Magnetic susceptibility of quark condensate via holography

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This talk is devoted to the calculation of magnetic susceptibility of quark condensate in the framework of the AdS/QCD model. This model uses the methods of AdS/CFT correspondence in order to describe the dynamics of QCD. The calculation concerned is a good test of the model and demonstrates its potential power.

References

The talk is based on the paper by A.K. and A.Gorsky:

arXiv:0902.1832 [hep-ph]

The model is introduced by J. Erlich et al. in

arXiv:hep-ph/0501128

The notation is used according to A.K.

arXiv:0801.4215 [hep-th]

The AdS/CFT conjecture

N=4 SYM on $Mink_4$	$\leftrightarrow$	**type IIB string theory** on $AdS_5 \times S_5$
global symmetries: $SO(2,4) \times SU(4)$	$\leftrightarrow$	isometries of the space: $SO(2,4) \times SO(6)$
coupling constant: $\lambda' = N_c g_{ym}^2$	$=$	string tension: $\frac{R^4}{4\pi\alpha'^2}$
sources of operators: $J_i(x_\mu)$	$=$	boundary values of the fields: $\Phi_i(x_\mu, 0)$
generation functional of correlators: $Z[J_i(x_\mu)]$	$=$	classical action: $iS_{str}[\Phi_{i\ cl}(z, x_\mu)]$

Space geometry in AdS/QCD

N=4 SYM	$\rightarrow$	**QCD**
R-symmetry breaking:		
S_5	$\rightarrow$	$\varnothing$
absense of conformality :		
AdS_5	$\rightarrow$	AdS_5 with cut off at $z = z_m$

Field content of the model

$$\left(\frac{2}{z}\right) X^{\alpha\beta} \leftrightarrow \bar{q}_R^{\alpha} q_L^{\beta} \qquad \text{scalar quark current}$$

$$A^a_{L\mu} \leftrightarrow \bar{q}_L \gamma^\mu t^a q_L \qquad \text{left quark current}$$

$$A^a_{R\mu} \leftrightarrow \bar{q}_R \gamma^\mu t^a q_R \qquad \text{right quark current}$$

$$\Downarrow$$

$$V^a_\mu \leftrightarrow \bar{q} \gamma^\mu t^a q \qquad \text{vector current}$$

$$A^a_\mu \leftrightarrow \bar{q} \gamma^\mu \gamma^5 t^a q \qquad \text{axial-vector current}$$

Action

$$S_5 = \int^{z_m} d^4x \; dz \sqrt{g} \, Tr \left\{ \Lambda^2 \left(|DX|^2 + \frac{3}{R^2} |X|^2 \right) - \frac{1}{4 g_5^2} (F_L^2 + F_R^2) \right\}$$

$$D_\mu X = \partial_\mu X - \imath L_\mu X + \imath X R_\mu$$

Parameters

The 5D coupling constant

$$\frac{g_5^2}{R^2} = \frac{12\pi^2}{N_c} \qquad \text{fixed by the 2-point function of vector currents}$$

The position of the IR boundary

$$z_m = \frac{1}{323 Mev} \qquad \text{related to the } \rho\text{-meson mass}$$

The X field has a vacuum profile

$$X_0(z) = \frac{1}{2} M z + \frac{1}{2} \Sigma z^3 .$$

M is the quark mass matrix, and Σ is proportional to $\langle \bar{q} q \rangle$

(the coefficient Λ is fixed by scalar 2-point function)

Magnetic susceptibility of the quark condensate

We consider the problem of the derivation of the magnetic susceptibility of the quark condensate, defined as

$$\langle \bar{q}\sigma_{\mu\nu}q\rangle_F = \chi\langle \bar{q}q\rangle F_{\mu\nu}.$$

Operator Product Expansion

- Consider the correlator

$$T_{\mu\gamma\nu} = -\int d^4x d^4y \ e^{iqx-iky} \langle 0| T\{j_\mu(x)\tilde{j}_\gamma(y) j^5_\nu(0)\} |0\rangle$$

(Vainstein: hep-ph/0212231)

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- Multiply it by the polarization of soft photon

$$T_{\mu\nu} = e^\gamma(k) T_{\mu\gamma\nu} = i \int d^4x \; e^{iqx-iky} \langle 0 | T\{j_\mu(x) j^5_\nu(0)\} | \gamma(k) \rangle$$

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- Perform the OPE

$$\langle 0| T\{j_\mu(x) j^5_\nu(0)\}|0\rangle_F \sim \frac{N_C}{2\pi^2}\tilde{F}_{\mu\nu} + \frac{4m_f\langle \bar{q}\gamma^5\sigma_{\mu\nu}q\rangle_F}{Q^2} + \dots$$

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- Extract the susceptibility

$$-i\langle \bar{q}\gamma^5\sigma_{\mu\nu}q\rangle_F = \epsilon_{\mu\nu\alpha\beta}\langle \bar{q}\sigma_{\alpha\beta}q\rangle_F = \chi\langle \bar{q}q\rangle \tilde{F}_{\mu\nu}$$

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Calculation of the correlator in AdS/QCD

- $$\langle 0|A_\nu V_\mu \tilde{V}_\gamma|0\rangle = \frac{\delta^3}{\delta\hat{A}_\nu \delta\hat{V}_\mu \delta\hat{\tilde{V}}_\gamma} S_{cl}(\hat{A}\hat{V}\hat{\tilde{V}}),$$

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- $$S_5 = \int \sqrt{g}\, Tr\left\{\Lambda^2\left(|DX|^2 + \frac{3}{R^2}|X|^2\right) - \frac{1}{4g_5^2}(F_L^2 + F_R^2)\right\} + \Delta S$$

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- $$\Delta S = S_{CS}(A_L) - S_{CS}(A_R),$$

$$S_{CS}(A) = \frac{N_C}{24\pi^2}\int Tr\left(AF^2 - \frac{1}{2}A^3F + \frac{1}{10}A^5\right).$$

Classical solution

For the 3-point correlator one obtains:

$$\langle A_{\|\mu}(k_1) V_\nu(k_2) \tilde{V}_\rho(k_3) \rangle =$$
$$= -\frac{N_C}{\pi^2} \langle T_A T_V T_{\tilde{V}} \rangle \, \delta^4(k_1+k_2+k_3) \epsilon^{\|}_{\mu\nu\rho\sigma} \int dz \, (ik_{2\sigma}) \varphi v \dot{\tilde{v}} - (ik_{3\sigma}) \varphi \dot{v} \tilde{v}$$

Classical solutions are:

$$v(Q,z) = Qz \left(K_1(Qz) + \frac{K_0(Qz_m)}{I_0(Qz_m)} I_1(Qz) \right)$$

$$\varphi(z) = F\alpha z[AI_{1/3}(z^3) + BK_{1/3}(z^3)] - 1.075 \; \frac{m\langle \bar{q}q \rangle}{f_\pi^2} z^2$$

in the region $200 Mev \ll Q \ll 1 Gev$

The result

$$\langle A_{\|\mu}(-Q)V_\nu(Q-k_3)\tilde{V}_\rho(k_3)\rangle = \langle T_A T_V T_{\tilde{V}}\rangle\ \epsilon^{\|}_{\mu\nu\rho\sigma}(ik_{3\sigma}) \left[\frac{N_C}{2\pi^2} - 1.075\frac{N_c}{\pi^2}\frac{m\langle\bar{q}q\rangle}{Q^2 f_\pi^2} + O(1/Q^4)\right]$$

In the AdS/QCD

$$\boxed{\chi = -2.15\frac{N_c}{8\pi^2}\frac{1}{f_\pi^2}}$$

In the pion dominance approach

$$\chi = -2\frac{N_c}{8\pi^2}\frac{1}{f_\pi^2}$$

Conclusion

The AdS/QCD model is far from completeness, but usually gives the results, unexpectedely close to the QCD calculations. Our result depends weekly on the position of the boundary. Although the model conserned is the simplest possible, the outcome can be treated as universal.

Equations of motion

$$\left[\partial_z\left(\frac{1}{z}\partial_z V_\mu^a\right)+\frac{q^2}{z}V_\mu^a\right]_\perp=0$$

$$\left[\partial_z\left(\frac{1}{z}\partial_z A_\mu^a\right)+\frac{q^2}{z}A_\mu^a-\frac{R^2g_5^2\Lambda^2v^2}{z^3}A_\mu^a\right]_\perp=0$$

$$\partial_z\left(\frac{1}{z}\partial_z\phi^a\right)+\frac{R^2g_5^2\Lambda^2v^2}{z^3}(\pi^a-\phi^a)=0$$

$$-q^2\partial_z\phi^a+\frac{R^2g_5^2\Lambda^2v^2}{z^2}\partial_z\pi^a=0$$

Chiral symmetry breaking

The classical solution for $X^{\alpha\beta}$ has the form

$$X_0(z) = \frac{1}{2}Mz + \frac{1}{2}\Sigma z^3.$$

By the AdS/CFT conjecture, one relates M to the quark mass matrix, and Σ to the VEV of operator $\langle \bar{q}q \rangle$, i.e. quark condensates. We choose normalization such as $M = m\mathbf{1}; \Sigma = \sigma\mathbf{1}$, assuming the equality of quark masses.

It is convenient to decompose:

$$X = X_0 e^{\imath 2\pi^a (t^a)} = \mathbf{1}\frac{v(z)}{2}e^{\imath 2\pi^a t^a} \qquad v(z) = mz + \sigma z^3$$

One can see, that in the quadratic order X interacts only with axial field $2A = L - R$ and not with vector one ($2V = R + L$). That means X breaks chiral symmetry.