Holographic Superconductors

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Outline – Lecture 1

1) Introduction to superconductivity
2) Simple model for a holographic superconductor
3) Probe limit (condensate and conductivity)

Outline – Lecture 2

1) Full solution
   (new insight into conductivity)
2) Ground state of the holographic superconductor
3) Embedding in string theory
4) Magnetic fields

References

Review articles:

1) S. Hartnoll, *Lectures on holographic methods for condensed matter physics,* 0903.3246
2) C. Herzog, *Lectures on Holographic Super-fluidity and Superconductivity*, 0904.1975

Some original papers:

S. Hartnoll, C. Herzog, G.H., 0803.3295 and 0810.1563

M. Roberts, G.H., 0810.1077 and 0908.3677

Superconductivity 101

In conventional superconductors (Al, Nb, Pb, …) pairs of elections with opposite spin can bind to form a charged boson called a Cooper pair.

Below a critical temperature T_c, there is a second order phase transition and these bosons condense.

The DC conductivity becomes infinite.

There is an energy gap Δ for charged excitations.

Landau and Ginzburg proposed a phenomenological description of superconductivity in 1950. Phase transition is put in by hand. There is a complex field ψ and the free energy is assumed to be

$$F = \alpha(T - T_c)|\psi|^2 + \beta|\psi|^4 + ...$$

Bardeen, Cooper, Schrieffer (BCS) proposed a microscopic theory of superconductivity in 1957. It predicts that the energy gap Δ is related to the critical temperature by $\Delta = 1.77\ T_c$.

The electron-phonon interaction is weak, and Cooper pairs are much larger than the interatomic spacing.

It was once thought that the highest T_c for a BCS superconductor was around 30^o K. But in 2001, MgB_2 was found to be superconducting at 40^o K and is believed to be described by BCS. Some people now speculate that BCS could describe a superconductor with T_c = 200^o.

The new high T_c superconductors were discovered in 1986. These cuprates (e.g. YBaCuO) are layered and superconductivity is along CuO_2 planes.

Highest T_c today (HgBaCuO) is $T_c = 134K$.

Another class of superconductors discovered March 08 based on iron and not copper FeAs(…) Highest $T_c = 56K$.

The pairing mechanism is not well understood. Unlike BCS theory, it involves strong coupling.

AdS/CFT is an ideal tool to study strongly coupled field theories.

Physics here described by CFT associated with quantum critical point x_m

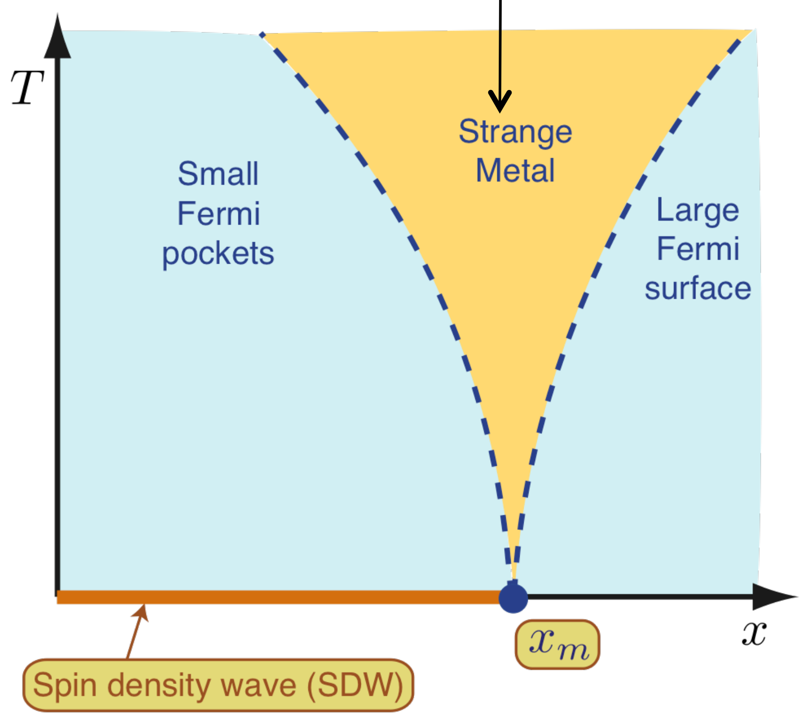


Figure 2 The SDW QCP in the metal and its crossover regimes.

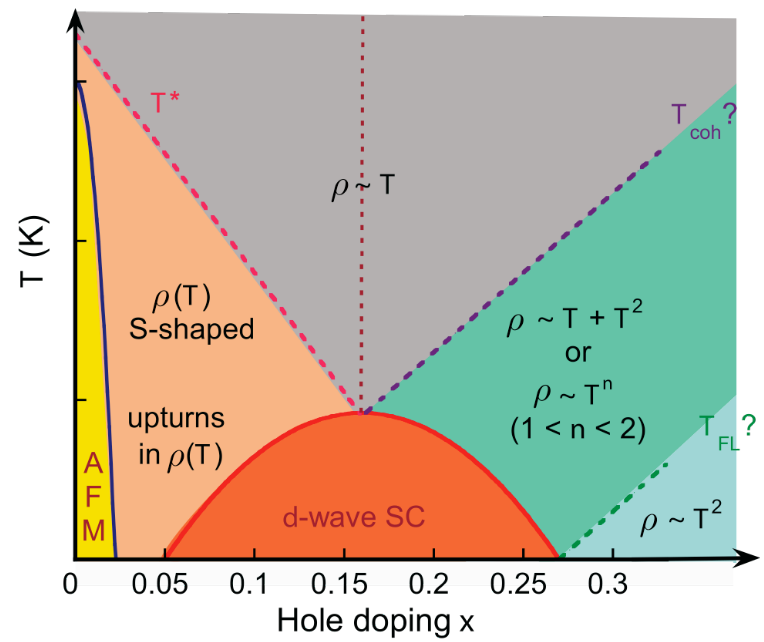


Figure 1 From Ref. [1]: Crossover phase diagram for the resistivity (ρ) in the hole doped cuprates. The strange metal phase is the regime above optimal doping where $\rho \sim T$.

Taken from Sachdev, 0907.0008

AdS/CFT Dictionary

Gravity	Superconductor
Black hole	Temperature
Charged scalar field	Condensate

Need to find a black hole that has scalar hair at low temperatures, but no hair at high temperatures.

This is not an easy task.

String theory has many “dilatonic” black holes with scalar hair, but this is a result of a coupling

$$e^{2a\phi} F_{\mu\nu} F^{\mu\nu}$$

F^2 is a source for ϕ, so all charged black holes have nonzero ϕ. This “secondary hair” is not what we want.

Hertog (2006) showed that for a real scalar field with arbitrary potential $V(\phi)$, neutral AdS black holes have scalar hair iff AdS is unstable.

Gubser (2008) argued that a charged scalar field around a charged black hole would have the desired property. Consider

$$\mathcal{L} = R + \frac{6}{L^2} - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} - |\partial\Psi - iqA\Psi|^2 - m^2|\Psi|^2$$

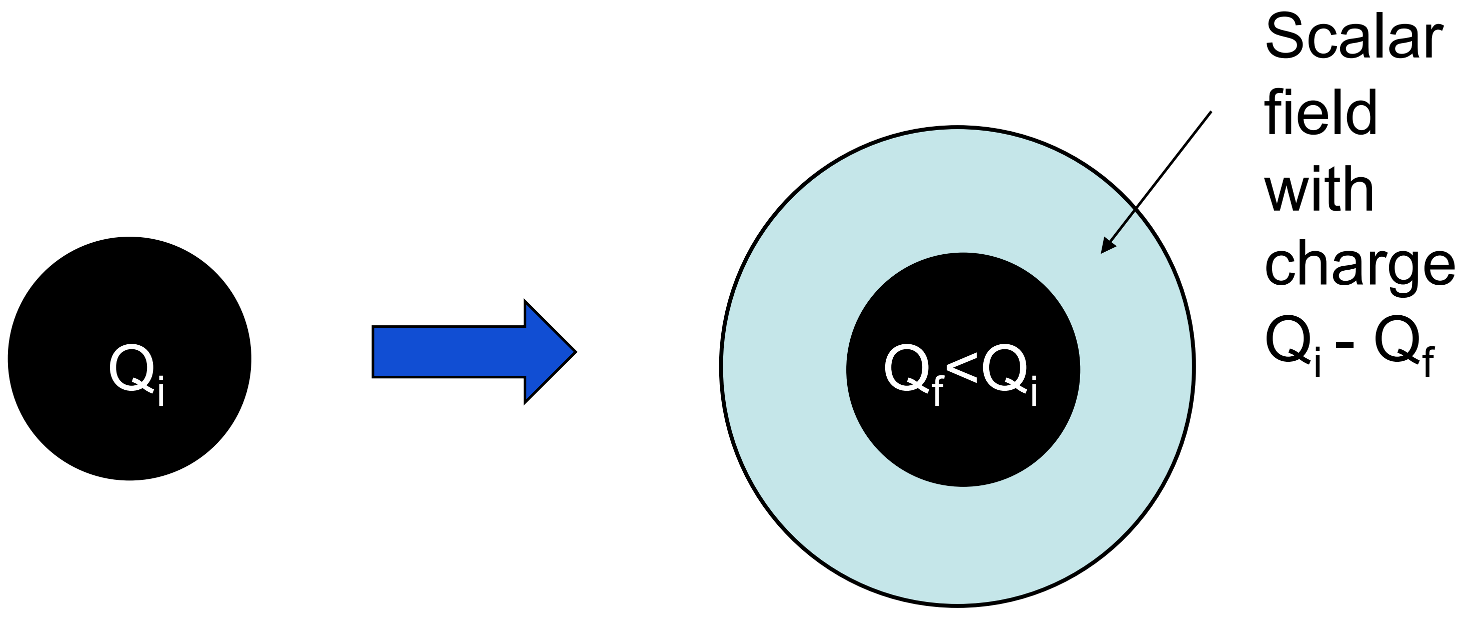
For an electrically charged black hole, the effective mass of Ψ is

$$m^2_{eff} = m^2 + q^2 g^{tt} A_t^2$$

But the last term is negative. There is a chance for nontrivial hair.

This does not work for asymptotically flat spacetimes, but does work for asymptotically AdS spacetimes.

Quantum picture: If qQ is large enough, even extremal black holes create pairs of charged particles. In AdS, the charged particles can't escape, and settle outside the horizon.



If you rescale $A \rightarrow A/q$ and $\Psi \rightarrow \Psi/q$, then the matter action has a $1/q^2$ in front, so that large q suppresses the backreaction on the metric.

We will first consider this large q (probe) limit for a 2+1 dimensional superconductor (four dimensional bulk). Then we will generalize to other cases.

Probe Limit

We use the planar neutral black hole

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2(dx^2 + dy^2)$$

where $$f = \frac{r^2}{L^2} - \frac{M}{r}$$

Hawking temperature $$T = \frac{3M^{1/3}}{4\pi L^{4/3}}$$

Assume a plane symmetric ansatz

$$A = \phi(r) dt, \qquad \Psi = \psi(r)$$

The Maxwell equations imply that the phase of ψ must be constant, so can assume ψ is real. The field equations then take the form

$$\psi'' + \left(\frac{f'}{f} + \frac{2}{r} \right) \psi' + \frac{\phi^2}{f^2} \psi - \frac{m^2}{f} \psi = 0,$$

$$\phi'' + \frac{2}{r} \phi' - \frac{2\psi^2}{f} \phi = 0.$$

We first consider the case $m^2 = -2/L^2$.

Although the mass squared is negative, it is above the Breitenlohner-Freedman bound and hence does not induce an instability. It arises in several contexts in which the AdS_4/CFT_3 correspondence is embedded into string theory.

Boundary conditions

The source for Maxwell's equations includes a term $|\psi|^2 A_\mu$.

At the horizon where $f(r_0) = 0$, ϕ must vanish in order for $A = \phi\, dt$ to have finite norm. The field equations then implies that ψ and ψ' are not independent.

So there are a two parameter family of solutions with regular horizons labeled by $\phi'(r_0)$, $\psi(r_0)$.

Asymptotically:

$$\phi = \mu - \frac{\rho}{r}, \quad \psi = \frac{\psi^{(1)}}{r} + \frac{\psi^{(2)}}{r^2}$$

For ψ, either falloff is normalizable. After imposing the condition that either $\psi^{(1)}$ or $\psi^{(2)}$ vanish we have a one parameter family of solutions.

Solutions look boring, but they always have T/μ smaller than some bound.

Dual field theory

Start with a 2+1 CFT with a global U(1) symmetry at temperature T.

Properties of the dual field theory are read off from the asymptotic behavior of the solution:

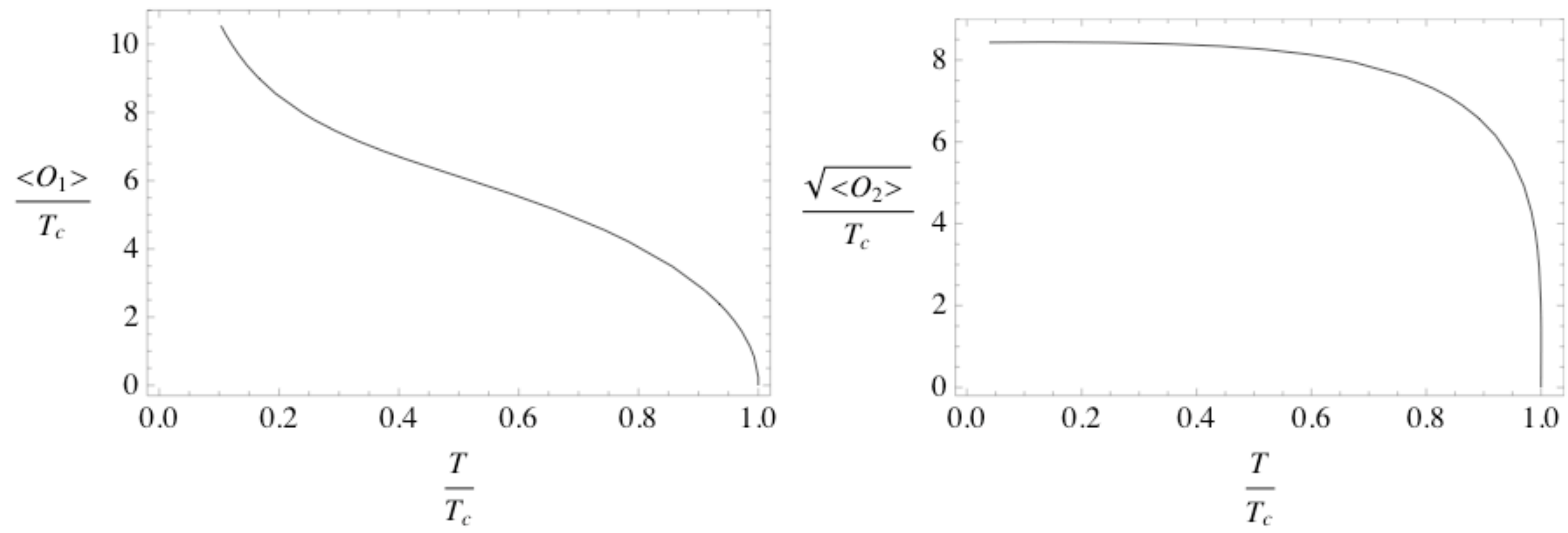
μ= chemical potential, ρ = charge density

If O is the operator dual to ψ, then

$$\langle \mathcal{O}_1 \rangle = \sqrt{2}\psi^{(1)} \quad \text{when } \psi_2 = 0$$

$$\langle \mathcal{O}_2 \rangle = \sqrt{2}\psi^{(2)} \quad \text{when } \psi_1 = 0.$$

O_i has dimension i, and μ has dimension one, so O_i / T^i and T/μ are dimensionless.



$$T_c \propto \mu$$

Condensate (hair) as a function of T

Near the transition, there is a square root behavior

$$\langle \mathcal{O}_1 \rangle \approx 9.3\, T_c \,(1 - T/T_c)^{1/2}\,, \quad \text{as} \quad T \to T_c$$

$$\langle \mathcal{O}_2 \rangle \approx 144\, T_c^2 \,(1 - T/T_c)^{1/2}\,, \quad \text{as} \quad T \to T_c$$

One can compute the free energy (euclidean action) of these hairy configurations and compare with the solution $\psi = 0$, $\phi = \mu - \rho/r$. The free energy is always lower for the hairy configurations and becomes equal as $T \to T_c$.

This is a second order phase transition.

Generalize to other dimensions and other masses

We use the planar neutral black hole (i = 1,…,d-1)

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 dx_i dx^i$$

where $$f(r) = \frac{r^2}{L^2}\left(1 - \frac{r_0^d}{r^d}\right)$$

Hawking temperature $$T = \frac{dr_0}{4\pi L^2}$$

As before, we assume

$$A = \phi(r) dt, \qquad \Psi = \psi(r)$$

The field equations are similar, but with a few d dependent coefficients

$$\psi'' + \left(\frac{f'}{f} + \frac{d-1}{r}\right)\psi' + \frac{\phi^2}{f^2}\psi - \frac{m^2}{f}\psi = 0,$$

$$\phi'' + \frac{d-1}{r}\phi' - \frac{2\psi^2}{f}\phi = 0.$$

Asymptotically:

$$\phi = \mu - \frac{\rho}{r^{d-2}} + \cdots \qquad \psi = \frac{\psi_-}{r^{\lambda_-}} + \frac{\psi_+}{r^{\lambda_+}} + \cdots$$

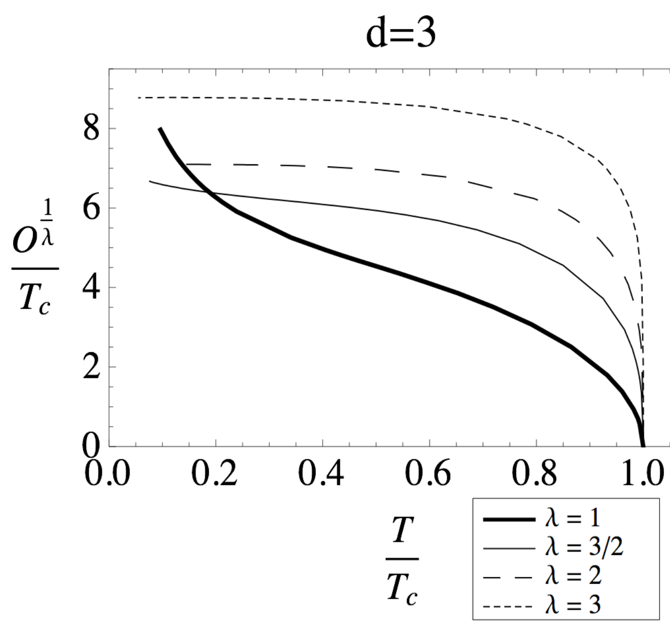
where

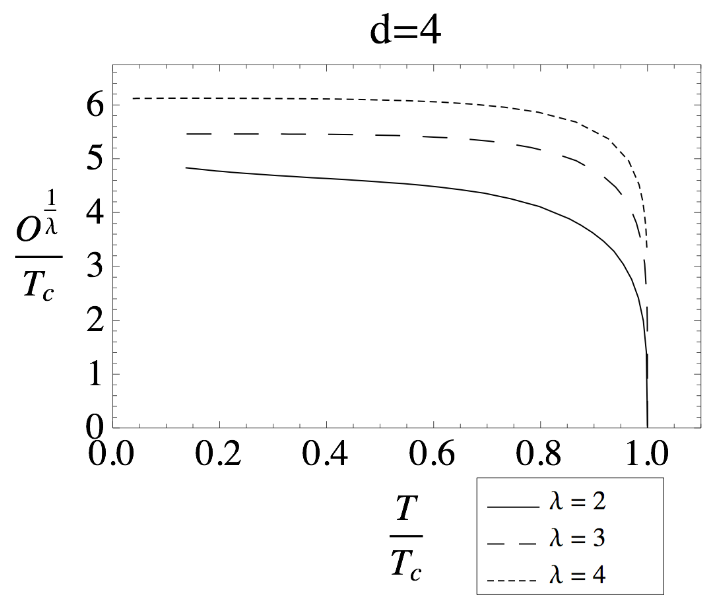
$$\lambda_\pm = \tfrac{1}{2}(d \pm \sqrt{d^2 + 4m^2}).$$

Typically, we need to impose $\psi_- = 0$, in order for ψ to be normalizable. This gives a one parameter family of solutions.

If O is the operator dual to ψ, then O has dimension λ_+ and $\psi_+ = \langle \mathcal{O} \rangle$

Condensate (hair) as a function of T





$$T_c \propto \mu$$

There is a second order phase transition at $T = T_c$.

Conductivity

We want to compute the conductivity as a function of frequency. Start by solving for fluctuations in A_x in the bulk. Maxwell's equation with zero spatial momentum and time dependence $e^{-i\omega t}$ gives

$$A_x'' + \left(\frac{f'}{f} + \frac{d-3}{r}\right) A_x' + \left(\frac{\omega^2}{f^2} - \frac{2\psi^2}{f}\right) A_x = 0$$

We want to solve this with ingoing wave boundary conditions at the horizon.

For d = 3, the asymptotic behavior is

$$A_x = A_x^{(0)} + \frac{A_x^{(1)}}{r} + \cdots$$

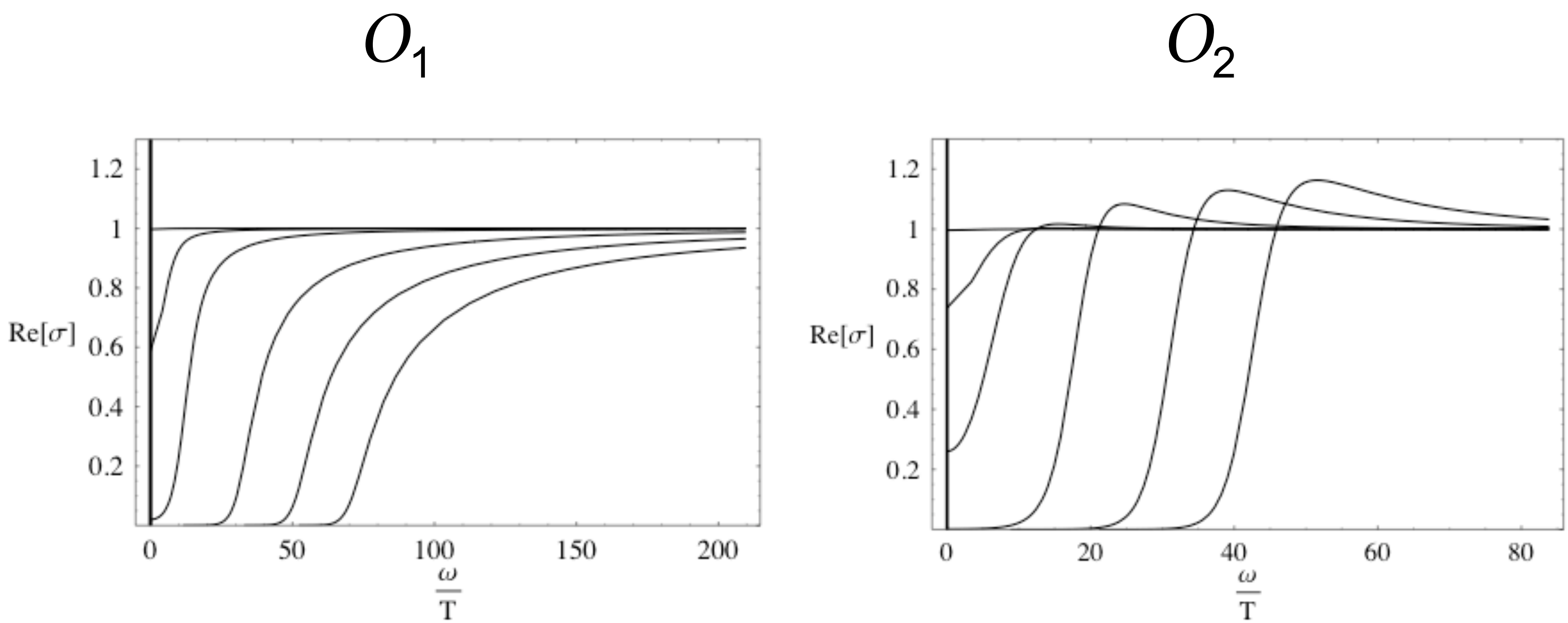
The AdS/CFT dictionary says

$$A_x = A_x^{(0)} \,, \qquad \langle J_x \rangle = A_x^{(1)}$$

From Ohm's law we obtain the conductivity

$$\sigma(\omega) = \frac{\langle J_x \rangle}{E_x} = -\frac{\langle J_x \rangle}{\dot{A}_x} = -\frac{i \langle J_x \rangle}{\omega A_x} = -\frac{i A_x^{(1)}}{\omega A_x^{(0)}}$$

Consider first λ = 1, 2 (d = 3)



Curves represent successively lower temperatures. Gap opens up for $T < T_c$.

There is a delta function at $\omega = 0$ for all $T < T_c$.

This can be seen by looking for a pole in Im[σ].

Simple derivation (Drude model): $$m\frac{dv}{dt} = eE - m\frac{v}{\tau}$$

If $E(t) = Ee^{-i\omega t}$, $\sigma(\omega) = \dfrac{k\tau}{1 - i\omega\tau}$ where $k = ne^2/m$

So

$$\mathrm{Re}[\sigma] = \frac{k\tau}{1+\omega^2\tau^2}, \quad \mathrm{Im}[\sigma] = \frac{k\omega\tau^2}{1+\omega^2\tau^2}$$

For superconductors, $\tau \rightarrow \infty$,

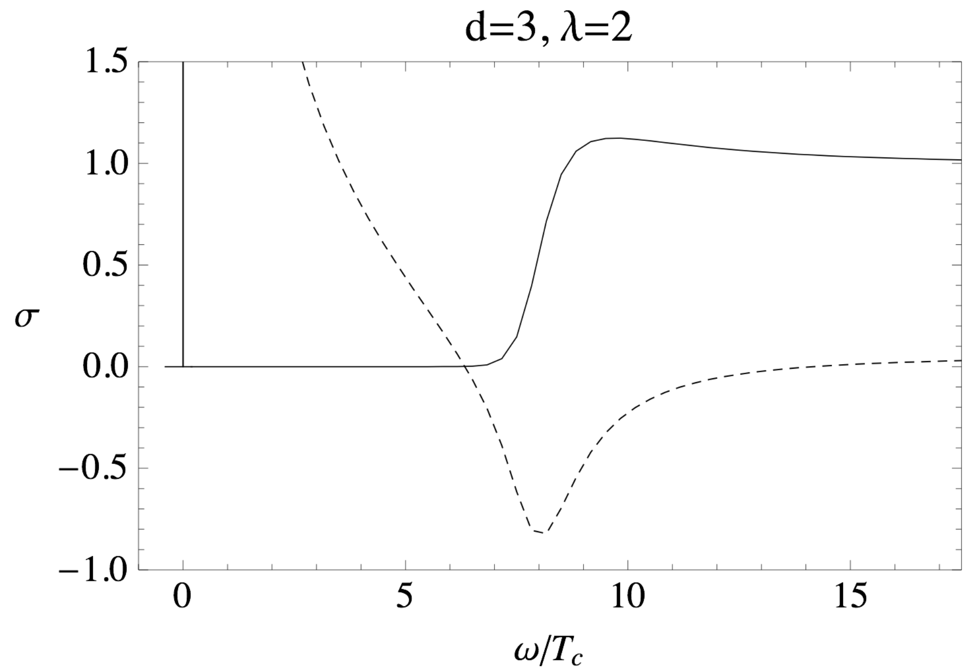
$$\mathrm{Re}[\sigma] \propto \delta(\omega), \quad \mathrm{Im}[\sigma] \propto 1/\omega$$

More general derivation:

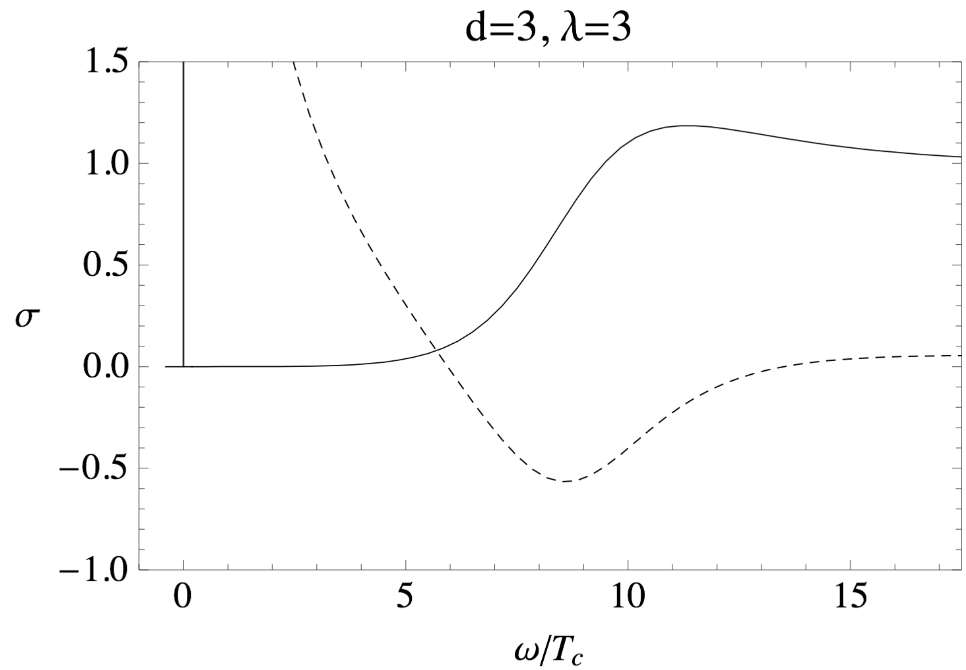
The Kramers-Kronig relations relate the real and imaginary parts of any causal quantity, such as the conductivity, when expressed in frequency space.

$$\mathrm{Im}[\sigma(\omega)] = -\frac{1}{\pi}\mathcal{P}\int_{-\infty}^{\infty}\frac{\mathrm{Re}[\sigma(\omega')]d\omega'}{\omega' - \omega}$$

So the real part of the conductivity contains a delta function if and only if the imaginary part has a pole. One indeed finds a pole in Im[σ] at $\omega = 0$ for all $T < T_c$.

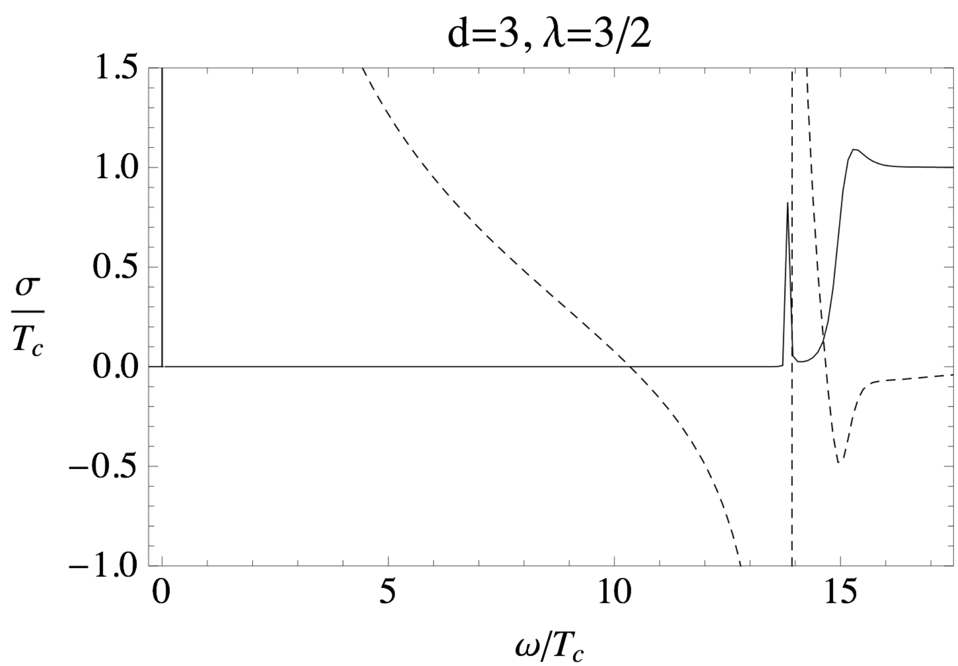


Low temperature limit of conductivity

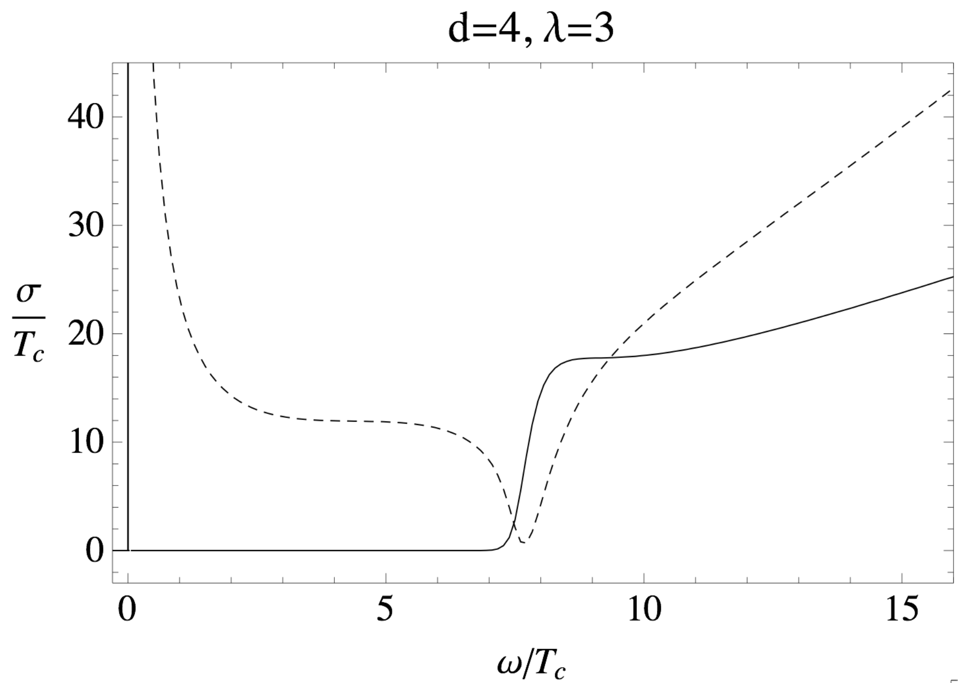


Solid line is real part.
Dashed line is imaginary part.

If the BF bound is saturated, something interesting happens.

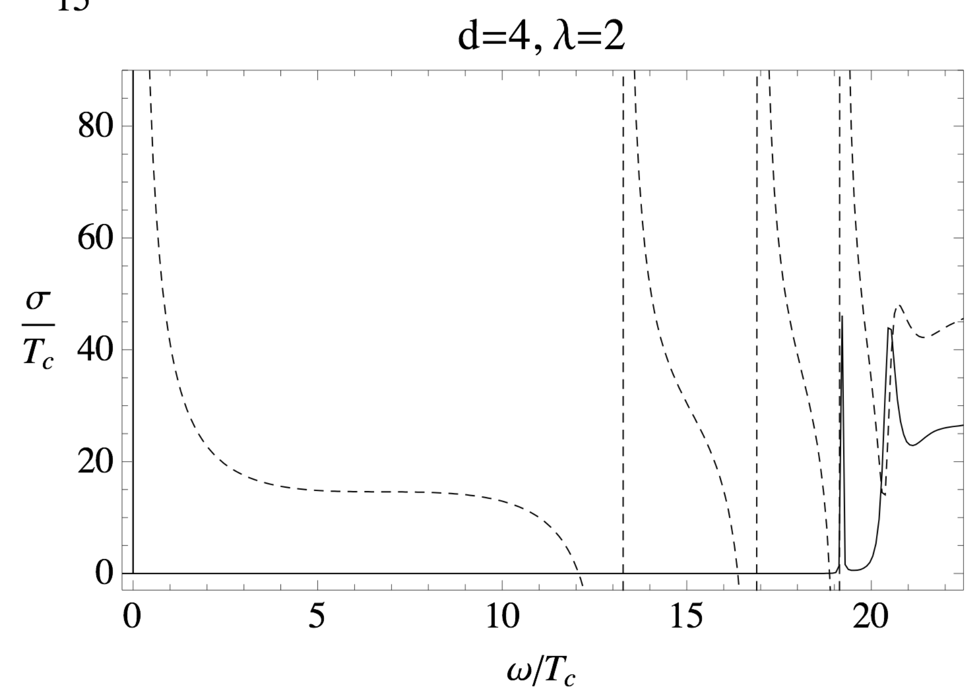


As you lower T, a new spike appears inside the gap. This looks like a new bound state of quasiparticles.



The d = 4 conductivities at low T are similar.

Once again, if the BF bound is saturated new spikes appear.



A robust feature

For both d=3 and d=4, and all $\lambda > \lambda_{BF}$

$$\frac{\omega_g}{T_c} \approx 8$$

with deviations of less than 10%. In BCS theory, this ratio is about 3.5. This shows that the energy to break apart the condensate is more than twice the weakly coupled value.

(This is modified by higher order curvature effects in bulk, Gregory et al. 0907.3203. See talk by Kanno this afternoon.)

We saw before that $$\sigma = -\frac{i}{\omega}\frac{A^{(1)}}{A^{(0)}}$$

But the coefficient of the pole in Im $\sigma(\omega)$ is the superfluid density n_s. Thus, we immediately get London's eq. in the dual theory:

$$J_\mu = - n_s A_\mu$$

Numerically, we find (for T close to T_c)

$$n_s \approx 20(T_c - T) \quad (d = 3), \qquad n_s \approx 100 T_c(T_c - T) \quad (d = 4)$$

with only a weak dependence on the dimension of the condensate.

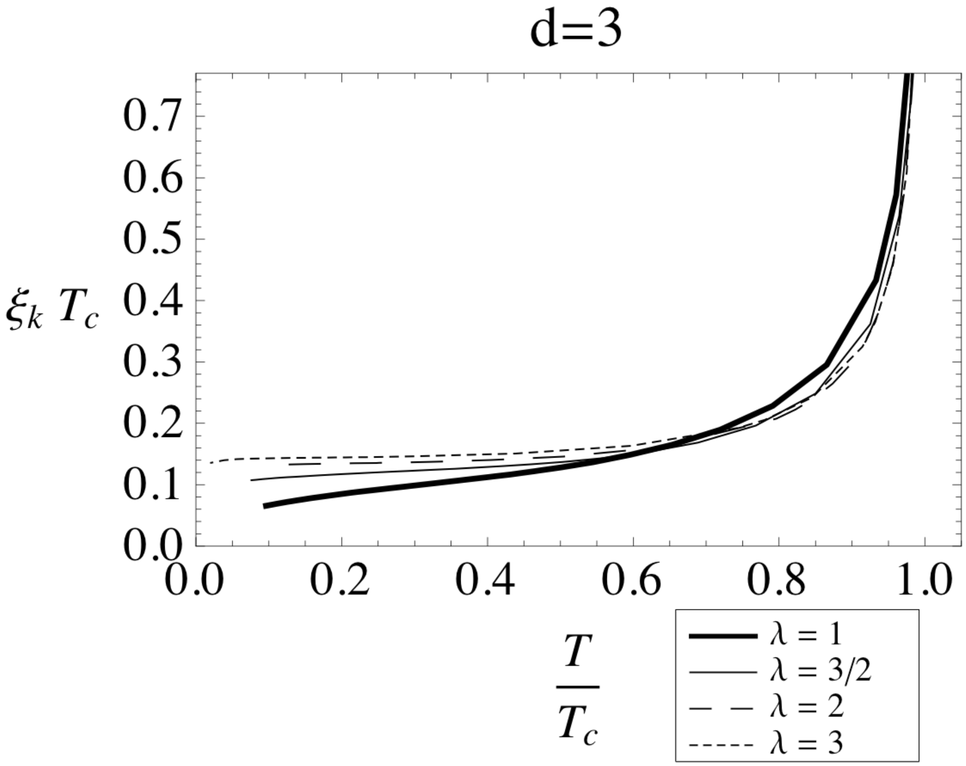
Correlation length

The retarded Green's function (current-current two point function) is

$$G_R(\omega, k) = \frac{A^{(1)}(\omega, k)}{A^{(0)}(\omega, k)}$$

Considering perturbations with e^{ikx} dependence, one can define a correlation length by expanding

$$Re\ G_R = -n_s(1 + \xi_k^2 k^2 + \cdots)$$



The d=3 correlation length. Near T = T_c:

$$\xi_k T_c = \frac{0.1}{(1 - T/T_c)^{1/2}}$$

Part 1 Summary

- A simple gravitational theory can reproduce basic properties of a superconductor in both d=3 and d=4 spacetime dimensions.
- When $\lambda = \lambda_{BF,}$ new spikes appear in Re $\sigma(\omega)$.
- When $\lambda > \lambda_{BF}$; $\omega_g/T_c \approx 8$ in all cases.

Including backreaction

Recall that our Lagrangian is

$$\mathcal{L} = R + \frac{6}{L^2} - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} - |\partial\Psi - iqA\Psi|^2 - m^2|\Psi|^2$$

To solve for the backreaction on the metric, set

$$ds^2 = -g(r)e^{-\chi(r)}dt^2 + \frac{dr^2}{g(r)} + r^2\left(dx^2 + dy^2\right)$$

$$A = \phi(r)dt, \quad \Psi = \psi(r)$$

Get four coupled nonlinear ODE's. At the horizon, r = r_0, g vanishes and χ is constant.

Equations are invariant under two scaling symmetries:

$$t \to at\,, \quad e^\chi \to a^2 e^\chi\,, \quad \phi \to \phi/a$$

$$r \to ar\,, \quad (t,x,y) \to (t,x,y)/a\,, \quad g \to a^2 g\,, \quad \phi \to a\phi$$

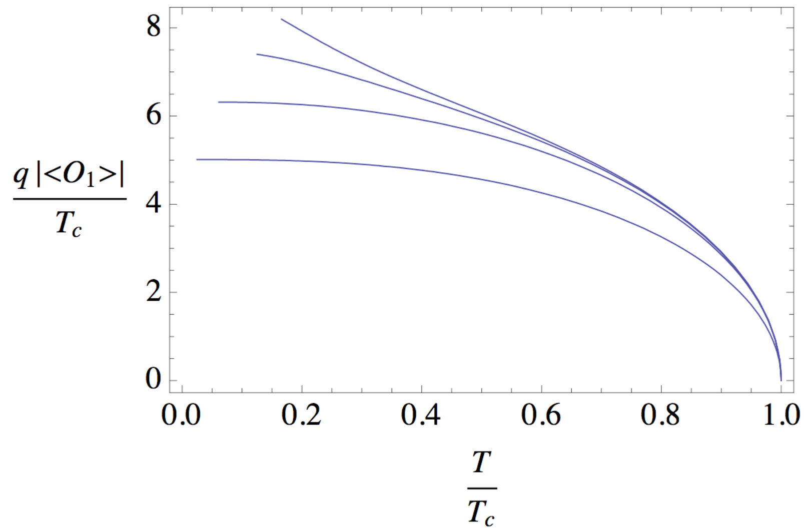
The first can be used to set $\chi = 0$ at infinity and the second can be used to set $r_0 = 1$ (as long as $T \neq 0$).

We have solved the bulk equations for finite q, including the backreaction on the metric for d = 3, and various $m^2 \leq 0$.

The qualitative behavior is unchanged (but the "robust feature" $\frac{\omega_g}{T_c} \approx 8$ is less robust for q < 3).

There are two main differences.

The divergence in the dimension one condensate at low T is gone.



From bottom to top, the curves correspond to q = 1, 3, 6, 12

For m^2 close to BF bound, T_c remains nonzero even when q=0

There is a new source of instability: nearly extremal charged AdS black holes are unstable to forming neutral scalar hair.

An extremal AdS black hole has a near horizon geometry $AdS_2 \times R^2$. The BF bound for AdS_{d+1} is $m^2_{BF} = - d^2/4$. Our scalar can be above the BF bound for AdS_4, but below the bound for AdS_2.

General argument for instability

Consider a scalar field with mass m and charge q in the near horizon geometry of an extremal Reissner-Nordstrom AdS black hole. Get a wave equation in AdS_2 with effective mass

$$m_{eff}^2 = \frac{m^2 - 2q^2}{6}$$

The extremal RN AdS black hole is unstable when this is below -1/4, the BF bound for AdS_2. The condition for instability is

$$m^2 - 2q^2 < -3/2$$

Conductivity

The perturbed Maxwell field A_x now couples to the perturbed metric component g_{tx}. One can solve for g_{tx} in terms of A_x and get

$$A_x'' + \left[\frac{g'}{g} - \frac{\chi'}{2}\right] A_x' + \left[\left(\frac{\omega^2}{g^2} - \frac{\phi'^2}{g}\right) e^{\chi} - \frac{2q^2\psi^2}{g}\right] A_x = 0$$

If $A_x = A_x^{(0)} + A_x^{(1)}/r$

The conductivity is again $\sigma(\omega) = -\frac{i}{\omega}\frac{A_x^{(1)}}{A_x^{(0)}}$

Reformulation of Conductivity Calculation

Introduce a new radial variable $dz = \frac{e^{\chi/2}}{g} dr$

Near the horizon $z \propto \log(r - r_0)$
and asymptotically $z = -1/r$

The equation for the perturbed vector potential takes the form of a standard Schrodinger equation

$$-A_{x,zz} + V(z)A_x = \omega^2 A_x$$

with $$V(z) = g[\phi_{,r}^2 + 2q^2\psi^2 e^{-\chi}]$$

Near the horizon, V vanishes exponentially. At z = 0, V vanishes if the condensate has dimension greater than one, and $V \propto z^{2(\lambda-1)}$ for $1/2 < \lambda \leq 1$

Want to solve this with ingoing wave boundary conditions at the horizon. Set V = 0 for z > 0, and send in a wave from the right

$$A_x = e^{-i\omega z} + Re^{i\omega z} \text{ for } z > 0$$

Recall that if: $A_x = A_x^{(0)} + A_x^{(1)}/r$

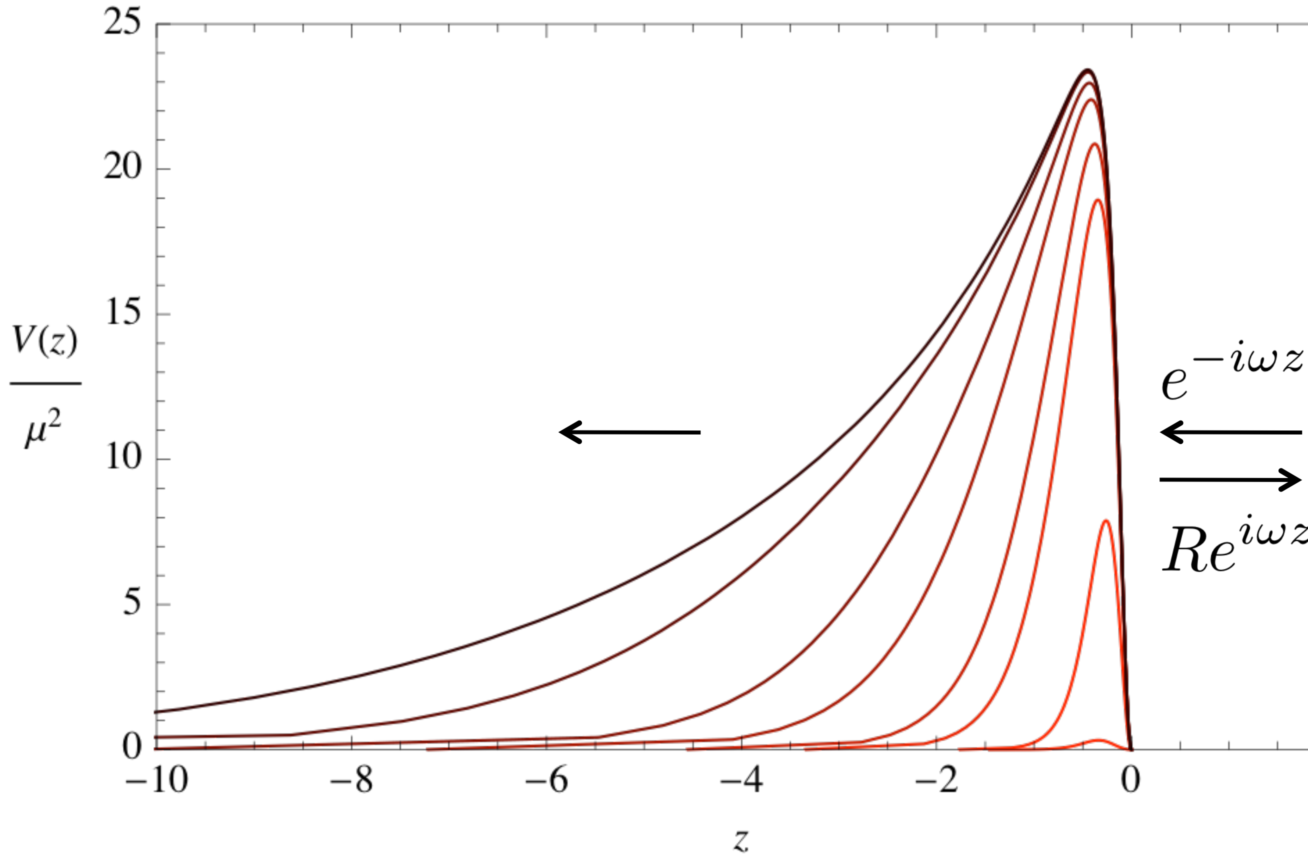
The conductivity is: $\sigma(\omega) = -\frac{i}{\omega}\frac{A_x^{(1)}}{A_x^{(0)}}$

In our reformulation: $A_x^{(0)} = A_x(0) = 1 + R$

and $A_x^{(1)} = -A_{x,z}(0) = i\omega(1 - R)$

so

$$\sigma(\omega) = \frac{1 - R}{1 + R}$$



The potential grows as T/T_c gets smaller
(for q= 10, λ = 2)

To see the delta function in Re σ at ω = 0, we need to have a pole in Im σ. From

$$\sigma(\omega) = -\frac{i}{\omega}\frac{A_x^{(1)}}{A_x^{(0)}}$$

it suffices that $A_x^{(0)}$ and $A_x^{(1)}$ are real and nonzero at ω = 0. This is true for any positive V(z) that vanishes at the horizon:

If ω = 0, Schrodinger's eq. implies $A_{x,zz} > 0$. So starting with $A_x = 1$ at z = - ∞, and integrating to z =0, $A_x^{(0)}$ and $A_x^{(1)}$ are indeed real and nonzero.

This also explains the spikes in Re σ at nonzero frequency:

If the potential is high enough, the reflected wave can interfere destructively with the incident wave causing $A_x^{(0)}$ to be very small.

Using standard WKB approximation, spikes occur when there exists ω satisfying

$$\int_{-z_0}^{0} \sqrt{\omega^2 - V(z)}dz + \frac{\pi}{4} = n\pi$$

for some integer n, where $V(-z_0) = \omega^2$.

Ground State of Holographic Superconductors

The extremal Reissner Nordstrom AdS black hole has large entropy and T =0. If this was dual to a condensed matter system, it would mean the ground state was highly degenerate.

The extremal limit of the hairy black holes is not like Reissner Nordstrom. It has zero horizon area ($r_0/\mu \longrightarrow 0$ as $T \longrightarrow 0$). It also has zero charge (except when q=0).

The near horizon behavior depends on m, q. Typically, the solution is not smooth at r = 0.

Schrodinger potential still vanishes at the horizon, so Re σ(ω) ≠ 0 at low frequency even at T = 0. There is no hard gap.

Typically, $V = c/z^2$ near the horizon, so

$$\int_{-\infty}^{0} \sqrt{V(z)}dz = \infty$$

Re σ(ω) vanishes at T = 0 as $\omega \longrightarrow 0$.

Case 1: m = 0 and $q^2 > 3/4$

Recall:

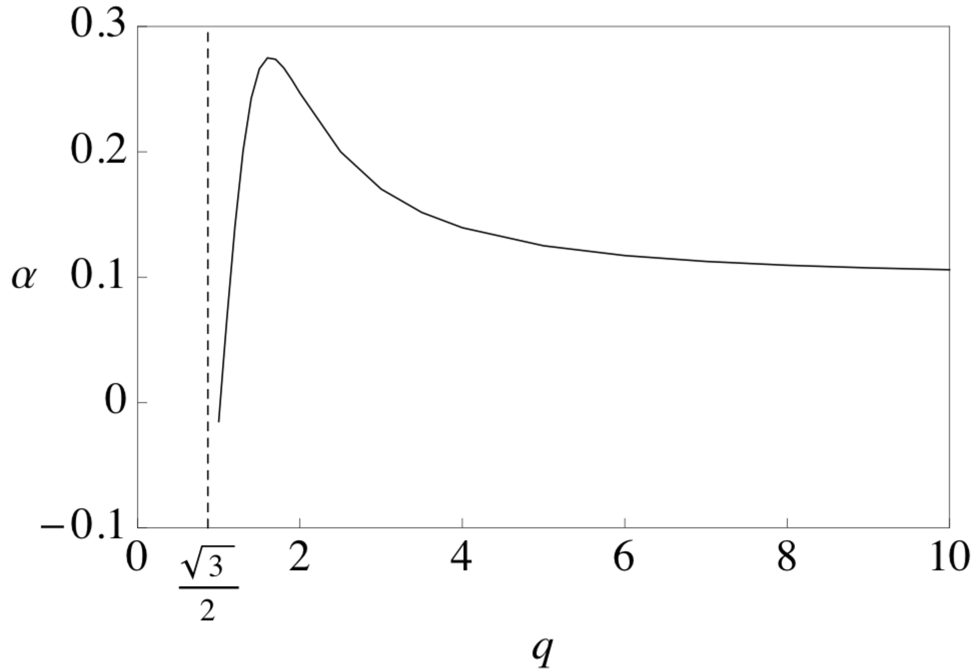
$$ds^2 = -g(r)e^{-\chi(r)}dt^2 + \frac{dr^2}{g(r)} + r^2\left(dx^2 + dy^2\right)$$

The near horizon solution is given by

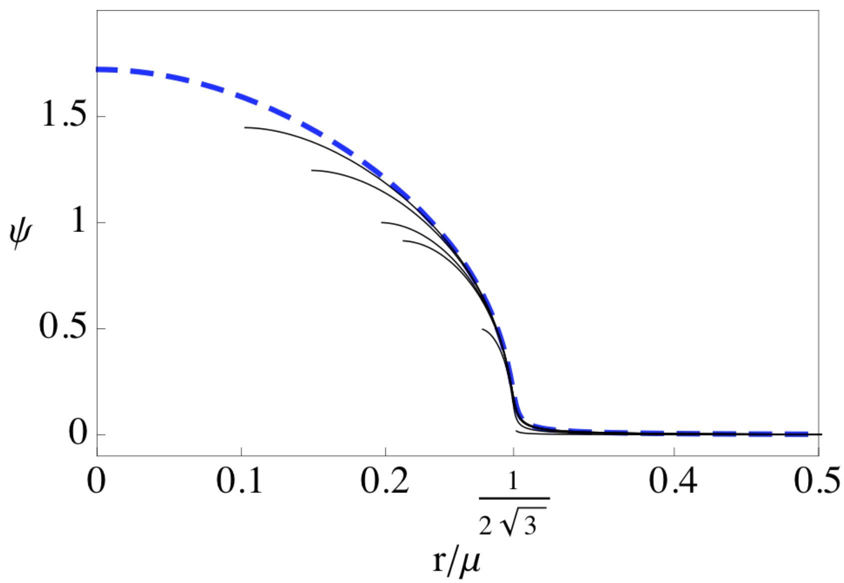
$$\phi = r^{2+\alpha}, \quad \psi = \psi_0 - \psi_1 r^{2(1+\alpha)}$$

$$\chi = \chi_0 - \chi_1 r^{2(1+\alpha)}, \quad g = r^2(1 - g_1 r^{2(1+\alpha)})$$

The coefficients are functions of q and α. The exponent α is chosen to satisfy the boundary condition at infinity.

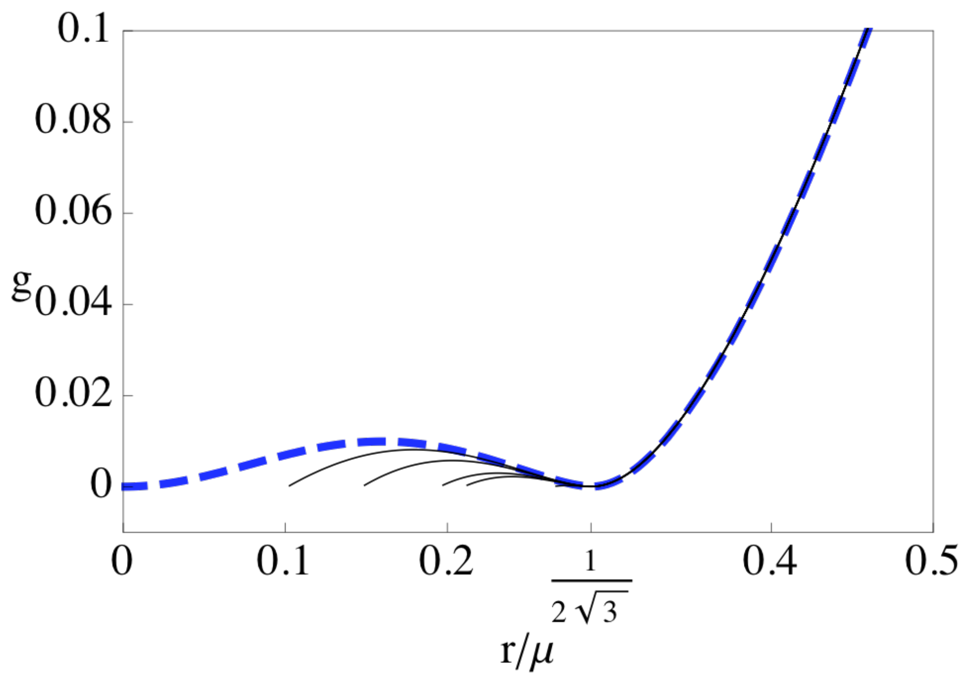


The exponent is always small, and vanishes for q = 1.02. The solution is nonsingular (curvature is always finite at r = 0) and is completely smooth when α = 0.



Low temperature solutions approach T=0 solution.

Dotted blue curve is T=0 solution with q=1. Black curves are successively lower temperature solutions.



These solutions approach AdS_4 near $r = 0$ (with the same value of the cosmological constant as infinity).

The holographic superconductor has emergent conformal symmetry in the infrared.

The bulk solutions describe charged scalar solitons.

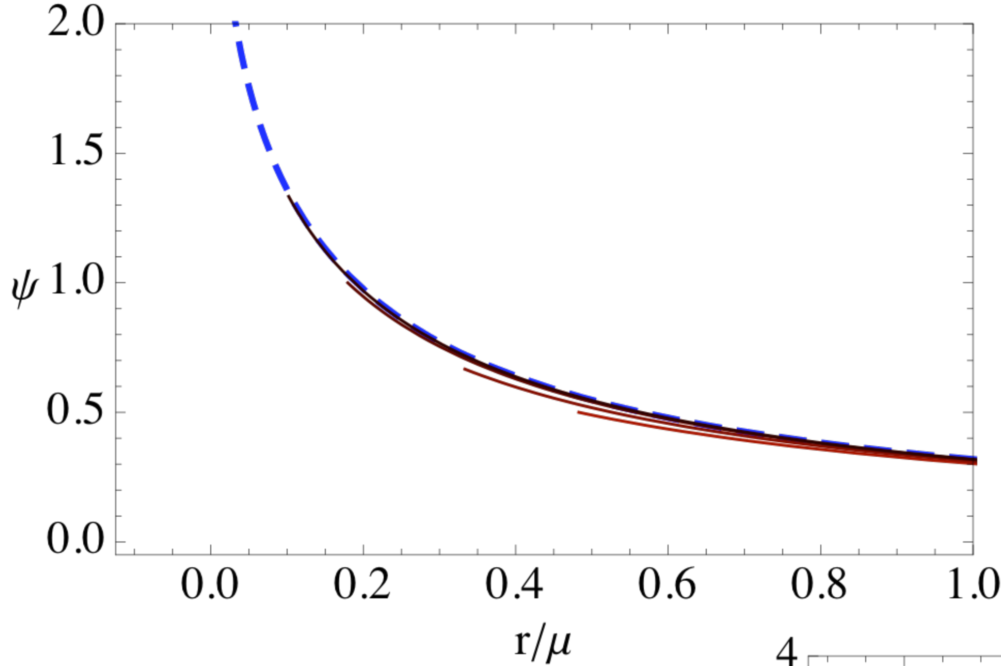
Case 2: $m^2 < 0$, $q^2 > |m^2|/6$

The near horizon solution is given by

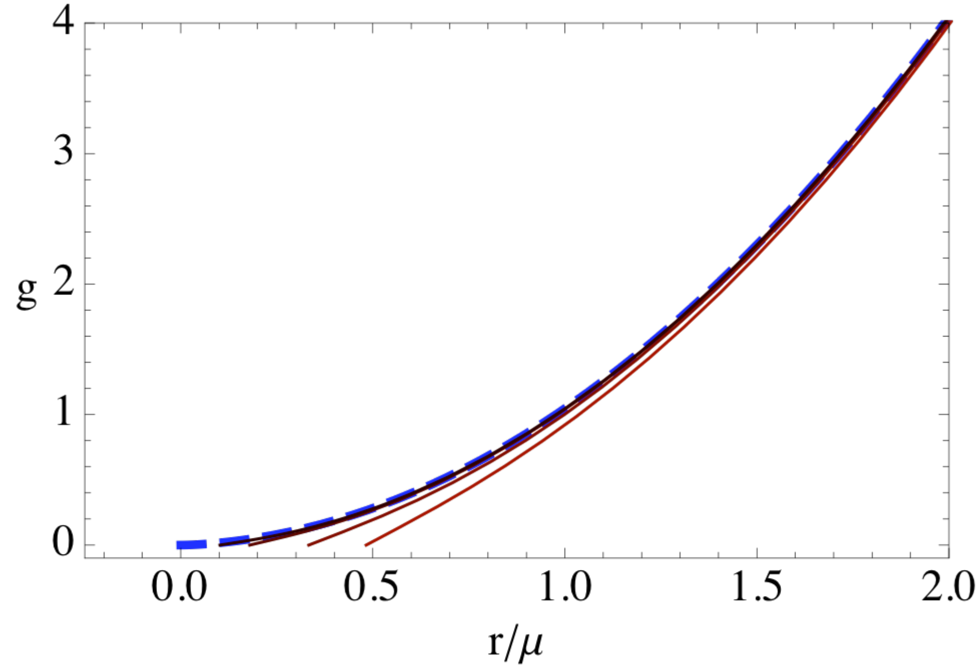
$$\psi = 2(-\log r)^{1/2}, \quad \phi = \phi_0 r^{\beta}(-\log r)^{1/2}$$

$$g = (2m^2/3)r^2 \log r, \quad e^{\chi} = -K \log r$$

β is a fixed function of m and q. φ_0 can be adjusted to satisfy boundary condition at infinity.



Dotted blue curve is T=0 solution with q=10, $m^2 = -2$.
Red curves are successively lower temperature solutions.



Near r = 0, the metric takes the form

$$ds^2 = r^2(-dt^2 + dx_i dx^i) + \frac{dr^2}{g_0 r^2(-\log r)}$$

There is a mild null singularity.

The holographic superconductor has emergent Poincare symmetry (but not conformal symmetry) in infrared.

Embedding in String Theory

Gubser et al. (0907.3510) realized the d = 4, m^2 = -3, q=2 model in type IIB string theory.

Gauntlet et al. (0907.3796) realized the d=3, m^2 = -2, q=2 model in M theory.

Both used Sasaki-Einstein compactifications, where the scalar is the size of the U(1) fibration.

In the d = 4 example, the condensate is bilinear in the fermions, like a Cooper pair.

The truncation of supergravity leads to potentials, $V(\psi)$, that have more than one extremum. There are smooth zero temperature solutions which interpolate between two AdS solutions with different radii of curvature (Gubser, Pufu, and Rocha, 0908.0011).

They find $\mathrm{Re}\, \sigma(\omega) = k\, \omega^{\delta}$ for small ω.

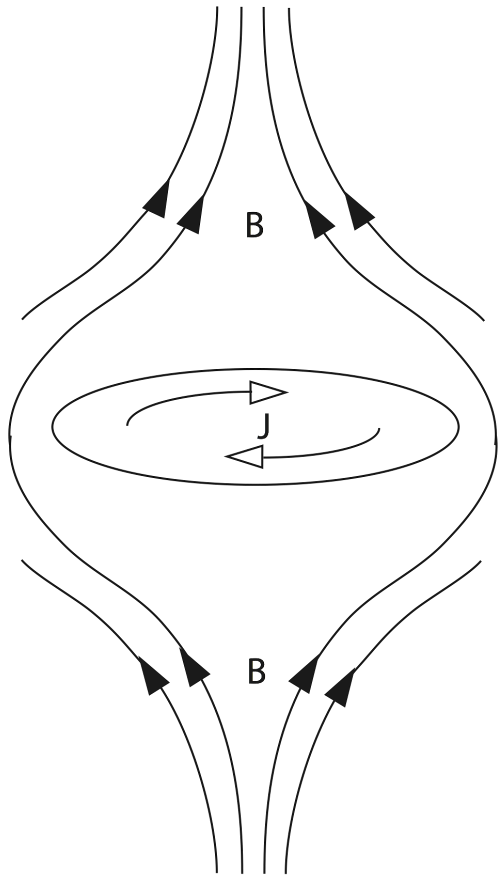
Adding magnetic fields (d=3)

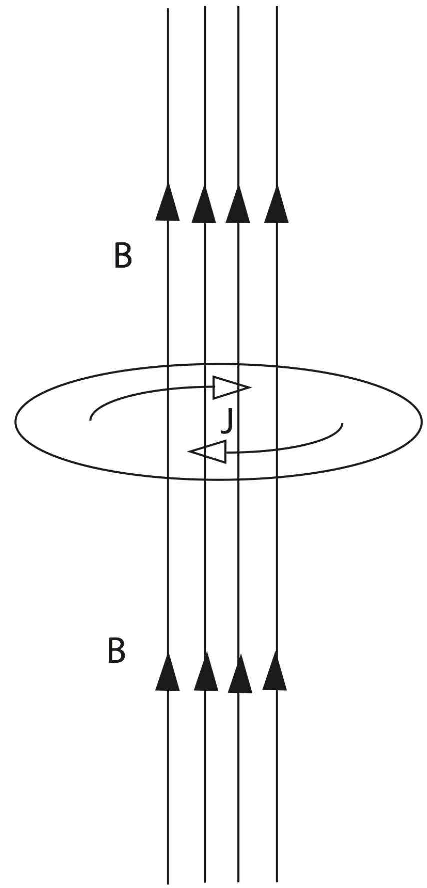
Large B fields destroy superconductivity. Superconductors must perform work $B^2V/8\pi$ to expel an applied field B from volume V. The thermodynamic critical field is

$$\frac{B_c^2(T)V}{8\pi} = F_n(T) - F_s(T)$$

where F is the free energy.

Claim: If we add a magnetic field perpendicular to our 2+1 superconductor, B_c (T) = 0.





To expel B from disk of radius R, the superconductor must do work ~ R^3. The difference in free energy is only ~ R^2.

Starting at low T and large B, the material is in the normal phase. Now lower B.

Type I superconductors have a first order phase transition at $B = B_c$ below which the material becomes superconducting everywhere and $B = 0$.

Type II superconductors have a second order phase transition at $B = B_{c2} > B_c$ where superconducting droplets form.

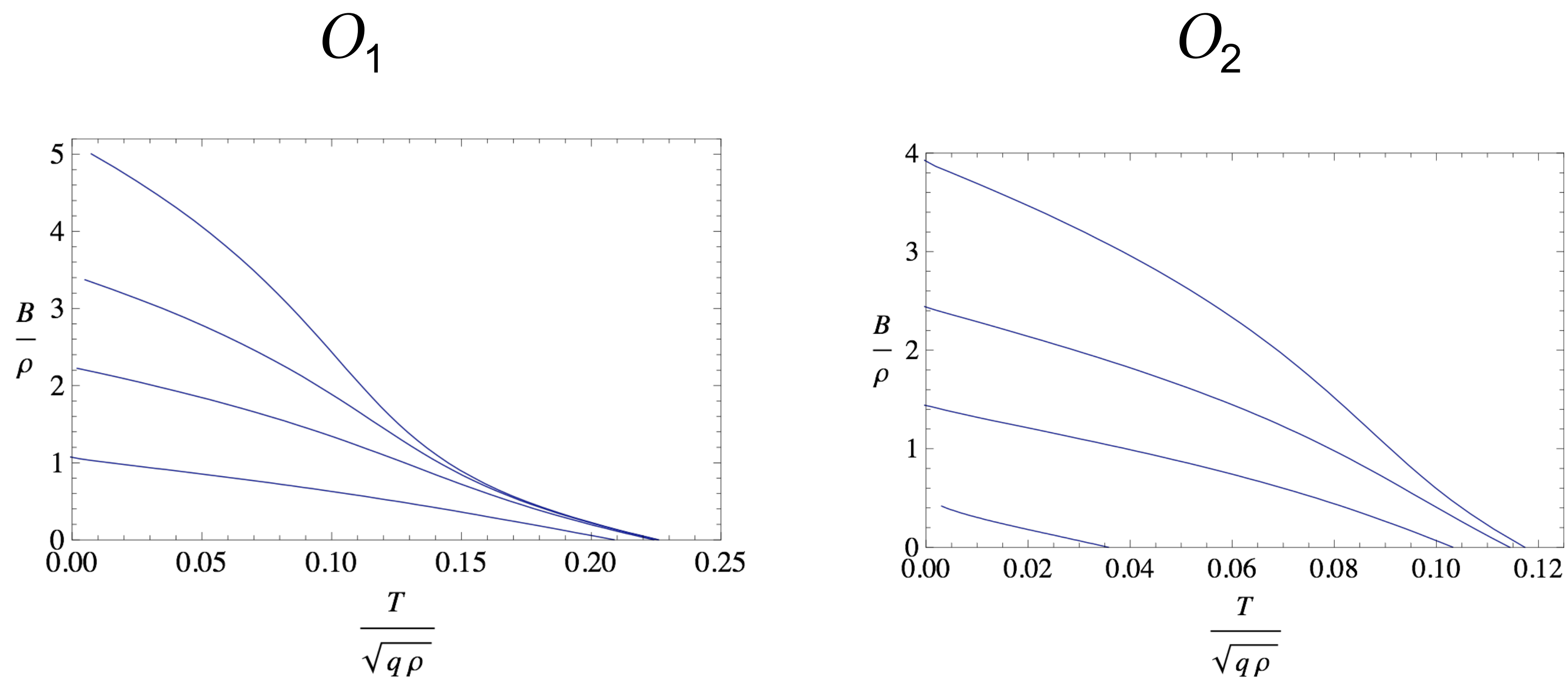
Holographic superconductors are Type II.

Start with Reissner-Nordstrom AdS metric with both electric and magnetic charges. Write $dx^2 + dy^2 = du^2 + u^2 d\varphi^2$. Then $A_\varphi = Bu^2/2$.

Since we are interested in the onset of superconductivity, we can assume ψ is small. The solution takes the form $\psi(r,u) = R(r)\, U(u)$.

U(u) satisfies the Schrodinger equation for a 2D harmonic oscillator, so $U(u) = e^{-qBu^2/4}$

Numerically, we find solutions for R(r) exist for B less than a critical value $B_{c2} > 0$.



The critical field B_{c2} as a function of T. From top to bottom the curves are q = 12, 6, 3, 1. Below the curves, there are superconducting droplets (cf. Albash and Johnson, 0804.3466).

Holographic superconductors cannot expel a B field since the U(1) symmetry is not gauged. But they do produce currents whose backreaction would cancel the B field.
(see also Maeda and Okamura, 0809.3079)

The superconducting droplets generate currents around their edge. One can see this by going to second order in ψ. Find:

$$A_\varphi = Bu^2/2 + A_\varphi^{(1)}/r$$

Vortex solution

Montull, Pomarol, Silva, 0906.2396;
Albash and Johnson, 0906.1795

When a magnetic field starts to penetrate a superconductor, it forms vortices which contain quantized flux: $B = 2\pi n$. These can be found by solving PDE's in the bulk.

Consider ansatz:

$$\psi = \psi(r,u)\, e^{i\varphi n}, \quad A_t = A_t(r,u), \quad A_\varphi = A_\varphi(r,u)$$

From the asymptotic behavior of ψ, one sees that superconductivity is suppressed inside the vortex:

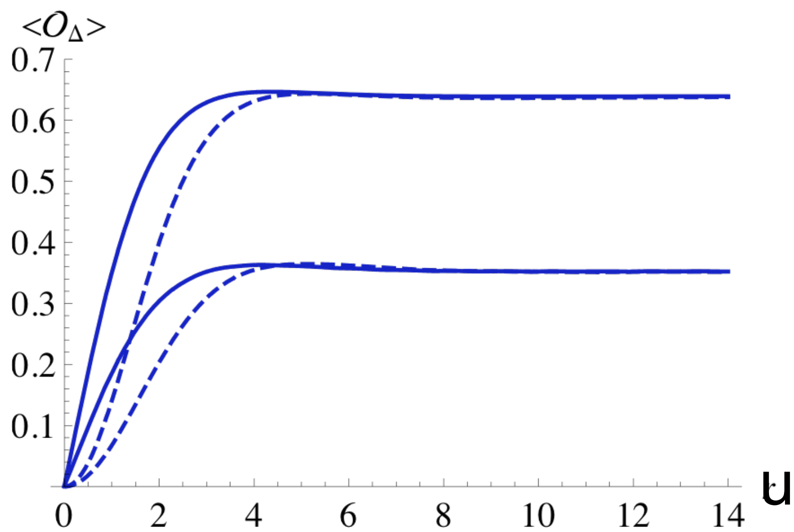
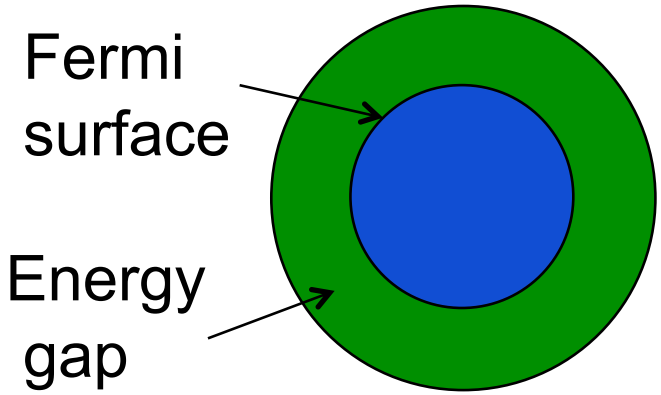


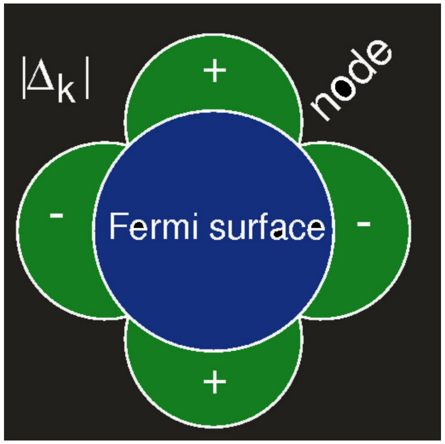
FIG. 1: *Order parameter* $\langle \mathcal{O}_\Delta \rangle$ *for the* $n = 1$ *(solid) and* $n = 2$ *(dashed) vortex configuration. The lower (upper) curves correspond to the case* $m^2 = 0$ (-2). *Presented in units of* $\sqrt{\rho} = 1$.

Taken from Montull, Pomarol, Silva, 0906.2396

The symmetry of a superconductor refers to the energy gap near the Fermi surface:



s-wave



d-wave

The fact that our bulk solution is rotationally invariant means that this is an s-wave superconductor.

Part 2 Summary

- There are two distinct instabilities which cause condensate (hair) to form at low T.
- The conductivity is simply related to a reflection coefficient in a scattering problem
- There is no hard gap (Re $\sigma(\omega) \neq 0$ at $T = 0$).
- Holographic superconductors are Type II.