Gravity Duals of Quark-Gluon Plasmas with Flavors

Aldo L. Cotrone
KU Leuven

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with Francesco Bigazzi (ULB), Javier Mas (Santiago U.), Angel Paredes (UB), Alfonso Ramallo (Santiago U.), Javier Tarrio (Santiago U.).

- Holographic description of Quark-Gluon Plasmas.
- Missing effects: e.g. enhancement of jet quenching.

A flavored $\mathcal{N} = 4$ SYM plasma

Flavors beyond quenched approximation

$$=$$

backreaction of D7-branes

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Tricks:

- Smear $N_f \gg 1$ D7 in transverse directions $\rightarrow$ ODE.
  Flavor group is Abelian.

  [Bigazzi-Casero-Cotrone-Kiritsis-Paredes 2005]

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- Solve in series of $\epsilon = \lambda \frac{N_f}{N_c} \longrightarrow$ analytic results (2nd order).
  Note: excellent approximation because $\lambda \frac{N_f}{N_c}$ must be small.
  [Gubser-Herzog-Klebanov-Tseytlin 2001]

A flavored $\mathcal{N}=4$ *SYM plasma*

Action: IIB + DBI + CS

Solution for massless flavors:

$$ds^2 = -\frac{r^2}{R^2}(1-\frac{r_h^4}{r^4})dt^2+\frac{r^2}{R^2}d\vec{x}_3^2+\tilde{S}^8\tilde{F}^2\frac{R^2}{r^2}\frac{dr^2}{(1-\frac{r_h^4}{r^4})}+\tilde{S}^2R^2ds^2_{CP^2}+\tilde{F}^2R^2(d\tau+A_{CP^2})^2$$

$$\begin{aligned} R^4 &= 4\pi g_s N_c \alpha'^2 \\ F_{(5)} &= 16\pi g_s N_c \alpha'^2 (1+*)\varepsilon(S^5) \\ F_{(1)} &= \frac{g_s}{2\pi}N_f\,(d\tau + A_{CP^2}) \end{aligned}$$

T=0 solution in [Benini-Canoura-Cremonesi-Nunez-Ramallo 2006].

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$$\tilde{F} = 1 - \frac{\epsilon}{24}\left(1 + \frac{2r^4 - r_h^4}{6r_*^4 - 3r_h^4}\right)$$

$$\tilde{S} = 1 + \frac{\epsilon}{24}\left(1 - \frac{2r^4 - r_h^4}{6r_*^4 - 3r_h^4}\right)$$

$$\Phi = \Phi_* + \epsilon \log \frac{r}{r_*}$$

A flavored $\mathcal{N}=4$ SYM plasma

$$\tilde{F} = 1 - \frac{\epsilon}{24}\left(1 + \frac{2r^4 - r_h^4}{6r_*^4 - 3r_h^4}\right) + \frac{\epsilon^2}{1152}\left(17 - \frac{94}{9}\frac{2r^4 - r_h^4}{2r_*^4 - r_h^4} + \frac{5}{9}\frac{(2r^4 - r_h^4)^2}{(2r_*^4 - r_h^4)^2} + \right.$$
$$\left. - \frac{8}{9}\frac{r_h^8(r_*^4 - r^4)}{(2r_*^4 - r_h^4)^3} - 48\log\left(\frac{r}{r_*}\right)\right)$$

$$\tilde{S} = 1 + \frac{\epsilon}{24}\left(1 - \frac{2r^4 - r_h^4}{6r_*^4 - 3r_h^4}\right) + \frac{\epsilon^2}{1152}\left(9 - \frac{106}{9}\frac{2r^4 - r_h^4}{2r_*^4 - r_h^4} + \frac{5}{9}\frac{(2r^4 - r_h^4)^2}{(2r_*^4 - r_h^4)^2} + \right.$$
$$\left. - \frac{8}{9}\frac{r_h^8(r_*^4 - r^4)}{(2r_*^4 - r_h^4)^3} + 48\log\left(\frac{r}{r_*}\right)\right)$$

$$\Phi = \Phi_* + \epsilon\log\frac{r}{r_*} + \frac{\epsilon^2}{72}\left(1 - \frac{2r^4 - r_h^4}{2r_*^4 - r_h^4} + 12\log\frac{r}{r_*} + 36\log^2\frac{r}{r_*} + \right.$$
$$\left. + \frac{9}{2}\left(Li_2\left(1 - \frac{r_h^4}{r^4}\right) - Li_2\left(1 - \frac{r_h^4}{r_*^4}\right)\right)\right)$$

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Comments:

- Landau pole Λ_{LP}: work in IR, below UV cutoff $\Lambda_* \sim r_*$: $\Lambda_* \ll \Lambda_{LP}$ if $\epsilon \ll 1$.

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- Full regime of validity:

$$N_c \gg 1 \qquad \lambda \gg 1 \qquad N_f \gg 1 \qquad \epsilon = \frac{\lambda}{8\pi^2}\frac{N_f}{N_c} \ll 1$$

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- Leading corrections if $\lambda^{-3/2} \ll \epsilon^2$.

Thermodynamics:

$$
\begin{aligned}
s &= \frac{1}{2}\pi^2 N_c^2 T^3 \left[1 + \frac{1}{2}\epsilon_h + \frac{7}{24}\epsilon_h^2\right] \\
\varepsilon &= \frac{E_{ADM}}{V_3} = \frac{3}{8}\pi^2 N_c^2 T^4 \left[1 + \frac{1}{2}\epsilon_h + \frac{1}{3}\epsilon_h^2\right] \\
p &= -\frac{F}{V_3} = Ts - \varepsilon = \frac{1}{8}\pi^2 N_c^2 T^4 \left[1 + \frac{1}{2}\epsilon_h + \frac{1}{6}\epsilon_h^2\right]
\end{aligned}
$$

Note: $\frac{d\epsilon_h}{dT} = \frac{\epsilon_h^2}{T} + O(\epsilon_h^3)$.

Check: fi rst order equal to probe computation [Mateos-Myers-Thomson 2007].

Thermodynamics

Breaking of conformality at second order:

$$\varepsilon - 3p = \frac{1}{16}\pi^2 N_c^2 T^4 \epsilon_h^2$$

$$v_s^2 = \frac{1}{3}\left[1 - \frac{1}{6}\epsilon_h^2\right]$$

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Running of central charge at second order and $a = p$.

Hydrodynamics

Shear viscosity [Kovtun-Son-Starinets 2004]:

$$\frac{\eta}{s} = \frac{1}{4\pi}$$

Bulk viscosity if Buchel's bound [Buchel 2007] is valid:

$$\zeta \geq \frac{\pi}{72} N_c^2 T^3 \epsilon_h^2$$

Jet quenching

$$\hat{q} = \frac{\pi^{3/2}\Gamma(\frac{3}{4})}{\Gamma(\frac{5}{4})}\sqrt{\lambda}\,T^3\left[1+\frac{1}{8}(2+\pi)\epsilon_h + 0.5565\,\epsilon_h^2\right]$$

Transport coefficient characterizing probe energy loss.
In string theory calculated by light-like Wilson loop.

[Liu-Rajagopal-Wiedemann 2006]

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- at fixed $N_c = 3, \lambda = 6\pi, T = 300$ MeV

[Liu-Rajagopal-Wiedemann 2006]

$\hat{q}_{\mathcal{N}=4} \sim 4.5$ (Gev)^2/fm $\quad \longrightarrow \quad$ $\hat{q}_{N_f=3} \sim 5.3$ (Gev)^2/fm

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- at fixed $s/T^3 = 17.5, T = 300$ MeV, $\alpha_s = 1/2$ (or λ)

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- at fixed $N_c, \varepsilon, \alpha_{qq}$ [Gubser 2006].

Jet quenching

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- Opposite to perturbative: do phenomenologists underestimate effect of flavors? [Muller-Nagle 2006]
- Intuitive reason?
- $\hat{q} \neq \sqrt{s}$ with flavors. [Liu-Rajagopal-Wiedemann 2006]

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Friction parameter enhanced as well:

[Herzog-Karch-Kovtun-Kozcaz-Yaffe, Gubser 2006]

$$\mu = \frac{\pi}{2M}\sqrt{\lambda}T^2\left[1+\frac{1}{8}(2-\log(1-v^2))\epsilon_h+\right.$$
$$\left.+\frac{1}{384}\left[44-20\log(1-v^2)+9\log^2(1-v^2)+12Li_2(v^2)\right]\epsilon_h^2\right]$$

Final comments

- Same construction for every Sasaki-Einstein space.
- Equations extended to massive flavors.
- Study of probes in these backgrounds.
- Conductivity [Karch-O'Bannon 2007].
- Meson spectra.

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