5th Aegeon Summer School, Islands of Milos, Greece, 25 Sep. 2009

AdS/CFT on the Brane

--- Holographic Cosmology ---

based on works with Sugumi Kanno

- **Braneworld Effective Action at Low Energies and AdS/CFT correspondence, Phys.Rev. D66, 043526 (2002);**
- **Radion and Holographic Brane Gravity, Phys.Rev. D66, 083506 (2002);**

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Goal of this talk

As you have already learned,
AdS/CFT has been applied to various phenomena on various scales

QCD Condensed matter Black holes Cosmology

Here, I would like to discuss braneworld cosmology from a view of AdS/CFT.
Namely, I will explain AdS/braneworld correspondence.

Admittedly, the role of AdS/CFT in braneworld has not been fully understood.
I just mention what I understand.

Goal of this talk is to convince you that AdS/CFT is related to brane cosmology.
In doing so, we show how the cosmological evolution on the brane is affected by the bulk geometry.

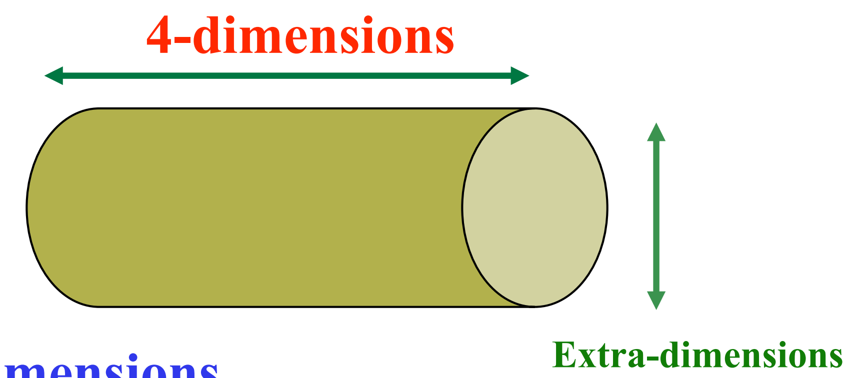
Introduction to higher dimensions

Unified theory such as string theory predicts that
our space-time is more than 4-dimensions.

How to reconcile extra-dimensions with our experience?

Traditional idea Kaluza-Klein mechanism

- Both matter and gravity live in the higher dimensions
- Extra-dimensions are compactified to the Planck scale

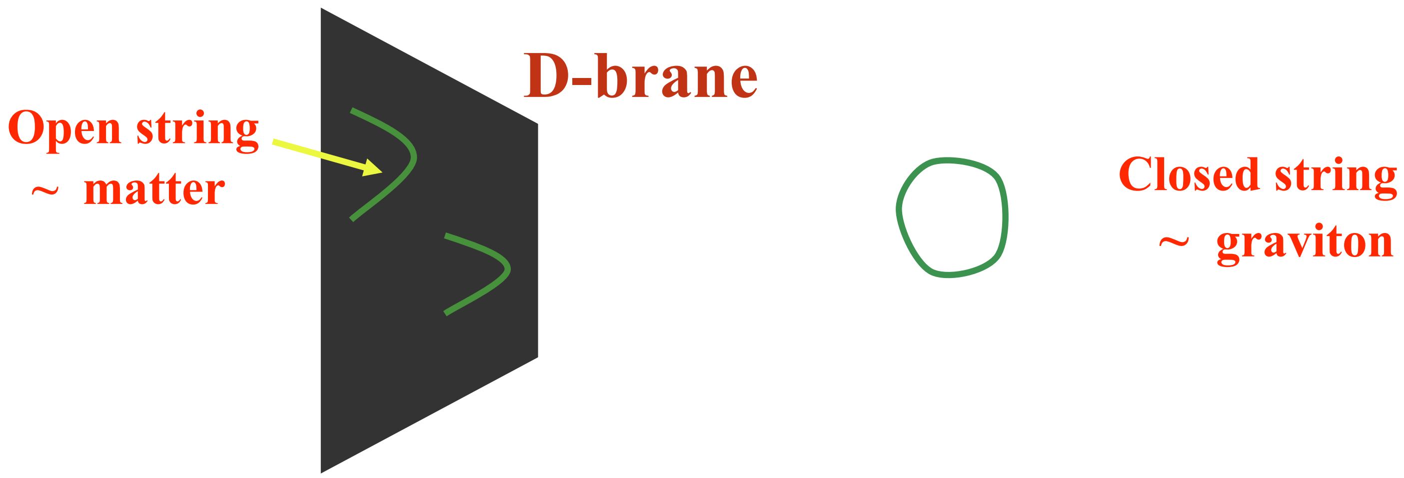


Hence, our universe is essentially 4-dimensions.

This is an attractive idea, but not a unique one.

New idea

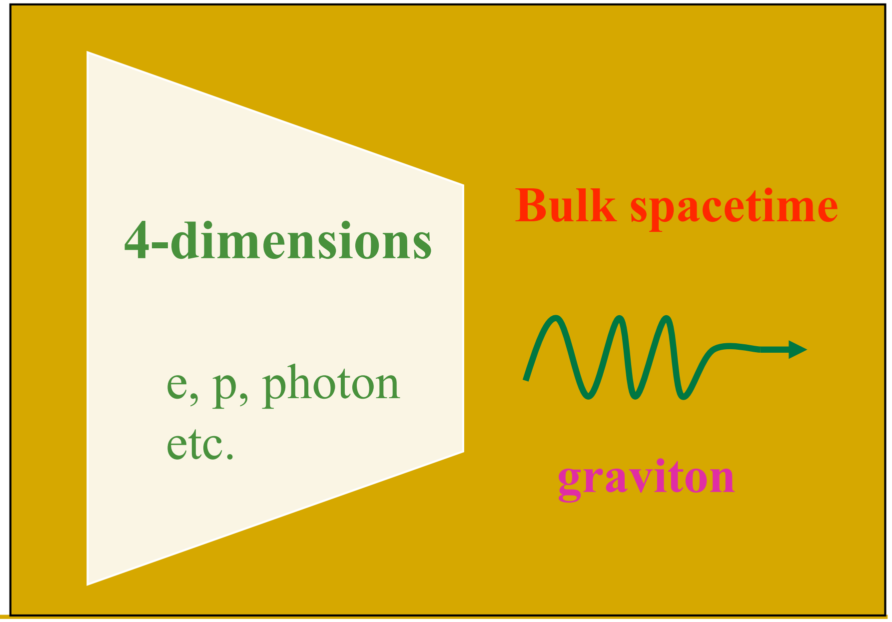
Akama (1982) , Rubakov & Shaposhnikov (1983)
Horava & Witten (1996), Arkani-Hamed et al (1998)



Braneworld

- **4-d (mem)brane is embedded in higher-dimensions**
- **Matter on the brane**
- **Gravity in the bulk**

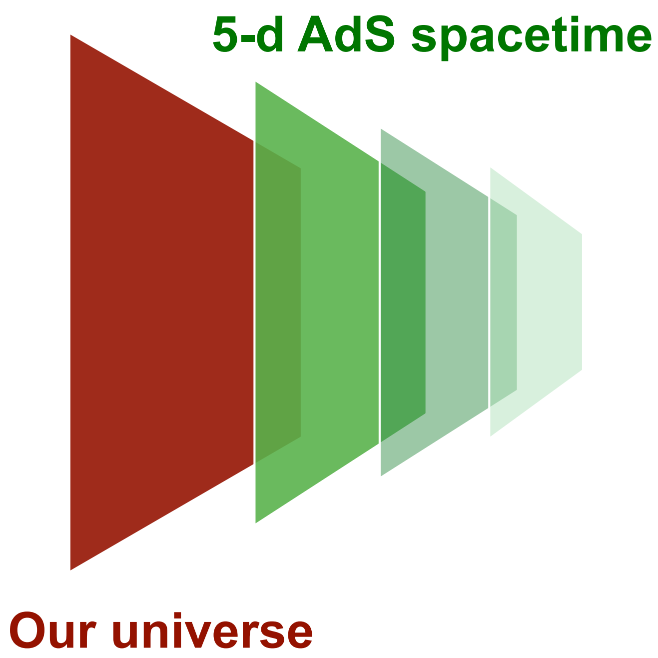
compactification scale > Planck scale
It could be 1mm!



Warped compactification: Randall-Sundrum models (RS)

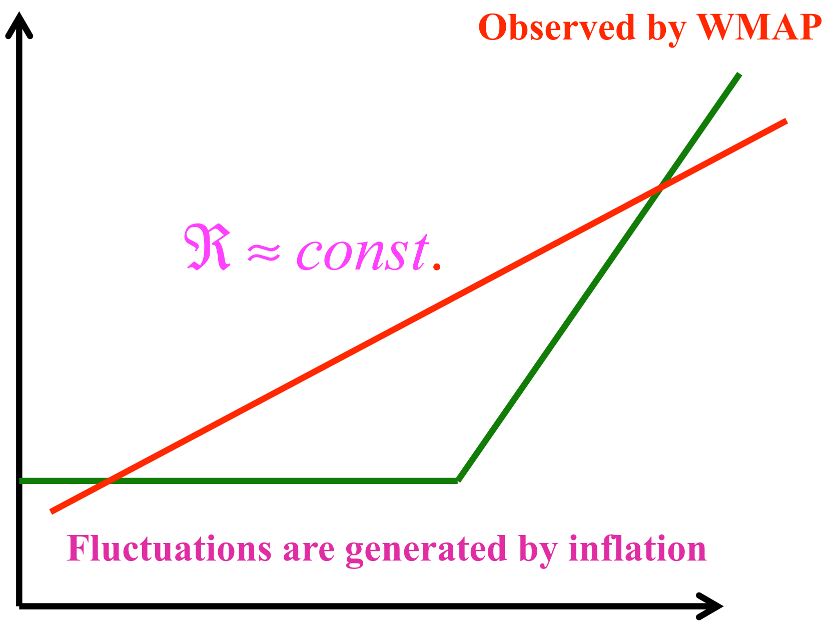
From string cosmology point of view, the warped compactification is more interesting and has relevance to AdS/CFT.

The spacetime is shrinking towards infinity. Hence, volume is finite although an extra-dimension is not compact.

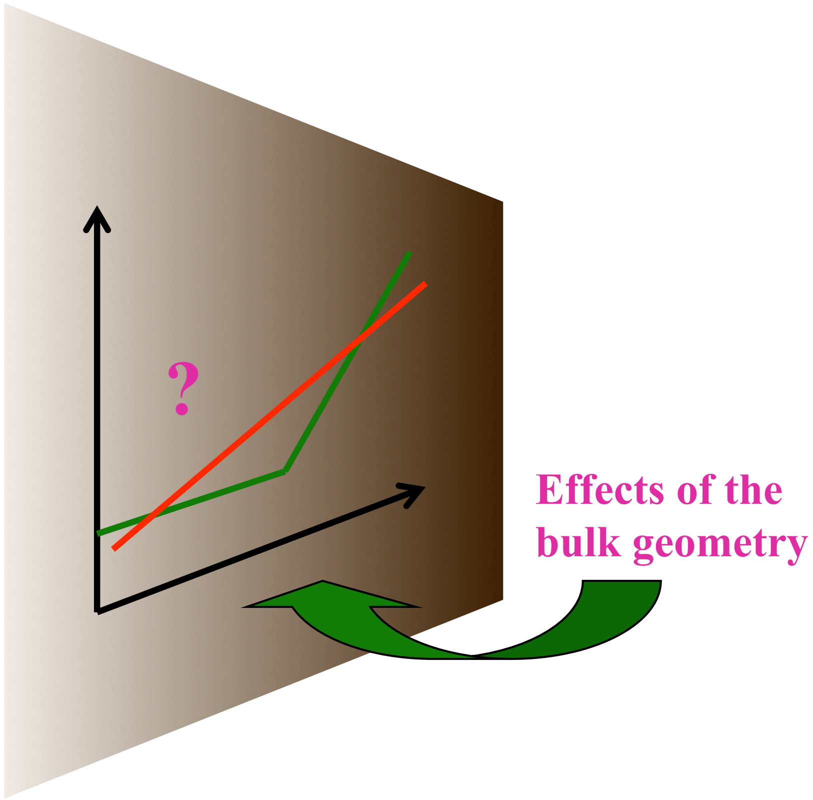


Issue in the cosmology side

Conventional picture of the evolution of cosmological structure.



How the evolution of cosmological structure is modified in the braneworld picture?



Plan of my talk

1. RSII single brane models
2. Geometrical Holography
3. RSII via AdS/CFT
4. Gradient expansion method
5. Effective equations and CFT
6. RSI two-brane models
7. Effective equations and radion
8. Conclusion

RSII: Single-brane Model

Randall & Sundrum (1999)

$$S = \frac{1}{2\kappa^2}\int d^5x\sqrt{-\gamma}\left[\overset{(5)}{\Re} + \frac{12}{\ell^2}\right] - \sigma\int d^4x\sqrt{-g} + \int d^4x\sqrt{-g}\ L_{matter}$$

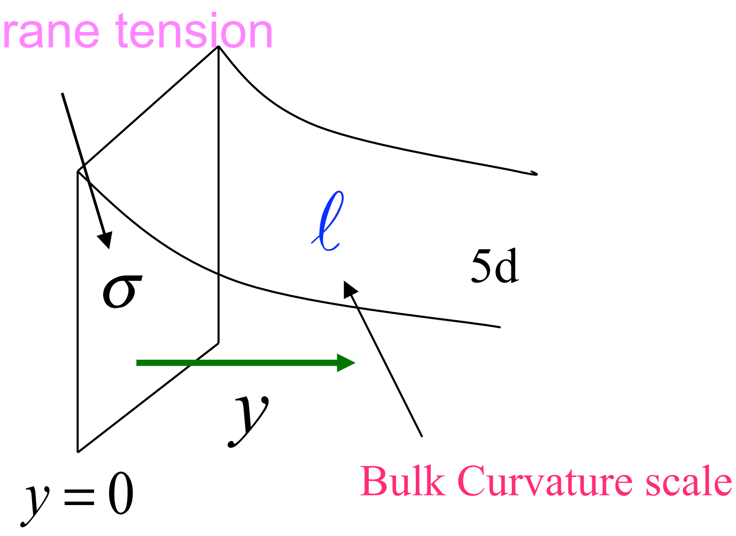
Coupling constant

$\overset{(5)}{\Re}$: 5-d Ricci scalar

γ_{AB} : 5-d metric

$g_{\mu\nu}$: 4-d induced metric

$$ds^2 = dy^2 + g_{\mu\nu}(y, x^\alpha)\,dx^\mu\,dx^\nu$$



Extrinsic curvature $K_{\mu\nu} = -\frac{1}{2}\frac{\partial}{\partial y} g_{\mu\nu} = \Sigma_{\mu\nu} + \frac{1}{4} g_{\mu\nu} K$ $\qquad \Sigma^{\mu}{}_{\mu} = 0$

Geometrical Holography

4-d components of 5-d Einstein tensor

$$\overset{(5)}{G}_{\mu\nu} = G_{\mu\nu} + K_{\mu\nu,y} - g_{\mu\nu}K_{,y} - KK_{\mu\nu} + 2K_{\mu}{}^{\lambda}K_{\lambda\nu} + \frac{1}{2}g_{\mu\nu}\left(K^2 + K^{\alpha\beta}K_{\alpha\beta}\right)$$

4-d Einstein tensor

``electric" part of Weyl tensor

$$C_{y\mu y\nu} = K_{\mu\nu,y} - g_{\mu\nu}K_{,y} + K_{\mu}{}^{\lambda}K_{\lambda\nu} + g_{\mu\nu}K^{\alpha\beta}K_{\alpha\beta} - \frac{3}{\ell^2}g_{\mu\nu}$$

junction condition

$$\left[\Sigma^{\mu}{}_{\nu} - \frac{3}{4}\delta^{\mu}{}_{\nu}K\right]\Big|_{y=0} = -\frac{\kappa^2\sigma}{2}\delta^{\mu}{}_{\nu} + \frac{\kappa^2}{2}T^{\mu}{}_{\nu}$$

Shiromizu-Maeda-Sasaki equation

Shiromizu, Maeda and Sasaki (2000)

high energy corrections

$$G_{\mu\nu} = \left(\frac{3}{\ell^2} - \frac{\kappa^4\sigma^2}{12}\right)g_{\mu\nu} + \frac{\kappa^4\sigma}{6}T_{\mu\nu} + \kappa^4\pi_{\mu\nu} - E_{\mu\nu}$$

bulk effects

$$\pi_{\mu\nu} = -\frac{1}{4}T_{\mu}{}^{\lambda}T_{\lambda\nu} + \frac{1}{12}TT_{\mu\nu} + \frac{1}{8}g_{\mu\nu}\left(T^{\alpha\beta}T_{\alpha\beta} - \frac{1}{3}T^2\right)$$

$$E_{\mu\nu} = C_{y\mu y\nu}\big|_{y=0}$$

Effective Friedman equation

$$\kappa^2 \sigma = \frac{6}{\ell} \qquad 8\pi G \equiv \frac{\kappa^2}{\ell}$$

$$G_{\mu\nu} = 8\pi G T_{\mu\nu} + \underbrace{\kappa^4 \pi_{\mu\nu}}_{High\ Energy\ Effect} - \underbrace{E_{\mu\nu}}_{Bulk\ Effec}$$

Unknown!

Homogeneous Cosmology

$$E^{\mu}{}_{\mu} = 0$$

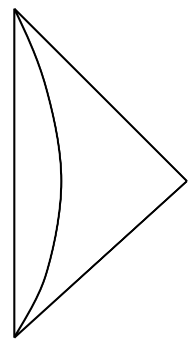
$$H^2 = \frac{8\pi G}{3}\rho + \underbrace{\kappa^4 \rho^2}_{High\ Energy\ Effect} + \frac{C}{\underbrace{a^4}_{Bulk\ Effec}}$$

Dark Radiation

AdS/CFT correspondence

Gubser (2001)

$$\exp\left[-S_{5d}(G|_b = g) - S_{ct}\right] = \left\langle \exp g\hat{T} \right\rangle_{CFT}$$



This part contains brane tension, 4-d gravity and higher curvature terms!

$$\underbrace{S_{5d\,gravity} + S_{brane\,tension}}_{RS\ model} = S_{4d\,gravity} + S_{CFT} + \{R^2 terms\}$$

Newtonian Gravity $\quad V(r) = \frac{1}{r} + \frac{\ell^2}{\underbrace{r^3}_{CFT}} + \cdots$

$$G_{\mu\nu} = 8\pi G T_{\mu\nu} + 8\pi G T_{\mu\nu}^{CFT} + \{R^2 terms\}$$

What is the precise meaning of AdS/CFT on the brane?

Geometrical holography

$$G_{\mu\nu} = 8\pi G T_{\mu\nu} + \kappa^4 \pi_{\mu\nu} - E_{\mu\nu}$$

How are these two pictures related?

AdS/CFT correspondence

$$G_{\mu\nu} = 8\pi G T_{\mu\nu} + 8\pi G T_{\mu\nu}^{CFT} + \{R^2\}$$

How can the high energy effect be recovered?

Strategy to reveal the relation

Since the brane is not located at the boundary of AdS,
CFT is a cut off theory which is a vague concept.

Geometrical holography is a kind of cheating because equations of motion have never been solved.

We now solve bulk geometry using the technique common in AdS/CFT
and derive the low energy effectve theory
without resorting to AdS/CFT correspondence.

The method we use is the gradient expansion method.

Basic Eqations

$$ds^2 = dy^2 + g_{\mu\nu}(y, x^\alpha)\, dx^\mu\, dx^\nu$$

Extrinsic curvature $K_{\mu\nu} = -\frac{1}{2}\frac{\partial}{\partial y} g_{\mu\nu} = \Sigma_{\mu\nu} + \frac{1}{4} g_{\mu\nu} K$ $\quad \Sigma^\mu{}_\mu = 0$

Evolution Equation $\frac{\partial}{\partial y}\Sigma^\mu{}_\nu - K\Sigma^\mu{}_\nu = -\left[R^\mu{}_\nu - \frac{1}{4}\delta^\mu{}_\nu R\right]$

Hamiltonian Constraint $\frac{3}{4}K^2 - \Sigma^\alpha{}_\beta \Sigma^\beta{}_\alpha = R + \frac{12}{\ell^2}$

Momentum Constraint $\nabla_\nu \Sigma^\nu{}_\mu - \frac{3}{4}\nabla_\mu K = 0$

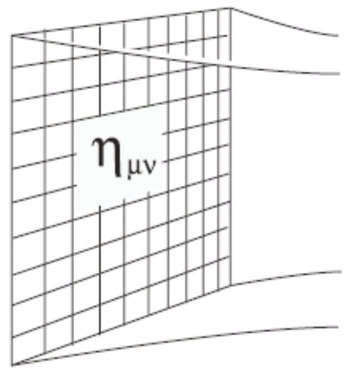
Junction condition $\left[\Sigma^\mu{}_\nu - \frac{3}{4}\delta^\mu{}_\nu K\right]\Big|_{y=0} = -\frac{\kappa^2\sigma}{2}\delta^\mu{}_\nu + \frac{\kappa^2}{2} T^\mu{}_\nu$

Z_2 symmetry

energy momentum tensor of matter

Gradient Expansion

Kanno & Soda (2002)



We assume:

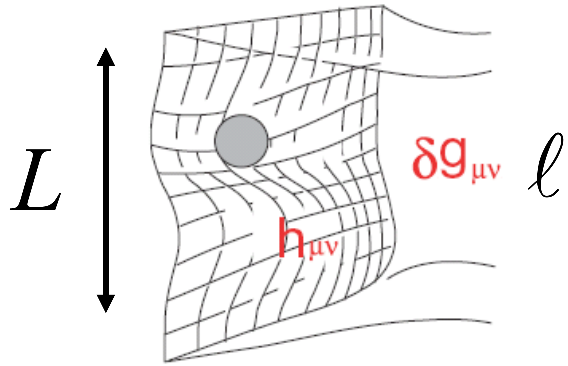
Energy density on the brane
<< brane tension

$$\varepsilon \equiv \frac{R/G}{\sigma} \approx R\ell^2 = \left(\frac{\ell}{L}\right)^2 << 1$$

$$\frac{\partial}{\partial x^\mu} = O\left(\frac{1}{L}\right) \quad \ll \quad \frac{\partial}{\partial y} = O\left(\frac{1}{\ell}\right)$$

$$\frac{3}{4}K^2 - \Sigma^\alpha{}_\beta \Sigma^\beta{}_\alpha = R + \frac{12}{\ell^2} \quad \xrightarrow{\varepsilon \to 0} \quad \overset{(0)}{K} = \frac{4}{\ell}$$

$$ds^2 = dy^2 + e^{-y/\ell} \, h_{\mu\nu}(x^\mu) \, dx^\mu \, dx^\nu$$



→ One can solve bulk Eqs. by expanding fields in the order of ε

Gradient Expansion Method

$$b(y) = e^{-y/\ell}$$

$$\varepsilon \qquad \varepsilon^2$$

$$g_{\mu\nu}(y, x^\mu) = b^2(y)\left[h_{\mu\nu}(x^\mu) + \overset{(1)}{g}_{\mu\nu}(y, x^\mu) + \overset{(2)}{g}_{\mu\nu}(y, x^\mu) + \cdots \right]$$

$$K = \frac{4}{\ell} + \overset{(1)}{K} + \overset{(2)}{K} + \cdots$$

$$\Sigma^\mu_\nu = \overset{(1)}{\Sigma}{}^\mu_\nu + \overset{(2)}{\Sigma}{}^\mu_\nu + \cdots$$

A Historical Remark

The idea, the so-called gradient expansion method, presented in this talk goes back to Lifshitz and Khalatnikov 1963, subsequently it has been developed by Tomita 1975, and later by Salopek 1992, Comer et al. 1994, Soda et al. 1995, and many others.

There, inflationary universe has been envisaged.

In the context of braneworld, a relation to AdS/CFT correspondence has been pointed out by Gubser 2001.

The gradient expansion method has been applied to braneworld cosmology by Kanno and Soda 2002.

First order Solutions

$$\frac{3}{4}K^2 - \Sigma^{\alpha}{}_{\beta}\Sigma^{\beta}{}_{\alpha} = R + \frac{12}{\ell^2} \longrightarrow \frac{3}{4}\times 2\overset{(0)}{K}\overset{(1)}{K} = R(g) = \frac{1}{b^2(y)}R(h)$$

$$\therefore \overset{(1)}{K} = \frac{\ell}{6b^2}R(h)$$

$$\frac{\partial}{\partial y}\Sigma^{\mu}{}_{\nu} - K\Sigma^{\mu}{}_{\nu} = -\left[R^{\mu}{}_{\nu} - \frac{1}{4}\delta^{\mu}{}_{\nu}R\right] \longrightarrow \frac{\partial}{\partial y}\overset{(1)}{\Sigma}{}^{\mu}{}_{\nu} - \frac{4}{\ell}\overset{(1)}{\Sigma}{}^{\mu}{}_{\nu} = -\frac{1}{b^2(y)}\left[R^{\mu}{}_{\nu}(h) - \frac{1}{4}\delta^{\mu}{}_{\nu}R(h)\right]$$

$$\therefore \overset{(1)}{\Sigma}{}^{\mu}{}_{\nu} = \frac{\ell}{2b^2}\left[R^{\mu}{}_{\nu} - \frac{1}{4}\delta^{\mu}_{\nu}R\right] + \frac{\ell}{2b^4}\chi^{\mu}{}_{\nu}(x^{\mu}) \qquad \chi^{\mu}{}_{\mu} = 0$$

constant of integration

$$\nabla_{\nu}\Sigma^{\nu}{}_{\mu} - \frac{3}{4}\nabla_{\mu}K = 0 \longrightarrow \chi_{\mu}{}^{\nu}{}_{|\nu} = 0$$

Hence, $\chi_{\mu\nu}$ can be interpreted as energy momentum tensor of CFT.

Einstein equation on the brane

Junction condition

$$\left[\overset{(1)}{\Sigma}{}^{\mu}{}_{\nu}-\frac{3}{4}\delta^{\mu}{}_{\nu}\overset{(1)}{K}\right]\Bigg|_{y=0}=\frac{\ell}{2}\left[R^{\mu}{}_{\nu}-\frac{1}{4}\delta^{\mu}_{\nu}R\right]+\frac{\ell}{2}\chi^{\mu}{}_{\nu}(x^{\mu})-\frac{3}{4}\delta^{\mu}{}_{\nu}\frac{\ell}{6}R(h)=\frac{\kappa^{2}}{2}T^{\mu}{}_{\nu}$$

Effective Eqs. on the Brane

$$G^{\mu}_{\nu}=\frac{\kappa^{2}}{\ell}T^{\mu}_{\nu}-\underbrace{\chi^{\mu}_{\nu}}_{\textit{Dark Radiation}}$$

Regularity requires $\chi^{\mu}{}_{\nu}=0$

If black holes exists in the bulk, we need to include dark radiation.
This direction is related to the recent hydrodynamics calculations.

Second order corrections

By solving Hamiltonian constraint and integrating evolution equation, we have

$$\overset{(2)}{K} = \frac{\ell^3}{8b^4}\left(R^{\alpha\beta}R_{\alpha\beta} - \frac{2}{9}R^2\right) - \frac{\ell^3}{12b^2}\left(R^{\alpha\beta}R_{\alpha\beta} - \frac{1}{6}R^2\right)$$

$$\overset{(2)}{\Sigma}{}^{\mu}{}_{\nu} = -\frac{\ell^3}{24b^2}\left(RR^{\mu}{}_{\nu} - \frac{1}{4}\delta^{\mu}{}_{\nu}R^2\right) - \frac{\ell^2}{2}\left(\frac{y}{b^4} + \frac{\ell}{2b^2}\right)S^{\mu}{}_{\nu}$$
$$+\frac{\ell^3}{b^4}\left[\frac{1}{32}\delta^{\mu}{}_{\nu}\left(R^{\alpha\beta}R_{\alpha\beta} - \frac{1}{3}R^2\right) + \frac{1}{24}\left(RR^{\mu}{}_{\nu} - \frac{1}{4}\delta^{\mu}{}_{\nu}R^2\right) - \tau^{\mu}{}_{\nu} - \alpha S^{\mu}{}_{\nu} - \frac{\beta}{3}K^{\mu}{}_{\nu}\right]$$

where we have solved momentum constraint which yields three constant of integration. One of which is a tensor field satisfying a conservation law

$$\tau^{\mu}{}_{\nu|\mu} = 0$$

Other two parameters are coeffcients of tensors

$$S_{\mu\nu} \equiv \frac{1}{2\sqrt{-g}}\frac{\delta}{\delta g^{\mu\nu}}\int d^4x\sqrt{-g}\left[R^{\alpha\beta}R_{\alpha\beta} - \frac{1}{3}R^2\right] \qquad S^{\mu}{}_{\nu|\mu} = 0 \qquad S^{\mu}{}_{\mu} = 0$$

$$K_{\mu\nu} \equiv \frac{1}{2\sqrt{-g}}\frac{\delta}{\delta g^{\mu\nu}}\int d^4x\sqrt{-g}R^2 \qquad K^{\mu}{}_{\nu|\mu} = 0$$

Effective equation on the brane and CFT

Junction condition

$$\left[\overset{(1)}{\Sigma}{}^{\mu}{}_{\nu}+\overset{(2)}{\Sigma}{}^{\mu}{}_{\nu}-\frac{3}{4}\delta^{\mu}{}_{\nu}\left(\overset{(1)}{K}+\overset{(2)}{K}\right)\right]\Bigg|_{y=0}=\frac{\kappa^2}{2}T^{\mu}{}_{\nu}$$

gives

Bulk gravitational waves

$$\underbrace{G_{\mu\nu}=\frac{\kappa^2}{\ell}T_{\mu\nu}+\frac{\kappa^2}{\ell}}_{Einstein\ Eq.}\underbrace{\frac{\ell^3}{\kappa^2}\tau^{\mu}{}_{\nu}}_{CFT\ matter}+\alpha S^{\mu}{}_{\nu}+\beta K^{\mu}{}_{\nu}$$

α,β : const. of integration

CFT = const. of integration

$$\tau^{\mu}{}_{\nu|\mu}=0 \qquad \tau^{\mu}{}_{\mu}=\frac{1}{4}\left(R^{\alpha\beta}R_{\alpha\beta}-\frac{1}{3}R^2\right)-\beta\Box R$$

Trace anomaly!

$$\Sigma^{\mu}{}_{\mu}=0$$

AdS/CFT can be related to geometrical holography

Projected Weyl tensor

$$E^{\mu}{}_{\nu} = \ell^2 \left[P^{\mu}{}_{\nu} - \tau^{\mu}{}_{\nu} - \alpha S^{\mu}{}_{\nu} - \beta K^{\mu}{}_{\nu} \right]$$

$$P^{\mu}{}_{\nu} = -\frac{1}{4} R^{\mu}{}_{\alpha} R^{\alpha}{}_{\nu} + \frac{1}{6} R R^{\mu}{}_{\nu} + \frac{1}{8} \delta^{\mu}_{\nu} R^{\alpha}{}_{\beta} R^{\beta}{}_{\alpha} - \frac{1}{16} \delta^{\mu}_{\nu} R^2$$

$$\approx \pi^{\mu}{}_{\nu}$$

$$E^{\mu}{}_{\mu} = 0 \longrightarrow \tau^{\mu}{}_{\mu} = \pi^{\mu}{}_{\mu}$$

Trace anomaly

Thus, one can reveal the relation between the geometrical holography and AdS/CFT correspondence.

$$G_{\mu\nu} \approx 8\pi G T_{\mu\nu} + \ell^2 \pi_{\mu\nu} - E_{\mu\nu}$$

Cosmological Perturbation Theory

The energy momentum tensor can be related to the effective action by

$$\left\langle T_{\mu\nu}^{CFT} \right\rangle = \frac{-2}{\sqrt{-g}} \frac{\delta S^{CFT}}{\delta g^{\mu\nu}}$$

Correlation function can be calculated as

$$\left\langle T_{\mu\nu}^{CFT}(x) T_{\lambda\rho}^{CFT}(y) \right\rangle = \frac{-2}{\sqrt{-g}} \frac{\delta \left\langle T_{\mu\nu}^{CFT}(x) \right\rangle}{\delta g^{\lambda\rho}(y)}$$

Then cosmological perturbation theory can be

$$\delta G_{\mu\nu} = 8\pi G \delta T_{\mu\nu}$$

$$-\frac{1}{2} \int d^4 y \sqrt{-g(y)} \left\langle T_{\mu\nu}^{CFT}(x) T^{CFT\lambda\rho}(y) \right\rangle \delta g_{\lambda\rho}(y) + \alpha \delta S_{\mu\nu} + \beta \delta K_{\mu\nu}$$

CFT generates a nonlocal effect on cosmological fluctuations.
Detailed analysis requires a numerical work.

RSI: Two-brane Model

Randall & Sundrum (1999)

$$S = \frac{1}{2\kappa^2}\int d^5x\sqrt{-\gamma}\;\left[\overset{(5)}{\Re} + \frac{12}{\ell^2}\right]$$

$$- \sum_{i=\oplus,\Theta} \sigma_i \int d^4x\sqrt{-g_i} \;+ \sum_{i=\oplus,\Theta} \int d^4x\sqrt{-g_i}\;\; L^i_{matter}$$

Coupling constant

$\overset{(5)}{\Re}$: 5-d Ricci scalar

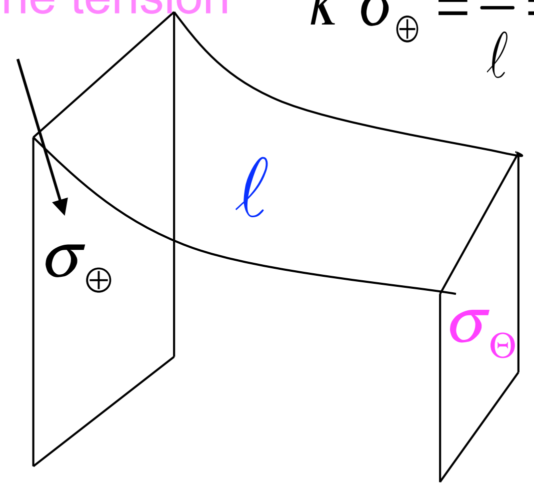
γ_{AB} : 5-d metric

$g^i_{\ \mu\nu}$: 4-d metric

RS fine tuning

$$\kappa^2\sigma_\oplus = \frac{6}{\ell} = -\kappa^2\sigma_\Theta$$

Brane tension



Is there any AdS/CFT description of RSI?

Geometrical holography

$$G_{\mu\nu} = 8\pi G T_{\mu\nu} + \ell^2 \pi_{\mu\nu} - E_{\mu\nu}$$

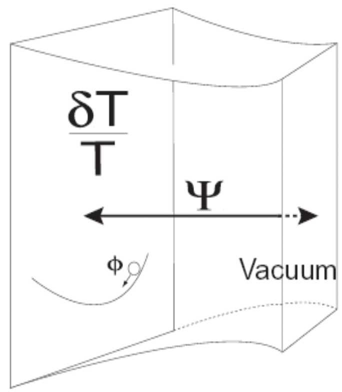
$\Updownarrow$ **How are these two pictures related?**

Linear analysis

$$\delta G_{\mu\nu} = 8\pi G \delta T_{\mu\nu} + \phi_{|\mu\nu} - \eta_{\mu\nu} \phi^{|\alpha}{}_{|\alpha}$$

Brans-Dicke Theory?

Gradient Expansion Method



Einstein Eq. on the positive tension brane

$$G^{\mu}_{\nu}(h_{\mu\nu}) = \frac{\kappa^2}{\ell} \overset{\oplus}{T}{}^{\mu}{}_{\nu} - \chi^{\mu}_{\nu}$$

Einstein Eq. on the negative tension brane

$$G^{\mu}{}_{\nu}(\Omega^2 h_{\mu\nu}) = -\frac{\kappa^2}{\ell} \overset{\ominus}{T}{}^{\mu}{}_{\nu} - \frac{\chi^{\mu}{}_{\nu}}{\Omega^4(x^{\mu})}$$

Warp factor

The constant of integrations can be fixed by boundary conditions.

$$\chi^{\mu}_{\nu} = -\frac{1}{\Psi}\left[\Psi^{|\mu}_{|\nu} - \delta^{\mu}_{\nu}\Psi^{|\alpha}_{|\alpha} + \cdots\right] - \frac{\kappa^2(1-\Psi)}{\ell\Psi}\left[\overset{\oplus}{T}{}^{\mu}{}_{\nu} - \cdots\right]$$

Radion

$$\Psi = 1 - \Omega^2$$

Scalar-Tensor Theory → Brans-Dicke Theory

Eq. of motion for the radion

$$\chi^{\mu}{}_{\mu} = 0$$

$$\to \quad \nabla^{\mu}\nabla_{\mu}\Psi = \frac{8\pi G}{2\omega+3}\left[\overset{\oplus}{T} + (1-\Psi)\overset{\ominus}{T}\right] - \frac{1}{2\omega+3}\frac{d\omega}{d\Psi}\nabla^{\mu}\Psi\nabla_{\mu}\Psi$$

Eq. of motion for the metric

$$G_{\mu\nu} = \frac{\kappa^2}{\ell\Psi}\overset{\oplus}{T}_{\mu\nu} + \frac{\kappa^2(1-\Psi)}{\ell\Psi}\overset{\ominus}{T}_{\mu\nu}$$

$$+\frac{1}{\Psi}\left(\Psi_{|\mu\nu} - g_{\mu\nu}\Psi^{|\alpha}{}_{|\alpha}\right) + \frac{\omega(\Psi)}{\Psi^2}\left(\Psi_{|\mu}\Psi_{|\nu} - \frac{1}{2}g_{\mu\nu}\Psi^{|\alpha}\Psi_{|\alpha}\right)$$

$$\omega = \frac{3}{2}\frac{\Psi}{1-\Psi}$$

Geometrical holography & Brans-Dicke Theory

$$G^{\mu}_{\nu} = \frac{\kappa^2}{\ell} \overset{\oplus}{T}{}^{\mu}_{\nu} - \chi^{\mu}_{\nu}$$

Two equivalent representations!

$$G_{\mu\nu} = \frac{\kappa^2}{\ell\Psi} \overset{\oplus}{T}_{\mu\nu} + \frac{\kappa^2(1-\Psi)}{\ell\Psi} \overset{\Theta}{T}_{\mu\nu} + \frac{1}{\Psi}\left(\Psi_{|\mu\nu} - g_{\mu\nu}\Psi^{|\alpha}{}_{|\alpha}\right) + \frac{\omega(\Psi)}{\Psi^2}\left(\Psi_{|\mu}\Psi_{|\nu} - \frac{1}{2} g_{\mu\nu}\Psi^{|\alpha}\Psi_{|\alpha}\right)$$

In the single brane limit $\Psi \to 1$, $\chi_{\mu\nu}$ vanish!

Effective action for two-brane system

Kanno & Soda (2002)

$$S=\frac{\ell}{2\kappa^2}\int d^4x\sqrt{-h}\,\left[\Psi R-\frac{3}{2(1-\Psi)}\Psi^{|\alpha}\Psi_{|\alpha}\right]$$
$$+\int d^4x\sqrt{-h}\,\overset{\oplus}{L}+\int d^4x\sqrt{-h}\,(1-\Psi)^2\,\overset{\ominus}{L}$$

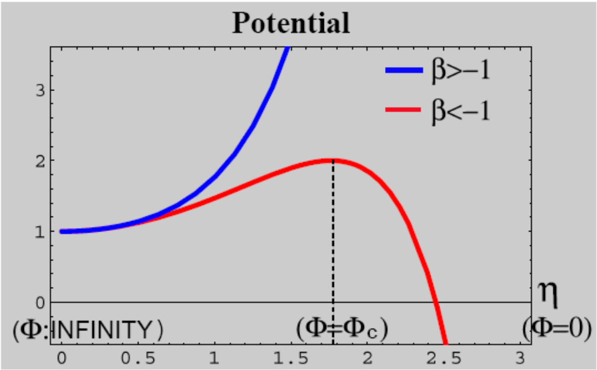
$$\Downarrow\quad \Psi=1-\varphi^2$$

$$S=\frac{6\ell}{\kappa^2}\int d^4x\sqrt{-h}\left[\frac{1}{12}R-\frac{1}{2}\varphi^{|\alpha}\varphi_{|\alpha}-\frac{1}{12}\varphi^2R+V_{eff}(\varphi)\right]$$

conformally invariant scalar

Einstein frame potential

$$V(\eta)=\delta\sigma_\oplus\left[\cosh^4\frac{\eta}{2}+\frac{\delta\sigma_\ominus}{\delta\sigma_\oplus}\sinh^4\frac{\eta}{2}\right]$$

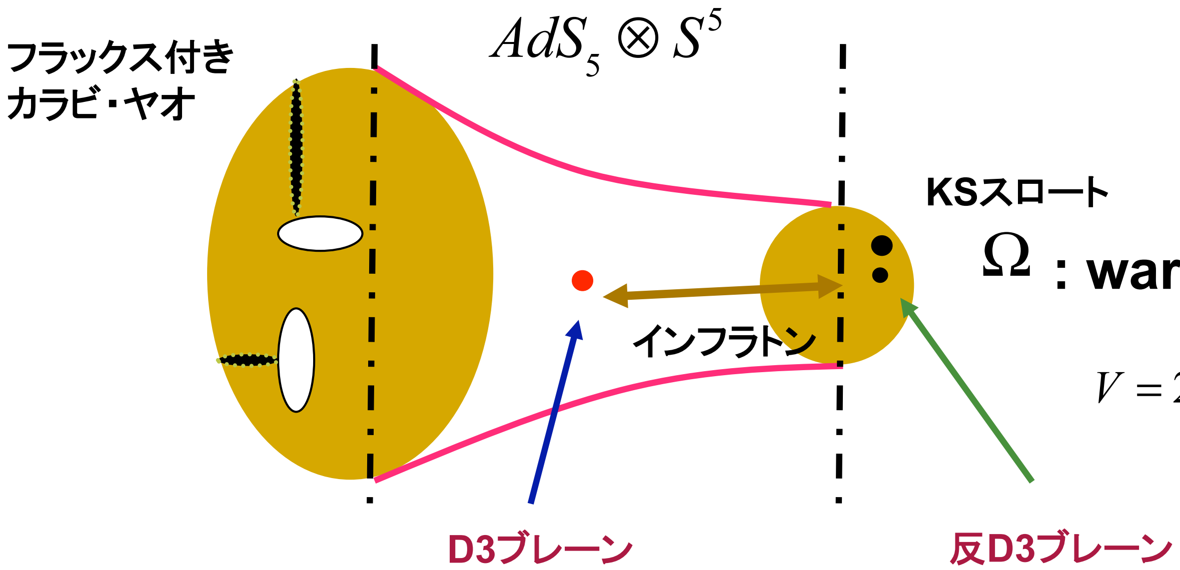


The brane is inflating in spite of the steep potential!

Warped brane Inflation

The previous result shows the difficulty in brane inflation.

Kachuru et al (KKLMMT 2003)



Ω **: warp factor**

$$V = 2T_3\Omega^4\left(1 - \frac{\Omega^4 A}{r^4}\right)$$

eta problem

$$S = \frac{6\ell}{\kappa^2}\int d^4x\sqrt{-h}\left[\frac{1}{12}R - \frac{1}{2}\varphi^{|\alpha}\varphi_{|\alpha} \underbrace{-\frac{1}{12}\varphi^2 R}_{H^2\varphi^2} + V_{eff}(\varphi)\right]$$

Summary

We have shown the equivalence between geometrical holography and AdS/CFT correspondence.

The leading correction of brane cosmology comes from CFT!

Two brane system can be described by conformaly invariant radion action. In this sense, AdS/CFT correspondence is still relevant.

What I do not understand is

How to formulate AdS-BH/braneworld correspondence?

AdS/CFT may play a more important role in brane cosmology.