Phase Structure of Defect Theories

Paolo Benincasa

CPT & Mathematical Science Dept, Durham

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P.B. - to appear

Outline

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- No complete picture for strongly coupled gauge theories

$$\downarrow$$

QCD/QGP investigated mainly via lattice simulation

- Strongly correlated systems for condensed matter

$$\downarrow$$

Quantum critical points
Superconductors
Superfluids
Quantum liquids
Quantum Hall Effect
Graphene

So far...

Intersecting brane models studied for:

Holographic duals to large-N QCD
- Kruczenski, Mateos, Myers, Winters
- Sakai, Sugimoto
- Burrington, Sonneschein

$\sim$ Holographic dual to CFL: Chen, Hashimoto, Matsuura

Models of Superconductors
- Ammon, Erdmenger, Kaminski, Kerner
- Peeters, Powell, Zamaklar

Quantum Hall Effect
- Myers, Wapler
- Davis, Kraus, Shah
- Fujita, Li, Ryu, Takayanagi

Holographic Quantum Liquids: Karch, Son, Starinets

Here...

Analytic study of the phase structure for *all* BPS intersections

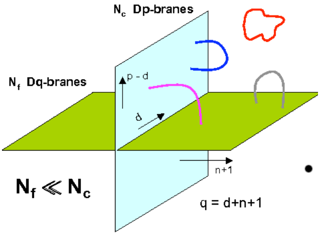
Focus on the plane (T, μ)

- First order phase transition $\longrightarrow$ QCD (?)
- Second order phase transition $\longrightarrow$ condensed matter
- Third order phase transition (less common)
  - Gross-Witten model ($\rightarrow$ also large-N)

Outline

Gauge/Gravity Correspondence

Introduction of flavour degrees of freedom [Karch, Katz]



Flavour degrees of freedom

BPS intersections $\longrightarrow$ Dp/D$(p+4-2k)$ branes

	0	1	...	p-3	p-2	p-1	p	p+1	p+2	p+3	p+4	...	8	9
Dp	X	X	...	X	X	X	X					...		
$k=0$	X	X	...	X	X	X	X	X	X	X	X	...		
$k=1$	X	X	...	X	X	X		X	X	X		...		
$k=2$	X	X	...	X	X			X	X			...		

Probe approximation:
M D$(p+4-2k)$-branes in a background generated by N Dp-branes ($M \ll N$).

$$\begin{aligned} ds_{\mathrm{T}}^2 &= d\rho^2 + \rho^2 \left(d\theta^2 + \sin^2\theta d\Omega_{3-k}^2 + \cos^2\theta d\Omega_{4-p+k}^2 \right) \\ &= d\varrho^2 + dy^2 + \varrho^2 d\Omega_{3-k}^2 + y^2 d\Omega_{4-p+k}^2, \end{aligned}$$

$$\downarrow$$

hypermultiplet in the fundamental representation

Two classes of embeddings for the D$(p+4-2k)$-branes

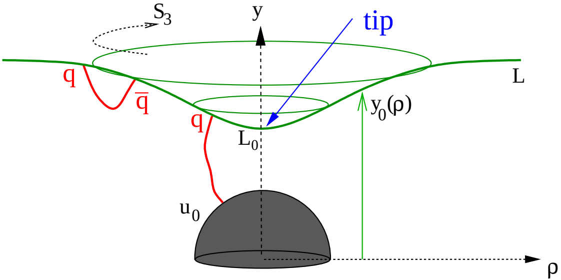
1. linear embedding $x^p \equiv z(\varrho)$ $(k \neq 0)$
   $\longrightarrow$ massless excitations;
   - $x^p \equiv z(\varrho)\,,\ \theta(\varrho) = 0$
     Susy broken, massless excitations;
   - $z(\varrho) = \text{const.},\ \theta(\varrho) = 0$
     Susy system.
2. angular embedding $\theta = \theta(\varrho) \longrightarrow$ massive excitations
   "quark"-mass = mass of the string stretching between the bulk and the boundary

Chemical potential $\longrightarrow$ non-trivial world-volume gauge field

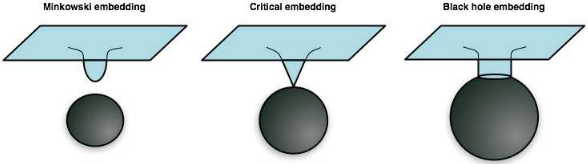
$$F_{[2]} = -f'(\varrho)\ dt \wedge d\varrho$$

Finite temperature $\longrightarrow$ Black Dp-brane background

Here: $z(\varrho) = 0$, $y = y(\varrho) \rightarrow$ massive excitations



Temperature axis $\rightarrow$ First order phase transition



Outline

1. Motivations
2. Holographic framework
3. Chemical Potential
4. Holographic Quantum Liquids
5. Conclusion

Chemical potential axis

Two first integral of motions: $f(\varrho) \to c_f$ and $y(\varrho) \to c_y$

Chemical potential & embedding function

$$\mu = \int_{\varrho_{\min}}^{\infty} d\varrho \frac{c_f}{\sqrt{\varrho^{2(3-k)} + c_f^2 - c_y^2}}, \qquad y(\varrho) = \int_{\varrho_{\min}}^{\varrho} d\varrho \frac{c_y}{\sqrt{\varrho^{2(3-k)} + c_f^2 - c_y^2}}$$

$$\varrho_{\min} = \begin{cases} \left(c_y^2 - c_f^2\right)^{\frac{1}{2(3-k)}} & \text{for } c_f^2 - c_y^2 < 0 \ \to \ \text{brane/anti-brane phase} \\ 0, & \text{for } c_f^2 - c_y^2 > 0 \ \to \ \text{black-hole crossing phase} \end{cases}$$

Order of the phase transition → Check the grand potential Ω

$$\Omega = -S^{(y)}_{\mathrm{D}(p+4-2k)}\Big|_{\text{ren}} = -\lim_{\Lambda\to\infty}\left[S^{(y)}_{\mathrm{D}(p+4-2k)}\Big|_{\text{on-shell}} + S^{(y)}_{\mathrm{D}(p+4-2k)}\Big|_{\text{ct}}\right]$$

Chemical potential axis

Grand-potential:

$$\Omega = \begin{cases} a_1 \left(\mu^2 - m^2\right)^{\frac{4-k}{2}}, & \mu > m \\ a_2 \left(m^2 - \mu^2\right)^{\frac{4-k}{2}}, & \mu < m \end{cases} \Rightarrow \begin{cases} \left.\frac{\partial \Omega}{\partial \mu}\right|_{\mu \to m} = 0 \\ \left.\frac{\partial^2 \Omega}{\partial \mu^2}\right|_{\mu \to m} \to \infty \end{cases}$$

Second order phase transition $\forall$ systems with $k = 0, 1$

$$k = 2\ ?$$

$$\downarrow$$

Different physical interpretation $\rightarrow$ non-trivial transverse embedding mode does not correspond to a mass-deformation.

Chemical Potential-Temperature plane

Equation of motion for the gauge field and the embedding function:

$$f'(\varrho) = c_f \frac{\mathrm{h}_-}{\mathrm{h}_+^{\frac{5-p}{7-p}}} \frac{\sqrt{1+(y')^2}}{\sqrt{c_f^2 + \varrho^{2(3-k)} \mathrm{h}_+^{4\frac{3-k}{7-p}}}}$$

$$0 = \frac{y''}{1+(y')^2} + \frac{3-k}{\varrho} y' + 2\left(\frac{\hat{\sigma}_\mathrm{h}}{\sigma}\right)^{7-p} \frac{\varrho y' - y}{\sigma^2 \mathrm{h}_- \mathrm{h}_+}\left[3+k-p+(4-k)\left(\frac{\hat{\sigma}_\mathrm{h}}{\sigma}\right)^{7-p}\right] -$$

$$- \frac{c_f^2}{c_f^2 + \varrho^{2(3-k)} \mathrm{h}_+^{4\frac{3-k}{7-p}}}\left[\frac{3-k}{\varrho} y' - 2(3-k)\left(\frac{\hat{\sigma}_\mathrm{h}}{\sigma}\right)^{7-p} \frac{\varrho y' - y}{\sigma^2 \mathrm{h}_+}\right]$$

Analytic approach: small density expansion $c_f \longrightarrow 0$
as for the D3/D7 system [Faulkner, Liu]

Chemical Potential-Temperature plane

Subtle perturbative expansion in $c_f \longrightarrow$ two regions:

1. Region 1: $\varrho \in [\varrho_\Lambda, \infty[$
2. Region 2: $\varrho \longrightarrow \tau = c_f^{-\frac{1}{2-k}} \varrho$, $\tau \in [\tau_h, \Lambda]$

Perturbative soln in the two region and match them in the limit $c_f \longrightarrow 0$, $\varrho_\Lambda \longrightarrow 0$, $\Lambda \longrightarrow \infty$

Scaling $c_f^{\frac{1}{2-k}}$? $\longleftarrow$ from the $T = 0$ case.

Chemical potential:

$$\mu = m(T) + \mathfrak{s}_1(T)\, c_f - \mathfrak{s}_2(T)\, c_f \log c_f + \mathcal{O}\left(c_f^2\right)$$

$$\mathfrak{s}_2(T)\big|_{k=1} = 0$$

Chemical Potential-Temperature plane

Phase transitions

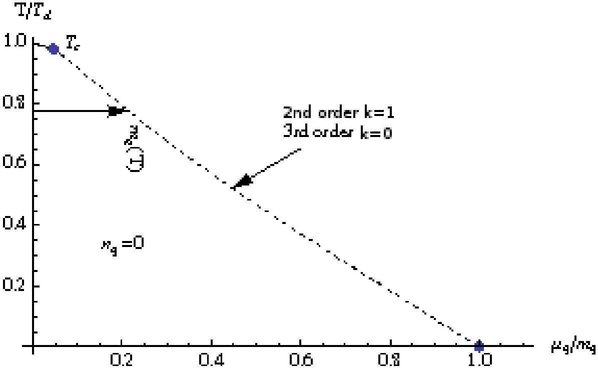
$$\frac{\partial^2\Omega}{\partial\mu^2} \sim \frac{1}{\mathfrak{s}_1 - \mathfrak{s}_2 - \mathfrak{s}_2 \log c_f} \xrightarrow{c_f \to 0} \begin{cases} 0 & k = 0, \\ \frac{1}{\mathfrak{s}_1} & k = 1, \mu > m \end{cases}$$

Second order phase transition for $k = 1$

$$\left.\frac{\partial^3\Omega}{\partial\mu^3}\right|_{k=0} \sim \frac{\mathfrak{s}_2\left(T\right) c_f^{-1}}{\left[\mathfrak{s}_1\left(T\right) - \mathfrak{s}_2\left(T\right) - \mathfrak{s}_2\left(T\right)\log c_f\right]^2} \longrightarrow \infty$$

Third order phase transition for $k = 0$

Phase Diagram



Outline

Massless Hypermultiplet: Holographic Quantum Liquids

Position of the probe branes in the transverse space fixed to wrap the maximal S^{3-k}

Non-trivial profile for $F_{[2]}$

Specific heat at low temperature:

$$c_v \overset{T\to 0}{\sim} T^{2(3-k)}$$

Quantum liquids:

$$c_v \overset{T\to 0}{\sim} \begin{cases} T^{p-k} & \text{Bose liquids} \\ T & \text{Fermi liquids} \\ T^{\alpha(K)} & \text{Luttinger liquids}, \quad \alpha(K) = \begin{cases} \frac{1}{2(K^{-1}-1)} & \frac{\text{repulsive}}{\text{interaction}} \\ 2(K-1) & \frac{\text{attractive}}{\text{interaction}} \end{cases} \end{cases}$$

Massless Hypermultiplet: Holographic Quantum Liquids

- Bose Liquids:

$$p = 6 - k \qquad \longrightarrow \qquad \text{D5/D7}$$

- Fermi Liquids: Never right scaling
- Tomonaga-Luttinger liquids: $(p - k = 1)$

$$K = \begin{cases} 4\frac{3-k}{13-4k} & \frac{\text{repulsive}}{\text{interaction}} \\ 4 - k & \frac{\text{attractive}}{\text{interaction}} \end{cases}$$

- Zero-sound mode:

$$\omega = \pm\frac{q}{\sqrt{3-k}} - i\frac{q^2}{2(3-k)\mu_0} + \mathcal{O}\left(q^3\right)$$

Outline

Conclusion

- Phase diagram for codimension-k defect theories with massive hypermultiplet
- First order phase transition on the temperature axis
  Second order phase transition on the chemical potential axis
- Transition line in the $(\mu,\ T)$:
  1. $k = 0$: third-order phase transition
  2. $k = 1$: second-order phase transition
- QCD would like a first order phase transition
  However:
  1. $\exists$ (few) examples of 3rd order phase transition:
     Gross-Witten model is at large N
  2. $\exists$ (several) examples of 2nd order phase transition:
     Codimension-1 systems for condensed matter physics?
     Graphene?

More to do

- D3/D7 system with codimension-1 intersection
  1. charge transport properties studied [Myers, Wapler]
  2. quantum hall plateau transition [Davis, Kraus, Shah]

  However:

  - system is completely unstable with no world-volume flux
  - issue of stability partially addressed only in [Myers, Wapler]: $\exists$ a window of possible stability with a self-dual instanton configuration ansatz for $F_{[2]}$ for some q_7.

  To do:

  - complete analysis of the stability of the system
  - massive hypermultiplet with chemical potential $\longrightarrow$ phase-diagram $(\mu,\ T)$
  - massless hypermultiplet with gauge fields $\longrightarrow$ quantum fluids?