# Low temperature properties of holographic matter

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# Introduction

- Gauge-gravity duality provides a tool for studying systems at strong coupling. Has been mainly applied to several problems of interest in nuclear physics. e.g. Quark gluon plasma and Hydrodynamics.

- Recently, a growing interest in applying gauge-gravity duality to condensed matter systems. This approach could be helpful in a qualitative or even quantitative manner, especially in extracting universal quantities. e.g. Hall effect and quantum critical phenomena.

# Introduction

**In light of these developments it is of interest to consider the physics of different holographic models and address questions such as**

- **How do these systems behave at low temperatures?**
- **Is their behaviour dependent on the details of the particular system and how?**
- **Can we make a detailed comparison with what is observed in nature?**

# Introduction

**Preview**:

Consider diverse brane setups and study them at strong coupling using holography. Examine the thermodynamic properties (e.g. specific heat, speed of sound etc) at low temperatures and the response of these systems to small perturbations induced by oscillations. Determine how the results vary from case to case.

Based on work in collaboration with A. Karch and A.Parnachev [0908.3493].

# Outline

- Review of the various brane systems under study
- Specific Heat
- Thermodynamic Sound
- Zero Sound
- Resistivity
- Conclusion and Comments
- Open Questions

# Dp/Dq Systems

Consider a Dp/Dq brane system where the Dq brane extends along $1 \leq d_s \leq q$ spatial directions parallel to Nc Dp branes with $p < 5$.

- Weak coupling: p+1 SYM theory interacting with low energy degrees of freedom from p-q strings.

  These generically involve fermions (and bosons) localized on the $d_s + 1$ dimensional defect.

  Stability: 4 or 6 ND directions.

- Strong Coupling: SYM $\leftrightarrow$ Gravity and Flavor fields $\leftrightarrow$ Induced brane geometry (DBI action)

# Dp/Dq systems

The near horizon solution of Dp branes at finite temperature is

$$
\begin{aligned}
ds^2 &= H^{-1/2}(-f dt^2 + dx_p^2) + H^{1/2}\left(\frac{du^2}{f} + u^2 d\Omega_{8-p}^2\right) \\
e^{\Phi} &= H^{\frac{3-p}{4}}, \qquad C_{01\ldots p} = H^{-1} \\
H(u) &= (L/u)^{7-p}, \qquad f(u) = 1 - (u_h/u)^{7-p}
\end{aligned}
$$

The temperature $T$ is given in terms of $u_h$ via

$$
u_h = \left(\frac{4\pi}{7-p}\right)^{2/(5-p)} T^{2/(5-p)}
$$

# Dp/Dq Systems

Study the system at finite temperature and chemical potential.

Finite temperature $\Rightarrow$ Black Hole Dp-background

Chemical potential for the flavors $\Rightarrow$ Temporal component of the gauge field along the Dq brane worldvolume.

$$\mu \longrightarrow A_0$$

Unless an additional source is introduced, only the configuration with the branes falling into the horizon of the background geometry can support a non-trivial gauge field and chemical potential (black hole embedding).

# Dp/Dq Systems

For massless flavors the brane profile is trivial $\partial_u X = 0$. The only field is turned on along the brane is $A_0(u)$.

$$S_{DBI} = -\mathcal{N} \int du\, u^{\nu} \sqrt{1 - A_0'^2}\,,$$

where

$$\nu = \frac{(p-7)(q-2d_s-4+p)}{4} + q - d_s - 1\,,$$

The electric field on the brane satisfies

$$A_0' = \frac{\tilde{d}}{\sqrt{u^{2\nu} + \tilde{d}^2}}$$

with $\tilde{d}$ proportional to the baryon number density $d$ as $d = (2\pi\alpha'\mathcal{N})\tilde{d}$.

# Dp/Dq Systems

The grand canonical potential $\Xi$ is given by $\Xi = -TS_{DBI}$ Evaluated on the specific solution for $A_0(u)$ along with the chemical potential $\mu$

$$\mu = \int_{u_h}^{\infty} A_t' = \left(\frac{\widetilde{d}}{\gamma}\right)^{\frac{1}{\nu}} - u_h + \mathcal{O}\left[u_h^{2\nu+1}\right] \Rightarrow \left(\frac{\widetilde{d}}{\gamma}\right)^{\frac{1}{\nu}} \simeq (\mu + u_h)$$

$$\Xi = \mathcal{N} \int_{u_h}^{\infty} \mathcal{L} = -\frac{\mathcal{N}}{1+\nu} \gamma \left(\frac{\widetilde{d}}{\gamma}\right)^{1+\frac{1}{\nu}} + \mathcal{O}\left[u_h^{2\nu+1}\right] \Rightarrow$$

$$\Xi \simeq -\frac{\mathcal{N}}{1+\nu} \widetilde{d}\mu - \mathcal{N}\widetilde{d}u_h.$$

The temperature dependent piece of the grand canonical potential is equal to the change of mass of a string in this background.

# Specific Heat

- Entropy density

$$s_{fluid} = -\left.\frac{\partial \Xi}{\partial T}\right|_{\mu} = \mathcal{N}\,\tilde{d}\,\frac{\partial u_h}{\partial T} = d\,\frac{\partial \Delta m}{\partial T}$$

**Note**: At finite density or chemical potential, the entropy vanishes at zero temperature except for the conformal case $p = 3$. No degeneracy.

- Specific Heat

$$c_V = T\left.\frac{\partial S}{\partial T}\right|_{d} = d\,T\,\frac{\partial^2(\Delta m)}{\partial T^2} \sim T^{\frac{2}{5-p}}$$

For $p = 4$ the specific heat varies linearly with temperature (Landau Fermi Liquid Theory). These results remain valid for massive embeddings.

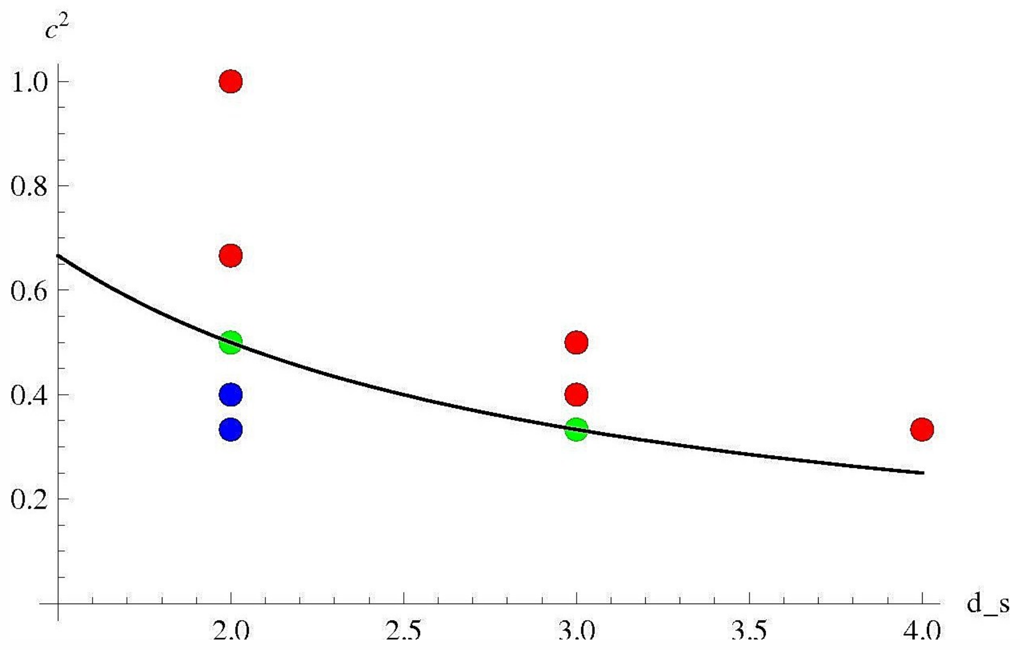
# Thermodynamic Sound

**Another interesting property of the thermodynamics is the speed of sound. Thermodynamic sound at low temperatures can be deduced from the equation of state.**

$$\left.\begin{aligned} &P = \frac{\mathcal{N}}{1+\nu}\gamma\mu^{\nu+1} \\ &\epsilon = \nu P \end{aligned}\right\} \Rightarrow \quad c_s^2 = \frac{\partial \epsilon}{\partial P} = 1/\nu$$

**For $p = 3$ we recover the conformal value $c_s^2 = \frac{1}{3}$. Unlike the specific heat the speed of thermodynamic sound depends on all integers $p, q, d_s$ characterizing the embedding.**

# Thermodynamic Sound




**Speech of sound squared $c^2$, for all Dp/Dq systems with p=2, 3 or 4 and 4 or 6 ND directions, as a function of $d_s$, the number of spatial dimensions. Color distinguishes p=4 (red), p=3 (green) and p=2 (blue).**

# Zero Sound

Another property of several Dp/Dq systems at low temperatures, is the existence of a mode with dispersion relation

$$\omega = u_0 q$$

This is a sound mode propagating in the collisionless regime as the characteristic zero sound of Landau Fermi Liquids. It appears as a pole in the density-density correlation function [Karch, Son, Starinets][M.K, Parnachev].

Furthermore, the speed of this "zero sound" is exactly equal to the speed of thermodynamic sound. In Fermi liquid theory this can be regarded as an indication of strong quasiparticle interactions.

# Conductivity

Electrical conductivity was computed for all Dp/Dq systems [Karch-O'Bannon]. For metrics of the general form

$$ds^2 = -g_{tt}dt^2 + g_{xx}dx^2 + g_{uu}du^2 + g_{SS}d\Omega^2_{k=(q-d_s-1)}$$

it is given by

$$\sigma_{full} = \sqrt{\sigma_0^2 + \sigma^2}$$

The density dependent part of the conductivity is

$$\sigma = d(2\pi l_s^2)g_{xx}^{-1}$$

with $g_{xx}$ evaluated at $u_*$ such that $g_{tt}g_{xx} = (2\pi l_s^2)^2 E^2$

$$u_* = u_h \left[ 1 + (2\pi l_s^2)^2 \frac{E^2}{u_h^{7-p}} \right]^{\frac{1}{7-p}}$$

# Resistivity

The density dependent resistivity is completely independent of the probe.

$$\frac{1}{\sigma} \equiv \rho = \lambda^{\frac{1}{5-p}} u_h^{\frac{7-p}{2}} \frac{\sqrt{1 + \frac{E^2}{u_h^{7-p}}}}{d}$$

It is easily related to the drag force experienced by a string in the Dp background [Gubser, Herzog, Karch, Kozcac, Yaffe]. When $F_{drag} = E$ quarks move with the steady state velocity

$$v_{steady} = \frac{E}{\lambda^{\frac{1}{5-p}} u_h^{\frac{7-p}{2}} \sqrt{\frac{E^2}{u_h^{7-p}} + 1}}$$

The current produced by a density $d$ of quarks $j_x = d v_{steady}$ results in the resistivity above.

# Resistivity

The density independent resistivity is however dependent on the details of the Dq probe.

$$\sigma_0 \simeq e^{-\Phi} g_{xx}^{(d_s-2)/2} g_{SS}^{k/2}$$

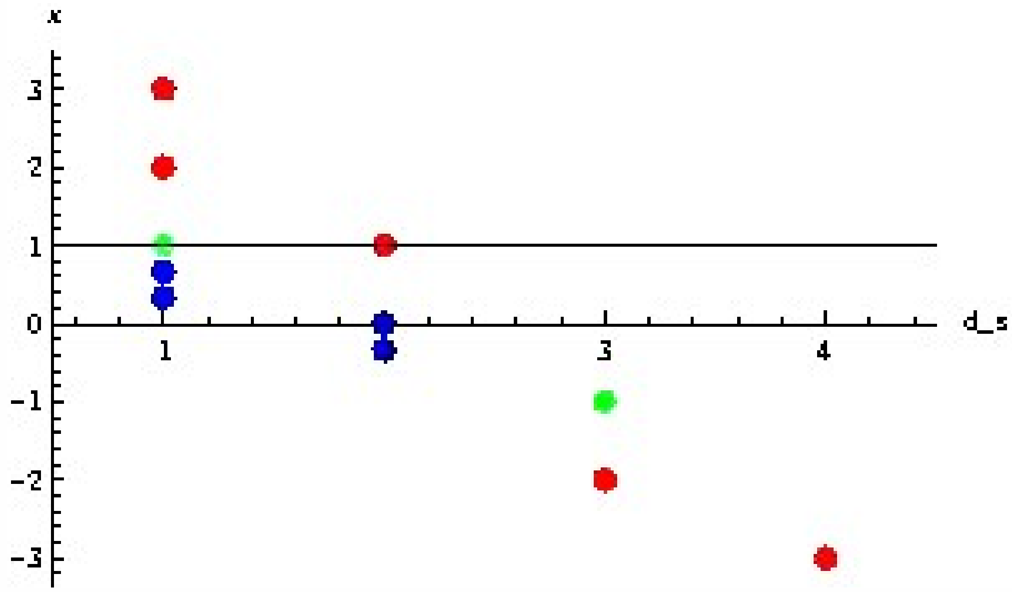
evaluated at $u^* \simeq u_h + \mathcal{O}\left(E^2\right)$.

The leading behavior at low temperatures is

$$\rho_0 \simeq T^x \quad x = -\frac{2}{5-p}\left[\frac{(p-7)(q-2d_s-2+p)}{4} + q - d_s - 1\right]$$

Resistivity linear in temperature only for two systems: $p = 3,\ d_s = 1$ and $p = q = 4,\ d_s = 2$.

# Resistivity




Resistivity scaling factor $x$, for all Dp/Dq systems with p=2, 3 or 4 and 4 or 6 ND directions, as a function of $d_s$, the number of spatial dimensions. Color distinguishes p=4 (red), p=3 (green) and p=2 (blue).

# Conclusions-Summary

- Several low temperature properties of the Dp/Dq systems are insensitive to the details of the embedding of the probe. Such are, the specific heat and the density dependent piece of the conductivity.
- The manifestly $q, d_s$ independent results for the specific heat and the conductivity can be understood from the dynamics of an external string in the background.
- The speed of thermodynamic sound and the density independent part of the resistivity exhibit dependence on the characteristics of the embedding.
- Zero sound propagation with a speed equal to that of thermodynamic sound.

# Open Questions

- Can we find a phenomenological theory consistent with the data?
- Do these results predict new types of quantum liquids that could be found in Nature?
- Is a fermionic quasiparticle description valid?

  Compute the Lorentz number

$$L_0 = \lim_{T \to 0} \frac{\kappa}{\sigma T}$$

# Open Questions

- Employ a similar analysis to understand the physics of Dq brane probes in the ABJM dual.
- Explore the implications of vanishing entropy at zero temperature and fixed density.
- Interpolation between collisionless and hydrodynamic regime.
- Further investigate the $p = q = 4$, $d_s = 2$ case (strange matter?).