Brane models at Finite Temperature: A Lattice Point of View

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Fifth Aegean Summer School
Milos, 21-26/09/2009

CONTENTS

- Introductory hints on the extra dimensional theories
- A short introduction to the Lattice Gauge Theories
- Pure U(1) model in 5D and the Layer phase; inclusion of anisotropic couplings in the action
- Finite Temperature: "absence" of the Layer phase
- Remarks & Conclusions

Extra dimensions

- ► As far as it is known, there is not any **fundamental principle** for which the space-time should be 3+1 dimensional…. Hence, "in principle" we can assume *extra* spatial dimensions.

- ► It can not be denied that, up to now, the theoretical exploration of extra dimensional models is rather *hypothetical* than based on well established facts.

- ► Nevertheless, the construction of the extra dimensional models is guided by the requirement of physical consistency in order to show possible ways on how the extra dimensions can be detectable (or why are *hidden*) and to connect the extra-dimensional models with a more **fundamental** theory in order to solve long-standing theoretical problems.

Kaluza-Klein proposal (1921, 1926):

- Consider a five-dimensional flat space with the fifth dimension compactified
- The photon resides in the extra dimension
- The unification of gravity with electromagnetism can be achieved

Extra dimensions *à la* Kaluza-Klein are hidden as a result of their size :

$l \sim 10^{-33}$ cm.

Need for energies as high as the Planck energy to make them detectable.

Brane models (1)

The modern idea of models with *Extra Dimensions* is connected with the

Brane world picture

The ordinary matter (with exception of gravitons) is localised to a 3-D submanifold (**brane**) embedded in a fundamental multi-dimensional space (**bulk**)

It is assumed that the brane picture is an *effective* model representation. It comes from a high-energy theory which appears at a fundamental scale M_*

The extra dimensions should be so large as to non contradict the observations

↓

For compactified extra dimensions their size can be as big as a fraction of millimeter dependent from the number of extra dimensions

Brane models (2)

One of the main **motivations** for the Brane picture used in higher dimensional models is to give an answer to the :

Hierarchy Problem

***Why* … $M_{Pl} >> M_{EW}$?**

Possible answer: The fundamental scale is $\mathbf{M_* \sim 1TeV}$ in a theory defined in a 4+n space

ADD model

[Arkani-Hamed, Dimopoulos & Dvali Phys.Lett.B 1998]

RS models

[Randall & Sundrum Phys.Rev.Lett. 1998]

RS model (1)

- Consider a five-dimensional Anti-DeSitter space (AdS_5)
  - the extra dimension obeys to
    $z = z + 2\pi r$ & $z \to -z$ (S^1/Z_2 orbifolded)
  - negative cosmological constant Λ in the bulk

- Assume the existence of two branes resided along the extra dimension and having opposite values of tension (energy density) σ.
  - Matter is assumed on the branes while the gravity can be spread in the bulk

It is shown that the metric admitted for this set-up takes the form:

$$ds^2 = a^2(z)\,\eta_{\mu\nu} dx^\mu dx^\nu - dz^2 \quad (\text{with } a(z) = e^{-kz})$$

The extra dimension becomes **warped** as a price of maintaining a 4D Lorentz space at every point of the extra dimension (non-factorizable geometry).

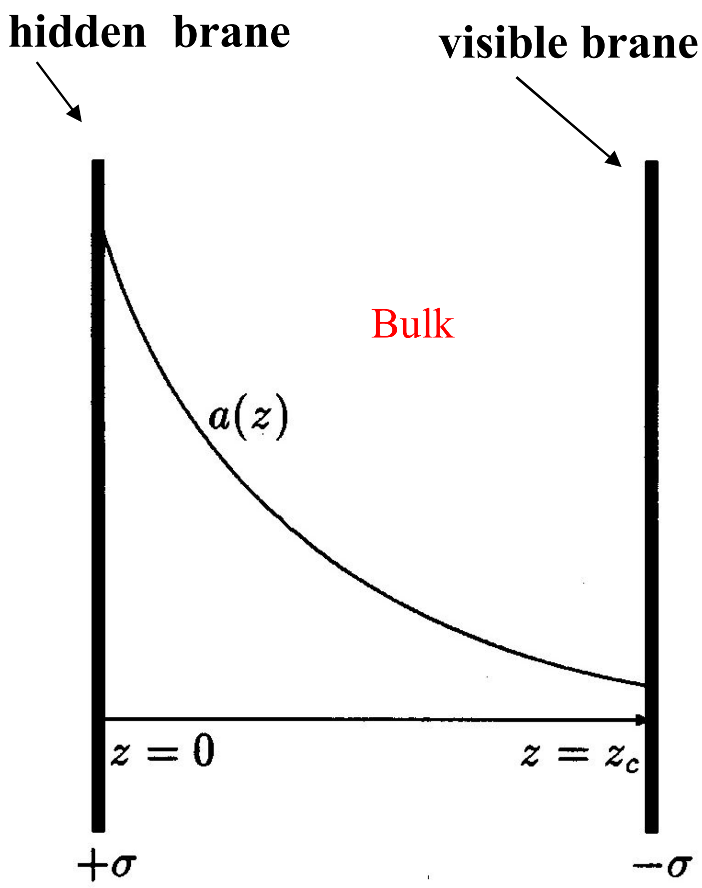
The model provides an ***exponential hierarchy*** expressed by the distance which separates the two branes along the extra coordinate.

RS model (2)

$$\mathbf{M}_{Pl}^{2} = \frac{\mathbf{M}_{*}^{3}}{k}(1 - \mathbf{e}^{k\mathbf{z}_{c}})$$

$$M_{Pl} \approx M_{*} \approx k$$

Physical scale $\approx e^{-kz_c} M_{Pl}$



Fine tunning

$$\sigma = 24 M_{*}^{3} k$$

$$\Lambda = -24 M_{*}^{3} k^{2}$$

A short introduction to the Lattice Gauge Theories

Lattice Gauge Theories (1)

- Total Action for the System

$$S_{total}^{M} = S_{gauge}(A) + S_{scalar}(\Phi, A) + S_{fermion}(\Psi, \bar{\Psi}, A)$$

- Quantization using the Functional Integral Formalism

$$Z = \int DAD\Phi D\Psi D\bar{\Psi} \exp(iS_{total}^{M})$$

Integrate over all possible Field configurations, we have a contribution from the classical solutions and the Quantum Fluctuations of the Fields.

- Calculate Mean Values for the Physical Quantities (Order Parameters) and Green Functions for the Field Propagation.

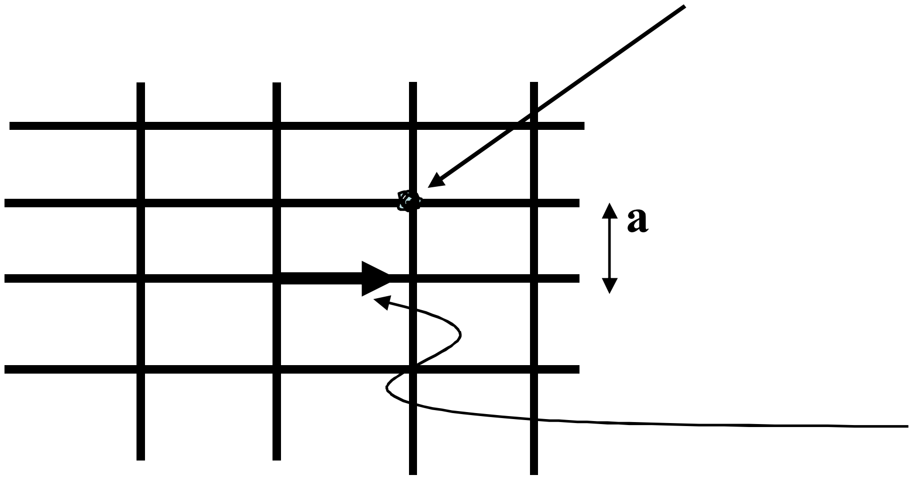
$$<\hat{T}> = \frac{1}{Z} \int DAD\Phi D\Psi D\bar{\Psi} T(A, \Phi, \Psi, \bar{\Psi}) \exp(iS_{total}^{M})$$

- $<\hat{T}>$ depend from the coupling constants of the Fields and the external conditions (Temperature, Magnetic Fields etc)

Lattice gauge theories (2)

- Wick rotation to Euclidean space time $t \rightarrow -i\tau$
- Discretize space time $x_\mu \rightarrow n_\mu a$, a is the lattice spacing

- Gauge Fields are defined in the links between the nodes of the lattice
- Matter Fields, Fermions *Ψ(x)* and Bosons *Φ(x)* are defined at the lattice nodes



$$U_\mu(x) = e^{igaA_\mu(x)}$$

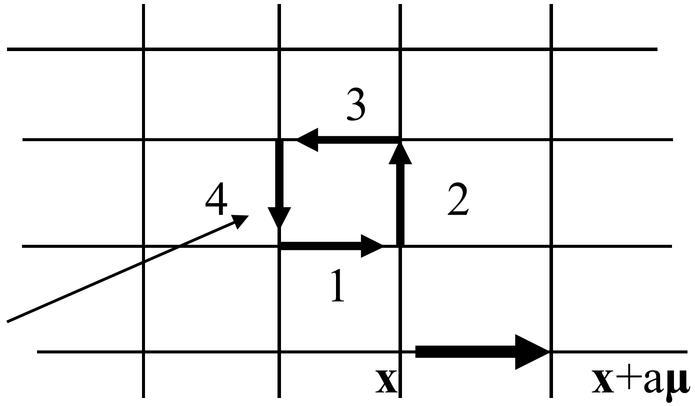
The Gauge Field it is an element of the gauge group

Lattice Gauge Theories (3)

$$\frac{1}{2}Tr[F_{\mu\nu}F^{\mu\nu}]$$

$$\downarrow$$

$$1-\frac{1}{2N}\{Tr[U_1U_2U^{\dagger}{}_3U^{\dagger}{}_4]+h.c.\}$$



We express the action term for the Gauge Field through the plaquettes.

$$\uparrow$$

$$\Phi^{\dagger}(x)U_{\mu}(x)\Phi(x+\hat{\mu})$$

$$\uparrow$$

Kinetic term for a Bosonic Field in the fundamental representation.

$$D_{\mu}\Phi^{\dagger}D_{\mu}\Phi$$

Local Gauge Transformations:

$$U_{\mu}(x)\rightarrow V(x)U_{\mu}(x)V^{\dagger}(x+a\hat{\mu})$$

$$\Phi(x)\rightarrow V(x)\Phi(x)$$

(4+1)-dimensional pure U(1) gauge model

The gauge field in extra dimensional space

- What about a possible extension of the gauge field in the bulk?
- Is there any possibility of gauge localisation on the brane in the RS set-up?

The answer given in terms of analytical work is *negative* …

Pomarol, Phys.Lett.B2000
Davoudiasl, Hewett & Rizzo, Phys.Lett.B2000

But …

since the localisation of the gauge field on the brane may involve *strong coupling dynamics* along the extra dimensions, the non-perturbative methods are necessary for giving an answer.

Abelian Gauge field in the RS background

- Assume the RS metric : $ds^2 = e^{-kz}\left(dx_0^2 - dx_1^2 - dx_2^2 - dx_3^2\right) - dz^2$

- Consider a 5-dim **abelian gauge model** in the background of the RS metric:

$$S_{gauge} = -\frac{1}{4g_5^2}\int d^5x\sqrt{-g}\, F_{MN}F_{KQ}\, g^{MK}g^{NQ}$$
$$= \int d^4x dz\left(-\frac{1}{4g_5^2}F_{\mu\nu}F_{\kappa\lambda}\eta^{\mu\kappa}\eta^{\nu\lambda} - \frac{1}{2g_5^2}e^{-kz}\, F_{\mu 5}F_{\nu 5}\eta^{\mu\nu}\right)$$

- In the Euclidean space the gauge action reads:

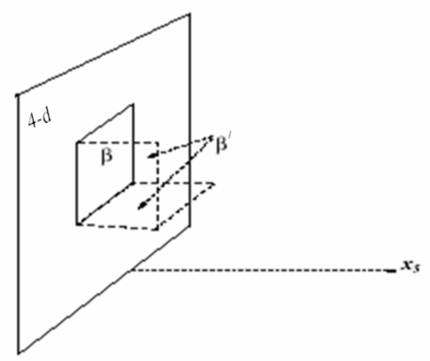
$$S^E_{gauge} = \int d^5x\left(\frac{1}{4g_5^2}F_{\mu\nu}F_{\kappa\lambda} + \frac{1}{2g_5^2}e^{-k|x5|}\, F_{\mu 5}F_{\mu 5}\right) \quad \mu,\nu = 1,\dots,4$$

⇨ We get a model with *two different couplings*, one defined on the 4D subspace and the second (***bigger***) along the extra dimension.

P. Dimopoulos, K.F., A. Kehagias and G. Koutsoumbas NPB 2001

On the lattice we define the pure U(1) Wilson gauge action with **anisotropic** couplings:

$$S_{gauge} = \beta \sum_{x,1\le\mu<\nu\le 4} \left(1 - Re U_{\mu\nu}(x)\right) + \beta' \sum_{x,1\le\mu\le 4} \left(1 - Re U_{\mu 5}(x)\right)$$



Link variables: $U_M = \left\{ U_\mu = e^{i a_s \bar{A}_\mu},\ U_5 = e^{i a_5 \bar{A}_5} \right\}$

Plaquettes:

$$U_{\mu\nu}(x) = U_\mu(x) U_\nu(x + a_s\hat{\mu}) U_\mu(x + a_s\hat{\nu}) U_\nu(x)$$

$$U_{\mu 5}(x) = U_\mu(x) U_5(x + a_s\hat{\mu}) U_\mu(x + a_5\hat{5}) U_5(x)$$

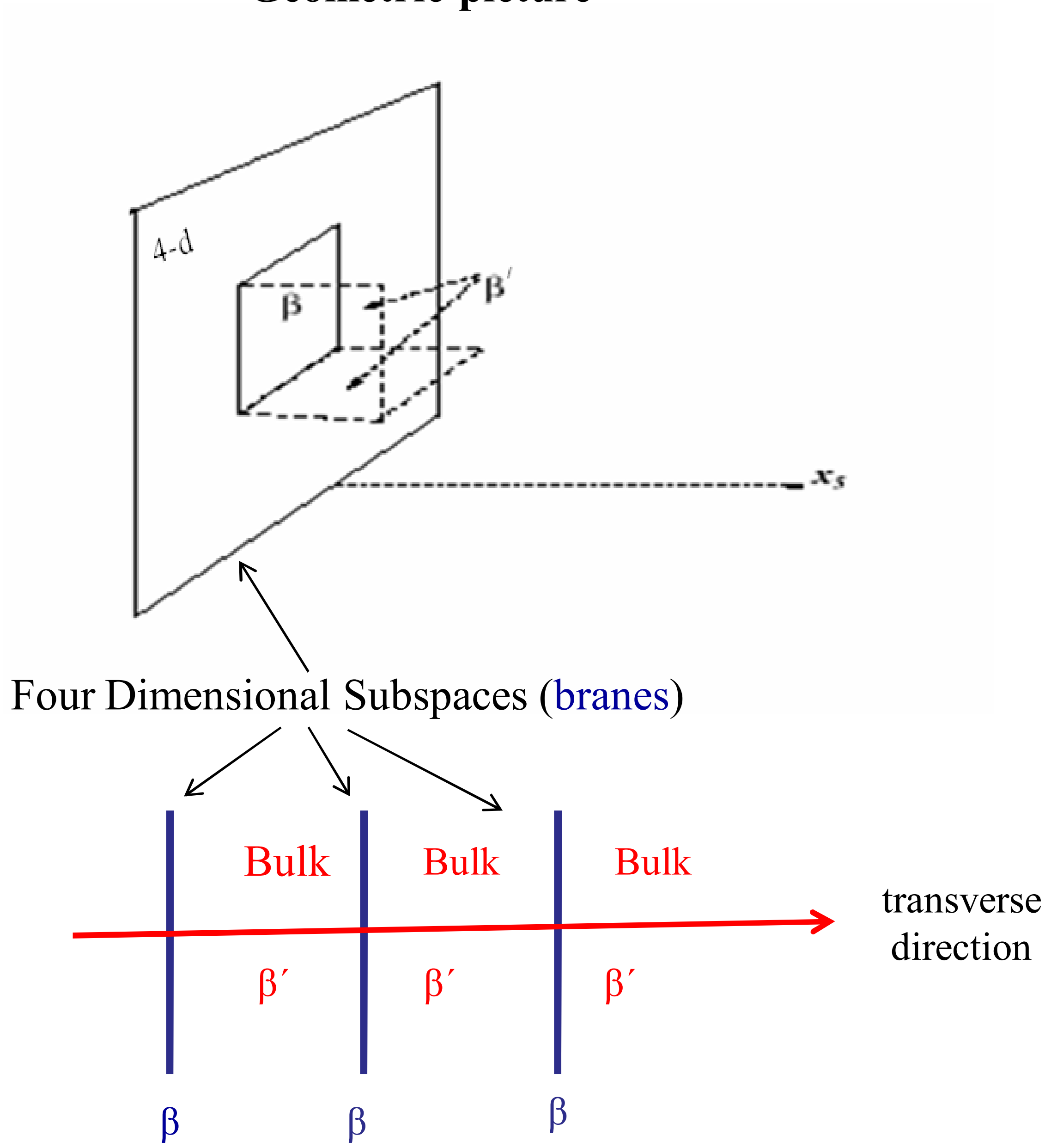
β it is inverse to the square of the gauge coupling

The **Fu-Nielsen** proposal for ***dimensional reduction*** from lattice anisotropic models (couplings $\beta \neq \beta'$)

Fu and Nielsen NPB 1984, 1985

Phase diagram becomes richer: A new phase appears …

Geometric picture



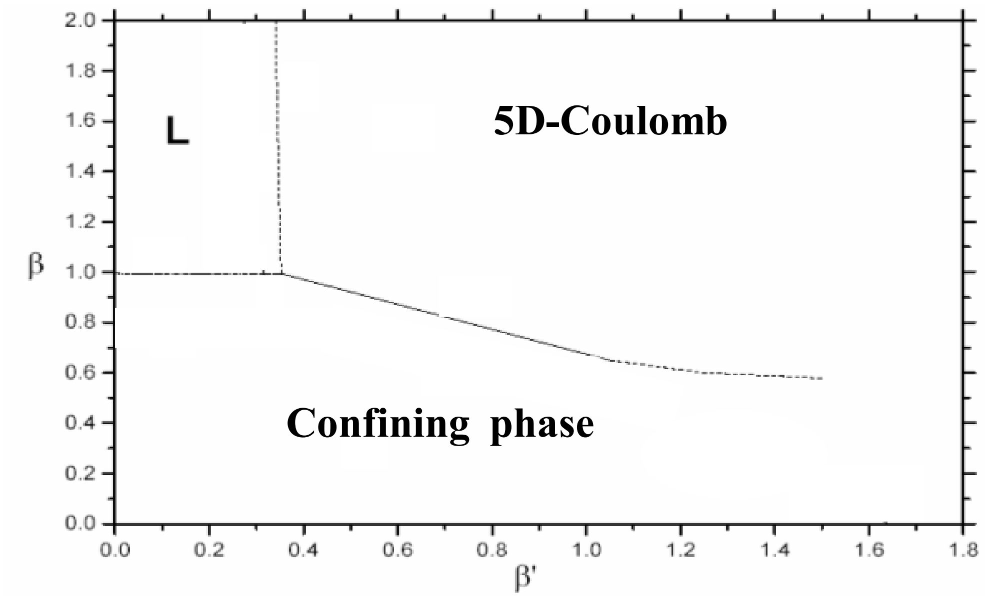
LAYER PHASE

The potential between heavy test charges is closely connected with the **Wilson loops**:

1. $W_{MN}(L_1,L_2) \approx \exp\left[-\sigma L_1 L_2\right]$ (Confining phase, $1 \le M, N \le 5$)
2. $W_{MN}(L_1,L_2) \approx \exp\left[-\tau(L_1 + L_2)\right]$ (Coulomb phase, $1 \le M, N \le 5$)
3. $W_{\mu\nu}(L_1,L_2) \approx \exp\left[-\tau(L_1 + L_2)\right]$
4. $W_{\mu 5}(L_1,L_2) \approx \exp\left[-\sigma'(L_1 L_2)\right]$

} Layer phase (3., 4.)

Phase Diagram from Mean Field analysis



Fu and Nielsen NPB 1984, 1985

Phase Diagram — Monte Carlo Study

Basic order parameters

- 4D (or space) plaquette : $\hat{P}_S \equiv \frac{1}{6N^5} \sum_{x,1\le\mu<\nu\le 4} \cos(F_{\mu\nu}(x))$ $\quad P_S = \langle \hat{P}_S \rangle$
- extra dimension (or transverse) plaquette : $\hat{P}_5 \equiv \frac{1}{4N^5} \sum_{x,1\le\mu\le 4} \cos(F_{\mu 5}(x))$ $\quad P_5 = \langle \hat{P}_5 \rangle$

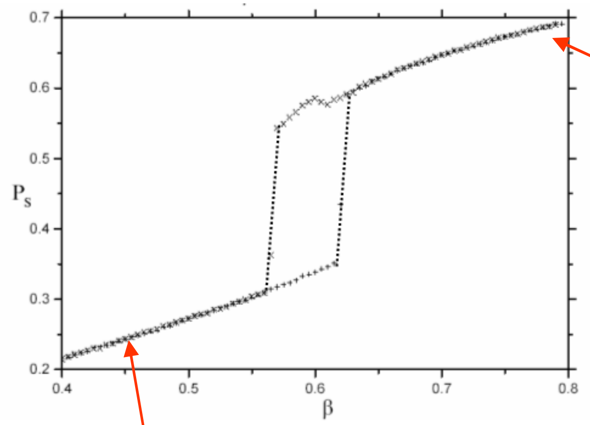
Behaviour of the Plaquette in the two limits of the coupling values

$$P = \begin{cases} \beta/2 + O(\beta^2) & [\ \beta \ll 1,\ \text{strong coupling}\ ] \\ 1 - \frac{1}{D\beta} + O(\beta^{-2}) & [\ \beta \gg 1,\ \text{weak coupling}\ ] \end{cases}$$

(D is the dimension of the space)

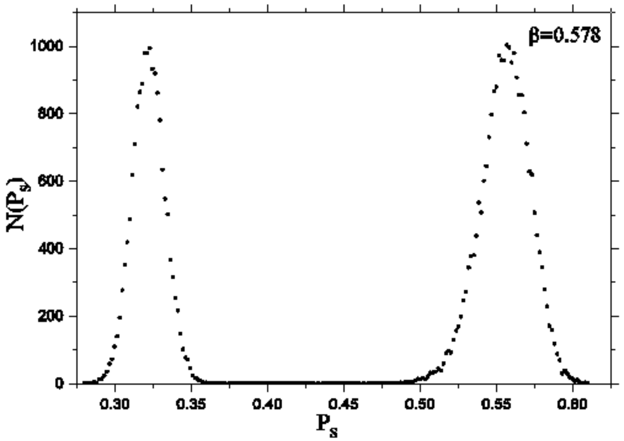
Confining – 5D Coulomb Phase Transition

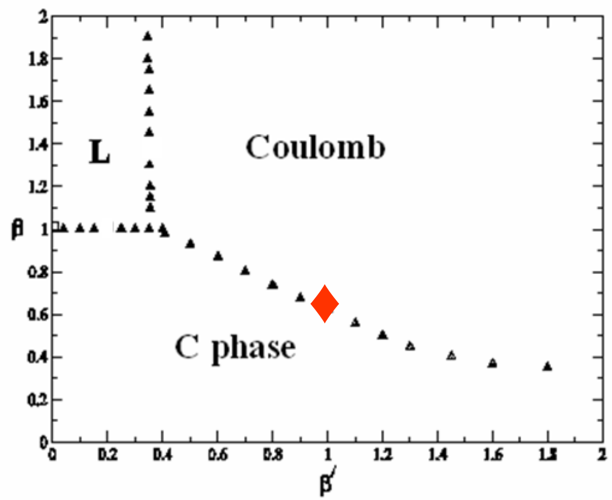
β′=1.0



~ [1-(1/5β)] : 5D-Coulomb phase

~ β /2: Confining phase



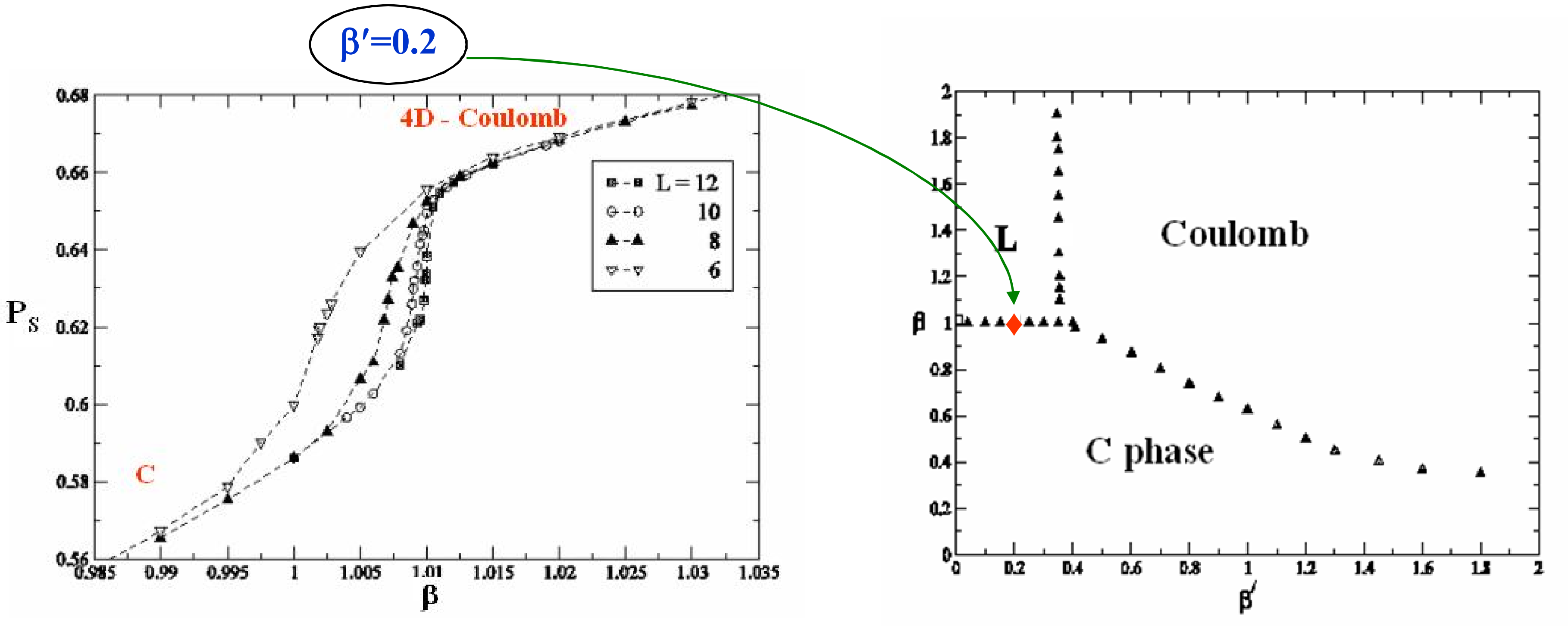


- **P. Dimopoulos, K.F., A. Kehagias and G. Koutsoumbas NPB 2001**
- **Hulsebos, Korthals-Altes and Nicolis NPB 1995**

Big hysteresis loop

Two-state signal → **1st order PT**

Confining – Layer Phase Transition



- As the lattice volume becomes bigger the transition becomes steeper
- For **β<<1, P_S → β/2** : Confining phase
- For **β>>1, P_S → (1-1/4β)** : 4D-Coulomb phase
- For **all** β, **$P_5 \approx \beta'/2=0.1$ (constant) → extra dimension is confined !!**

P.Dimopoulos, K.F., S.Vrentzos
Phys.Rev.D 2006

Identification of the phases using the helicity modulus

The **helicity modulus** is an order parameter; it gives the response of the system to an external elecrtomagnetic flux. It is defined through the second derivative of the free energy: [Vettorazzo and P. de Forcrand, NPB 2004]

$$h(\beta) = \left.\frac{\partial^2 F(\Phi)}{\partial \Phi^2}\right|_{\Phi=0}$$

In the confinement phase the system does not feel any changes due to the external flux. On the contrary in the Coulomb phase the system reveals a response.

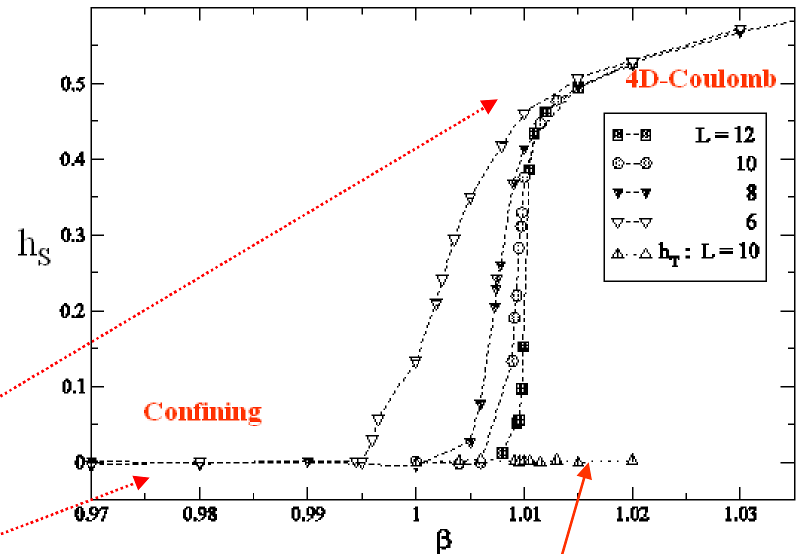
- $h(\beta) \neq 0$ in the Coulomb phase
- $h(\beta) = 0$ in the confining phase

The space-like and the transverse-like helicity modulus are:

$$h_S(\beta) = \frac{1}{(L_\mu L_\nu)^2}\left(\left\langle \sum_P (\beta \cos(F_{\mu\nu}))\right\rangle - \left\langle \left(\sum_P (\beta \sin(F_{\mu\nu}))\right)^2 \right\rangle\right)$$

$$h_5(\beta') = \frac{1}{(L_\mu L_5)^2}\left(\left\langle \sum_{P'} (\beta' \cos(F_{\mu 5}))\right\rangle - \left\langle \left(\sum_{P'} (\beta' \sin(F_{\mu 5}))\right)^2 \right\rangle\right)$$

$\beta'=0.2$



Helicity modulus in the transverse direction, h_5 takes zero value for all values of β.

Confinement along the extra dimension

- We impose an external flux Φ changing the gauge field in a set of links (twisted boundary conditions)

$$U_2(x_1+L, x_2=1, x_3, x_4, x_5) = U_2(x_1, x_2=1, x_3, x_4, x_5)e^{i\Phi}$$

- Confinement phase: The correlation length ξ is small (one or two a), so the Free Energy of the system is independent from the flux that we impose changing the boundary conditions → h(β)=0.

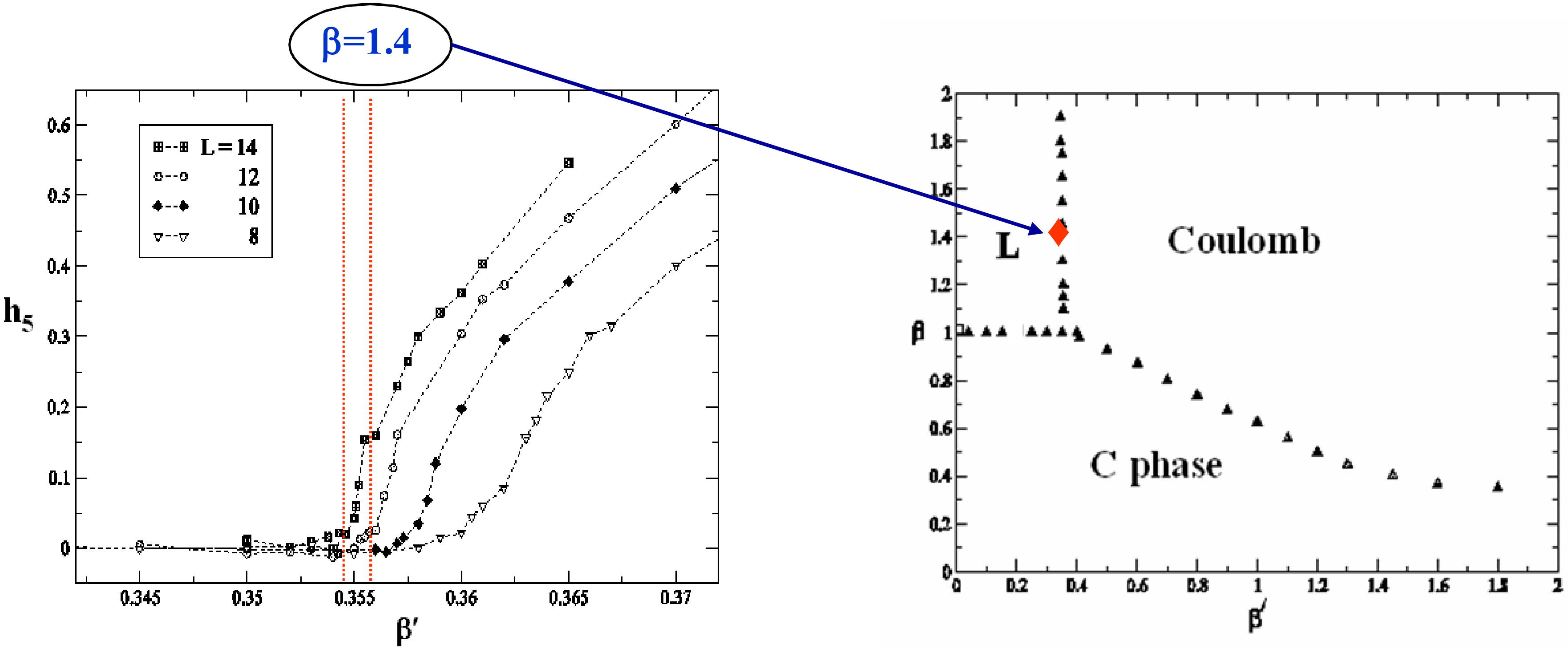
- Coulomb phase: Infinite correlation length (massless photon), the boundary conditions act to the whole system → the Free Energy dependent from the flux → h(β) is non zero.

- We can spread the flux Φ to the (1,2) plain by a change in the links, then we have flux $\Phi / L_\mu L_\nu$ through each plaquette in the (1,2) plane.

- Coulomb phase: From an expansion in the lattice gauge action,

$$F_\beta(\Phi) - F_\beta(0) = \frac{\beta_R}{2}\Phi^2 \frac{V}{(L_\mu L_\nu)^2} + \dots$$ where V is the volume of the system.

Layer – 5D Coulomb Phase Transition



► As the system passes to the 5D-Coulomb phase the extra fifth dimension ceases to be confined : h_5 passes from zero to a non-zero value

- *Finite size scaling analysis gives evidence for a second order Phase Transition*

Layer – 5D Coulomb Phase Transition

- A finite size scaling analysis assumes:

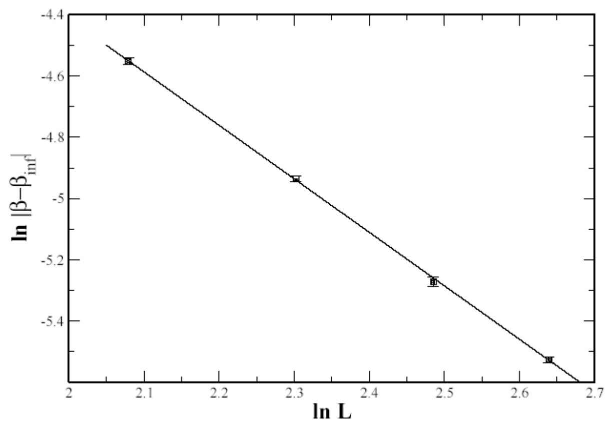
$$\beta'(L) = \beta'_\infty(1 + C_1 L^{-(1/\nu)})$$

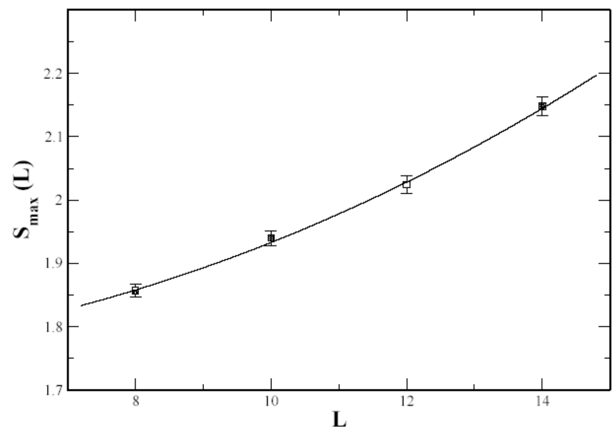
$$S_{max} = C_0 + C_1 L^{\gamma/\nu}$$

$$\beta'_\infty = 0.35028(53),\ \delta = 0.44(15)$$

	5D percolation	U(1) anisotropic	Gaussian
ν	0.57	0.57(5)	0.50
γ	1.18	1.24(44)	1.00

Evidence for a 2nd PT





* Volumes bigger than $V=14^5$ have to be used in order to confirm this evidence

It can be shown numerically on the lattice:

- how it can be identified a phase (Layer) which is coulombic on the four-dimensional subspace while it exhibits confinement along the extra dimension
- The Layer phase is stable and it is well separated from the confinement phase and the five-dimensional Coulomb phase

(P.Dimopoulos, K.F., S.Vrentzos Phys.Rev.D 2006)

- the potential between two test charges (on a single layer) in the layer phase is that of a 4D Coulomb interaction ($\sim 1/r$) and it is distinguishable from the potential of the 5D Coulomb phase (which goes as $\sim 1/r^2$)

(K.F. and S.Vrentzos, Phys.Rev.D 2008)

Finite Temperature, pure U(1) 5D model:
Absence of the Layer Phase

Finite Temperature

The physical system is defined in a Eucledian space time with compactified the temporal direction. Periodic boundary conditions for Bosons, antiperiodic for Fermions.

$$\int_0^{1/T} dt \int d^d x \rightarrow T \sum_{n=-\infty}^{\infty} \int d^d k$$

Matsubara modes

$k_0 \rightarrow 2\pi n T$

Lattice size: $V = L_t L_S^3 L_5 \ , \quad L_S = L_5$

Couplings: $\beta_t = \beta_S = \beta, \ \beta_5 = \beta'$

Temperature: $T = \dfrac{1}{L_t a}$

$$L_t << L_S$$

Zero Temperature case

$$L_t = L_S$$

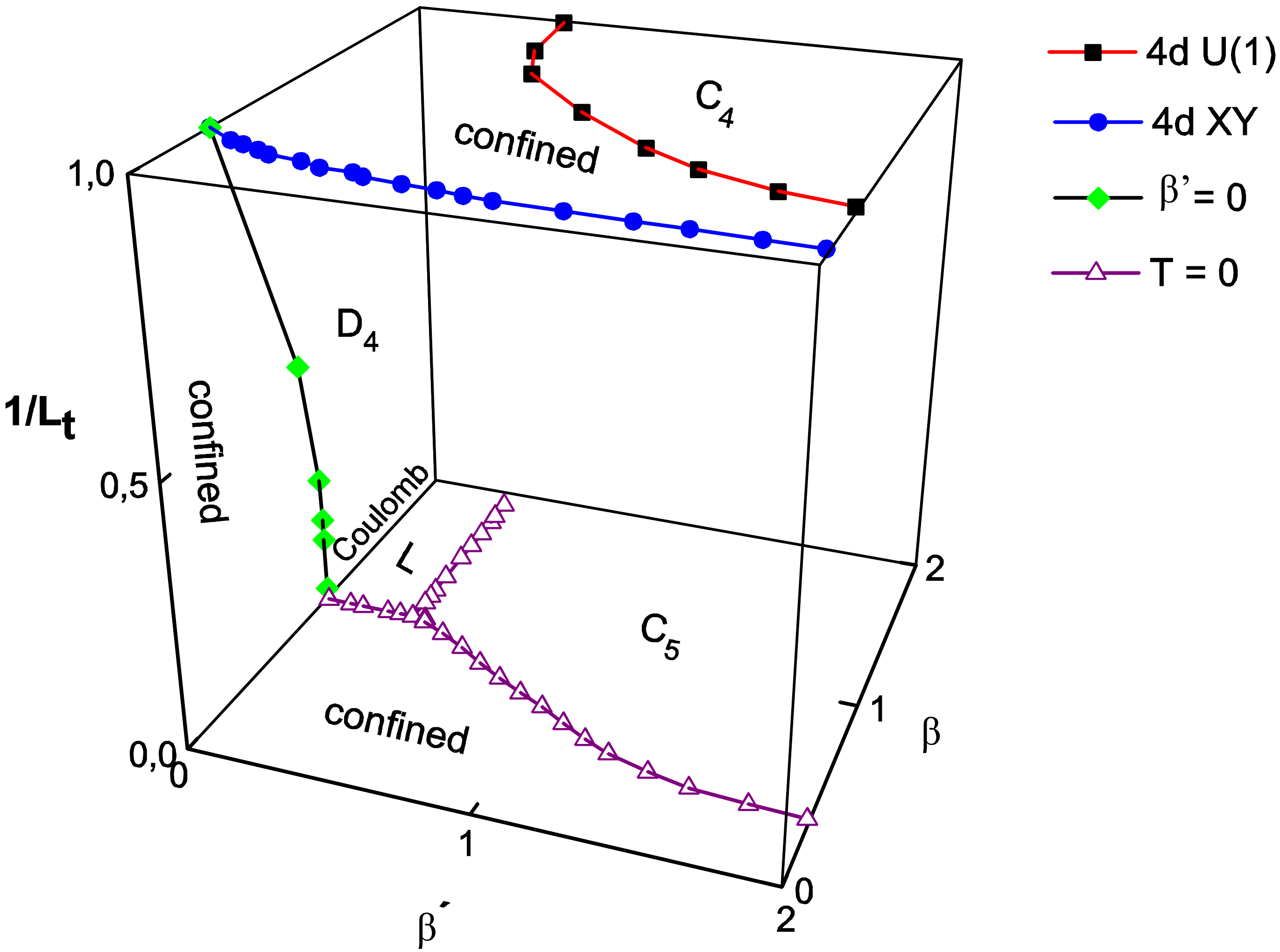
Infinite Temperature, ***Dimensional Reduction***

$$L_t = 1$$

Limiting Planes in the Three Dimensional Phase Diagram:

- $\beta' = 0$ [Vettorazzo and de Forcrand, NPB 2004 and PLB 2004]
- $T = 0 \; (L_t = L_s, \; 1/L_t \to 0)$ [P.Dimopoulos, K.F. and S.Vrentzos, Phys.Rev.D 2006]
- $L_t = 1$ [K.F. and S.Vrentzos, Phys.Rev.D 2008]

Three Dimensional Phase Diagram



Order Parameters

- **helicity modulus**: h_S, h_t, h_{S5}, h_{t5}

$$h_S(\beta) = \frac{1}{(L_\mu L_\nu)^2}\{<\sum_P(\beta\cos\theta_{\mu\nu})> - <(\sum_P(\beta\sin\theta_{\mu\nu}))^2>\}$$

$$h_t(\beta) = \frac{1}{(L_\mu L_t)^2}\{<\sum_P(\beta\cos\theta_{\mu t})> - <(\sum_P(\beta\sin\theta_{\mu t}))^2>\}$$

$$h_{S5}(\beta') = \frac{1}{(L_\mu L_5)^2}\{<\sum_{P'}(\beta'\cos\theta_{\mu 5})> - <(\sum_{P'}(\beta'\sin\theta_{\mu 5}))^2>\}$$

$$h_{t5}(\beta') = \frac{1}{(L_t L_5)^2}\{<\sum_{P'}(\beta'\cos\theta_{t5})> - <(\sum_{P'}(\beta'\sin\theta_{t5}))^2>\}$$

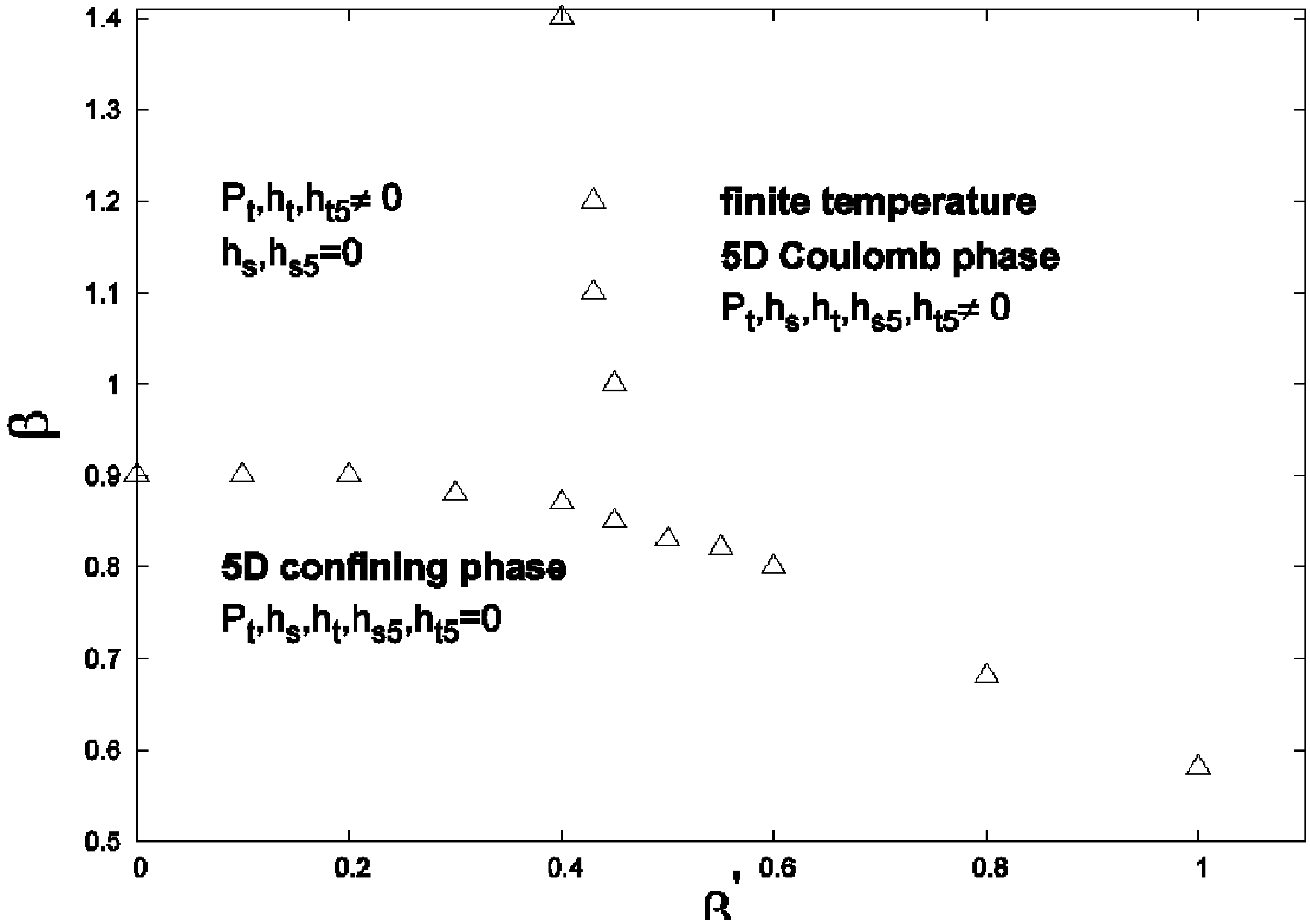
- **Polyakov loop**: $P_t(n_s, n_5) = \prod_{n_t=1}^{L_t} U_t(n_s, n_t, n_5)$, $P_t = \frac{1}{L_S^4}\sum_{n_s, n_5} P_t(n_s, n_5)$

$$<|P_t|> = e^{-L_t F_q}$$

F_q = Free energy of a static charge

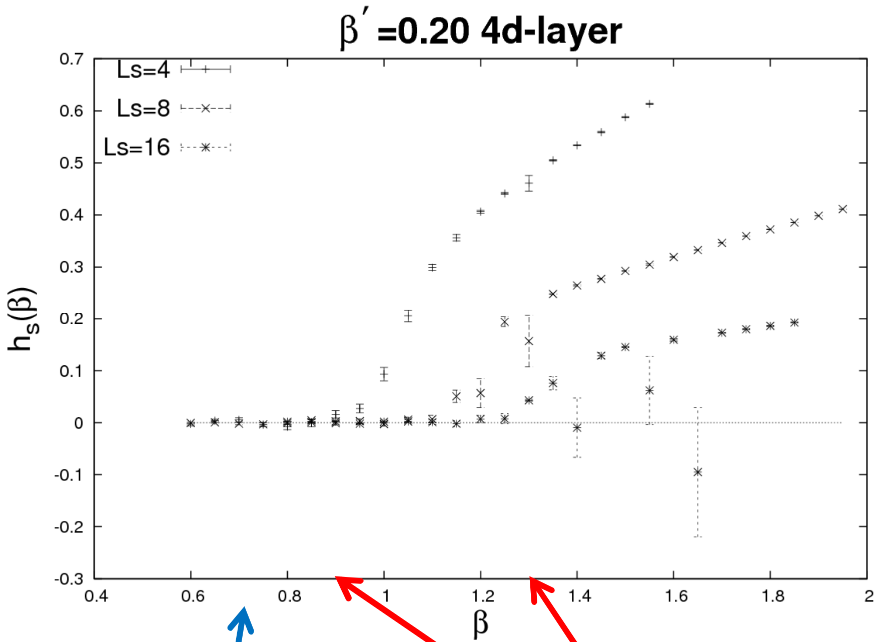
Confining Phase F_q → *Infinite*, $<|P_t|>$ → *0*

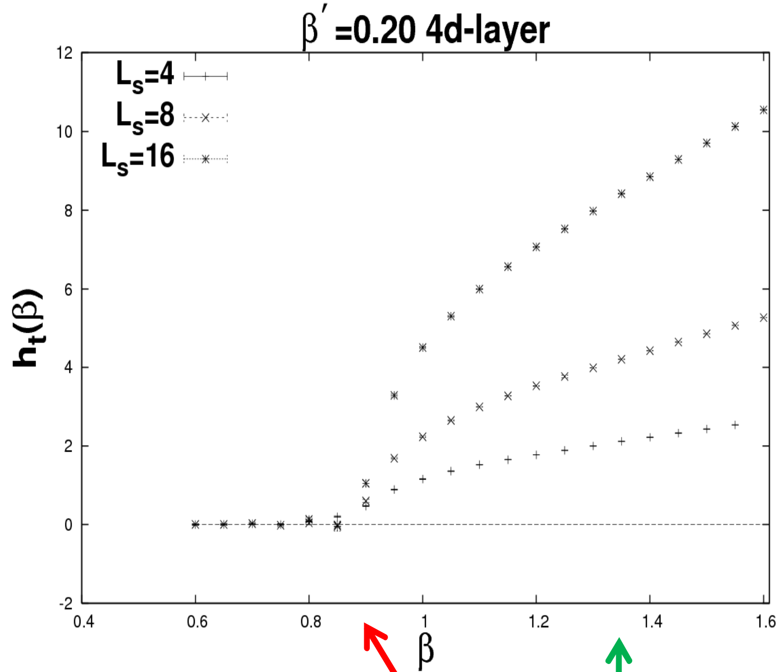
Phase Diagram for $L_t=2$



[K.F. and S.Vrentzos, Phys.Rev.D 2008]

$L_t=2$, *Confining – "Layer" Phase Transition*





$\beta_{PseudoCritical}(L_S) \to \infty$

$when\ L_S \to \infty$

$\beta_{critical} \sim 0.85$

$h_S(\beta)=0 \quad for \quad \beta<\beta_{PC}(L_S)$ Confining Phase

Temporal Coulomb Phase

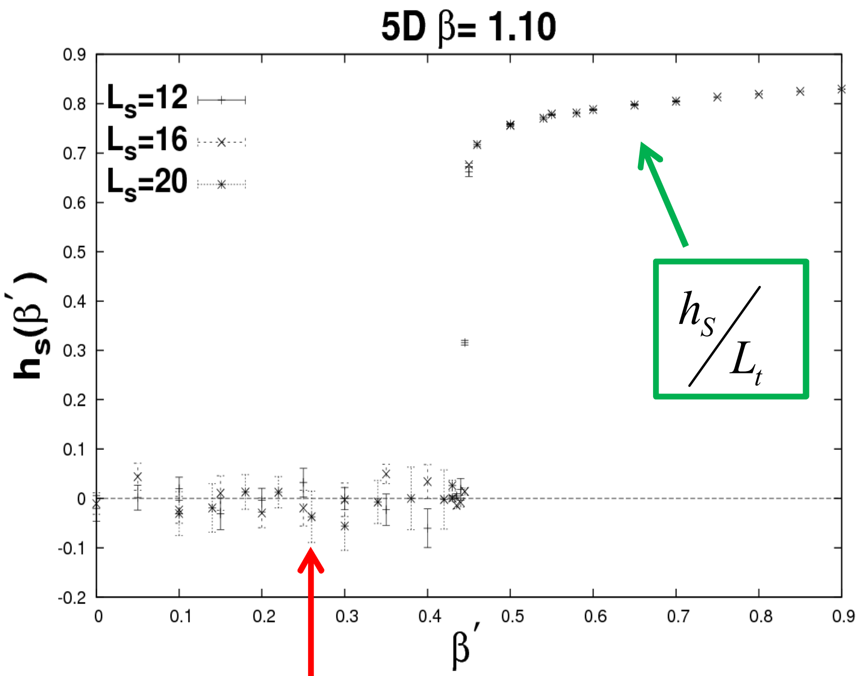
$h_t, h_{t5} \neq 0$

$For \quad \beta > \beta_{PC}(L_S) \quad h_S(\beta) \sim \dfrac{1}{L_S} \to 0 \quad when\ L_S \to \infty,\ h_{S5}(\beta)=0 \quad \forall \beta$

4d space = 3_space dimensions + 1_transverse dimension

"4d Confining Phase" in the thermodynamic limit

L_t=2, "Layer" – 5D Coulomb Phase Transition



$$\beta = 1.1 > \beta critical \simeq 0.85$$

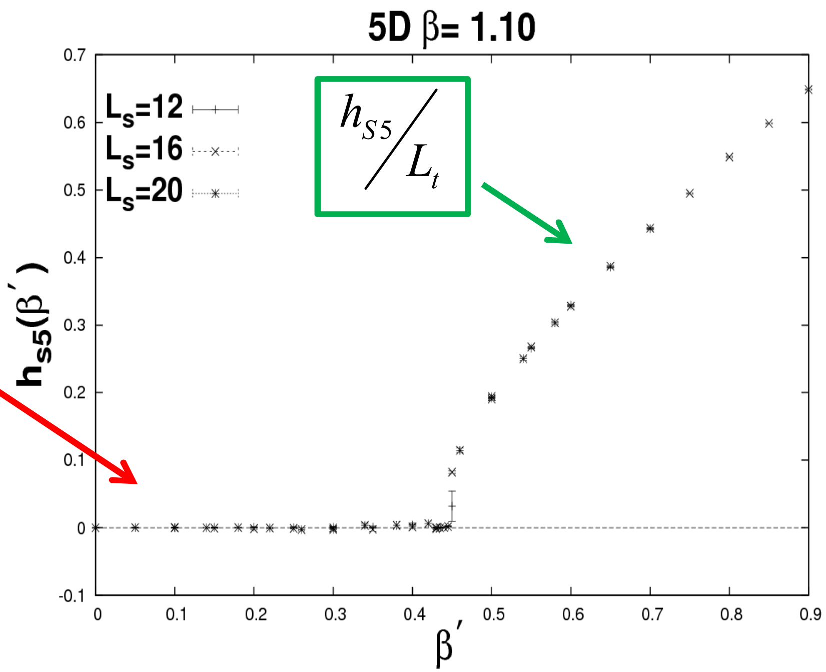
$$\beta' > \beta'_{critical} \approx 0.43$$

Finite Temperature
5D Coulomb Phase

$$h_S, h_{S5} \ are \ zero \ for \ \beta' < \beta'_{critical} \approx 0.43$$

Dimensionally reduced
4d confining phase - temporal Coulomb

$$P_t \neq 0, h_t \neq 0, h_{t5} \neq 0 \ and \ h_S, h_{S5} = 0$$



- The layer phase for zero temperature (with a massless photon on the brane and confinement in the extra *transverse* dimensions) gives its place to a deconfined phase at non-zero temperature.

- In this new phase the three spatial dimensions and the transverse one form a 4d subspace with confining properties, while the temporal direction shows a Coulombic behaviour.

- The critical temperature for the deconfinement is the temperature where the four dimensional Coulomb phase disappears.

- The brane models are well studied mainly in the zero temperature case, but if we imagine that our brane world is a part of the Universe history then a study of the brane models at high temperature is required.

- In this toy model of five dimensional U(1) anisotropic lattice gauge theory we believe that we have all the required essential characteristics, though neglecting the gravity effects.

Remarks & Conclusions

- **(D+1)-dimensional gauge models with anisotropic couplings can reveal a new kind of D-dimensional phase which we call Layer. It is a D-dimensional Coulomb phase accompanied by confinement along the extra dimension.**

- **The necessary condition for the layer phase formation is that the D-dimensional gauge model must already have two distinct phases. Hence the minimum dimensionality is D=4 for the pure U(1) model and D=5 for the pure SU(2) model.**

- **Extra dimensional lattice gauge models with anisotropic couplings can be "inspired" in an extra dimensional space described by the RS metric.**

- **The study of the 5D-Abelian Higgs model with anisotropic couplings shows that a Layer phase exists in the broken phase: we get a set of 4-dimensional subspaces in the Higgs phase which do not comunicate due to confinement along the extra direction.**

- **The 5D anisotropic SU(2)-Higgs model in the adjoint representation shows two main features:**
  - **► The inclusion of the scalar field is responsible for the formation of a 4–dimensional layer (higgs) phase in a model with non-abelian dynamics**
  - **► The confinement along the extra dimension is of the non-abelian type**

- **The existence of the layer phase in the phase diagram of (4+1)D lattice gauge models with anisotropic couplings supports the conjecture of an effectively four-dimensional world embedded in a bulk of extra dimensions.**