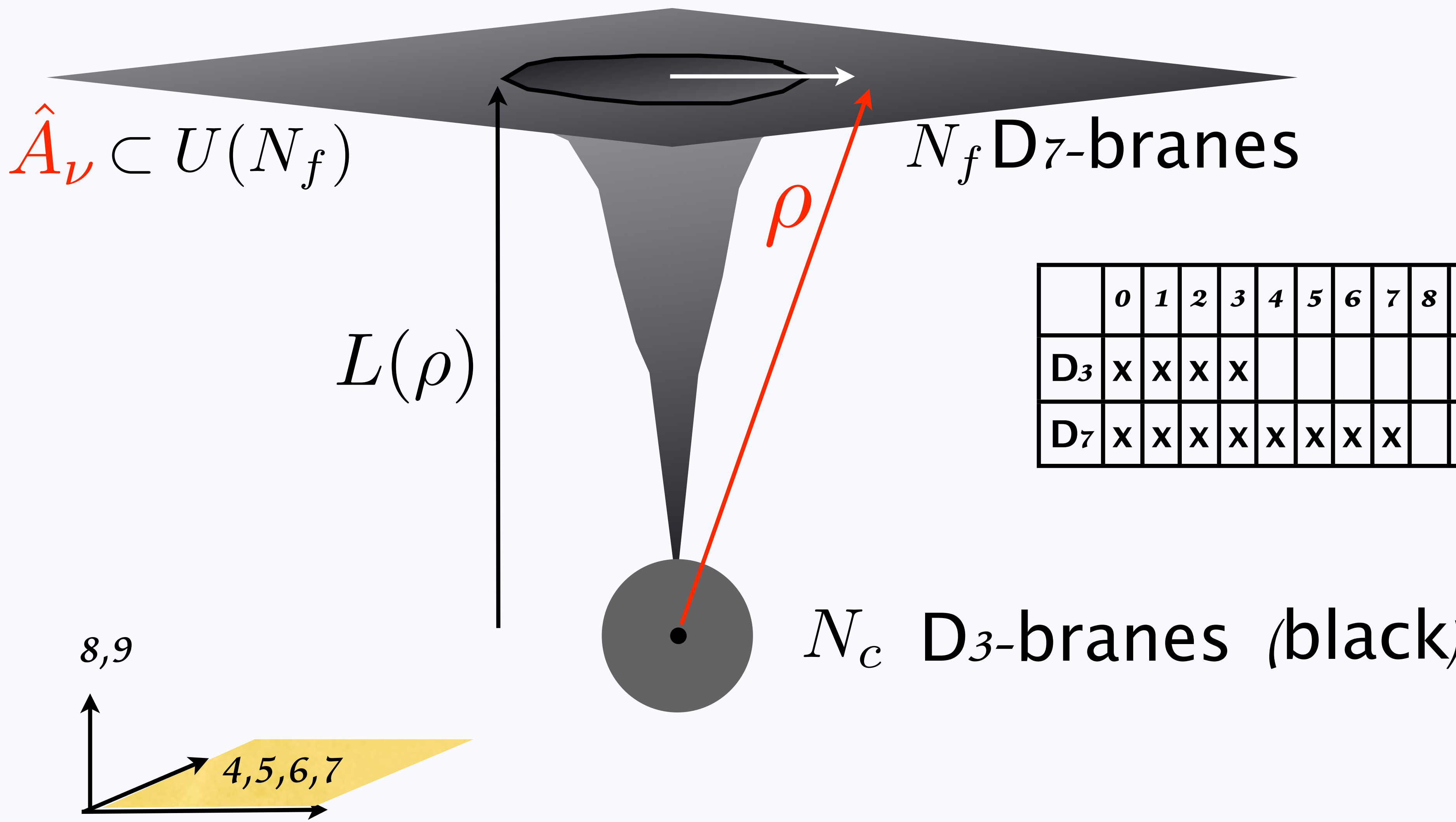

	0	1	2	3	4	5	6	7	8	9
D3	x	x	x	x						
D7	x	x	x	x	x	x	x	x		

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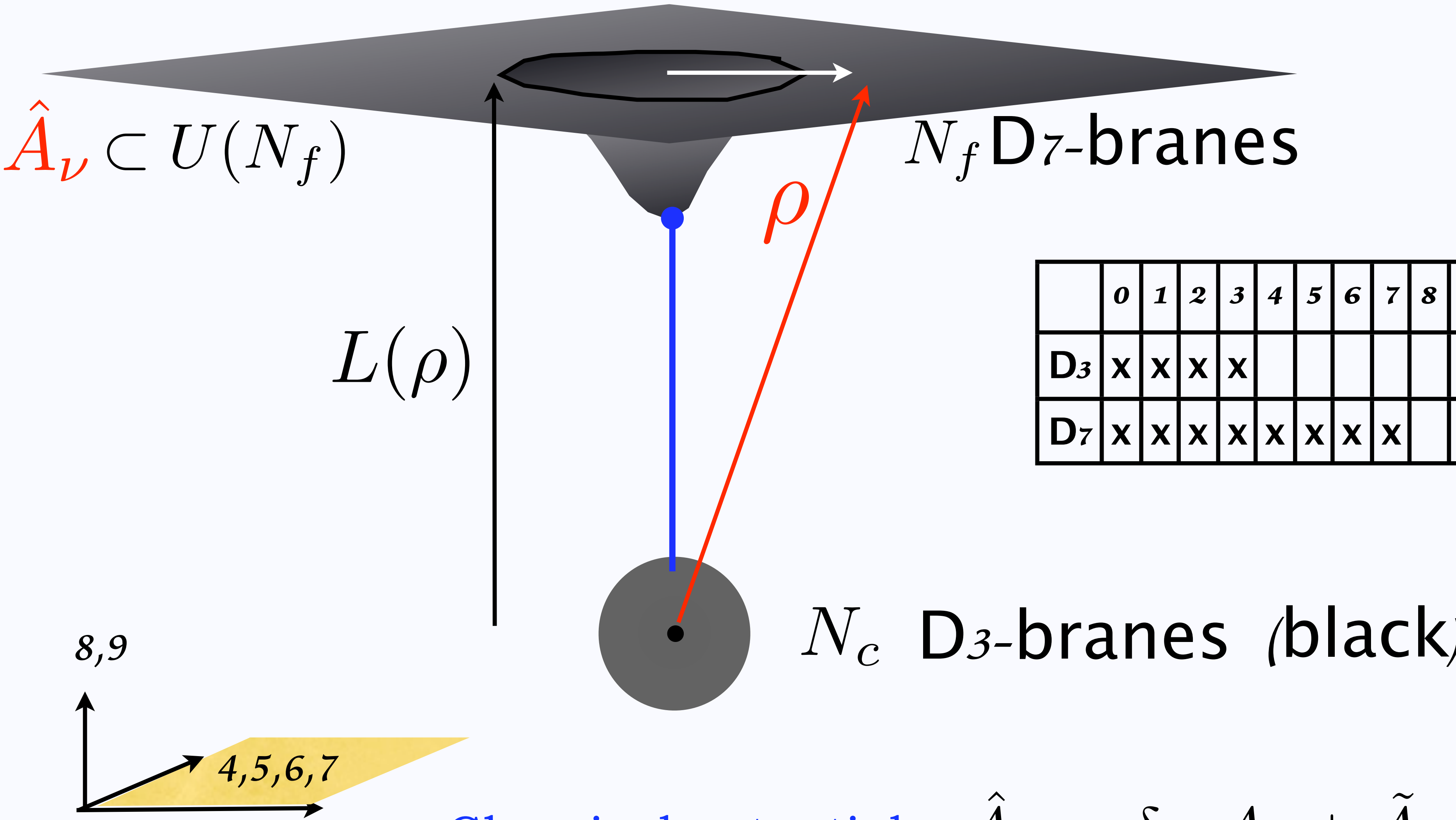

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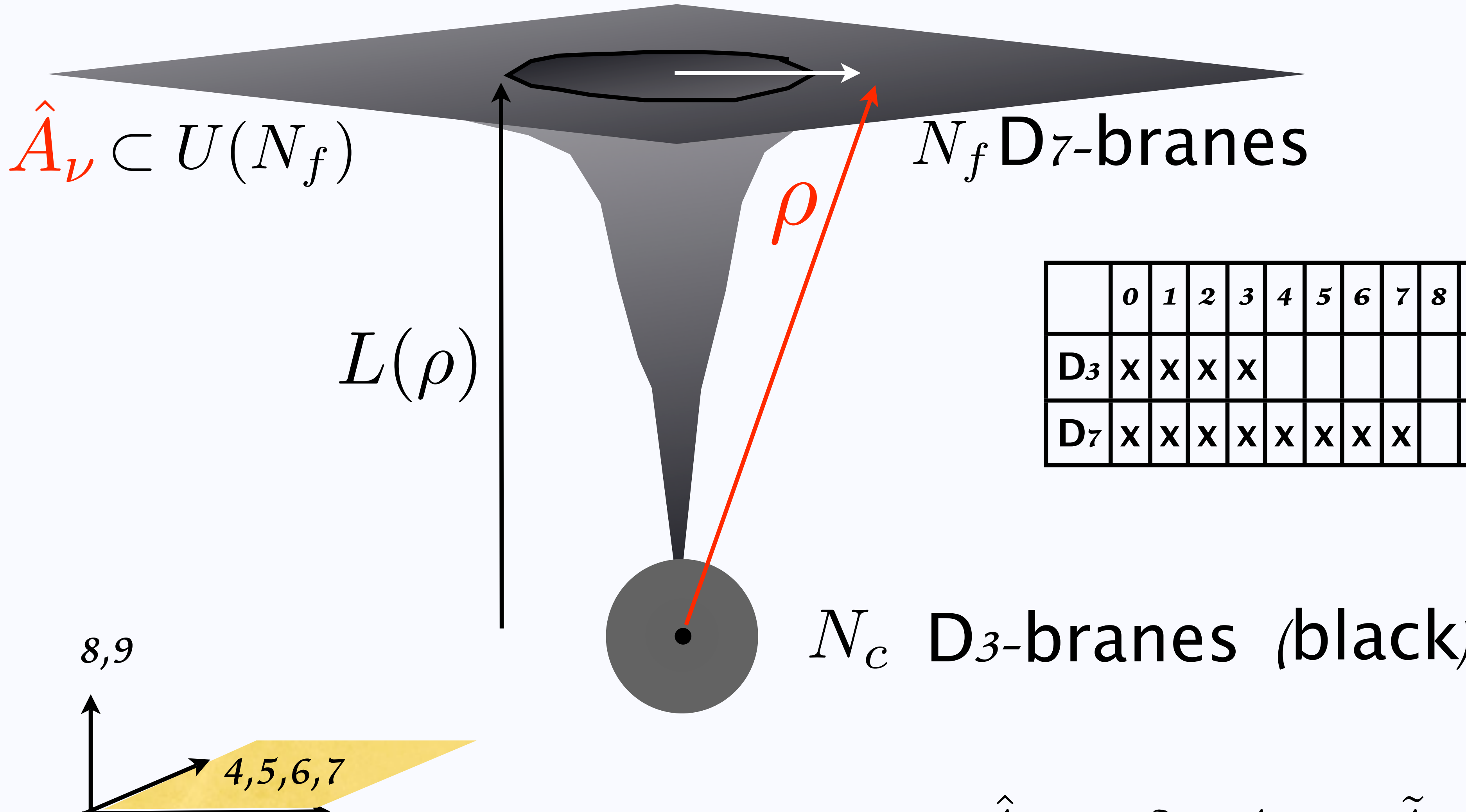
D3	x	x	x	x						
D7	x	x	x	x	x	x	x	x		

Chemical potential: $\hat{A}_\mu = \delta_{\mu 0} A_0 + \tilde{A}_\mu$

[Kobayashi et al. hep-th/0611099]
[Mateos et al. 0709.1225]

(cf. therm. FT)

II. Review: D3/D7-Brane Setup (2)



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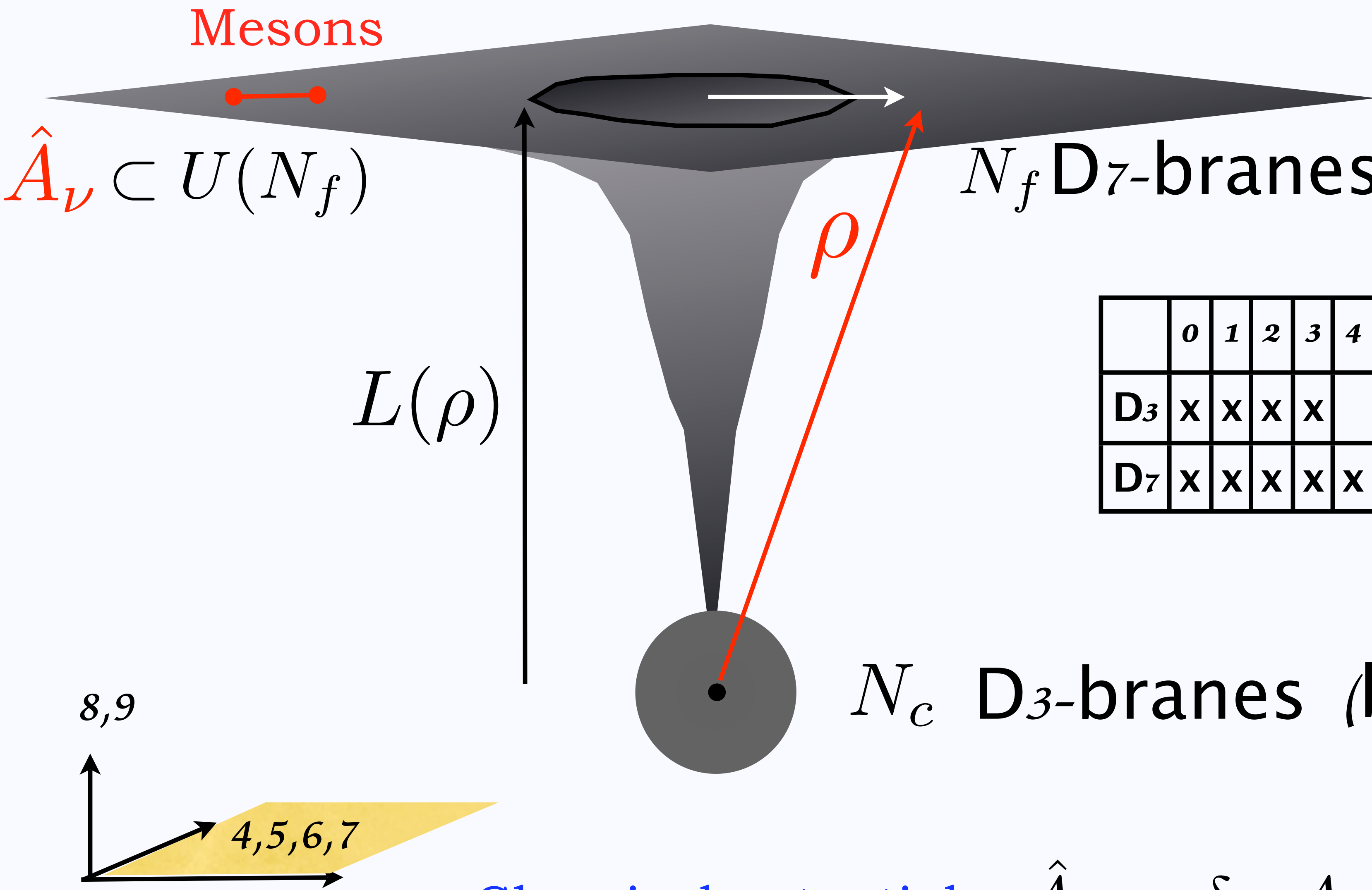

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Navigator

✓ Invitation: Superconductivity & Holography

✓ Review: Holographic Concepts

III. Details: Flavored Plasma (D3/D7)

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V. Discussion

III. Abelian Chemical Potential: Background

[Myers et al. hep-th/0611099]

AdS black hole metric

↓ pullback

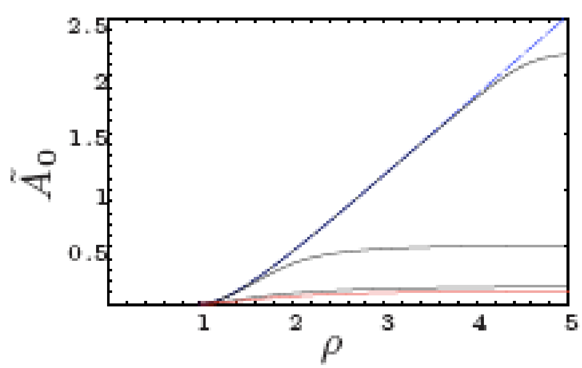
Induced metric on D7-brane

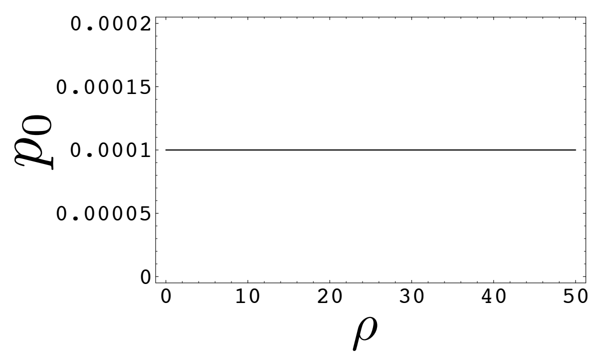
$$ds^2 = \frac{1}{2}\left(\frac{\varrho}{L}\right)^2\left[-\frac{f^2}{\tilde{f}}dt^2 + \tilde{f}dx_3^2\right] + \frac{L^2}{\varrho^2}\left[\frac{1-\chi^2+\varrho^2(\partial_\varrho\chi)^2}{1-\chi^2}\right]d\varrho^2 + L^2(1-\chi^2)d\Omega_3^2$$

$$f(\varrho) = 1 - \frac{{\varrho_H}^4}{\varrho^4}\,,\quad \tilde{f}(\varrho) = 1 + \frac{{\varrho_H}^4}{\varrho^4}\,,\quad \chi = \cos(\theta)\,,\quad \varrho^2 = r^2 + \sqrt{r^4 - {r_H}^4}$$

DBI action

$$I_{\mathrm{D7}} = -N_{\mathrm{f}}T_{\mathrm{D7}}\int d^8\sigma\,\frac{\varrho^3}{4}f\tilde{f}(1-\chi^2)\sqrt{1-\chi^2+\varrho^2(\partial_\varrho\chi)^2 - 2(2\pi\ell_{\mathrm{s}}^2)^2\frac{\tilde{f}}{f^2}(1-\chi^2)F_{\varrho t}^2}$$





↓

Can be rewritten as constant of motion $\tilde{d}$.

III. Abelian Chemical Potential: Fluctuations

[Myers,Starinets,Thomson 0710.0334]

DBI action: $$S_{\mathrm{D7}} = \int \mathrm{d}^8x \sqrt{\left|\det\{\underbrace{[g+F]}_{G} + \tilde{F}\}\right|}\,, \quad F_{\mu\nu} = \partial_{[\mu}A_{\nu]}$$

Equation of motion: $$0 = \tilde{A}'' + \frac{\partial_\rho[\sqrt{|\det G|}G^{22}G^{44}]}{\sqrt{|\det G|}G^{22}G^{44}}\tilde{A}' - \frac{G^{00}}{G^{44}}{\varrho_H}^2\omega^2\tilde{A}$$

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Equation of motion:

`Curved' Maxwell equations:

$$\partial_\mu F^{\mu\nu} = 0$$

$$\partial_\mu \left(\sqrt{-G} G^{\mu\nu} G^{\rho\sigma} F_{\nu\sigma}\right) = 0$$

$$\partial_\mu \left(\sqrt{-G} G^{\mu\nu} G^{\rho\sigma} \partial_{[\nu} \tilde{A}_{\sigma]}\right) = 0$$

III. Abelian Chemical Potential: Fluctuations

[Myers,Starinets,Thomson 0710.0334]

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Boundary conditions: $\tilde{A} = (\varrho - \varrho_H)^{-i\mathfrak{w}}[1 + \frac{i\mathfrak{w}}{2}(\varrho - \varrho_H) + \dots]$

→ shooting from horizon

Translation to gauge theory by duality: $\underset{\text{(source)}}{A_\mu} \overset{\text{AdS/CFT}}{\longleftrightarrow} J^\mu$

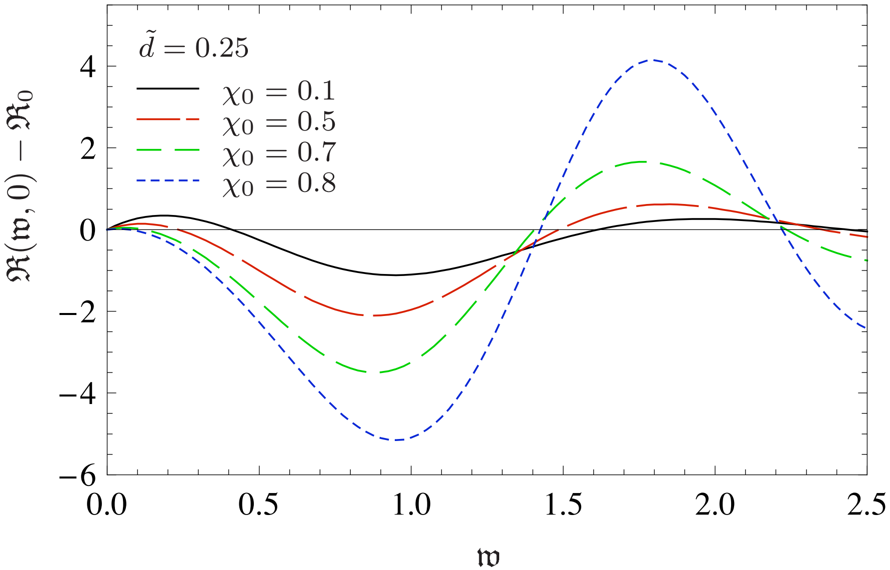
Gauge correlator: *[Son,Starinets hep-th/0205051]*

$$G^{\text{ret}} = \frac{N_f N_c T^2}{8} \lim_{\rho\to\rho_{\text{bdy}}} \left(\rho^3 \frac{\partial_\rho \tilde{A}(\rho)}{\tilde{A}(\rho)}\right)$$

III. Abelian Chemical Potential: Spectral F's

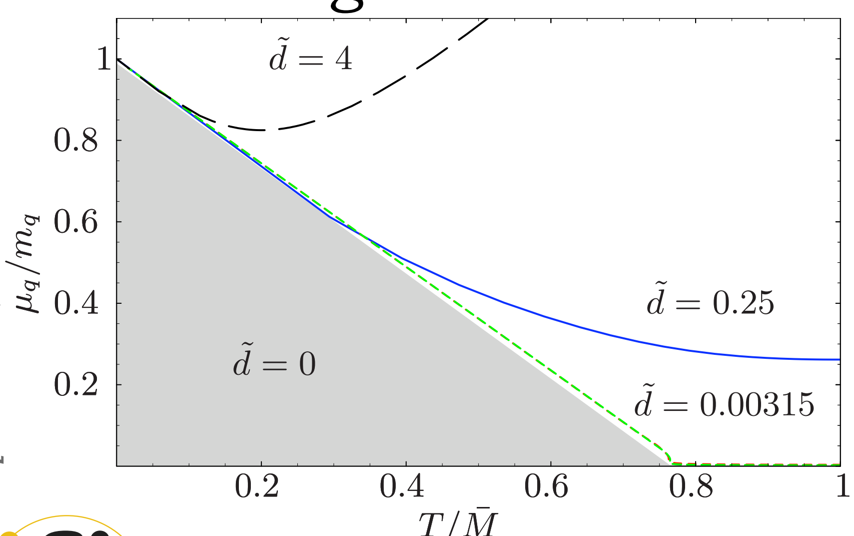
[Erdmenger, M.K., Rust 0710.0334]

Finite baryon density:



[Karch, O'Bannon '07]

Phase diagram:



$$L(\varrho) = \varrho\, \chi(\varrho)$$

$$\chi_0 = \chi(\rho)\big|_{\rho \to \rho_H} \sim \frac{m_{\text{quark}}}{T}$$

$$\chi = \chi(\tilde{d}, \rho)$$

III. Abelian Chemical Potential: Spectral F's

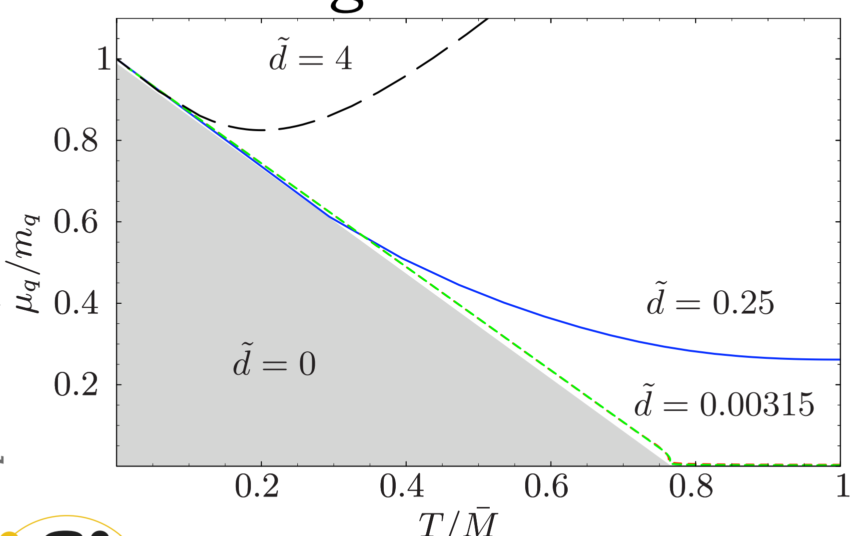
[Erdmenger, M.K., Rust 0710.0334]

Finite baryon density:

Lower temperature

Phase diagram:

[Karch, O'Bannon '07]



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[Erdmenger, M.K., Rust 0710.0334]

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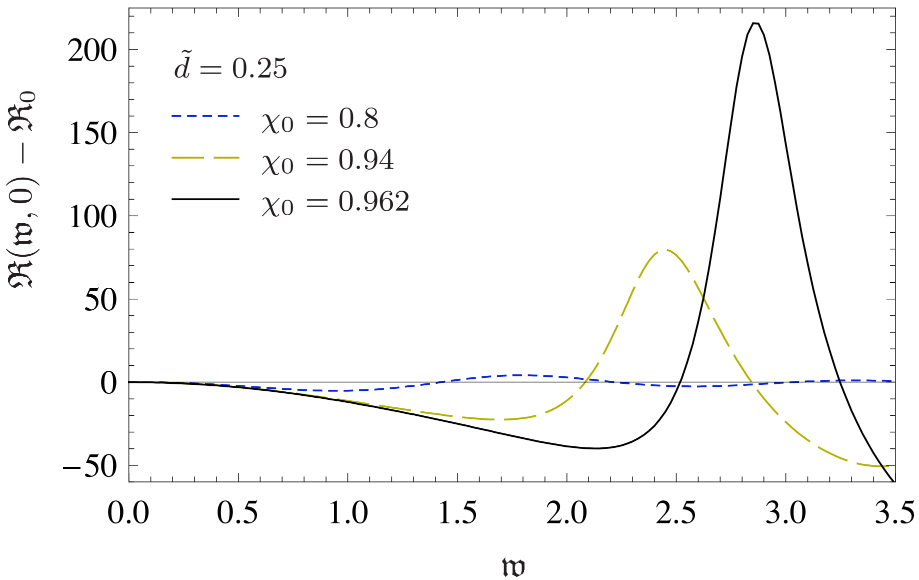

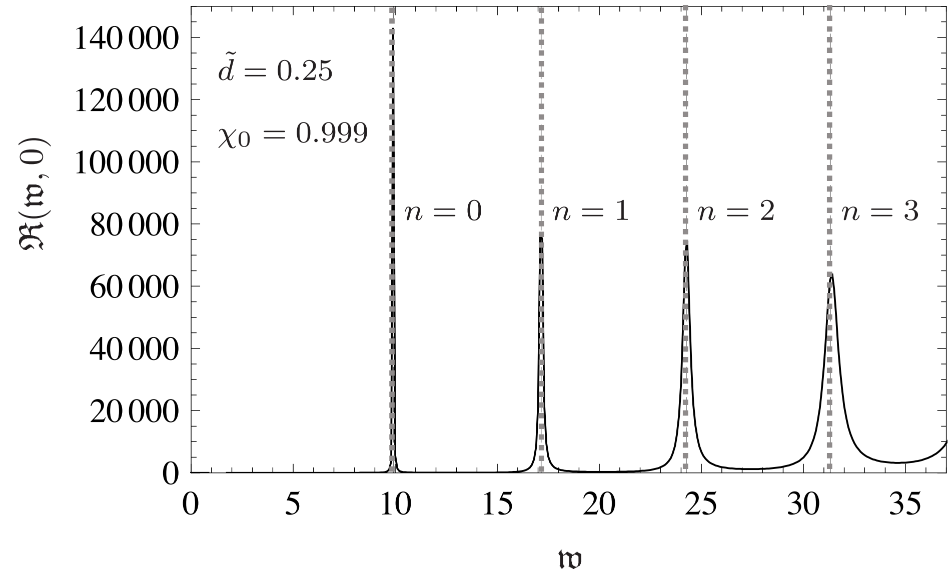

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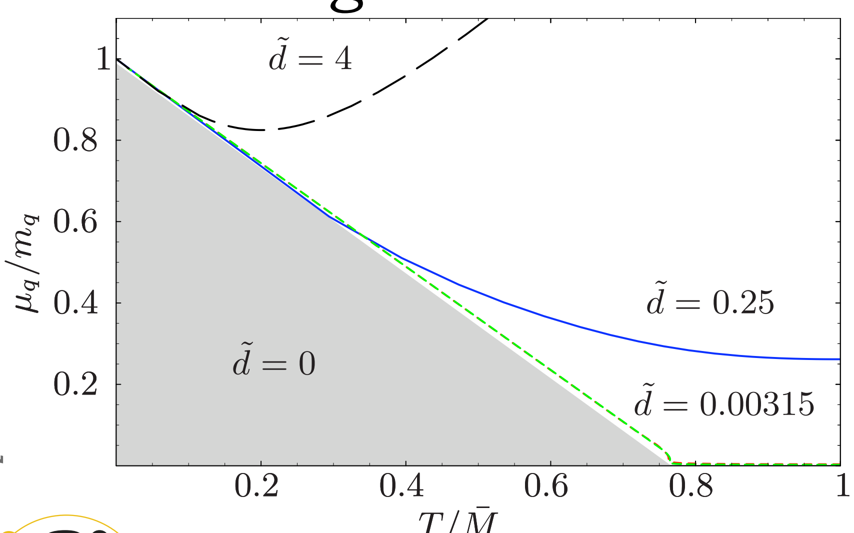

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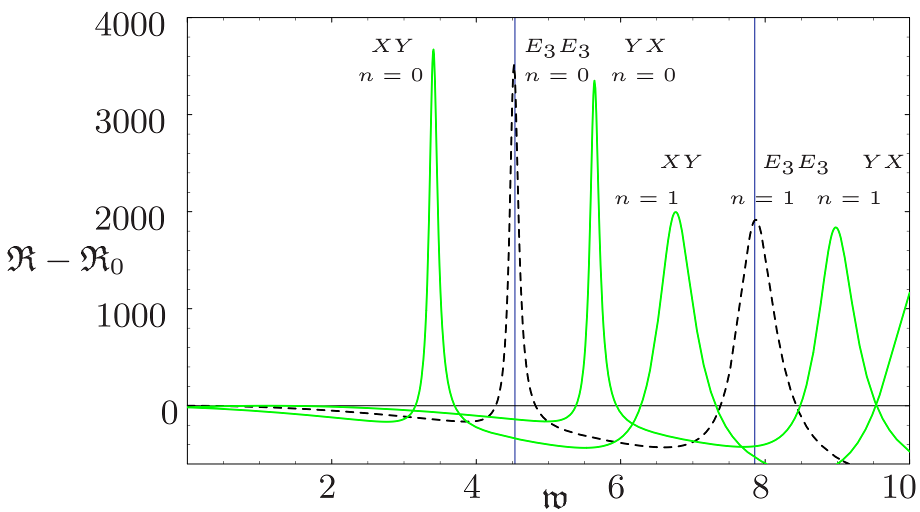
III. Isospin Chemical Potential: Spectral F's

[Erdmenger, M.K., Rust 0710.0334]

Finite isospin density



- triplet splitting
- analog to Rho-vector meson

analytical results & interpretation: [M.K. 0808.1114]

What has changed?

Background action contains

$$\mathrm{tr}(\sqrt{g + F^a T^a}) \sim N_f \times \text{Abelian}$$

↓

Diagonalize action in flavor space (X, Y, E)

↓

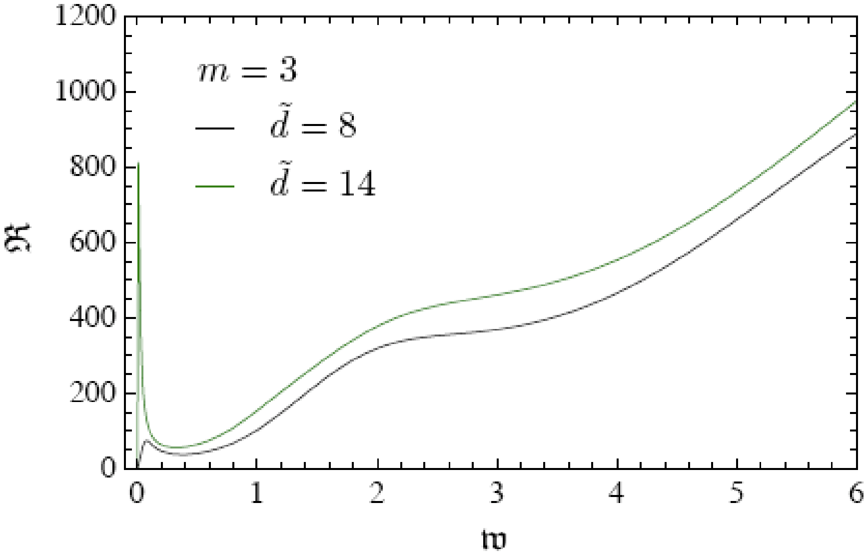
Fluctuations in three flavor directions, so two new: X, Y (orthogonal to isospin)

$$0 = X'' + \ldots X' + \ldots (\omega - \mu) X$$
$$0 = Y'' + \ldots Y' + \ldots (\omega + \mu) Y$$

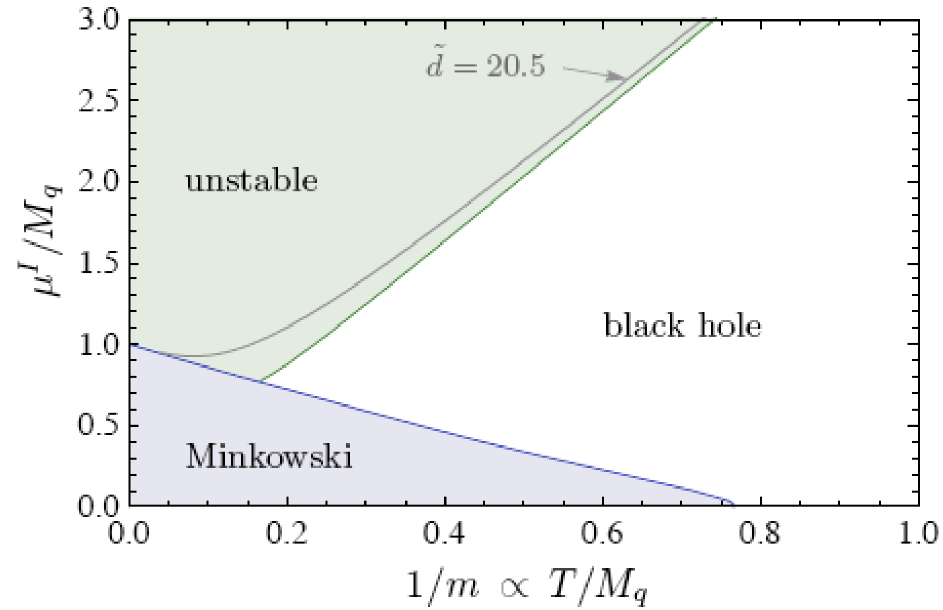
III. High isospin densities: Instability!

[Erdmenger, M.K., Kerner, Rust 0807.2663]

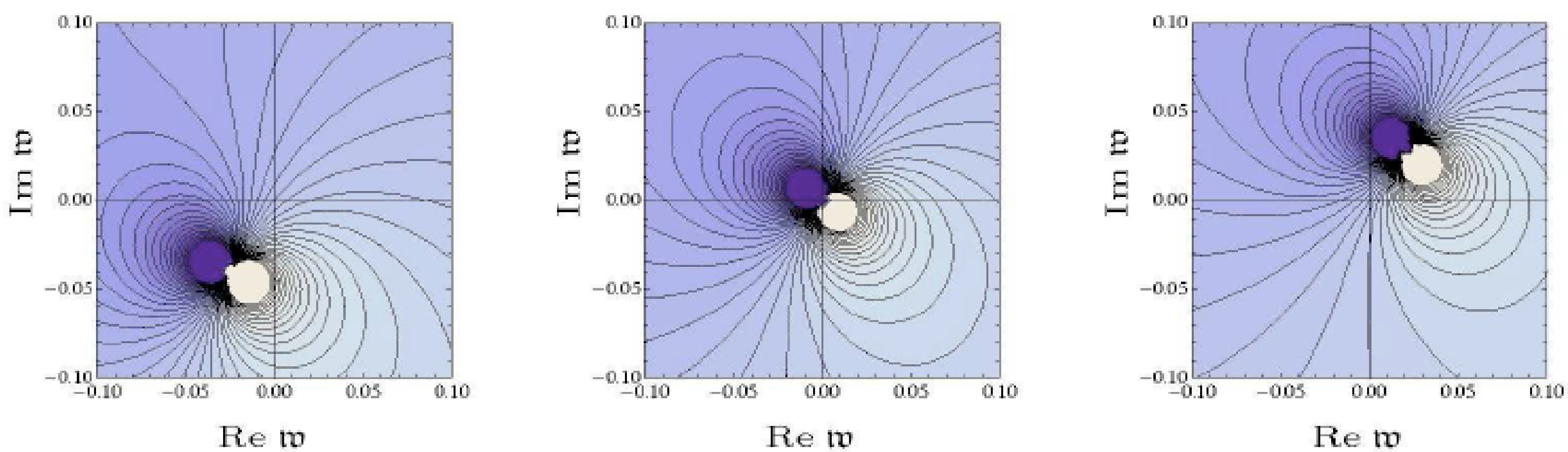
New (lowest) mesonic excitation:



New phase:



Stability:



First summary: QGP Phenomenology

✓ stable mesons survive deconfinement

Lattice QCD: [Umeda et al.'02, ...]

✓ meson mass changes with temperature

Lattice QCD: [Umeda et al.'02, ...]

✓ vec-mesons are isospin triplets (QCD's Rho-meson)

✓ new excitation/phase: instability

2-flavor QCD: [Splittorff et al.'03] ; Sakai-Sugimoto: [Aharony et al.'07]

☐ stabilize the new phase, new ground state

Navigator

✓ Invitation: Superconductivity & Holography

✓ Review: Holographic Concepts

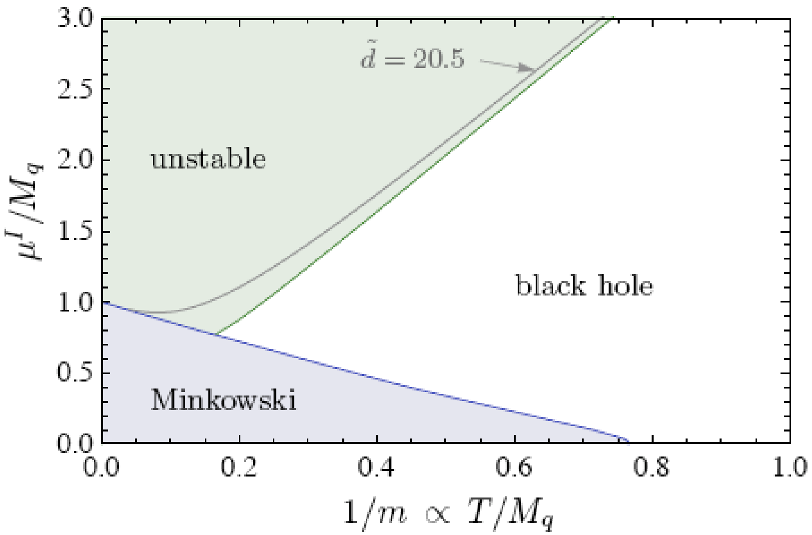
✓ Details: Flavored Plasma (D3/D7)

IV. Results: Flavor Superconducting Phase (D3/D7)

V. Discussion

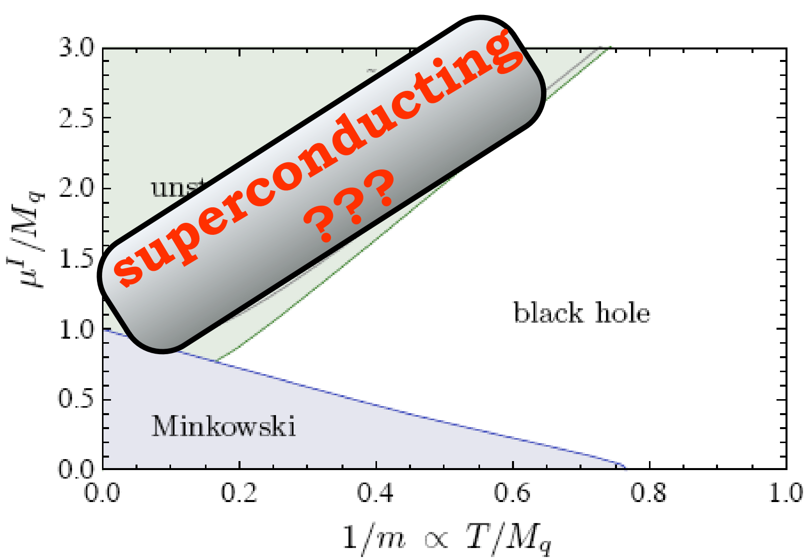
IV. Flavor Superconducting Phase

General idea



[Erdmenger, M.K., Kerner, Rust 0807.2663]

$$A_0^3 = \mu + \frac{d}{\rho^2} + \ldots$$



[Ammon, Erdmenger, M.K., Kerner 0810.2316]

$$A_0^3 = \mu + \frac{d_0^3}{\rho^2} + \ldots$$

$$A_3^1 = \frac{d_3^1}{\rho^2} + \ldots$$

← spont. breaks U(1)

[Gubser, Pufu 0805.2960]

IV. Field Theory Picture

Gravity field $A_0^3 = \mu + \frac{d_0^3}{\rho^2} + \dots$

dual to current $J_0^3 \propto \bar{\psi}\tau^3\gamma_0\psi + \phi\tau^3\partial_0\phi = n_u - n_d$

explicitly breaks $U(2) \sim U(1)_B \times SU(2)_I \rightarrow U(1)_B \times U(1)_3$

New field $A_3^1 = \frac{d_3^1}{\rho^2} + \dots$

dual to current
$$J_3^1 \propto \bar{\psi}\tau^1\gamma_3\psi + \phi\tau^1\partial_3\phi$$
$$= \bar{\psi}_u\gamma_3\psi_d + \bar{\psi}_d\gamma_3\psi_u + \text{bosons}$$

spontaneously breaks $U(1)_3 \leftrightarrow U(1)_{\text{em}}$

Exercise 1: Derive the background EOMs.

$$S_{\mathrm{DBI}} = -T_{D7}\int d^8\xi \,\mathrm{Str}\left(\sqrt{\det Q}\sqrt{\det\left(P_{ab}\left[E_{\mu\nu}+E_{\mu i}(Q^{-1}-\delta)^{ij}E_{j\nu}\right]+2\pi\alpha' F_{ab}\right)}\right)$$

$$Q^i{}_j = \delta^i{}_j + i2\pi\alpha'[\Phi^i,\Phi^k]E_{kj}\,, \qquad E_{\mu\nu} = g_{\mu\nu}+B_{\mu\nu}$$

$$(B\equiv 0)$$

$$\mu,\,\nu = 0,\ldots,9; \quad a,\,b = 0,\ldots,7; \quad i,\,j = 8,9$$

8,9 rotation: $\Phi^9 \equiv 0 \quad \Longrightarrow \quad Q^i_j = \delta^i_j$

Choose: $A = A_0^3\,dt\,\tau^3 + A_3^1\,dz\,\tau^1$ and $\Phi^8 \,||\, \tau^0$

$\Longleftrightarrow$ FT: charge eigenstates are also mass eigenstates

$\Longrightarrow$ scalars Φ^8, Φ^9 decouple from vectors A
(set to zero from now on, i.e. massless quarks)

Exercise 1: Derive the background EOMs.

$$S_{\rm DBI} = -T_{D7} \int d^8\xi \, {\rm Str}\{\sqrt{\det[g + 2\pi\alpha' F]}\}$$

$$= -T_{D7} N_f \int d^8\xi \, \sqrt{-g} \, {\rm Str} \sqrt{1 + g^{tt} g^{\rho\rho} (F^3_{\rho 0})^2 (\sigma^3)^2 + g^{33} g^{44} (F^1_{\rho 3})^2 (\sigma^1)^2 - c^2 g^{tt} g^{33} (F^2_{03})^2 (\sigma^2)^2}$$

$$F = dA + [A, A]$$

(e.g. [Myers et al. hep-th/0611099])

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$$F = dA + [A, A]$$

$$= \partial_\rho A^3_0 \, \tau^3 \, d\rho \wedge dt + \partial_\rho A^1_3 \, \tau^1 \, d\rho \wedge dz + i\epsilon^{231} A^3_0 A^1_3 \, dt \wedge dz$$

(e.g. [Myers et al. hep-th/0611099])

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Problem 1: How to evaluate the symmetrized trace of the square root exactly?

Problem 2: Non-Abelian DBI-action only known to fourth order in α'.

(e.g. [Myers et al. hep-th/0611099])

Exercise 1: Derive the background EOMs.

$$S_{\mathrm{DBI}} = -T_{D7}\int d^8\xi\, \mathrm{Str}\{\sqrt{\det[g+2\pi\alpha' F]}\}$$

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Problem 1: How to evaluate the symmetrized trace of the square root exactly?

Solution: Set commutators zero, set $(\sigma^i)^2=1$ inside symmetrized trace.

Problem 2: Non-Abelian DBI-action only known to fourth order in α'.

Solution: Expand square root to fourth order in α'.

(e.g. [Myers et al. hep-th/0611099])

Exercise 1: Derive the background EOMs.

$$S_{\rm DBI} = -T_{D7}\int d^8\xi\,{\rm Str}\{\sqrt{\det[g+2\pi\alpha' F]}\}$$

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$\Longrightarrow$ equations of motion

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Legendre transformed:

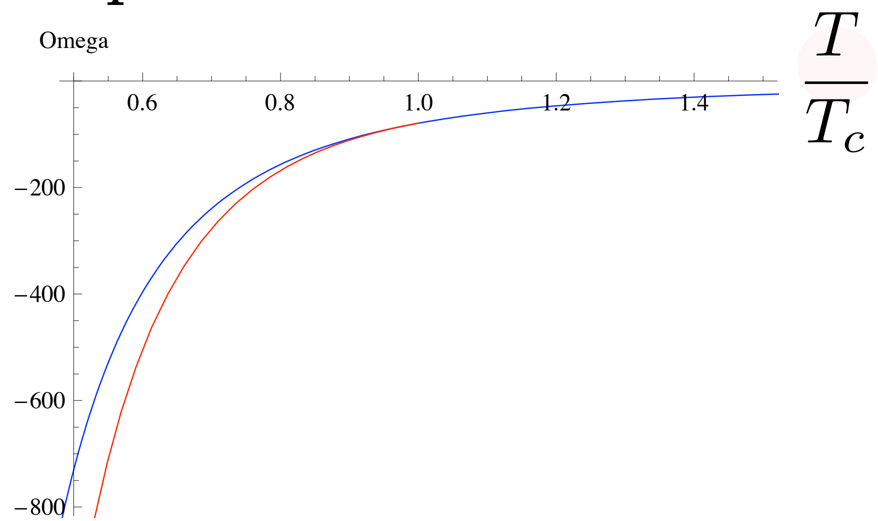
$$\tilde{S}_{\rm DBI} = -N_fT_{D7}\int d^8\xi\sqrt{-g}\left[(1-\frac{2c^2(A^3_0A^1_3)^2}{\pi^2\rho^4f^2})(1+\frac{8(p^3_0)^2}{\rho^6 f^3}-\frac{8(p^1_3)^2}{\rho^6\tilde{f}f^2})\right]^{\frac{1}{2}}$$

= factor * grand potential

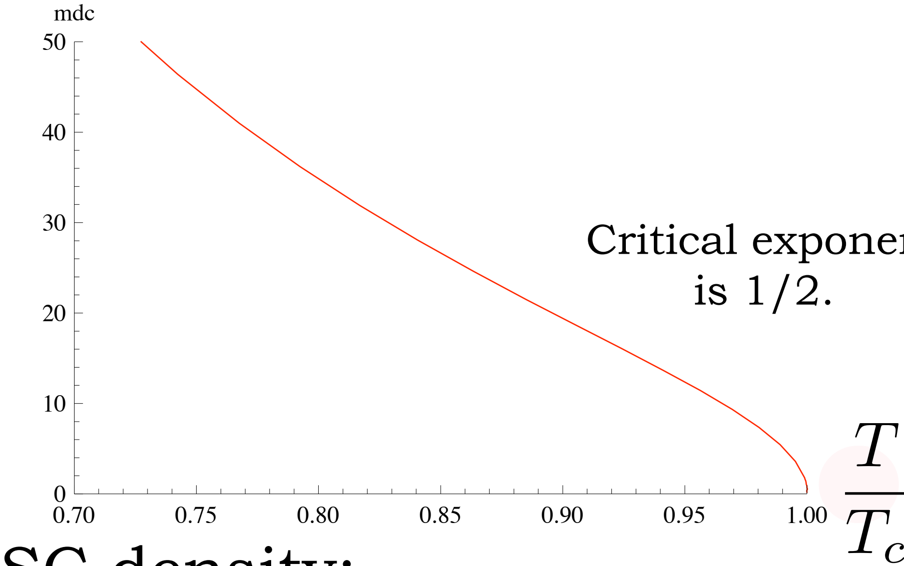
(e.g. [Myers et al. hep-th/0611099])

IV. Thermodynamics

Grand potential:

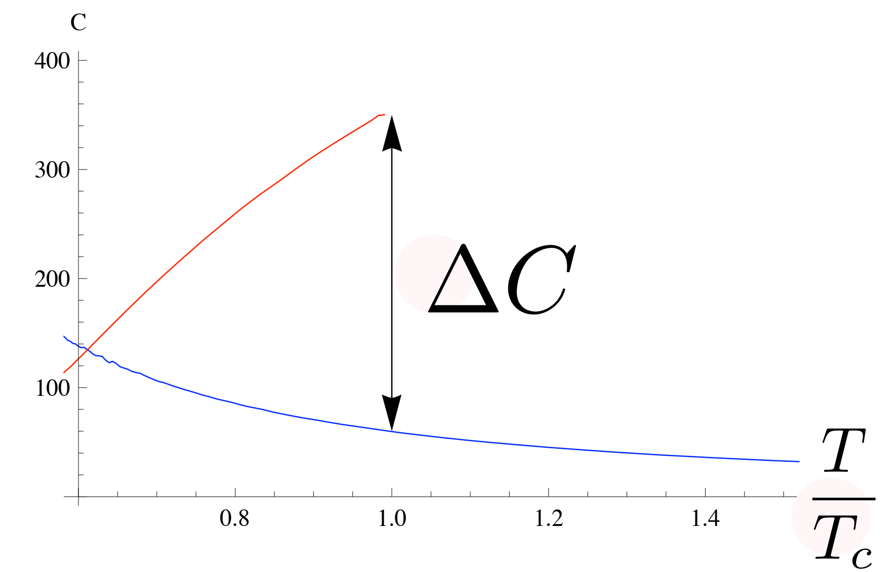


Order parameter:

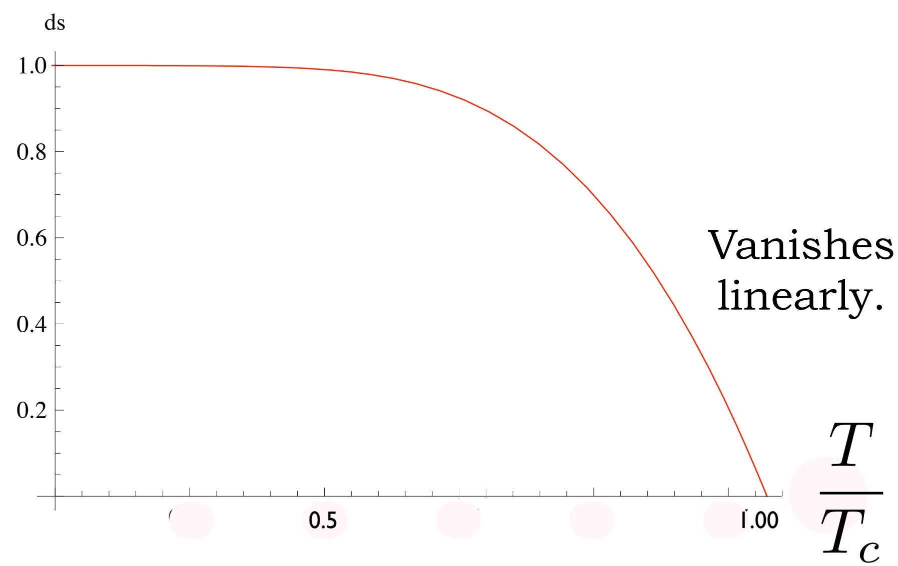


Critical exponent is 1/2.

Specific heat:



SC density:

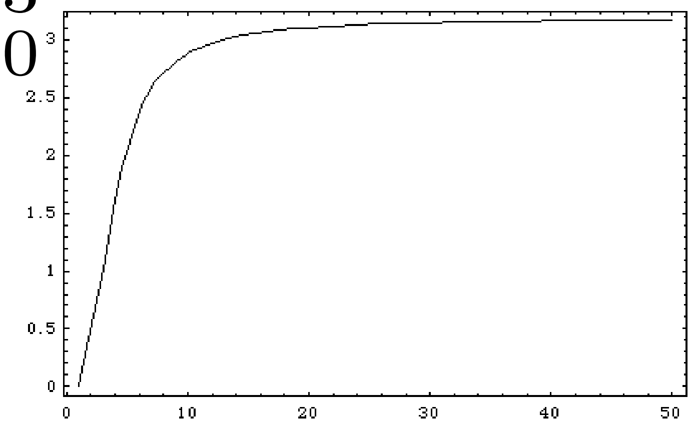


Vanishes linearly.

IV. Background field configuration

Gravity fields:

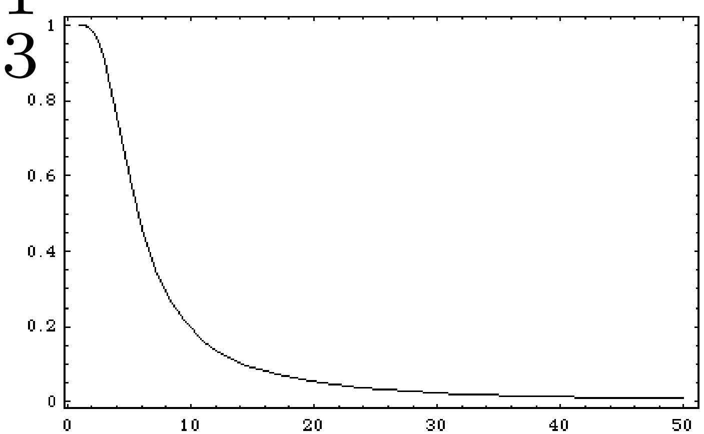
A_0^3



$\longrightarrow \mu_0^3$

ρ

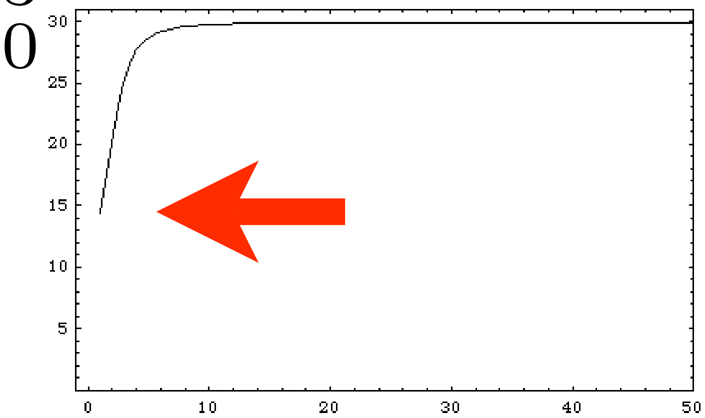
A_3^1



ρ

Conjugate momenta:

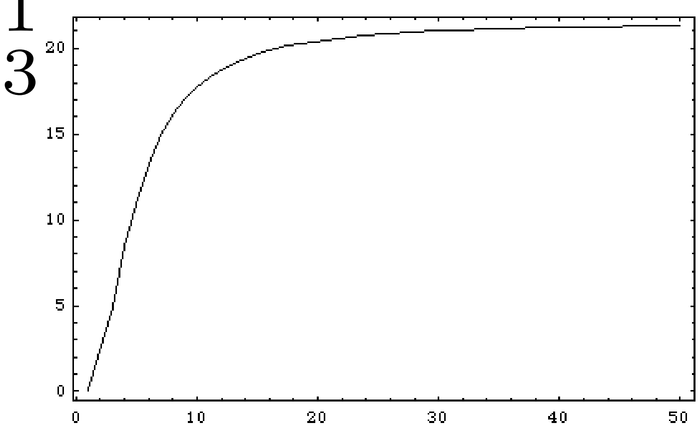
p_0^3



$\longrightarrow d_0^3$

ρ

p_3^1



$\longrightarrow d_3^1$

ρ

Exercise 2: Derive the fluctuation EOMs.

$$A = A^3_0\, dt\, \tau^3 + A^1_3\, dz\, \tau^1 + \tilde{A}^a_m dx^m \tau^a$$

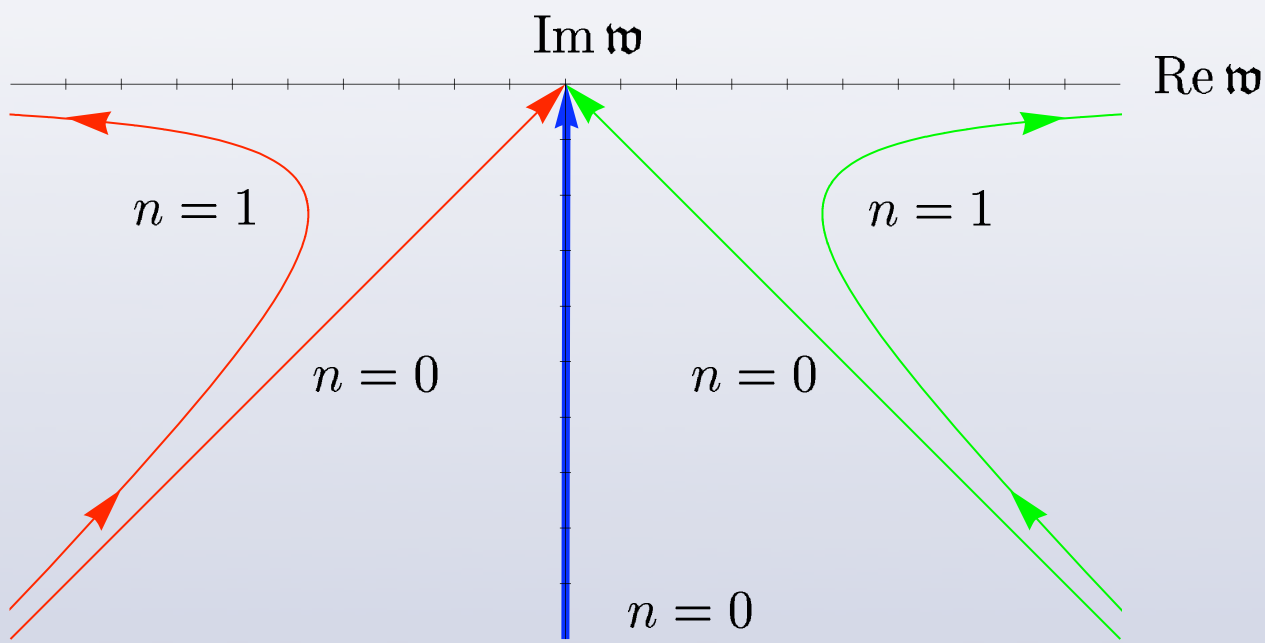
Linearized fluctuation equation of motion:

$$(\tilde{A}^3_2)'' + \frac{\partial_\rho H}{H}(\tilde{A}^3_2)' - \left[\frac{4\varrho_H^4}{R^4}\left(\frac{\mathcal{G}^{33}}{\mathcal{G}^{44}}(\mathfrak{m}^1_3)^2 + \frac{\mathcal{G}^{00}}{\mathcal{G}^{44}}\mathfrak{w}^2\right) - 16\frac{\partial_\rho\left(\frac{H}{\rho^4 f^2}A^3_0(\partial_\rho A^3_0)(\mathfrak{m}^1_3)^2\right)}{H\left(1 - \frac{2c^2}{\pi^2\rho^4 f^2}(A^1_3 \tilde{A}^3_0)^2\right)}\right]\tilde{A}^3_2 = 0$$

$$\mathfrak{m}^1_3 = \frac{c}{2\sqrt{2}\pi}A^1_3$$

IV. Stability

Poles of X, Y, $\tilde{A}_2^3$ in complex plane:

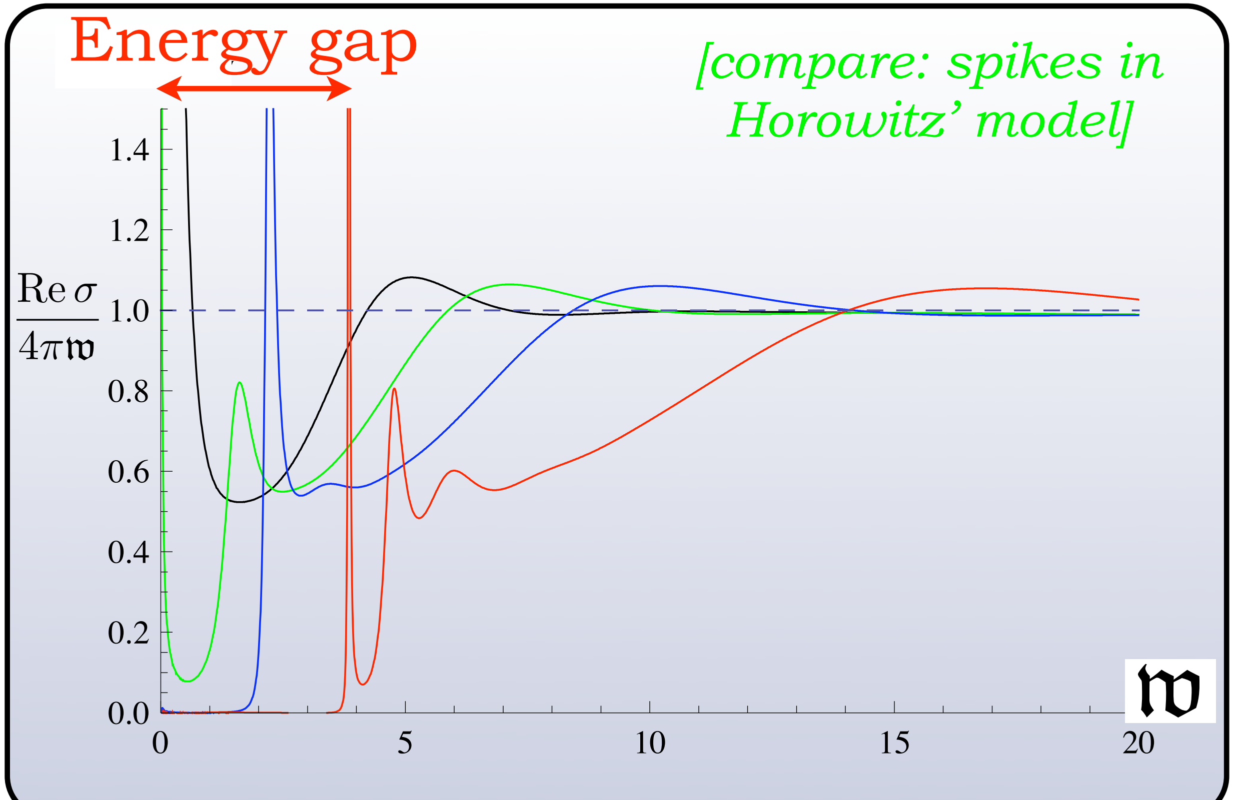


IV. Conductivity

Conductivity:

$$\sigma = \frac{J}{E} = \frac{A^{(2)}}{\partial_t A^{(0)}} \sim \frac{i A^{(2)}}{\omega A^{(0)}} = -\frac{i}{\omega}\frac{\rho^3 A'}{2\,A} = \frac{i}{\omega} G^{\mathrm{ret}}(\omega, \mathbf{q}=0)$$

with flavorelectric current $J_m \longleftrightarrow A_m \in U(1)$



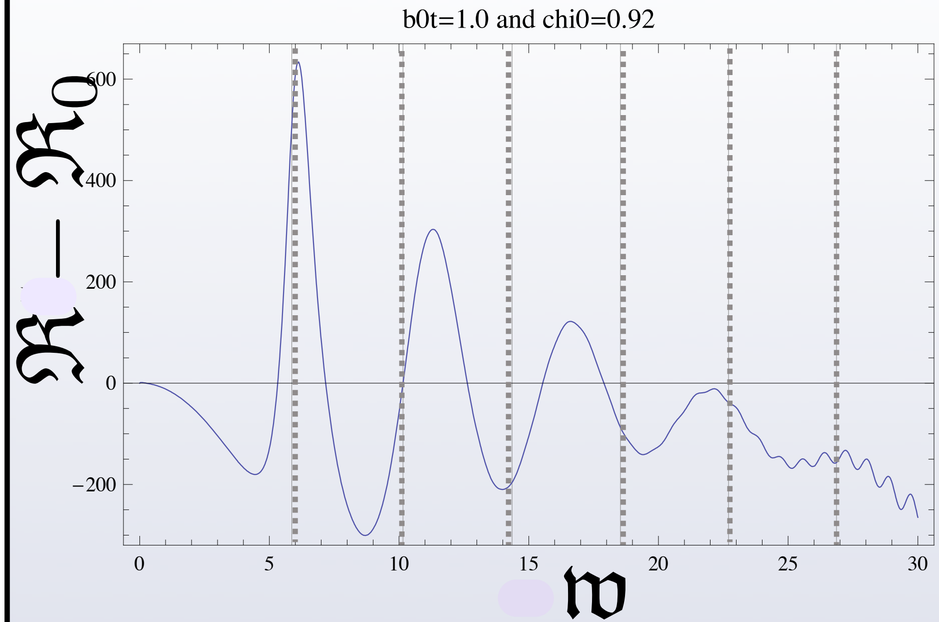
Only the first of these peaks was seen to second order in F. *[Basu et al. 0810.3970]*

Our expansion to fourth order shows all peaks **at zero quark mass!**

→ Peaks are higher order effect in F .

IV. Higgs mechanism & Meissner effect

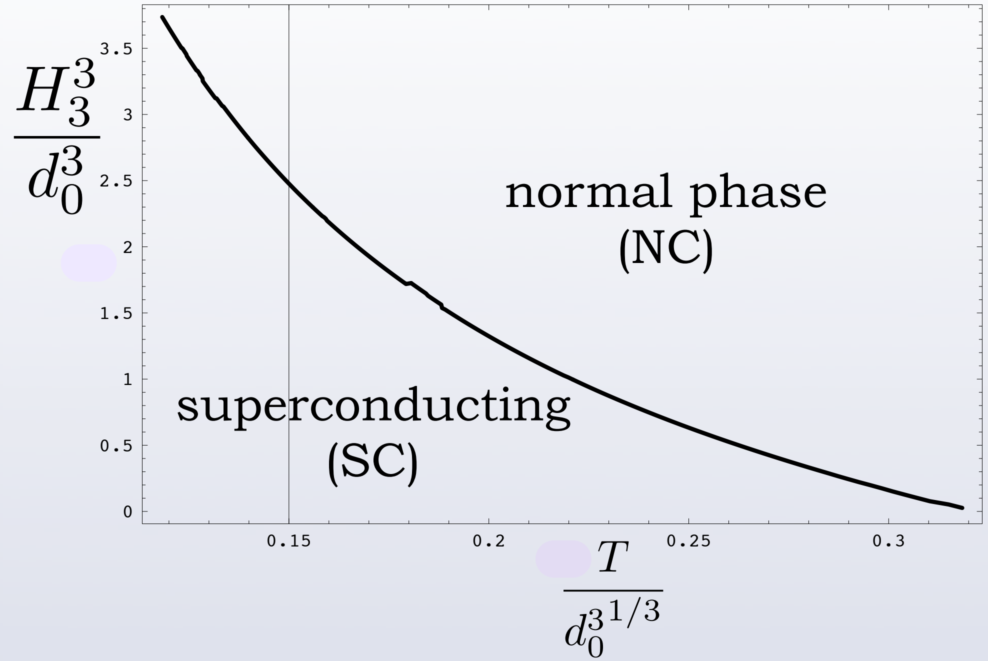
Peaks at finite mass:



Peaks in conductivity/ spectral function approach SUSY vector meson spectrum at large quark mass.

➔ Bulk Higgs Mechanism generates meson mass

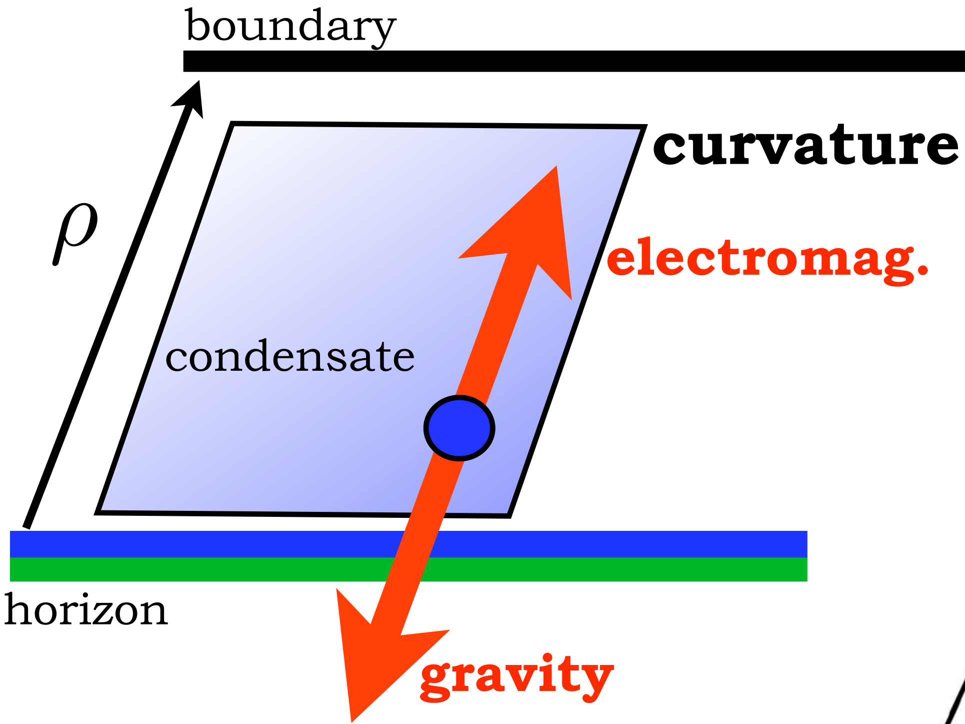
Finite magnetic field:



Add background field component: $H_3^3 = F_{12}^3 = \partial_1 A_2^3$

➔ Induced currents in SC phase with H

IV. String Theory Picture

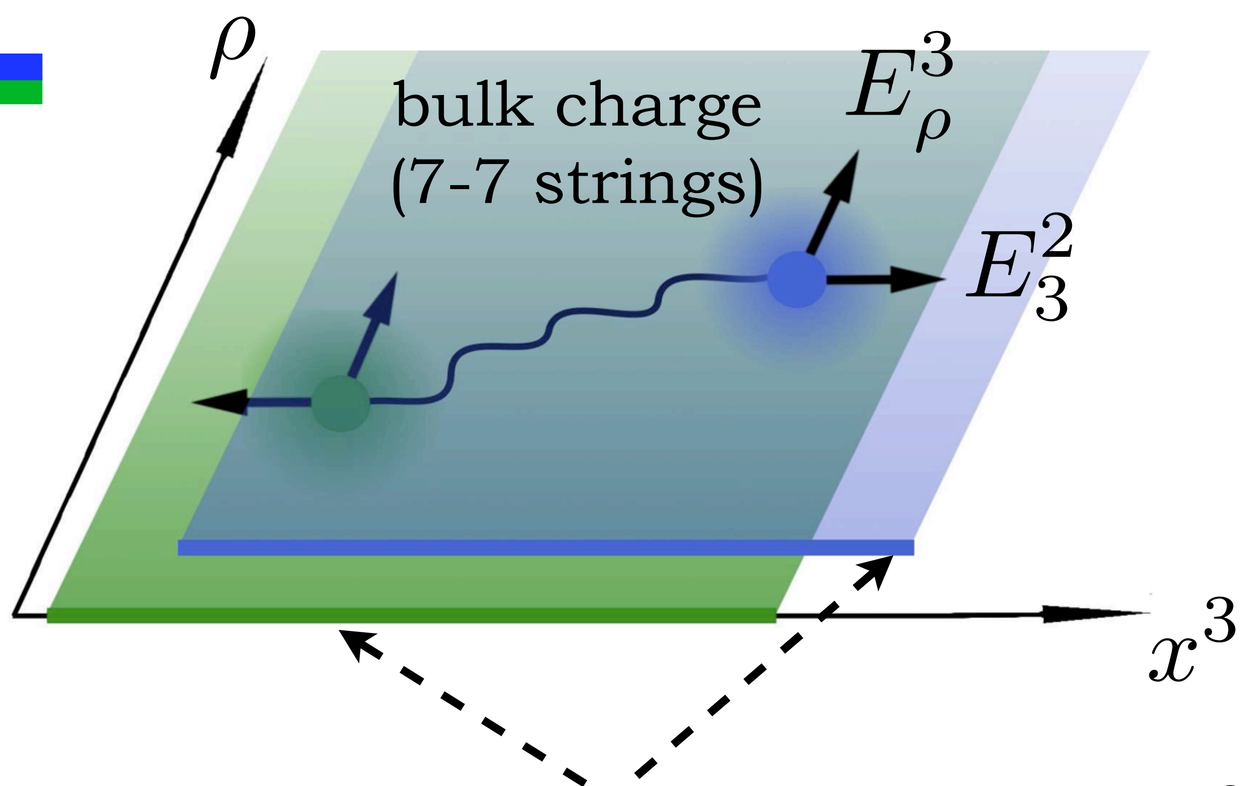


$$E^3_\rho = F^3_{\rho 0} = \partial_\rho A^3_0$$

$$B^1_{\rho 3} = F^1_{\rho 3} = \partial_\rho A^1_3$$

$$E^2_3 = F^2_{30} = A^1_3 A^3_0$$

7-7 strings generate A^1_3
i.e. they break the U(1)
and are thus dual to
Cooper pairs.



charged horizon (3-7 strings generate A^3_0)

IV. Discussion

✓ Rich strong coupling phenomenology (QGP)

- Resonances are vector mesons (analogous to rho-meson)
- Vector mesons survive deconfinement

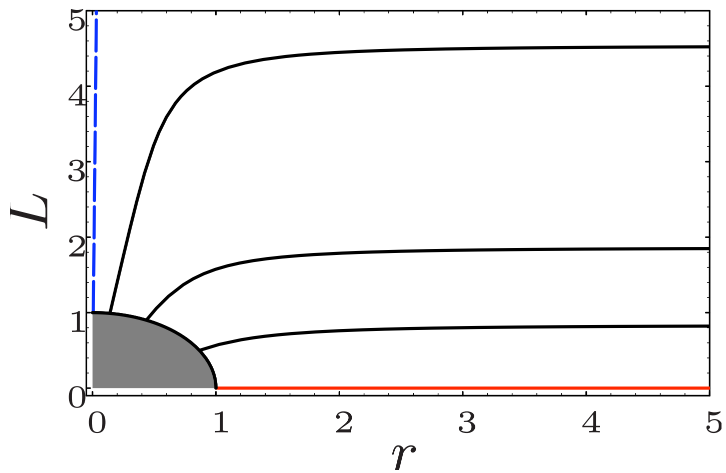
✓ Top-down approach: direct identification of d.o.f.

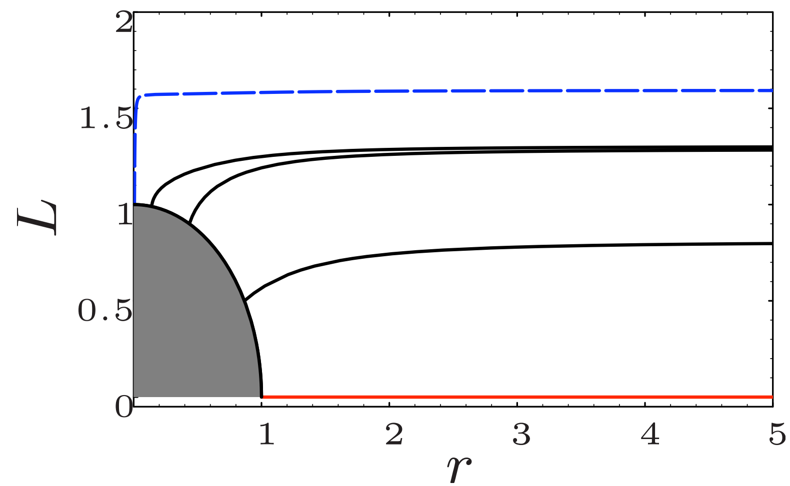
✓ Energy gap, Meissner effect, Higgs mechanism

✓ Stringy picture of pairing mechanism

☐ Critical exponents
☐ Speed of second, fourth sound (backreact)
☐ Drag on D7-D7 strings
☐ Fermi surface?

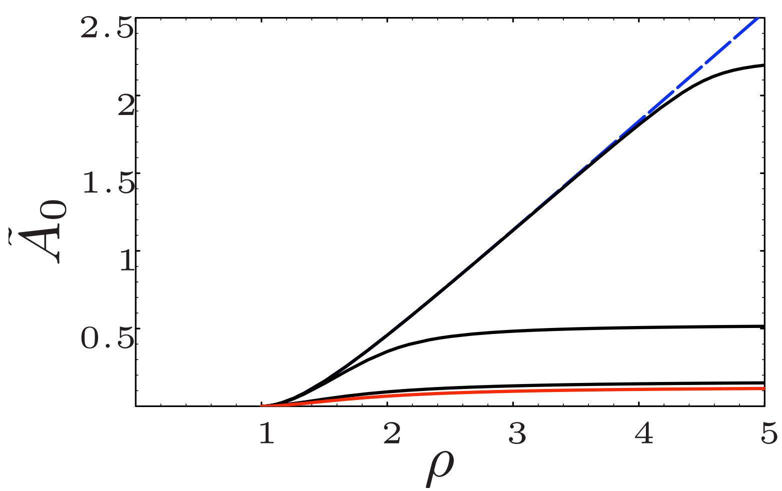
APPENDIX: Embeddings

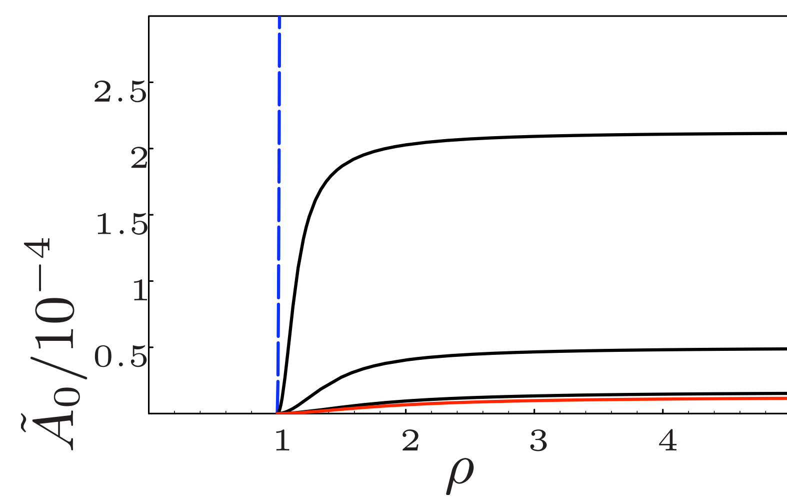




$\tilde{d} = 0.25$

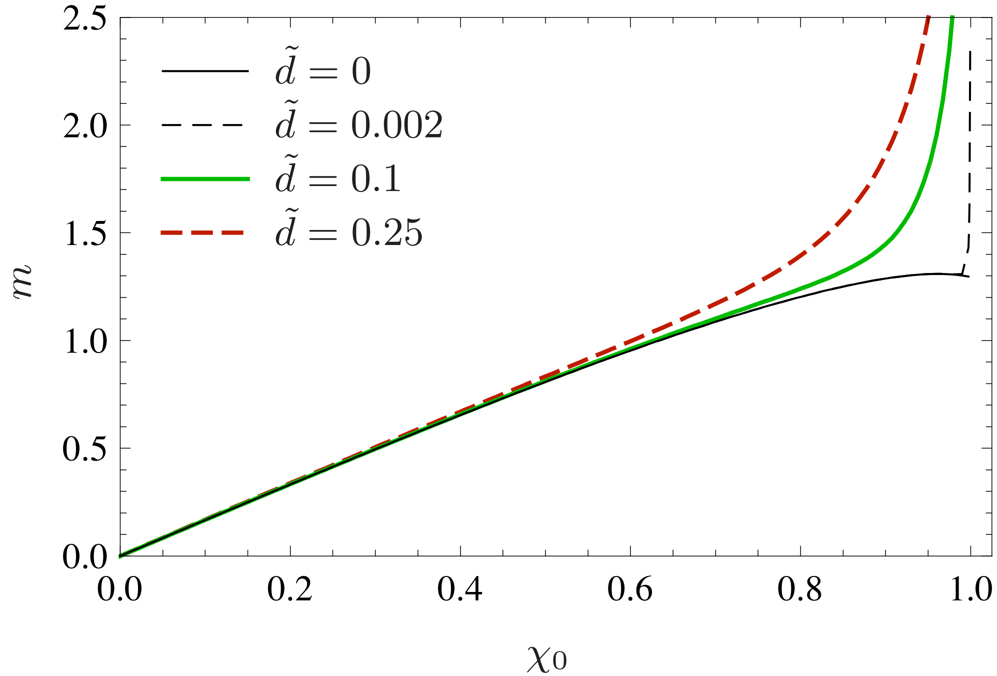
$\tilde{d} = \frac{10^{-4}}{4}$





APPENDIX: Parameters

The mass parameter m depending on the parameter χ_0.



$$\chi_0 = \chi(\rho)\Big|_{\rho\to\rho_H}$$

$$m = \lim_{\rho\to\rho_{\rm bdy}} \rho\,\chi(\rho) = \frac{2m_{\rm quark}}{\sqrt{\lambda}T}$$

Near-boundary expansions:

$$\chi(\rho) = \frac{m}{\rho} + \frac{c}{\rho^3} + \dots$$

$$A_0 = \mu - \frac{1}{\rho^2}\frac{\tilde{d}}{2\pi\alpha'} + \dots$$

Other relations:

$$L(\varrho) = \varrho\,\chi(\varrho)\,, \quad \rho = \frac{\varrho}{\varrho_H}$$

APPENDIX: Fluctuations

Equation of motion written out:

$$0=\tilde{A}''+\partial_\rho \ln\left(\frac{1}{8}\tilde{f}^2 f\rho^3(1-\chi^2+\rho^2\chi'^2)^{3/2}\times\sqrt{1-\frac{2\tilde{f}(1-\chi^2)(\partial_\rho A_0)^2}{f^2(1-\chi^2+\rho^2\chi'^2)}}\right)\tilde{A}'+8\mathfrak{w}^2\frac{\tilde{f}}{f^2}\frac{1-\chi^2+\rho^2\chi'^2}{\rho^4(1-\chi^2)}\tilde{A}$$

$$\rho=\frac{\varrho}{\varrho_H}\quad,\quad \tilde{f}(\varrho)=1+\frac{{\varrho_H}^4}{\varrho^4}\,,\; f(\varrho)=1-\frac{{\varrho_H}^4}{\varrho^4}\quad,\quad L(\varrho)=\varrho\,\chi(\varrho)\;\;,\;\mathfrak{w}=\frac{\omega}{2\pi T}$$

The D7-brane embedding $\chi(\rho)$ and gauge field component $A_0(\rho)$ are given numerically.

$$\partial_\rho A_t=2\tilde{d}\frac{f^2\sqrt{1-\chi^2+\rho^2\dot{\chi}^2}}{\sqrt{\tilde{f}(1-\chi^2)[\rho^6\tilde{f}^3(1-\chi^2)^3+8\tilde{d}^2]}}$$

$$A_0\equiv A_t$$

APPENDIX: Extension of the correspondence

		Universality	Original AdS/CFT correspondence	AdS Schwarzschild black hole (D3/D7)
Gauge		QCD	$\mathcal{N}=4$ SuperYangMills	thermal Yang-Mills
Gravity		?	Type II Sugra in AdS	TypeII Sugra in AdS Schwarzschild b.h.
Gauge theory symmetry	non-conf.	✓	⊙	✓
	non-SUSY	✓	⊙	✓
Relations				$T \leftrightarrow$ horizon $\mu_B\,,\ \mu_I \leftrightarrow A_0(\rho)$

$$g_{YM}{}^2 = g_s \qquad \frac{R^4}{(\alpha')^2} = 4\pi N_c g_s \equiv \lambda$$