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A Holographic Approach to QCD

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Introduction

- QCD is by now a confirmed theory for the strong interactions

- Its coupling is strong in the IR and the theory confines color

♠ Solving the theory in this regime has only been approached for some observable only by numerical lattice techniques

♠ A new arena for the theory is at high temperatures where the theory is expected to deconfine.

- An (initially vague) expectation exists since 30 years ago : that heavy ion collisions can be used to study this regime for QCD.

Where we fail so far in theory

• Interactions of hadrons at medium or low energy (little or no help from lattice, partial help from chiral perturbation theory)

• Transport coefficients of the deconfined phase (not computable directly from lattice, crucial for understanding current (RHIC) and future (LHC) heavy-ion data)

• The phase structure and properties of dense matter (not computable from lattice, important for understanding properties of nuclei, and dense nuclear matter, like neutron stars)

• Exploring the strong dynamics of other QCD-like theories, eg.

♠ N=1 super- QCD. (a very interesting toy model and may be relevant for nature)

♠ Technicolor theories

The heavy Ion experiments

- The energy density corresponding to the deconfinement transition is $E_c \sim 1\text{GeV/fm}^3$ (1 fm=10^{-15} m, radius of a proton $\simeq 0.8$ fm)

- The idea is to collide heavy-ion nuclei with the hope that for a short while they will create enough density and thermalize to probe the deconfined phase. A series of experimental efforts was devised:

- The first attempt: 1 Gev/nucleon at LBL's Bevalac. No signals.

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• The second attempt was made in AGS (Brookhaven) by sending 10 GeV/nucleon Si and Au nuclei on a fixed target. That amounted to 5 GeV/nucleon in the collision rest frame that was not enough!

• The next attempt was made at SPS (CERN). S and Pb nuclei were accelerated and collided on fixed target with 17 Gev/nucleon in the collision rest frame. That was still not enough!

• The CERN experiments after 15 years of running (in 2000) saw some hints of collective behavior beyond the known hadronic interactions.

Relativistic Heavy Ion Collider (RHIC)

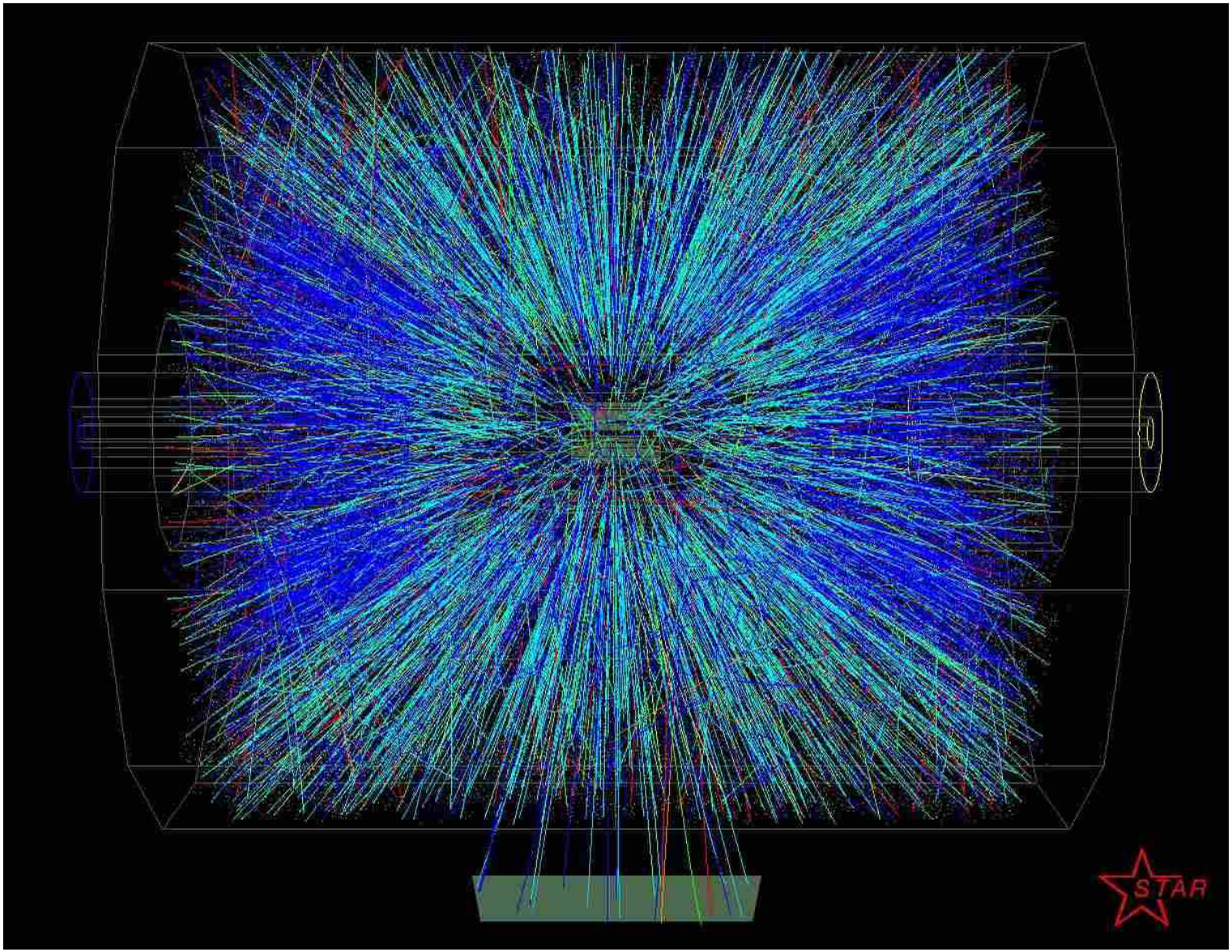
- The major breakthrough came at RHIC: two beams of Au or Cu nuclei colliding at 200 GeV/nucleon at the center-of-mass frame.

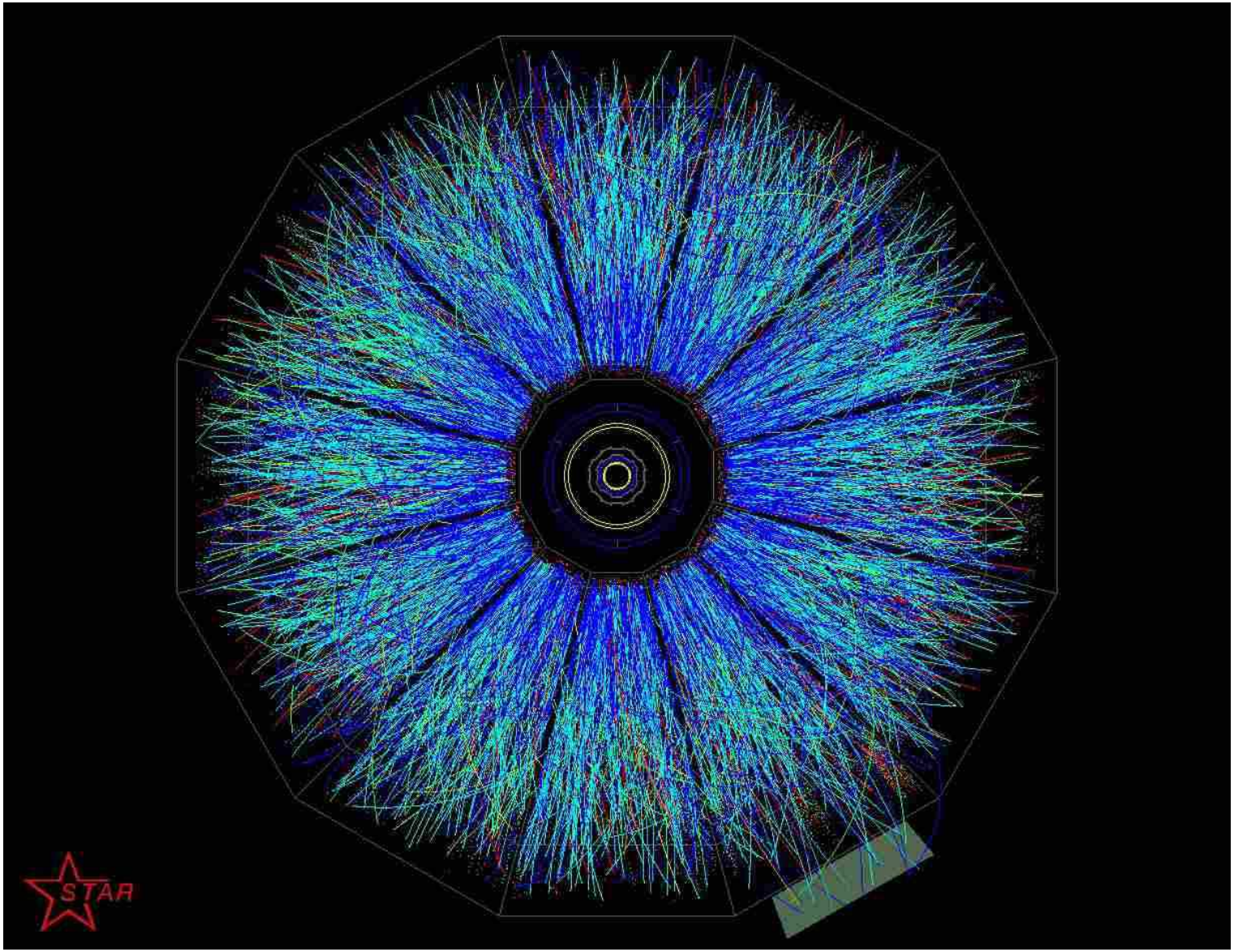
- Four experimental collaborations: BRAHMS, PHENIX, PHOBOS, STAR.

- For every almost central Au+Au collision we get about 7000 particles (fragments, most of them mesons).

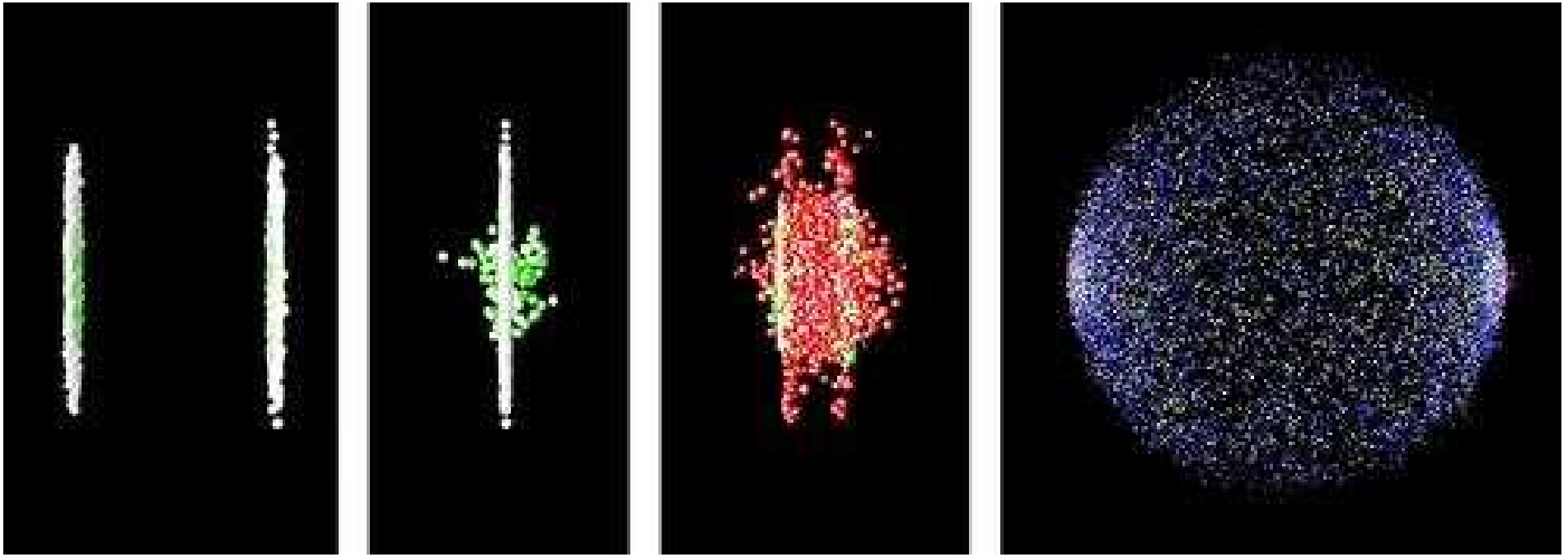
A Holographic Approach to QCD, Elias Kiritsis

RHIC head-on collision

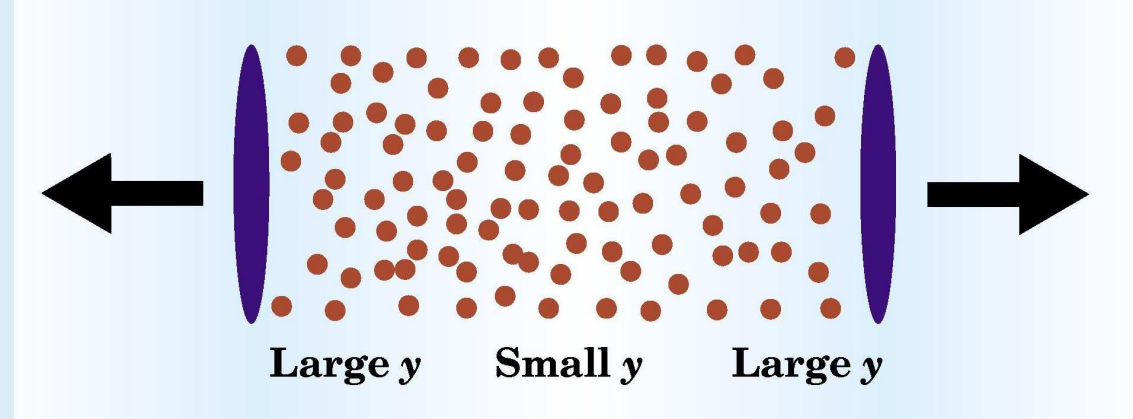



STAR

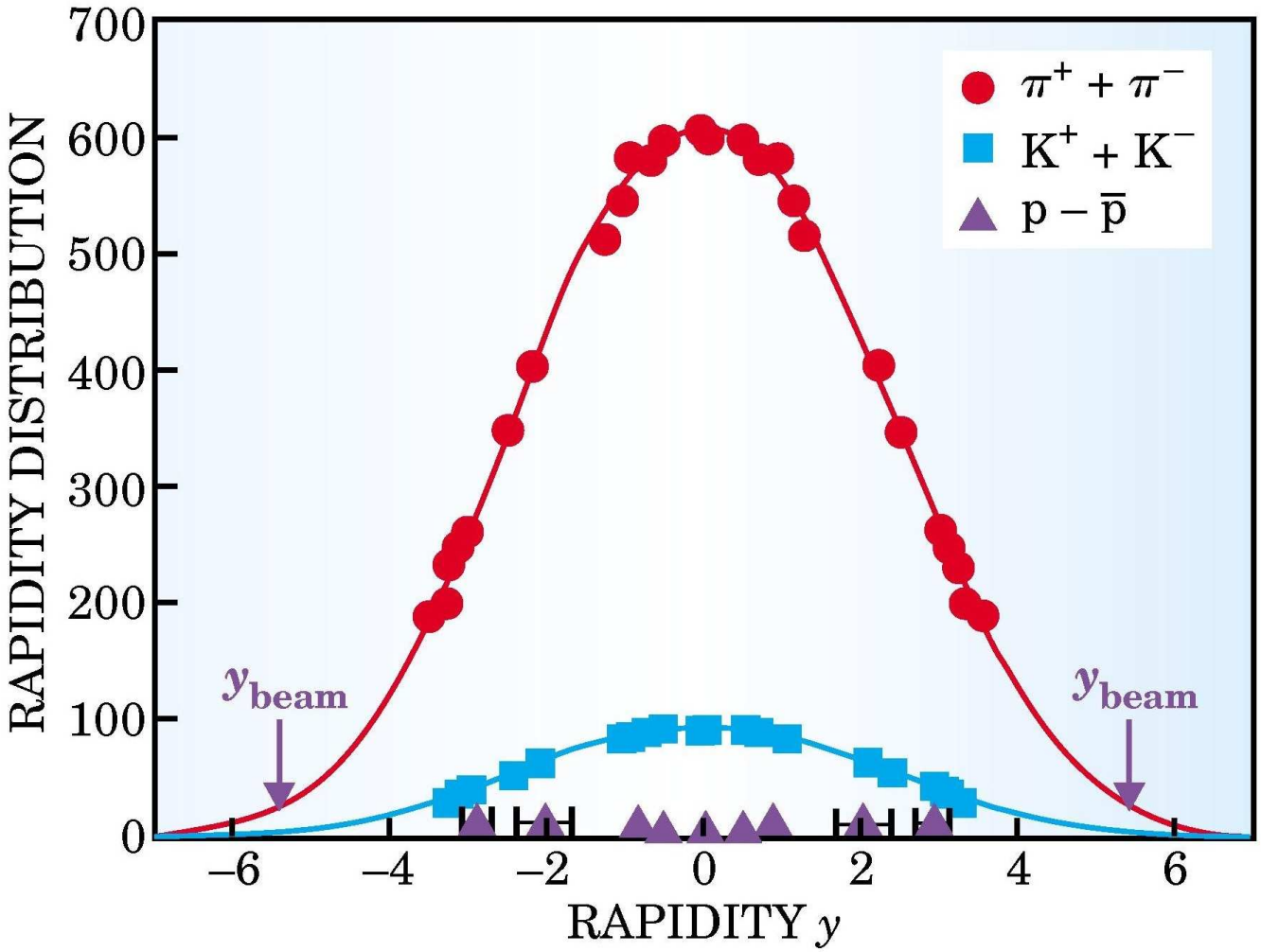
RHIC collision



The mid-rapidity range



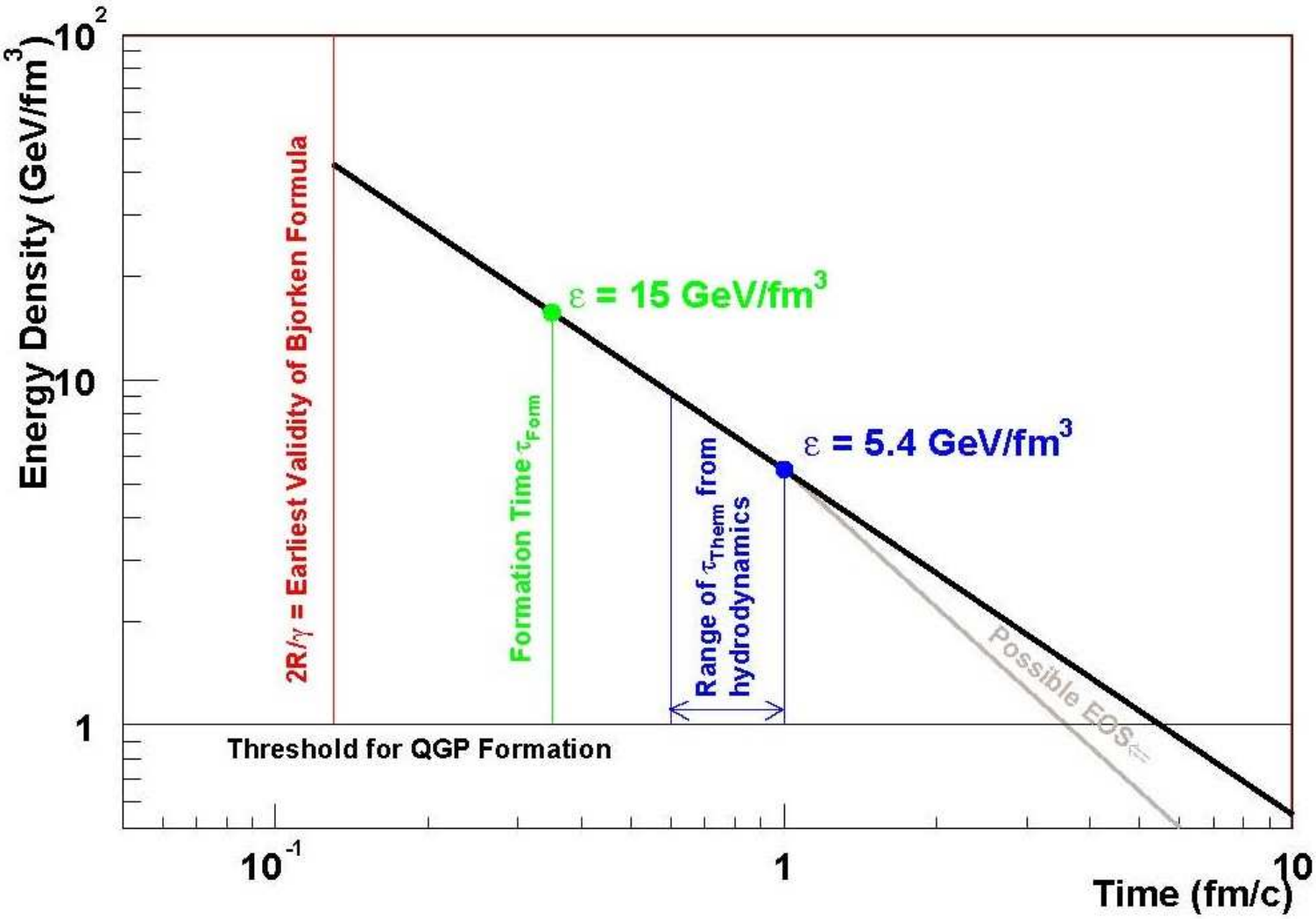
- The crossing time for Au nuclei (with radius 8 fm) is ~ 0.1fm/c $\simeq 3\times 10^{-25}$ seconds.
- The particles with small v_L are produced after 1 fm/c $\simeq 3\times 10^{-24}$ seconds. Those with higher v_L are produced later due to time dilation.
- Use the rapidity variable $y = \frac{1}{2}\log\left[\frac{1+\frac{v_L}{c}}{1-\frac{v_L}{c}}\right]$. Δy is Lorentz invariant.
- The "new matter" (free of fragments) is produced near $y \simeq 0$. This is what we are looking for.
- This can be tested by looking at how much "baryon" number is at mid-rapidity



• Distribution of zero baryon number and net baryon number particles as a function of rapidity (from BRAHMS)

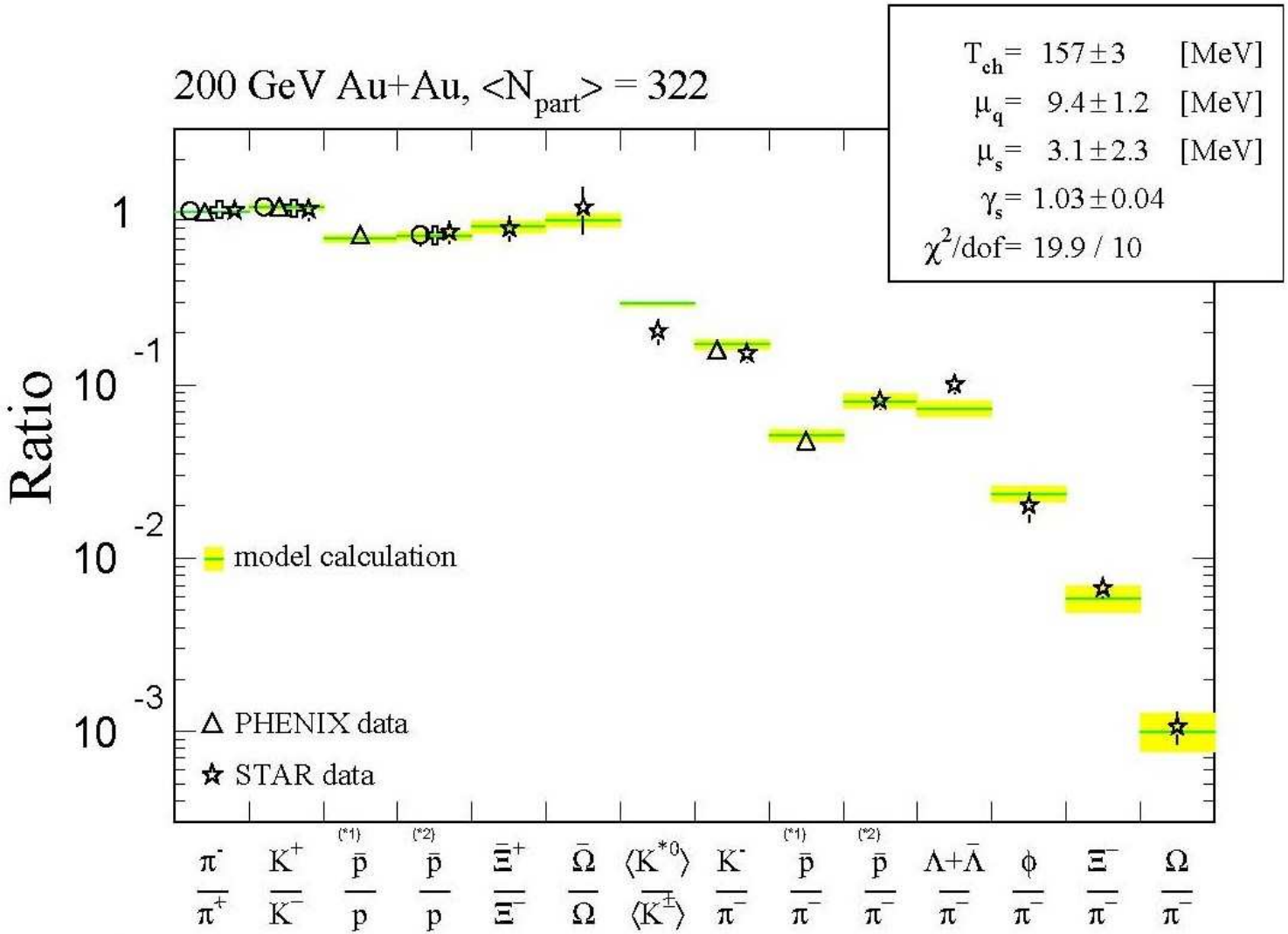
• Each beam nucleon looses 73 ± 6 GeV on the average that goes into creating new particles. Therefore there is 26 TeV worth of energy available for particle production.

Phases of a collision



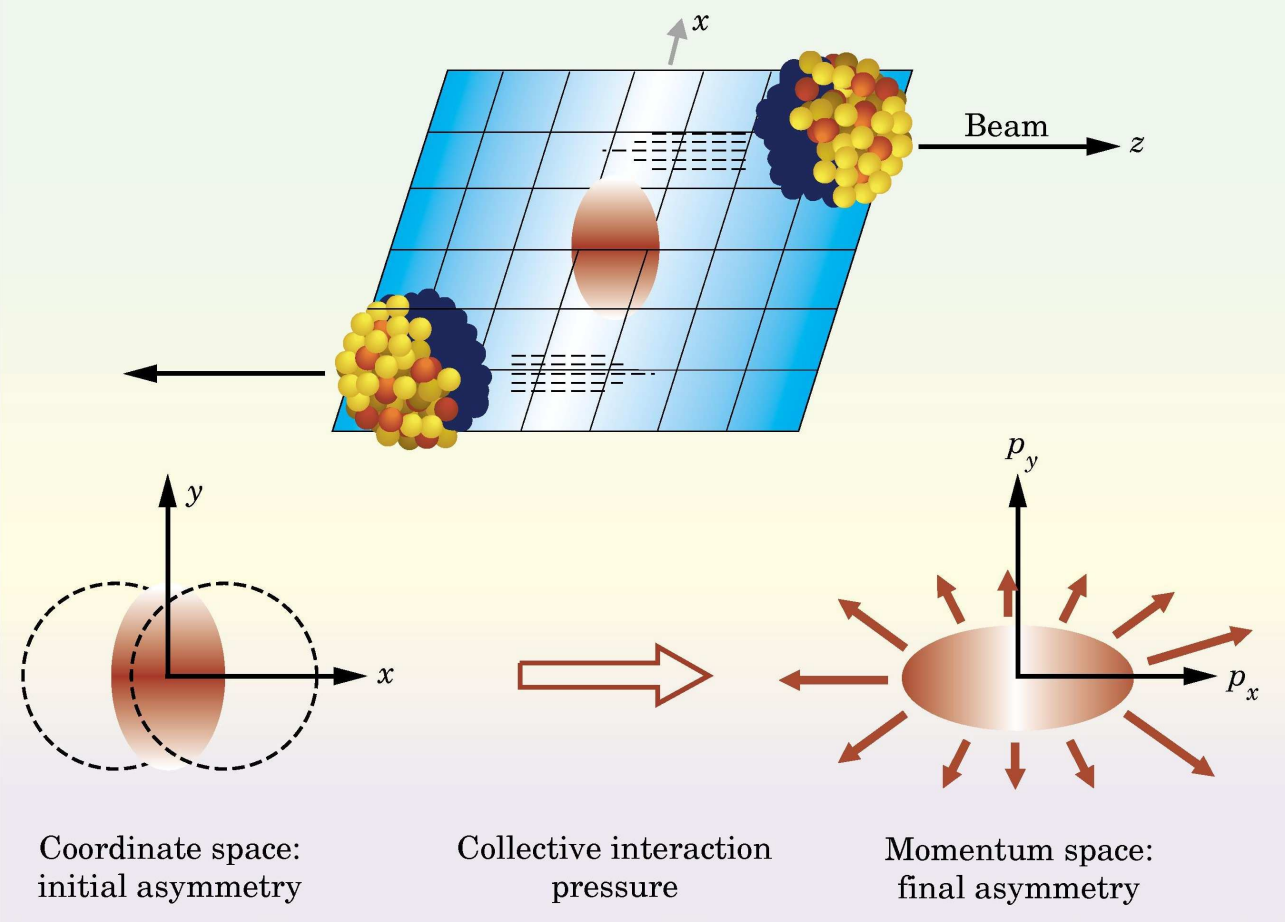
The "initial" energy density is given by the Bjorken formula

Is there thermal equilibrium?



PHENIX (triangles), STAR(stars), BRAHMS (circles) PHOBOS (crosses) particle ratios, at Au+Au (s=200 GeV) at mid-rapidity vs thermal ensemble predictions.

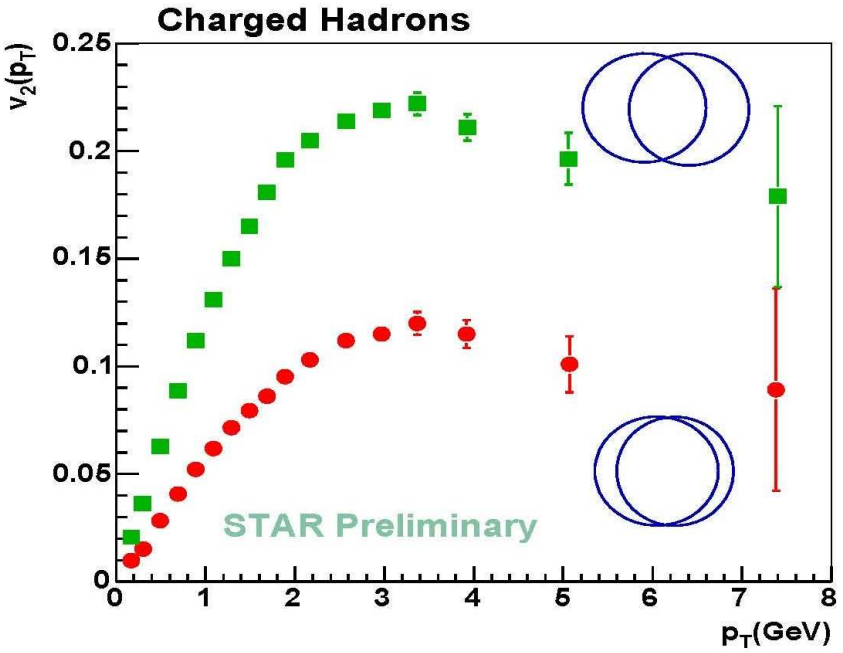
Ellipticity

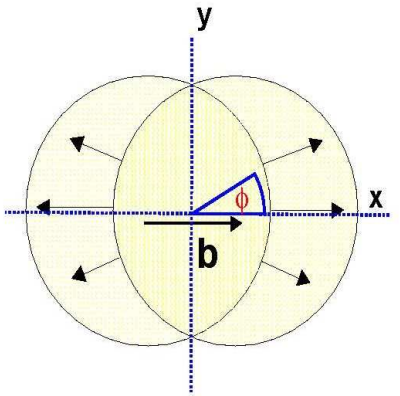


- In an off-center collision, an initial elliptic pattern is produced.
- If the subsequent interactions are weak particles are free streaming and this elliptic pattern is wiped-out
- If the interactions are strong, this pattern persists and is visible in the detectors.

Elliptic flow

$$\frac{1}{p_T}\frac{dN}{dp_T\,d\phi} = \frac{1}{p_T}\frac{dN}{dp_T}\,(1 + 2\,v_2(p_T)\cos(2\,\phi) + ...)$$



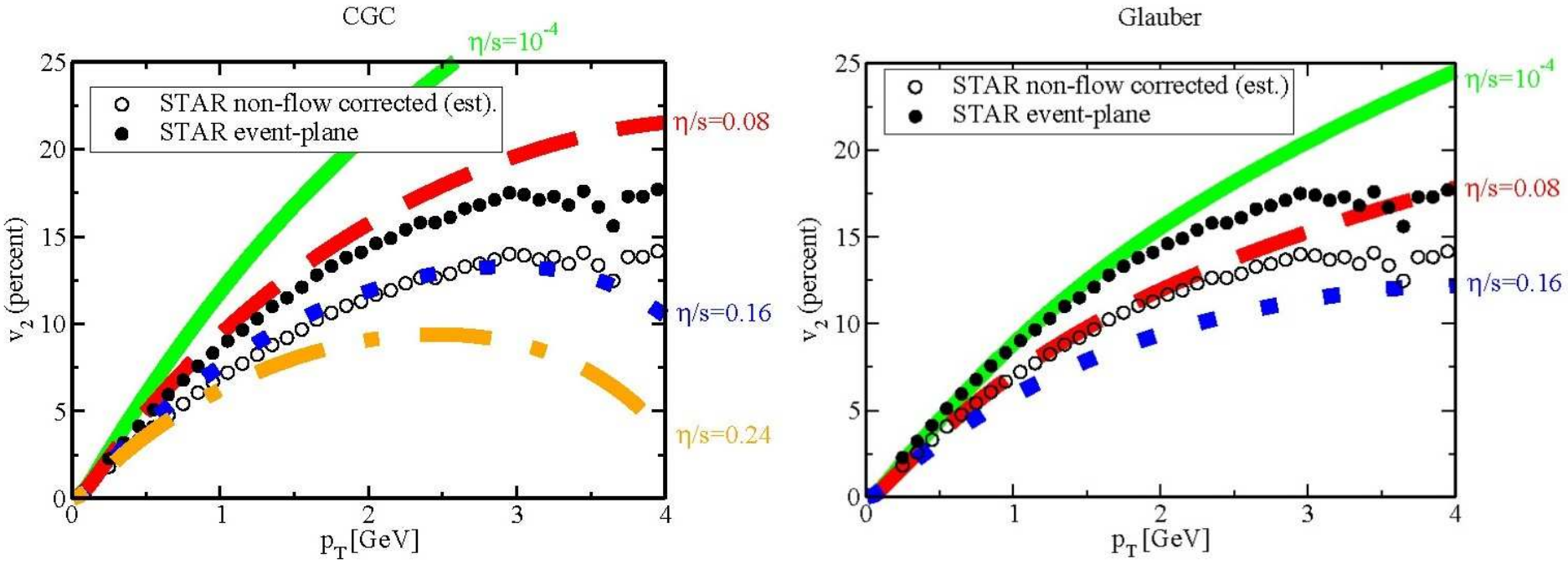


$$\text{X:Y} = (1 + \underbrace{2v_2}_{\sim 0.4} : 1 - \underbrace{2v_2}_{\sim 0.4})$$

Elliptic flow is large X:Y $\sim 2.0 : 1$

- Such Elliptic flow has been observed recently in strongly coupled cold gases.

Hydrodynamic elliptic flow



Elliptic flow data from STAR as a function of p_T (right) compared to relativistic hydrodynamics calculations with non-zero shear viscosity, from Luzum+Romanschke (2008).
(see also PHOBOS data)

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• Finite-temperature (equilibrium) relativistic hydrodynamics describes well the data with

$$\frac{\eta}{s} \simeq (0.05 - 0.12)\ \hbar$$

• Perturbative (weak-coupling) QCD gives a large ratio: $\frac{\eta}{s} \simeq \frac{1}{g^4 \log(1/g)} \sim (5 - 10)\ \hbar$.

• Conclusion :The QGP produced is strongly coupled.

• We need techniques to understand its physics: dynamics at finite temperature.

A string theory for QCD:(Very) basic expectations

• Pure SU(N_c) d=4 YM at large N_c is expected to be dual to a string theory in 5 dimensions only. Essentially a single adjoint field $\rightarrow$ a single extra dimension.

♠ The four vector components are related by the expected Lorentz invariance of the vacuum.

♠ Therefore: a single eigenvalue distribution $\rightarrow$ an extra dimension

♠ Intuition well tested in several matrix models including the "old-ones".

♠ The counting of dimensions can become complicated by the presence of several fields, "evanescent dimensions" and the knowledge/structure of RG topography.

- The theory becomes asymptotically free and conformal at high energy
- Following on N=4 intuition we might expect that $\ell_{AdS} \to 0 \quad \to \quad$ singularity.
- There are several possibilities for such singularities:

(a) They are "mirage": the geometry stabilizes at $\ell \sim \ell_s$. (different examples from WZW models and DBI actions).

(b) The singularity is resolved by the stringy or higher dimensional physics. The true string metric is regular (some examples from higher dimensional resolutions)

(c) The singularity remains (not our case we think)

- The N=4 relation $\ell^4 \sim \lambda \sim \frac{1}{\log r}$. seems to indicate a naked singularity.

- Another possibility is that the classical saddle point solution should asymptote to a regular but stringy ($\ell = \ell_s$) AdS_5 in the Einstein frame . This option has several advantages and provides a lot of mileage:

♠ It allows in principle the machinery of holography to be applied

♠ It realizes the geometrical implementation of the asymptotic conformal symmetry of YM theory in the UV.

The low energy spectrum

♠ In YM only $Tr[FF]$ and maybe $Tr[F \wedge F]$ have a source. However many operators can have a vev. We expect $\langle O_\Delta \rangle \sim (\Lambda_{QCD})^\Delta$.

♠ If that is the case this implies that many stringy states will have non-trivial profiles in the vacuum solution.

♠ Operators of higher dimension are not important in the UV (that's why we can truncate the RG flow). In the bulk, they have positive m^2, that suppresses their solutions.

These are scalar YM operators with $\Delta_{UV} > 4 \rightarrow m^2 > 0$ or higher spin fields.

• Although higher dimension operators may become important in the IR, indications from SVZ sum rules plus data suggest that the coefficients of higher dimension operators are "unaturally" small.

• It seems a reasonable assumption to neglect all $\Delta > 4$ fields when looking for the vacuum solution.

The minimal effective string theory spectrum

- NS-NS $\rightarrow$ $g_{\mu\nu} \leftrightarrow T_{\mu\nu}$, $B_{\mu\nu} \leftrightarrow Tr[F]^3$, $\phi \leftrightarrow Tr[F^2]$

- RR $\rightarrow$ $\text{Spinor}_5 \times \text{Spinor}_5 = F_0 + F_1 + F_2 + (F_3 + F_4 + F_5)$

♠ $F_0 \leftrightarrow F_5 \rightarrow$ C_4, background flux $\rightarrow$ no propagating degrees of freedom.

♠ $F_1 \leftrightarrow F_4 \rightarrow$ $C_3 \leftrightarrow C_0$: C_0 is the axion, C_3 its 5d dual that couples to domain walls separating oblique confinement vacua.

♠ $F_2 \leftrightarrow F_3 \rightarrow$ $C_1 \leftrightarrow C_2$: C_2 mixes with B_2 because of the C_4 flux, and is massive. C_1 is associated with baryon number (as we can see when we add flavor).

- In an ISO(3,1) invariant vacuum solution, only $g_{\mu\nu}, \phi, C_0 = a$ can be non-trivial.

$$ds^2 = e^{2A(r)}(dr^2 + dx_4^2) \quad , \quad a(r), \phi(r)$$

(Many) holographic approaches to QCD

♠ Witten (1998): Start from N=4 d=5 superYM $\rightarrow$ the near horizon solution is known (D4 brane in 10d) Put antiperiodic boundary conditions around 5th circle to fermions: they acquire masses$m_f \sim 1/R_5$.

Scalars get one loop masses $m_s^2 \sim g_5^2/R_5^3$

• At low energies the theory is expected to be pure YM in 4d.

• The dual solution that describes this theory is an AdS-like soliton solution: It has a cigar in the $r - x^5$ plane. This caps-off the geometry and imposes confinement.

• The solution is strongly coupled in the UV. This leads to the black-M5 solution that regulates the UV singularity.

• In the regime where the solution is reliable the glueball masses are of the same order as the KK masses ($\sim 1/R_5$)

♠ Klebanov-Strassler (2000). This is a theory that describes a cascading set of gauge theories (the number of colors is "cutoff dependent") .

• In the IR asymptotes to a confining pure gauge theory.

• In the UV it has a rather complicated behavior that defies a simple description.

♠ Maldacena-Nunez (2000). This is based on a gauge supergravity solution of Chamseddine and Volkov. It describes an M5-brane (little string theory) wrapped on a two cycle in a CY manifold down to four-dimensions with N=1 supersymmetry.

• In the IR the theory is similar to N=1 sYM

• The UV asymptotics coincide with 6d CFT based on M5-branes.

AdS/QCD

♠ A basic phenomenological approach: use a slice of AdS_5, with a UV cutoff, and an IR cutoff.

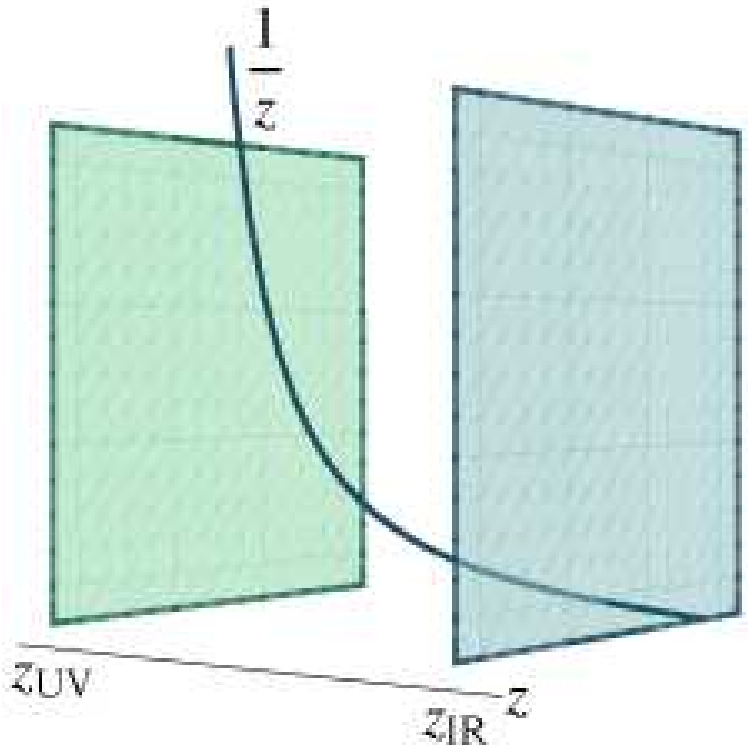
Polchinski+Strassler

♠ It successfully exhibits confinement (trivially via IR cutoff), and power-like behavior in hard scattering amplitudes

♠ It may be equipped with a bifundamental scalar, T, and $U(N_f)_L \times U(N_f)_R$, gauge fields to describe mesons.

Erlich+Katz+Son+Stepanov, DaRold+Pomarol

Chiral symmetry is broken by hand, via IR boundary conditions. The low-lying meson spectrum looks "reasonable".



♠ Shortcomings:

- The glueball spectrum does not fit very well the lattice calculations. It has the wrong asymptotic behavior $m_n^2 \sim n^2$ at large n.

- Magnetic quarks are confined instead of screened.

- Chiral symmetry breaking is input by hand.

- The meson spectrum has also the wrong UV asymptotics $m_n^2 \sim n^2$.

- at finite temperature there is a deconfining transition but the equation of state is trivial (conformal) (e-2p) and the speed of sound is $c_s^2 = \frac{1}{3}$.

Improved Holographic QCD: a model

• We would like to write down a model that captures the holographic behavior of YM and in particular the running of the coupling constant:

• The basic fields will be $g_{\mu\nu}, \phi, a$. We can neglect a when studying the basic vacuum solution (down by N_c^{-2}).

• In the IR the action should have two derivatives and admit solutions with weak curvature (in the string frame)

$$S_{\text{Einstein}} = M^3 N_c^2 \int d^5x \sqrt{g} \left[R - \frac{4}{3} \frac{(\partial\lambda)^2}{\lambda^2} + V(\lambda) \right] \quad , \quad \lambda = N_c \, e^{\phi}$$

• Although in the UV we expect higher derivatives to be important we will extend this by demanding that the solution is asymptotically AdS_5 and the 't Hooft coupling will run logarithmically.

• Although we do not expect this simple model to capture all aspects of YM dynamics we will se that it goes a long way.

The UV solution

• In order to obtain an asymptotically AdS_5 solution V should become a constant when $\lambda \to 0$.

• We therefore write an expansion for the potential in the UV as

$$\lim_{\lambda \to 0} V(\lambda) = \frac{12}{\ell^2}\left(1 + \sum_{n=1}^{\infty} c_n \lambda^n\right)$$

• The potential should be strictly monotonic to drive the theory to strong coupling without IR fixed points.

• In particular, the UV fixed point should NOT be a minimum of the potential.

• The vacuum solution ansatz is

$$ds^2 = e^{2A(r)}(dr^2 + dx^\mu dx_\mu) \quad , \quad \lambda(r)$$

and is the most general one that preserves 4d Poincaré invariance.

• We may choose the holographic "energy" scale (renormalization group scale) as the scale factor in the Einstein frame

$$E = e^{A_E}$$

This asymptotes properly in the UV, $E \sim 1/r$, is everywhere monotonic and becomes zero in the IR. This is a choice (scheme). Physical quantities do not depend on this choice.

• We may now solve the equations perturbatively in λ around $\lambda = 0$ and $r = 0$ (this is a weak coupling expansion) to find

$$\frac{1}{\lambda} = L - \frac{b_1}{b_0} \log L + \cdots \quad , \quad L \equiv -b_0 \log(r\Lambda)$$

$$e^A = \frac{\ell}{r}\left[1 + \frac{4}{9\log r\Lambda} + \mathcal{O}\left(\frac{\log\log r\Lambda}{\log^2 r\Lambda}\right)\right]$$

The identification is

$$c_1 = \frac{8}{9} b_0 \quad , \quad c_2 = \frac{23\ b_0^2 - 36\ b_1}{3^4} \quad , \quad c_3 = -2\frac{324\ b_2 + 124\ b_0^3 + 189\ b_1 b_0}{3^7}$$

with

$$V = \frac{12}{\ell^2}\left[1 + c_1\lambda + c_2\lambda^2 + c_3\lambda^3 + \cdots\right]$$

$$\frac{d\lambda}{d\log E} \equiv \beta(\lambda) = -b_0\lambda^2 + b_1\lambda^3 + b_2\lambda^4 + \cdots$$

♠ The asymptotic expansion of the potential is in one-to-one correspondence with the perturbative β-function. This can be verified by a more rigorous renormalization procedure.

Gursoy+Papadimitriou

An assessment of IR asymptotics

- We define the superpotential W as

$$V(\lambda) = \frac{4}{3}\lambda^2 \left(\frac{dW}{d\lambda}\right)^2 + \frac{64}{27}W^2$$

- We parameterize the UV ($\lambda \to 0$) and IR asymptotics ($\lambda \to \infty$) as

$$V(\lambda) \sim \lambda^Q (\log \lambda)^P$$

- We characterize the presence of confinement by calculating the heavy quark potential via the Wilson loop.

$$\lim_{\substack{R\to\infty, T\to\infty \\ R/T\to 0}} \langle \mathcal{W}(R,T) \rangle = e^{-TV(L)} \quad , \quad V(L) = \sigma_s \; L + \cdots$$

- All confining solutions have an IR singularity.

There are several types of solution for W:

- There is only one "Good type" solution.

$$W(\lambda) \sim (\log \lambda)^{\frac{P}{2}} \; \lambda^{\frac{Q}{2}}$$

It leads to a "good" IR singularity, confinement, a mass gap, discrete spectrum of glueballs and screening of magnetic charges if

$$\frac{4\sqrt{2}}{3} > Q > \frac{4}{3} \quad or \quad Q = \frac{4}{3} \quad and \quad P > 0$$

.

- The asymptotic spectrum of glueballs is linear if $Q = \frac{4}{3}$ and $P = \frac{1}{2}$.

- All other families of solutions are "bad" in that they either have no confinement or lead to a a bad singularity.

Selecting the IR asymptotics

The $Q = 4/3$, $0 \leq P < 1$ solutions have a singularity at $r = \infty$. They are compatible with

- Confinement (it happens non-trivially: a minimum in the string frame scale factor)
- Mass gap+discrete spectrum (except P=0)
- "good+repulsive" singularity (like in the linear dilaton vacuum).
- $R_{\text{string}} \to 0$ justifying the original assumption. More precisely: the string frame metric becomes flat at the IR .

♠ It is interesting that the lower endpoint: P=0 corresponds to linear dilaton and flat space (string frame). It is confining with a mass gap but has continuous spectrum.

- For linear asymptotic trajectories for fluctuations (glueballs) we must choose $P = 1/2$

$$V(\lambda) = \sim \lambda^{\frac{4}{3}} \sqrt{\log \lambda} + \text{subleading} \qquad \text{as} \qquad \lambda \to \infty$$

Summary

• We argued that an Einstein dilaton system with a potential can capture some important properties of YM: asymptotic freedom in the UV and confinement in the IR

$$S \sim \int \left[R - \frac{4}{3}(\partial\phi)^2 + V(\phi) \right]$$

• The potential is regular in the UV

$$V \to \frac{12}{\ell^2}\left[1 + c_1\lambda + c_2\lambda^2 + \cdots\right]$$

• In the IR it should behave as

$$V \sim \lambda^{\frac{4}{3}}(\log\lambda)^P$$

for linear trajectories $P = 1/2$.

• We can solve the equations of motion with $\lambda \to 0$ in the UV.

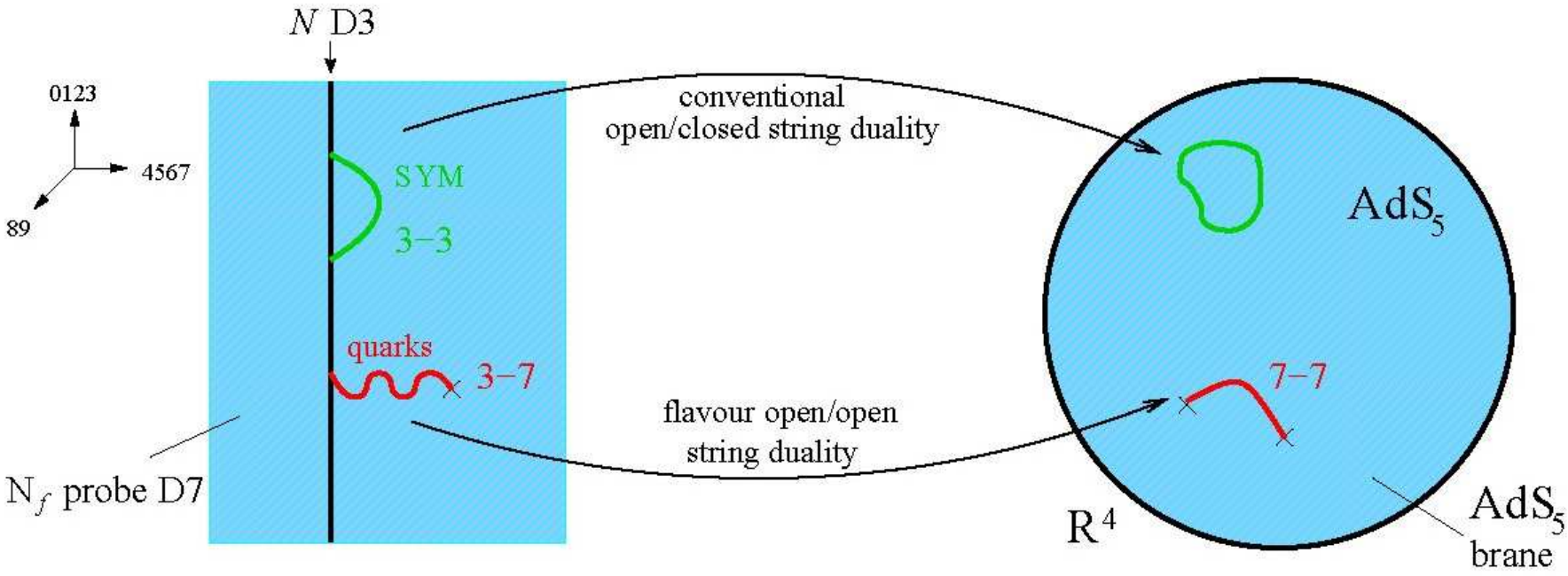
• The intermediate behavior of the potential is not fixed (phenomenological parameters).

• Once the potential is fixed, the solutions have only one parameter: Λ_{QCD}

Adding flavor

• To add N_f quarks q_L^I and antiquarks $q_R^{\bar{I}}$ we must add (in 5d) space-filling N_f D_4 and N_f $\bar{D}_4$ branes.
(tadpole cancellation=gauge anomaly cancellation)

• The q_L^I should be the "zero modes" of the $D_3 - D_4$ strings while $q_R^{\bar{I}}$ are the "zero modes" of the $D_3 - \bar{D}_4$



• The low-lying fields on the D_4 branes ($D_4 - D_4$ strings) are $\mathrm{U}(N_f)_L$ gauge fields A_μ^L. The low-lying fields on the $\bar{D}_4$ branes ($\bar{D}_4 - \bar{D}_4$ strings) are $\mathrm{U}(N_f)_R$ gauge fields A_μ^R. They are dual to the J_L^μ and J_μ^R

$$\delta S_A \sim \bar{q}_L^I \ \gamma^\mu \ (A_\mu^L)^{IJ} \ q_L^J + \bar{q}_R^{\bar{I}} \ \gamma^\mu \ (A_\mu^R)^{\bar{I}\bar{J}} \ q_R^{\bar{J}} = Tr[J_L^\mu \ A_\mu^L + J_R^\mu \ A_\mu^R]$$

• There are also the low lying fields of the ($D_4 - \bar{D}_4$ strings), essentially the string-theory "tachyon" $T_{I\bar{J}}$ transforming as $(N_f, \bar{N}_f)$ under the chiral symmetry $U(N_f)_L \times U(N_f)_R$. It is dual to the quark mass terms

$$\delta S_T \sim \bar{q}_L^I \ T_{I\bar{J}} \ q_R^{\bar{J}} + \text{complex congugate}$$

• The interactions on the flavor branes are weak, so that $A_\mu^{L,R}, T$ are as sources for the quarks.

• Integrating out the quarks, generates an effective action $S_{flavor}(A_\mu^{L,R}, T)$, so that $A_\mu^{L,R}, T$ can be thought as effective $q\bar{q}$ composites, that is : mesons

• On the string theory side: integrating out $D_3 - D_4$ and $D_3 - \bar{D}_4$ strings gives rise to the DBI action for the $D_4 - \bar{D}_4$ branes in the D_3 background:

$$S_{flavor}(A_\mu^{L,R}, T) \quad \longleftrightarrow \quad S_{DBI}(A_\mu^{L,R}, T) \quad \text{holographically}$$

• In the "vacuum" only T can have a non-trivial profile: $T^{I\bar{J}}(r)$. Near the AdS_5 boundary ($r \to 0$)

$$T^{I\bar{J}}(r) = M_{I\bar{J}}\, r + \cdots + \langle \bar{q}_L^I \; q_R^{\bar{J}} \rangle r^3 + \cdots$$

Casero+Kiritsis+Paredes

• A typical solution is T vanishing in the UV and $T \to \infty$ in the IR. At the point $r = r_*$ where $T = \infty$, the D_4 and $\bar{D}_4$ branes "fuse". The true vacuum is a brane that enters folds on itself and goes back to the boundary. A non-zero T breaks chiral symmetry.

• When $m_q = 0$, the meson spectrum contains N_f^2 massless pseudoscalars, the $U(N_f)_A$ Goldstone bosons.

• The WZ part of the flavor brane action gives the Adler-Bell-Jackiw $U(1)_A$ axial anomaly and an associated Stuckelberg mechanism gives an $O\left(\frac{N_f}{N_c}\right)$ mass to the would-be Goldstone boson η', in accordance with the Veneziano-Witten formula.

• We can derive formulae for the anomalous divergences of flavor currents, when they are coupled to an external source.

• T=0 is always a solution. However it is excluded from the absence of IR boundary for the flavor branes: Holographic Coleman-Witten theorem.

• Fluctuations around the T solution for $T, A_\mu^{L,R}$ give the spectra (and interactions) of various meson trajectories.

• A GOR relation is satisfied (for an asymptotic AdS_5 space)

$$m_\pi^2 = -2\frac{m_q}{f_\pi^2}\,\langle \bar{q}q\rangle \quad , \quad m_q \to 0$$

Shortcomings

Not everything is perfect: There are some shortcomings localized at the UV

- The conformal anomaly (proportional to the curvature) is incorrect.

- Shear viscosity ratio is constant and equal to that of N=4 sYM.

(This is not expected to be a serious error in the experimentally interesting $T_c \leq T \leq 4T_c$ range.)

Both of the above need Riemann curvature corrections.

- Many other observables come out very well both at T=0 and finite T

Finite temperature

The theory at finite temperature can be described by:
(1) The "thermal vacuum solution". This is the zero-temperature solution we described so far with time periodically identified with period β.

(2) "black-hole" solutions

$$ds^2 = b(r)^2 \left[\frac{dr^2}{f(r)} - f(r) dt^2 + dx^i dx^i \right], \qquad \lambda = \lambda(r)$$

♠ We need VERY UNUSUAL boundary conditions: The dilaton (scalar) is diverging at the boundary so that $\lambda \sim e^{\phi} \rightarrow \frac{1}{\log r} \rightarrow 0$

♠ The boundary AdS is NOT at a minimum of the potential.

• Such type of solutions have not been analyzed so far in the literature.

General phase structure

- For a general potential (with no minimum) the following can be shown :

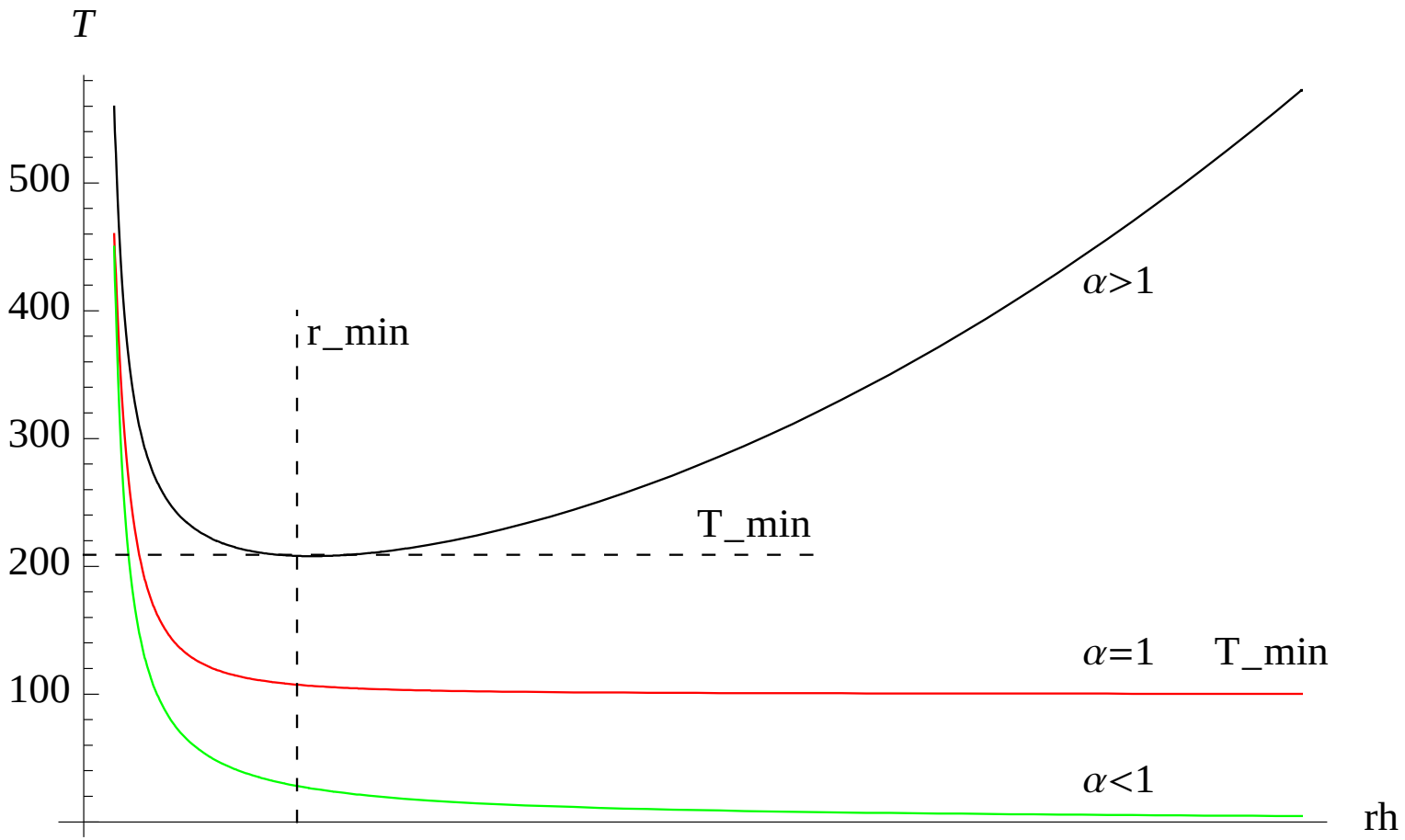
i. *There exists a phase transition at finite $T = T_c$, if and only if the zero-T theory confines.*

ii. *This transition is of the first order for all of the confining geometries, with a single exception described in iii:*

iii. *In the limit confining geometry $b_0(r) \to e^{-Cr}$, $\lambda_0 \to e^{\frac{3}{2}Cr}$, (as $r \to \infty$), the phase transition is of the second order and happens at $T = 3C/4\pi$.*
This is the linear dilaton vacuum solution in the IR.

iv. *All of the non-confining geometries at zero T are always in the black hole phase at finite T. They exhibit a second order phase transition at $T = 0^+$.*

Temperature versus horizon position



We plot the relation $T(r_h)$ for various potentials parameterized by a. $a = 1$ is the critical value below which there is only one branch of black-hole solutions.

The free energy

• The free energy is calculated from the action as a boundary term for both the black-holes and the thermal vacuum solution. They are all UV divergent but their differences are finite.

$$\frac{\mathcal{F}}{M_p^3 V_3} = 12\mathcal{G}(T) - T\ S(T)$$

• $\mathcal{G}$ is the temperature-depended gluon condensate $\langle Tr[F^2]\rangle_T - \langle Tr[F^2]\rangle_{T=0}$ defined as

$$\lim_{r\to 0}\ \lambda_T(r) - \lambda_{T=0}(r) = \mathcal{G}(T)\ r^4 + \cdots$$

• It is $\mathcal{G}$ the breaks conformal invariance essentially and leads to a non-trivial deconfining transition (as $S > 0$ always)

The pressure from the lattice at different N

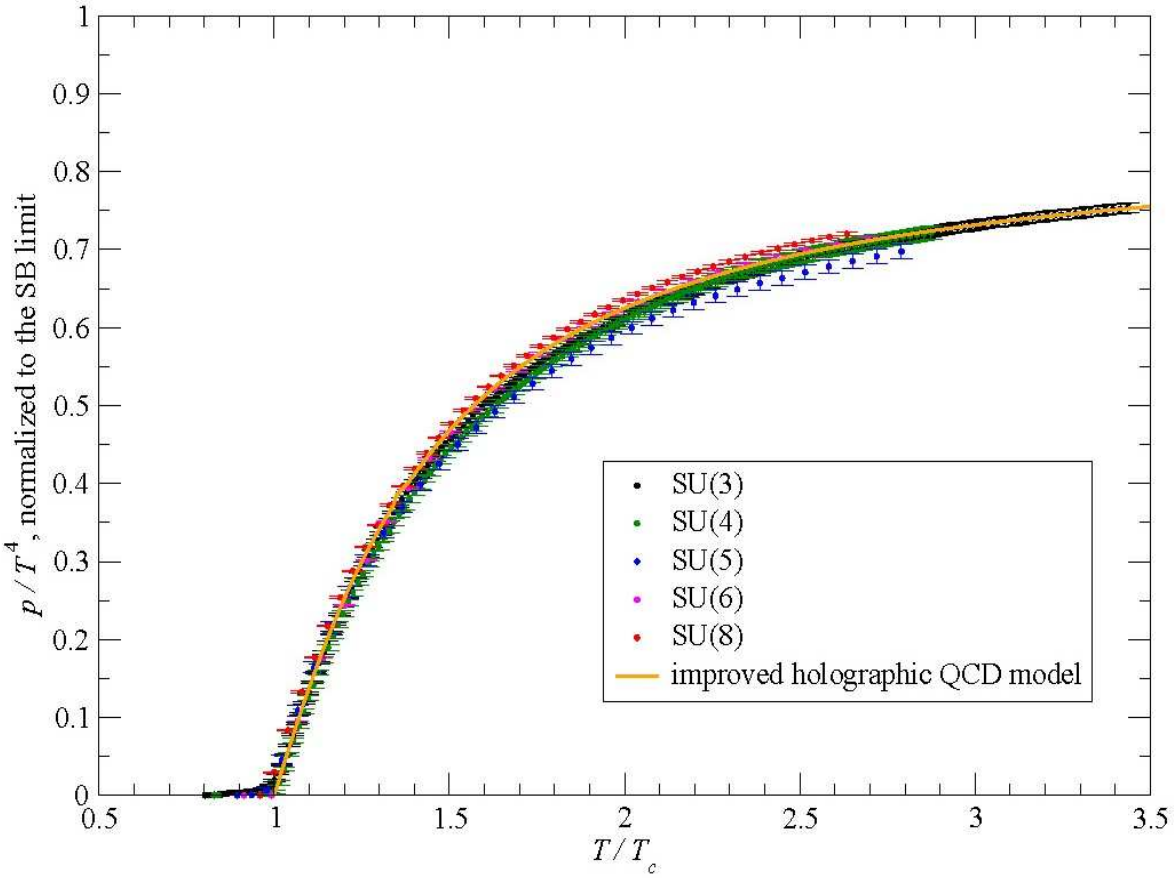


Figure 1: (Color online) The dimensionless ratio p/T^4, normalized to the lattice SB limit $\pi^2(N^2-1)R_I(N_t)/45$, *versus* T/T_c, as obtained from simulations of SU(N) lattice gauge theories on $N_t = 5$ lattices. Errorbars denote statistical uncertainties only. The results corresponding to different gauge groups are denoted by different colors, according to the legend. The yellow solid line denotes the prediction from the improved holographic QCD model from ref. [75] (with a trivial, parameter-free rescaling to our normalization).

Marco Panero arXiv: 0907.3719

The entropy from the lattice at different N

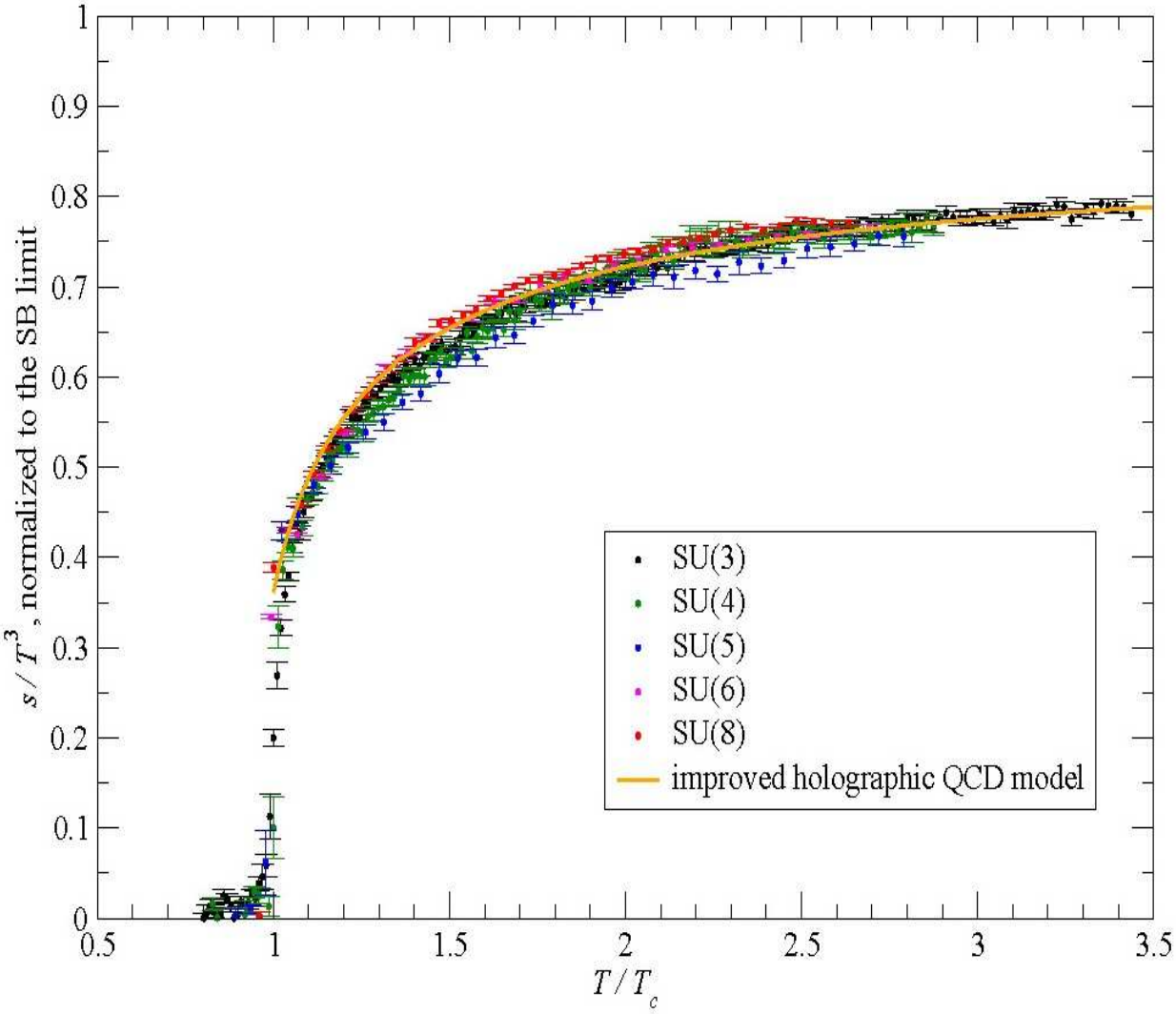


Figure 4: (Color online) Same as in fig. 1, but for the s/T^3 ratio, normalized to the SB limit.

Marco Panero arXiv: 0907.3719

The trace from the lattice at different N

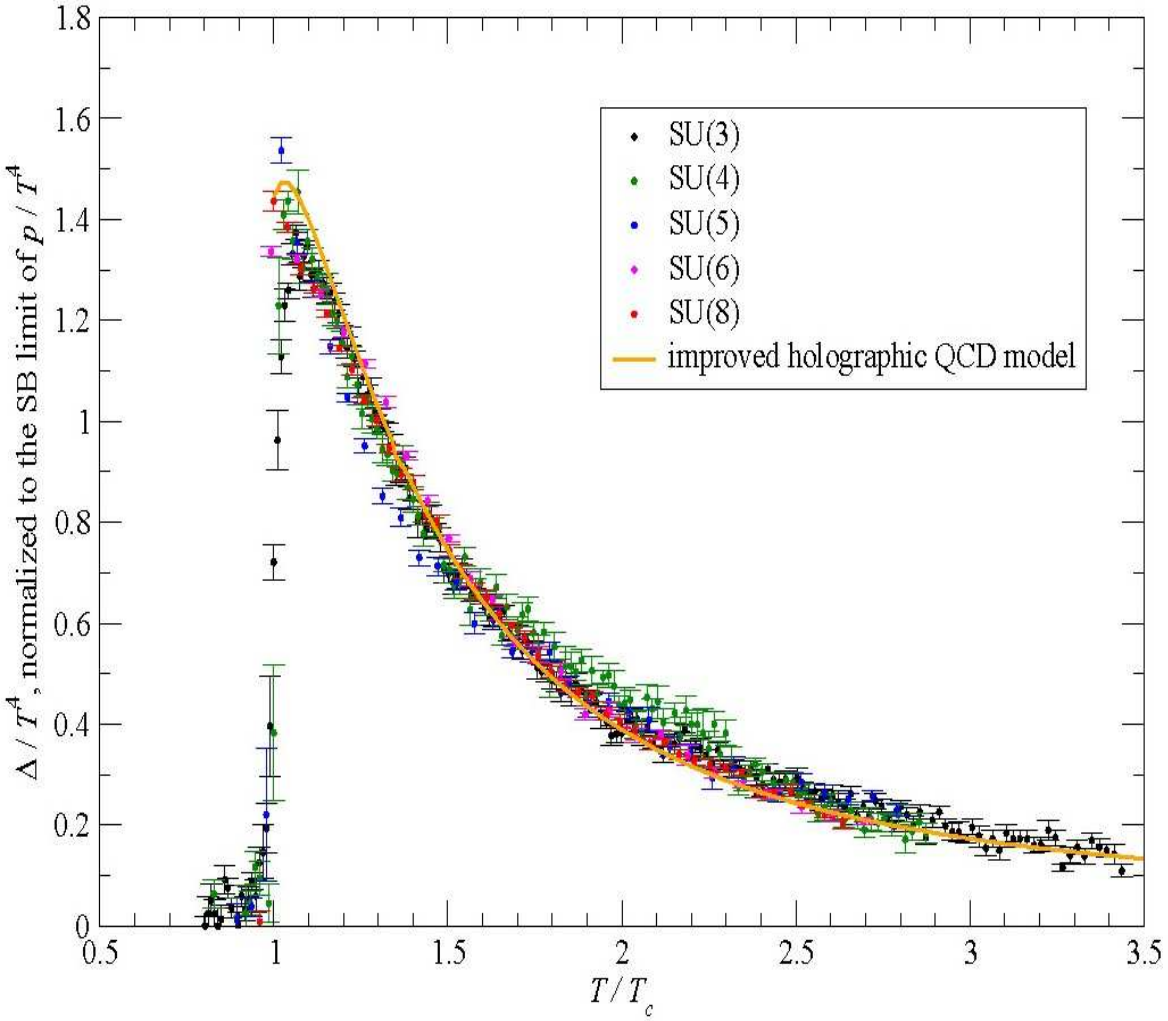
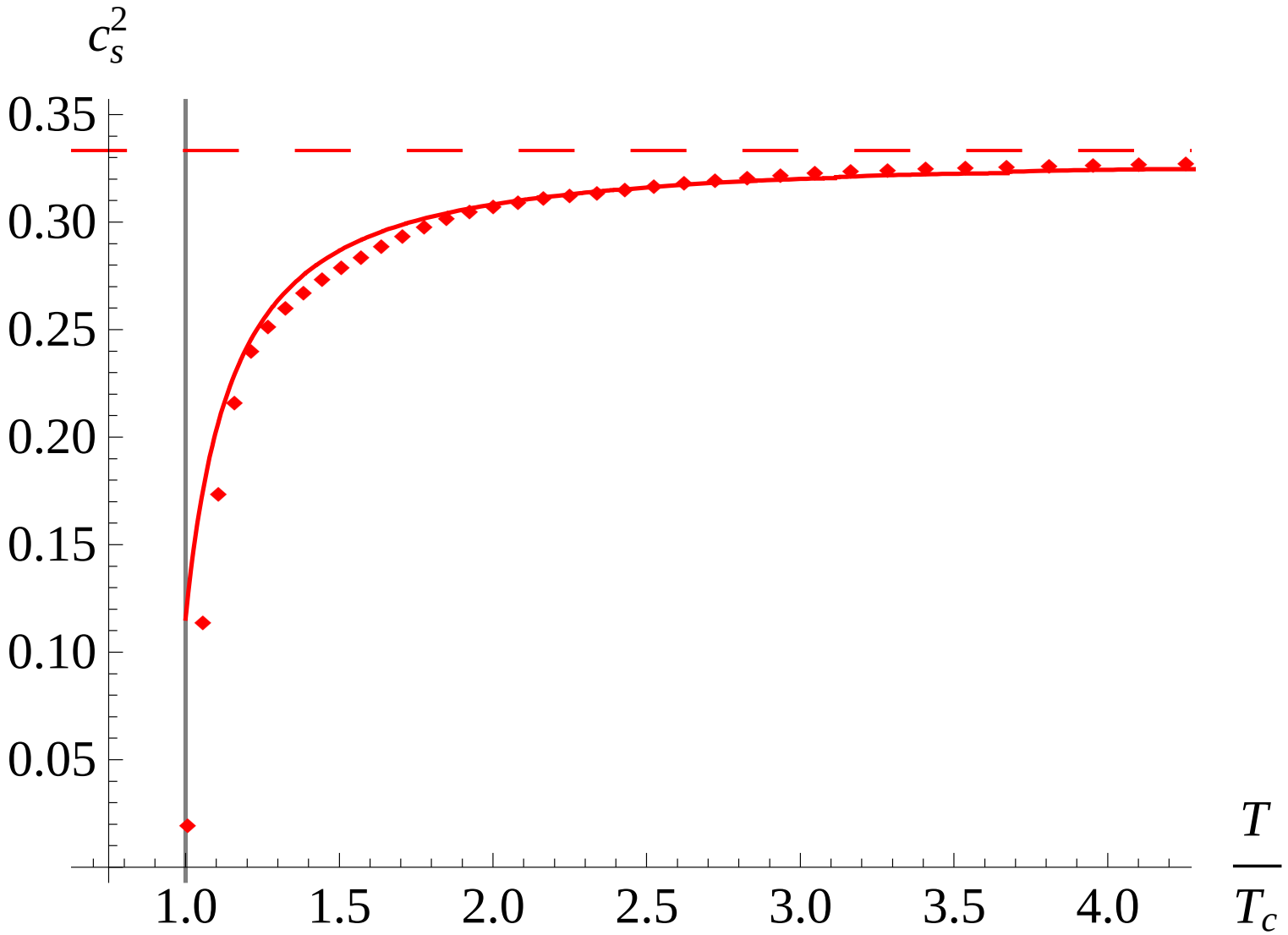


Figure 2: (Color online) Same as in fig. 1, but for the Δ/T^4 ratio, normalized to the SB limit of p/T^4.

Marco Panero arXiv: 0907.3719

The speed of sound



Viscosity

- Viscosity (shear and bulk) is related to dissipation and entropy production

$$\frac{\partial s}{\partial t} = \frac{\eta}{T}\left[\partial_i v_j + \partial_j v_i - \frac{2}{3}\delta_{ij}\partial\cdot v\right]^2 + \frac{\zeta}{T}(\partial\cdot v)^2$$

$$T^{\mu\nu} = (E+p)u^\mu u^\nu + pg^{\mu\nu} + Z^{\mu\alpha}Z^{\nu\beta}\left[\eta\left(\nabla_\alpha u_\beta + \nabla_\beta u_\alpha) - \frac{2}{3}g_{\alpha\beta}\nabla_\gamma u^\gamma\right) + \zeta g_{\alpha\beta}\nabla_\gamma u^\gamma\right]$$

$$Z^{\mu\nu} = g^{\mu\nu} + u^\mu u^\nu$$

- Hydrodynamics is valid as an effective description when relevant length scales $\gg$ mean-free-path:

- Conformal invariance imposes that $\zeta = 0$.

• Viscosity can be calculated from a Kubo-like formula (fluctuation-dissipation) as the linear terms at low frequencies

$$\eta\left(\delta_{ik}\delta_{jl}+\delta_{il}\delta_{jk}-\frac{2}{3}\delta_{ij}\delta_{kl}\right)+\zeta\delta_{ij}\delta_{kl}=-\lim_{\omega\to 0}\frac{Im\ G^R_{ij;kl}(\omega)}{\omega}$$

$$G^R_{ij;kl}(\omega)=-i\int d^3x\int dt\ e^{i\omega t}\theta(t)\ \langle 0|[T_{ij}(\vec{x},t),T_{kl}(\vec{0},0)]|0\rangle$$

• In all theories with gravity duals ($\lambda\to\infty$) at two-derivative level

$$\frac{\eta}{s}=\frac{1}{4\pi}$$

Policastro+Starinets+Son 2001, Kovtun+Son+Starinets 2003, Buchel+Liu 2003

• In Einstein-dilaton gravity shear viscosity is equal to the universal value.

The sum rule method

- Define the (subtracted) spectral density and relate its moment to the Euclidean density

$$\rho(\omega) = -\frac{1}{\pi} Im\ G^R(\omega) \quad , \quad \mathcal{G} \equiv \lim_{\omega \to 0} G^E(\omega) = 2\int_0^\infty \frac{\rho(u)}{u}\, du$$

Karsch+Kharzeev+Tuchin, 2008, Romatschke+Son 2009

- Using Ward identities we obtain the sum rule

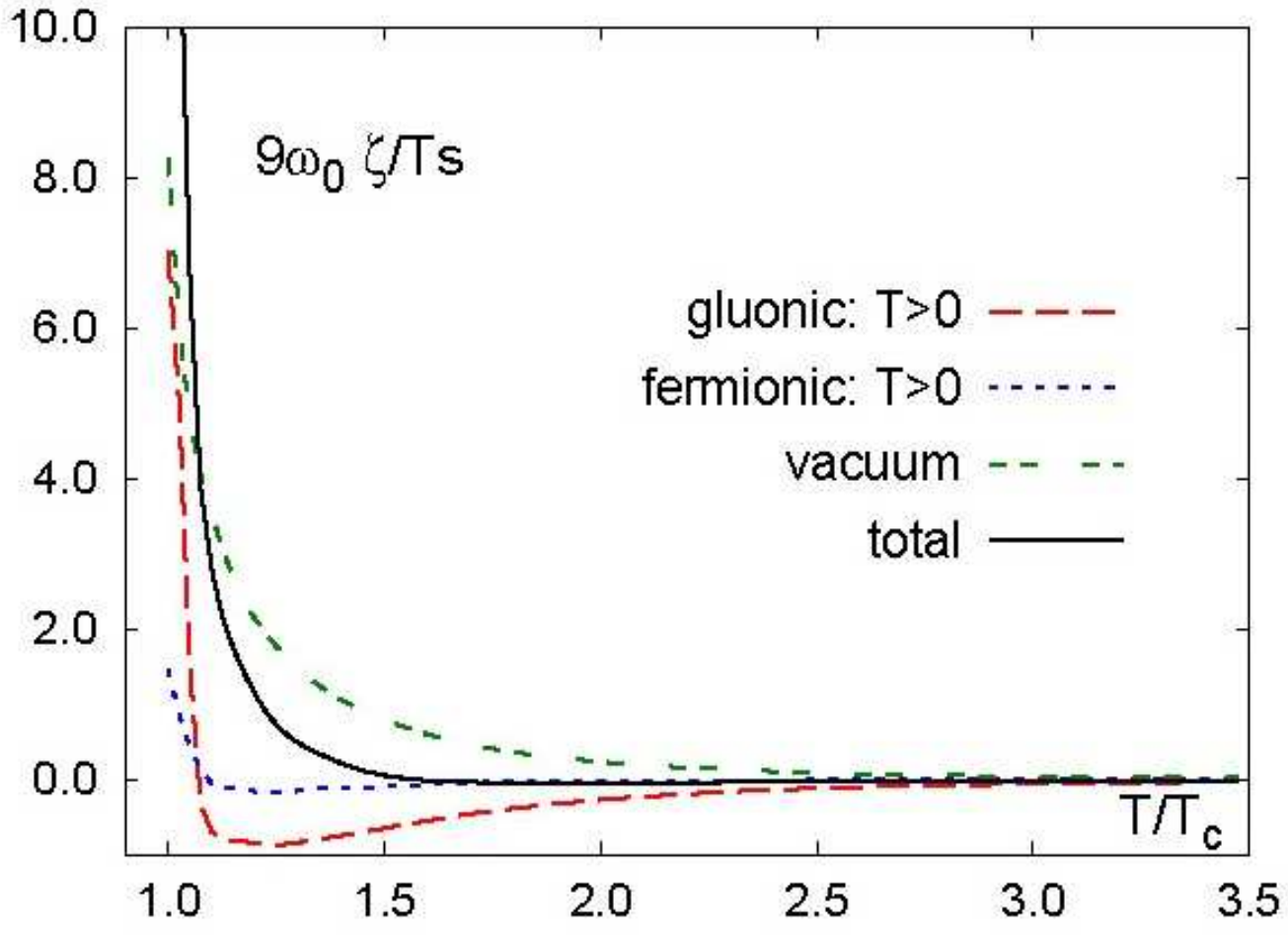
$$\mathcal{G} = \left(T\frac{\partial}{\partial T} - 4\right)(E - 3P + \langle\Theta\rangle_0) + \left(T\frac{\partial}{\partial T} - 2\right)(m\langle\bar{q}q\rangle_T + \langle\Theta_F\rangle_0)$$

with

$$\langle\Theta_F\rangle_0 = m\langle\bar{q}q\rangle_0 \simeq -m_\pi^2\ f_\pi^2 - m_K^2\ f_K^2$$

- Assume a density

$$\frac{\rho(\omega)}{\omega} = \frac{9\zeta(T)}{\pi}\frac{\omega_0(T)^2}{\omega^2 + \omega_0(T)^2}$$

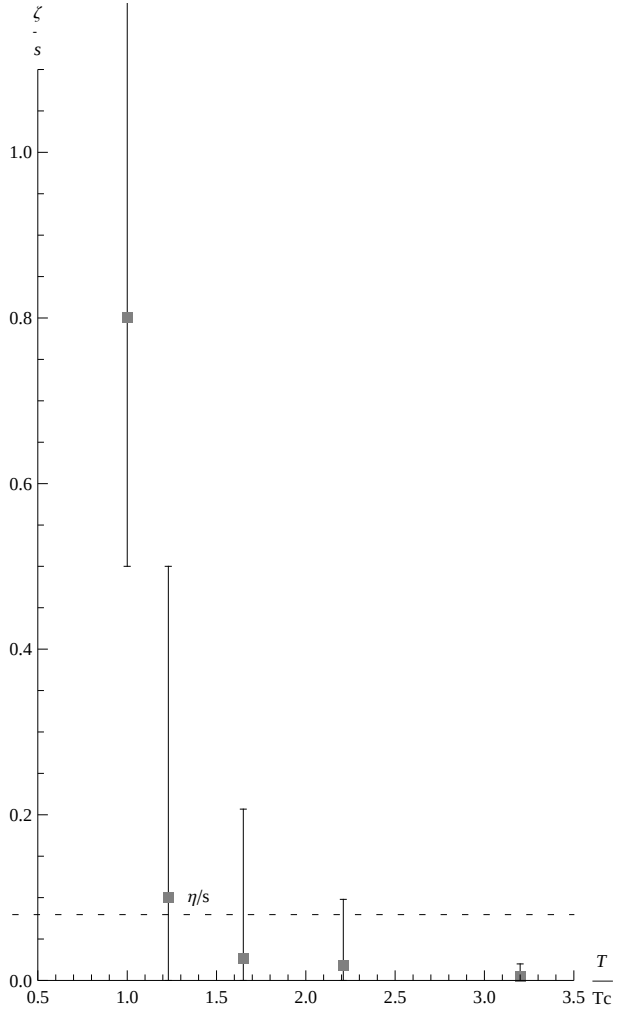


Karsch+Kharzeev+Tuchin, 2008

• A seeming rise near the phase transition but the (temperature-dependent) scale cannot be fixed.

The bulk viscosity in lattice SU(3) YM

H. Meyer 2007



Pure YM only. Error bar are statistical only.

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- If the lattice result is taken at phase value,

$$\frac{\zeta(T_c)}{s(T_c)} \quad \sim \quad 10\frac{\eta(T_c)}{s(T_c)} \quad = \quad 10\ \frac{1}{4\pi}$$

- Such a large value renders hydrodynamic codes unstable.

Heinz+Song (unpublished)

- At large values of viscosity, cavitation ($p < 0$) happens, signaling a breakdown of hydrodynamics.

- This was studied carefully and confirmed very recently

Rajagopal+Tripuraneni 2009

- Both estimates were challenged recently

Moore+Saremi, 2008

The bulk viscosity in HQCD: theory

- This is harder to calculate compared to the shear viscosity.

- Using a parametrization $ds^2 = e^{2A}(f dt^2 + d\vec{x}^2 + \frac{dr^2}{f})$ in a special gauge $\phi = r$ the relevant metric perturbation decouples *Gubser+Nellore+Pufu+Rocha 2008, Gubser+Pufu+Rocha,2008*

$$h_{11}'' = -\left(-\frac{1}{3A'} - A' - \frac{f'}{f}\right) h_{11}' + \left(-\frac{\omega^2}{f^2} + \frac{f'}{6fA'} - \frac{f'}{f}A'\right) h_{11}$$

with

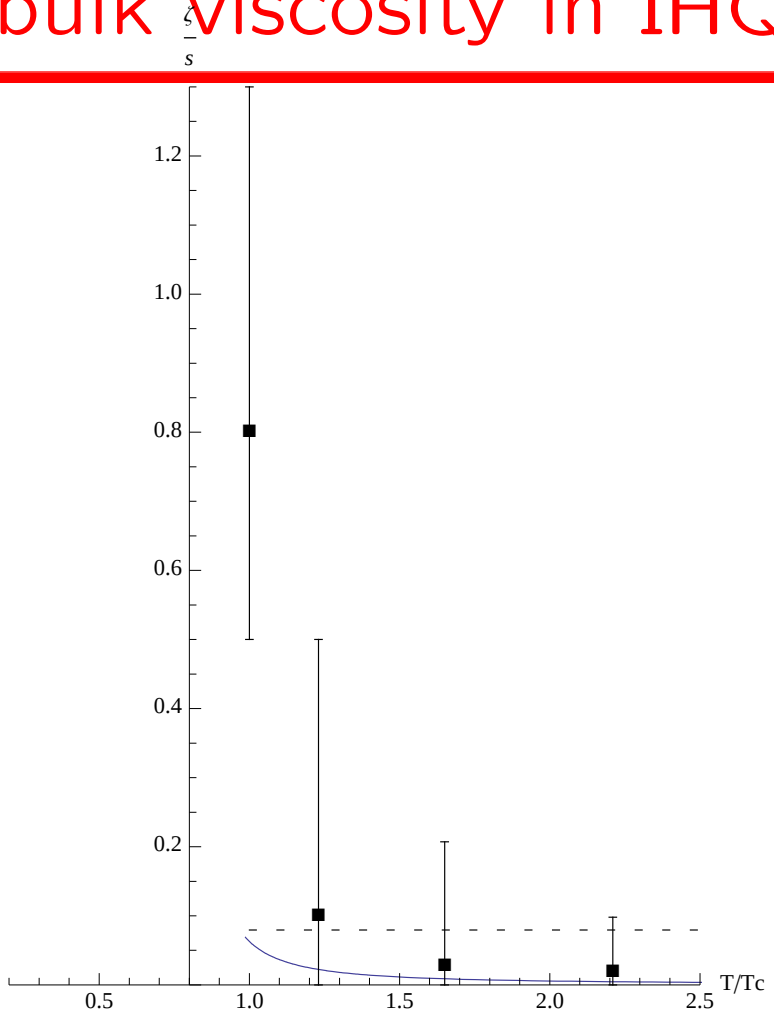
$$h_{11}(0) = 1 \quad , \quad h_{11}(r_h) \simeq C\ e^{i\omega t} \left| \log \frac{\lambda}{\lambda_h} \right|^{-\frac{i\omega}{4\pi T}}$$

The correlator is given by the conserved number of h-quanta

$$Im\ G_R(\omega) = -4M^3 \mathcal{G}(\omega) \quad , \quad \mathcal{G}(\omega) = \frac{e^{3A} f}{4A'^2} |Im[h_{11}^* h_{11}']|$$

$$\frac{\zeta}{s} = \frac{C^2}{4\pi} \left(\frac{V'(\lambda_h)}{V(\lambda_h)}\right)^2$$

The bulk viscosity in IHQCD

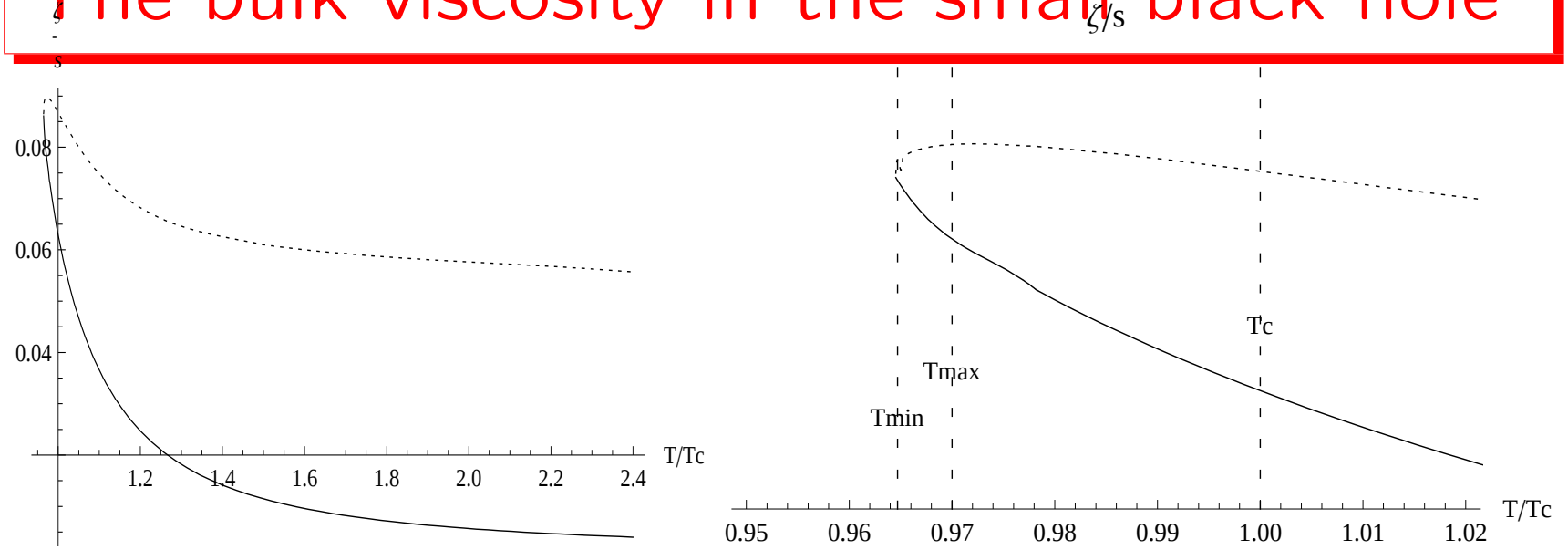


Gursoy+Kiritsis+Michalogiorgakis+Nitti, 2009

- Pure glue only.
- Calculations with other potentials show robustness

Gubser+Pufu+Rocha 2008, Cherman+Nellore 2009

The bulk viscosity in the small black hole



Gursoy+Kiritsis+Michalogiorgakis+Nitti, 2009

- At the turning point the behavior, $C_V \to \infty$ and ζ behaves similar to that observed in the N=2* theory

Buchel+Pagnutti, 2008

- The small black-hole bulk viscosity ratio asymptotes to a constant as $T \to \infty$.

$$\lim_{T\to\infty} \left.\frac{\zeta(T)}{s(T)}\right|_{\text{small}} = \frac{1}{6\pi}$$

The adiabatic approximation

- Adiabatic approximation: $\frac{V'(\phi)}{V(\phi)}$ is a slowly varying function.

Extremes: UV$\rightarrow V$ is constant, IR : $V \sim e^{a\phi}$.

$$\log s = -\frac{8}{3}\int^{\Phi_h} d\Phi \frac{V}{V'} + \cdots \quad , \quad \log T = \int^{\Phi_h} d\Phi \left(\frac{1}{2}\frac{V'}{V} - \frac{8}{9}\frac{V}{V'}\right) \cdots ,$$

- In this approximation there is no "scattering": $c_b \simeq 1$.

- Therefore,

$$\frac{\zeta}{s} \simeq \frac{3}{32\pi}\left(\frac{V'(\phi_h)}{V(\phi_h)}\right)^2$$

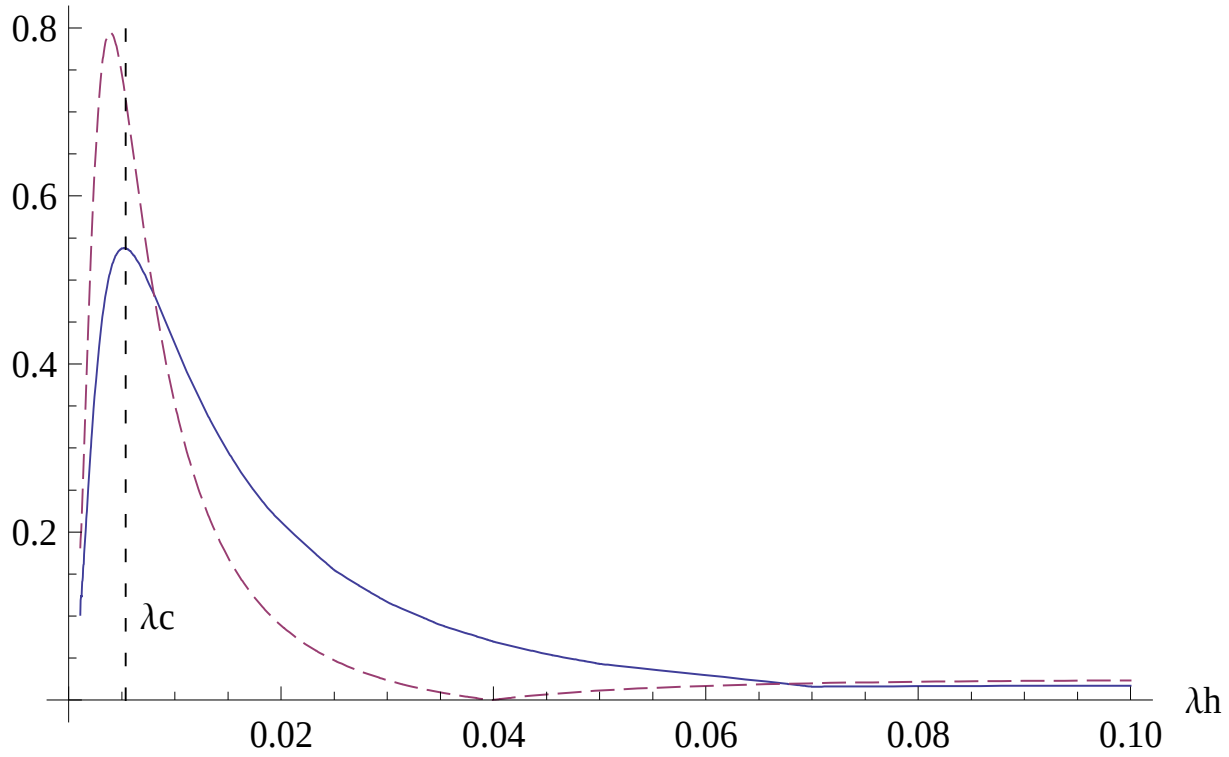
- In the UV (large BH), $\zeta/s \rightarrow 0$

- In the Small BH branch,

$$\frac{\zeta}{s} \simeq \frac{3Q^2}{32\pi} \quad , \quad V \sim e^{Q\phi}$$

- This gives upper bounds on bulk viscosity

.



Validity of the adiabatic approximation. Solid(blue) curve is the difference between the true numerical result and the adiabatic approximation (normalized by the true value) $(\zeta/s(true) - \zeta/s(adb))/(\zeta/s)(true)$, and the dashed(red) curve is the criterion $|(V'/V)'(\Phi)|$ for the validity of approximation.

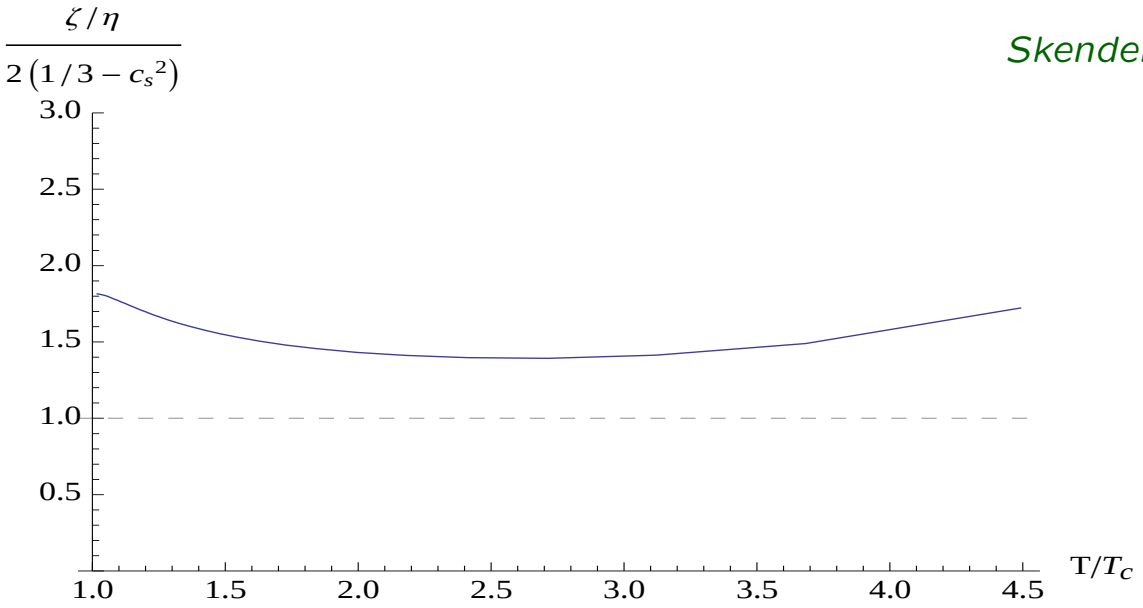
The Buchel parametrization (conjectured bound)

$$\frac{\zeta}{\eta} \geq 2\left(\frac{1}{3} - c_s^2\right)$$

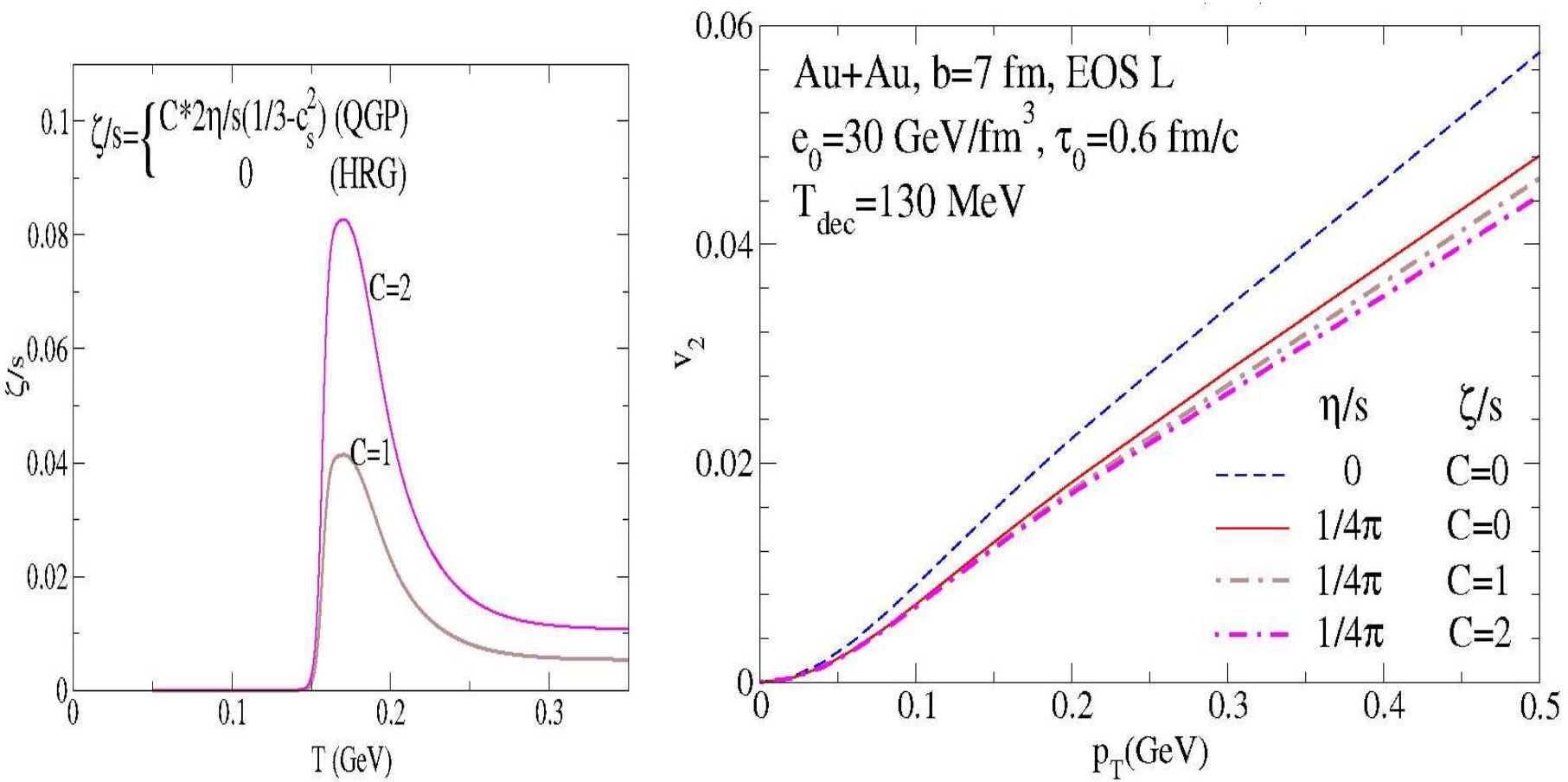
Buchel 2007

• For Dp branes, equality is a consequence of conformal invariance and dimensional reduction.

Skenderis+Kanitscheider 2009



Elliptic Flow vs bulk viscosity



U Heinz+H.Song 2008

Open problems

THEORETICAL:

• Investigate further the structure of the string dual of QCD. Try to control the UV physics (to which RR flux plays little role).

MORE PRACTICAL:

• Second order transport coefficients (matter of principle)

• Holographic calculation of two-point correlators of the stress tensor in the non-conformal (IhQCD) case. Application to lattice extraction techniques via sum rules (that may include fermions)

• Evaluation of the Langevin correlator in IhQCD and use as input for langevin MonteCarlo (both CFT and non-conformal)

• Implementation of a more realistic structure for the quarks in QGP: this will involve a more realistic holographic theory of flavor using $D_4\bar{D}_4$ branes

• Study of early thermalization procedure, as well as entropy production

• The search for other heavy-ion observables that may be calculable using holographic techniques.

Collaborators

My Collaborators

- Umut Gursoy (Utrecht)
- Ioannis Iatrakis (U. of Crete)
- Liuba Mazzanti (Ecole Polytechnique $\rightarrow$ Santiago de Compostella)
- George Michalogiorgakis (Purdue)
- Fransesco Nitti (APC, Paris)
- Angel Paredes (Barcelona)

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Thank you for your Patience

Parameters

- We have 3 initial conditions in the system of graviton-dilaton equations:

♠ One is fixed by picking the branch that corresponds asymptotically to $\lambda \sim \frac{1}{\log(r\Lambda)}$

♠ The other fixes $\Lambda \to \Lambda_{QCD}$.

♠ The third is a gauge artifact as it corresponds to a choice of the origin of the radial coordinate.

- We parameterize the potential as

$$V(\lambda) = \frac{12}{\ell^2}\left\{1 + V_0\lambda + V_1\lambda^{4/3}\left[\log\left(1 + V_2\lambda^{4/3} + V_3\lambda^2\right)\right]^{1/2}\right\},$$

- We fix the one and two loop β-function coefficients:

$$V_0 = \frac{8}{9}b_0 \quad , \quad V_2 = b_0^4\left(\frac{23 + 36b_1/b_0^2}{81V_1^2}\right)^2 , \quad \frac{b_1}{b_0^2} = \frac{51}{121}.$$

and remain with two leftover arbitrary (phenomenological) coefficients.

• We also have the Planck scale M_p

Asking for correct $T \to \infty$ thermodynamics (free gas) fixes

$$(M_p \ell)^3 = \frac{1}{45\pi^2} \quad , \quad M_{\text{physical}} = M_p N_c^{\frac{2}{3}} = \left(\frac{8}{45\pi^2 \ell^3}\right)^{\frac{1}{3}} \simeq 4.6 \text{ GeV}$$

• The fundamental string scale. It can be fixed by comparing with lattice string tension

$$\sigma = \frac{b^2(r_*)\lambda^{4/3}(r_*)}{2\pi \ell_s^2},$$

$\ell/\ell_s \sim \mathcal{O}(1)$.

• ℓ is not a parameter due to a special "scaling" pseudosymmetry:

$$e^{\phi} \to \kappa\, e^{\phi} \quad , \quad g_{\mu\nu} \to \kappa^{\frac{4}{3}}\, g_{\mu\nu} \quad , \quad \ell \to \kappa^{\frac{2}{3}}\, \ell \quad , \quad \ell_s \to \kappa^{\frac{2}{3}}\, \ell_s \quad , \quad V(e^{\phi}) \to V(\kappa\, e^{\phi})$$

But... $\frac{\ell}{\ell_S}$ is "observable" (and of order one).

Fit and comparison

	HQCD	lattice $N_c = 3$	lattice $N_c \to \infty$	Parameter
$[p/(N_c^2T^4)]_{T=2T_c}$	**1.2**	**1.2**	-	$V1 = 14$
$L_h/(N_c^2T_c^4)$	**0.31**	0.28 (Karsch)	**0.31** (Teper+Lucini)	$V3 = 170$
$[p/(N_c^2T^4)]_{T\to+\infty}$	$\pi^2/45$	$\pi^2/45$	$\pi^2/45$	$M_p\ell = [45\pi^2]^{-1/3}$
$m_{0^{++}}/\sqrt{\sigma}$	**3.37**	3.56 (Chen)	**3.37** (Teper+Lucini)	$\ell_s/\ell = 0.92$
$m_{0^{-+}}/m_{0^{++}}$	**1.49**	**1.49** (Chen)	-	$c_a = 0.26$
χ	$(191MeV)^4$	$(191MeV)^4$ (DelDebbio)	-	$Z_0 = 133$
$T_c/m_{0^{++}}$	0.167	-	0.177(7)	
$m_{0^{*++}}/m_{0^{++}}$	1.61	1.56(11)	1.90(17)	
$m_{2^{++}}/m_{0^{++}}$	1.36	1.40(4)	1.46(11)	
$m_{0^{*-+}}/m_{0^{++}}$	2.10	2.12(10)	-	

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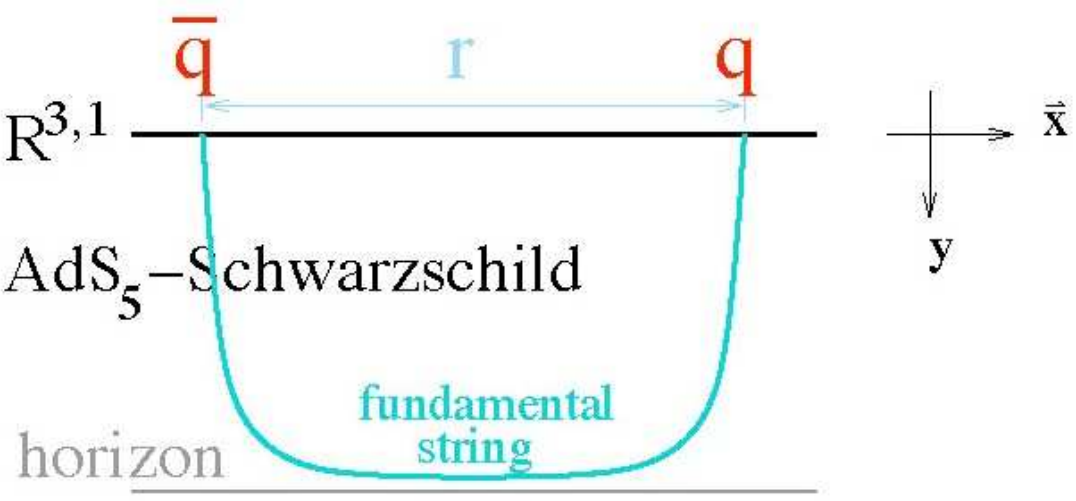
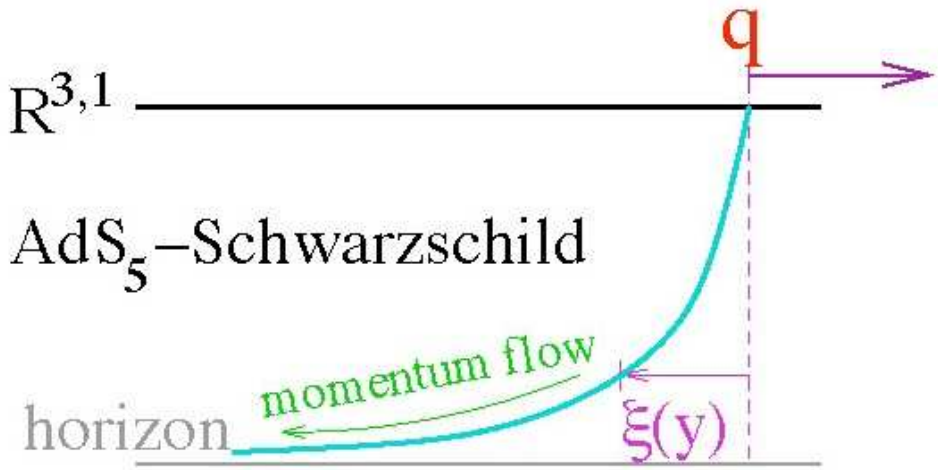
Heavy quarks and the drag force





- The dynamics is determined by the Nambu-Goto action.

$$S_{NG} = -\frac{1}{2\pi\ell_s^2}\int d\sigma d\tau \sqrt{det\left(-g_{MN}\partial_\alpha X^M \partial_\beta X^N\right)}\,,$$

- We must find a solution to the string equations with

$$x^1 = vt + \xi(r) \quad , \quad x^{2,3} = 0 \quad , \quad \sigma^1 = t \quad , \quad \sigma^2 = r$$

The spacetime metric is a black-hole metric (in string frame)

$$ds^2 = b(r)^2 \left[\frac{dr^2}{f(r)} - f(r)dt^2 + d\vec{x} \cdot d\vec{x} \right]$$

- The "momentum" conjugate to ξ is conserved

$$\pi_\xi = -\frac{1}{2\pi\ell_s^2} \frac{g_{00}g_{11}\xi'}{\sqrt{-g_{00}g_{rr} - g_{00}g_{11}\xi'^2 - g_{11}g_{rr}v^2}} \, .$$

We solve for ξ' to obtain

$$\xi' = \frac{\sqrt{-g_{00}g_{rr} - g_{11}g_{rr}v^2}}{\sqrt{g_{00}g_{11}\left(1 + g_{00}g_{11}/(2\pi\ell_s^2\pi_\xi)^2\right)}} \, .$$

- The solution profile is

$$\xi'(r) = \frac{C}{f(r)} \sqrt{\frac{f(r) - v^2}{b^4(r)f(r) - C^2}} \quad , \quad C = -(2\pi\ell_s^2)\ \pi_\xi = vb(r_s)^2 \quad , \quad f(r_s) = v^2$$

with r_s the turning point.

• The induced metric on the world-sheet is a 2d black-hole with horizon at the turning point $r = r_s$ ($t = \tau + \zeta(r)$).

$$ds^2 = b^2(r)\left[-(f(r)-v^2)d\tau^2 + \frac{1}{(f(r)-\frac{b^4(r_s)}{b^4(r)}v^2)}dr^2\right]$$

• We can calculate the drag force:

$$F_{\text{drag}} = \pi_\xi = -\frac{b^2(r_s)\sqrt{f(r_s)}}{2\pi\ell_s^2}$$

• In $\mathcal{N}=4$ sYM it is given by

$$F_{\text{drag}} = -\frac{\pi}{2}\sqrt{\lambda}\,T^2\frac{v}{\sqrt{1-v^2}} = -\frac{1}{\tau}\frac{p}{M} \quad , \quad \tau = \frac{2M}{\pi\sqrt{\lambda}\,T^2}$$

with τ the diffusion time.

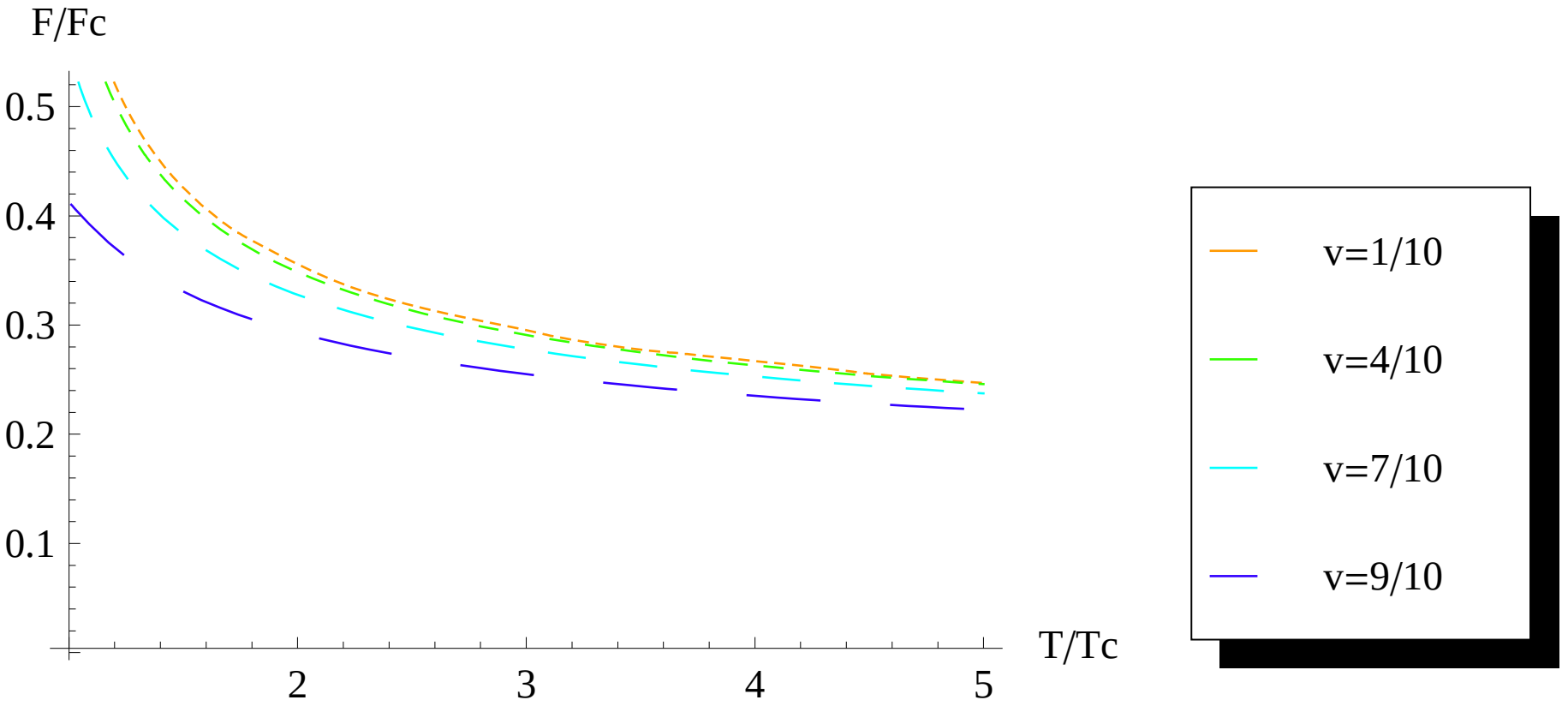
• For non-conformal theories it is a more complicated function of momentum and temperature.

The drag force in IhQCD

Systematic errors:

(a) Flavor description (heavy quark)

(b) Ignore light fermionic degrees of freedom in plasma



Gursoy+Kiritsis+Michalogiorgakis+Nitti, 2009

- F_{conf} calculated with $\lambda = 5.5$

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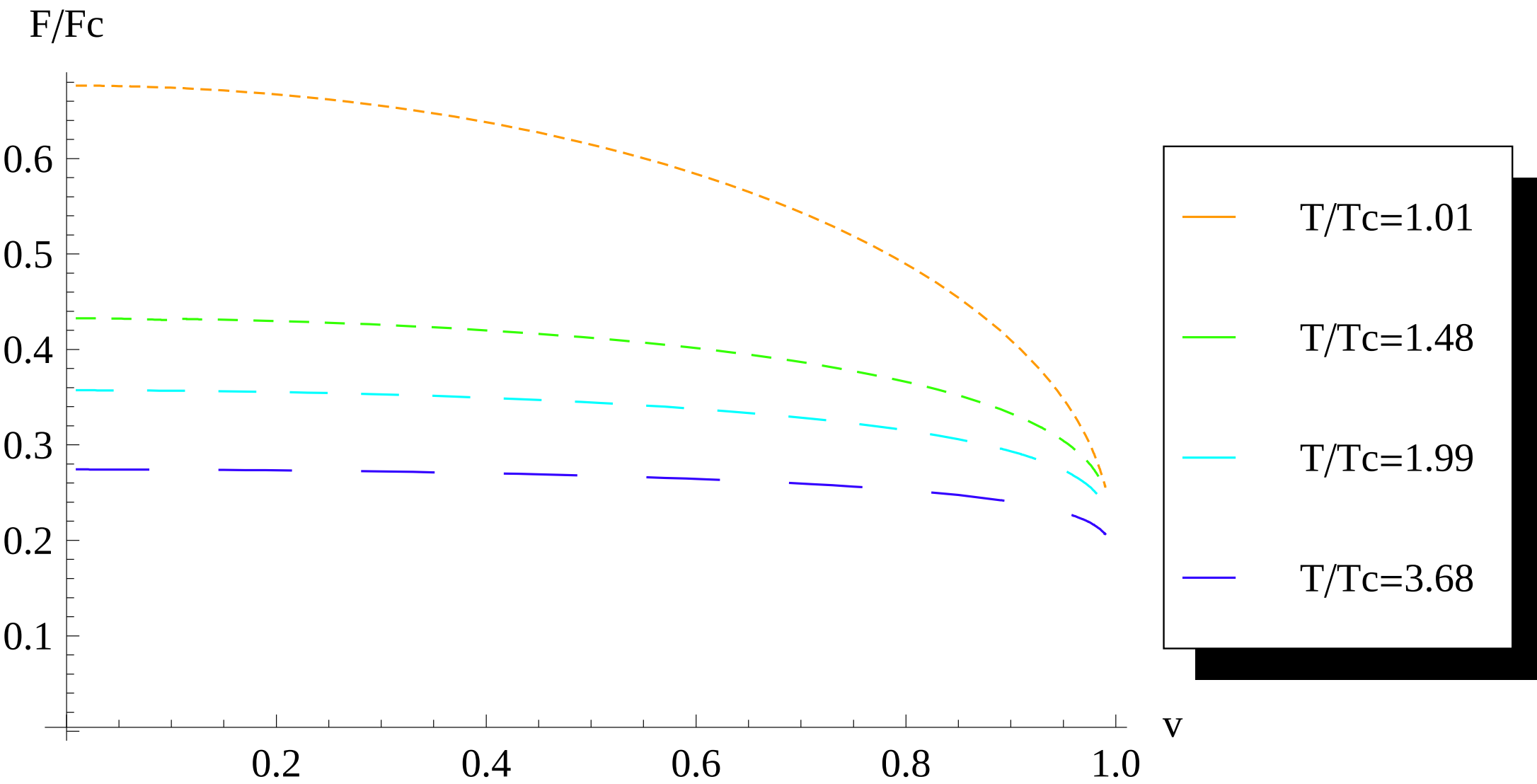
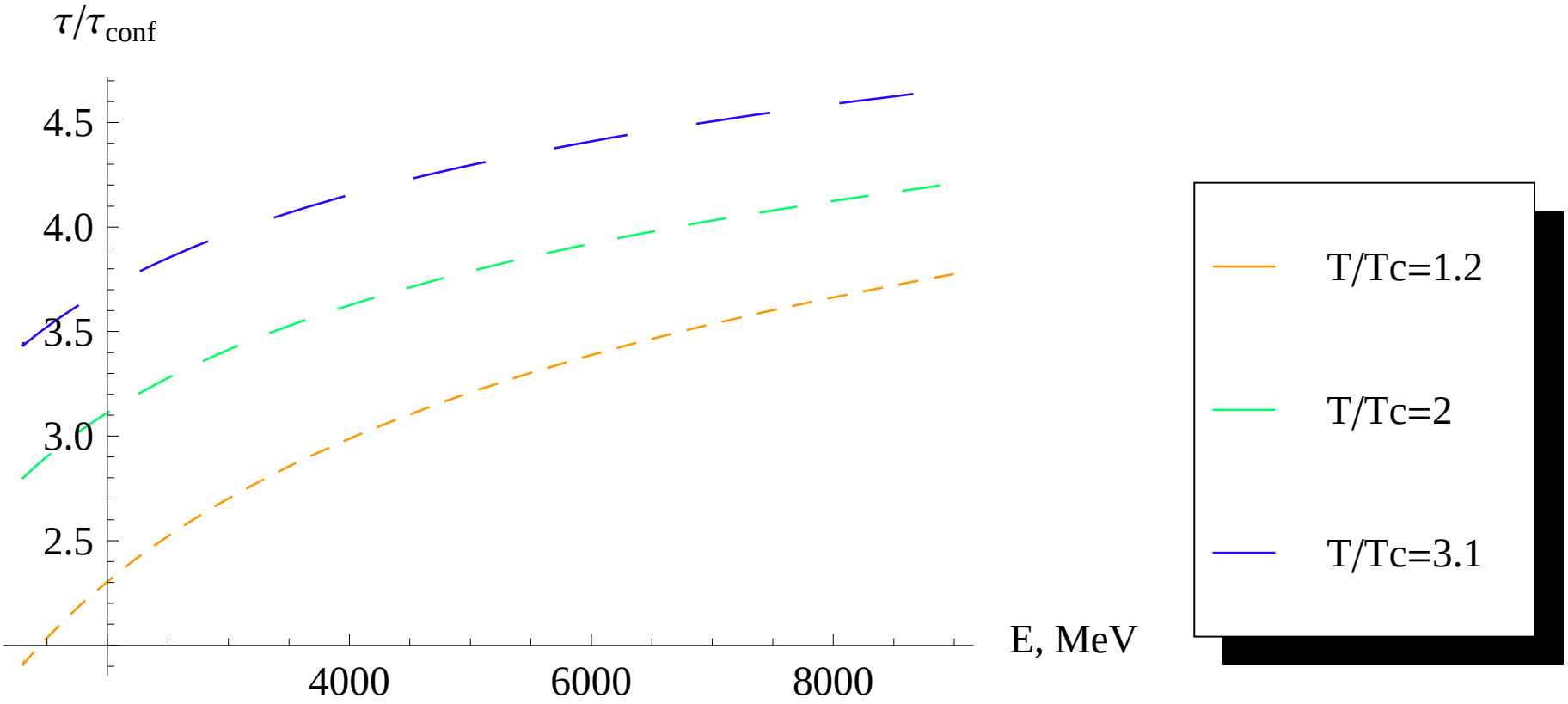


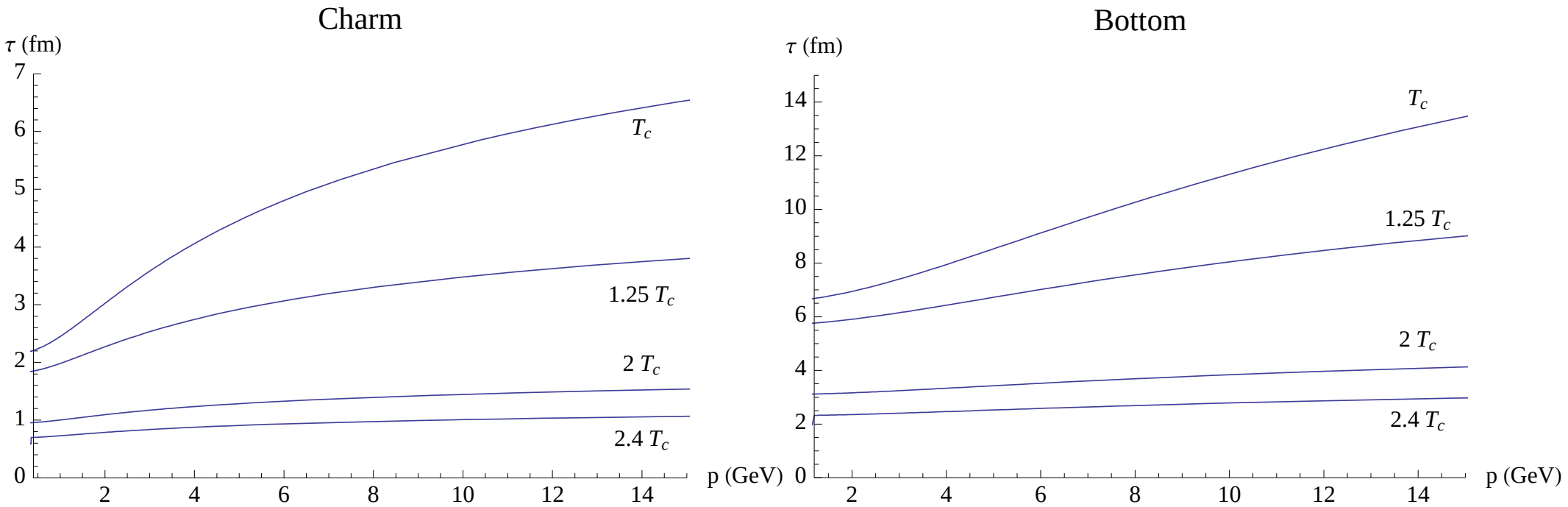
Gursoy+Kiritsis+Michalogiorgakis+Nitti, 2009

The diffusion time



Gursoy+Kiritsis+Michalogiorgakis+Nitti, 2009

$$\frac{dp}{dt} = -\frac{p}{\tau(p)}$$



Gursoy+Kiritsis+Michalogiorgakis+Nitti, 2009

	$\gamma = 0.3$	$\gamma = 1$	$\gamma = 3$
τ_c[fm]	22	6.7	2.2
τ_b[fm]	72	21	7.2

thermalized

not thermalized

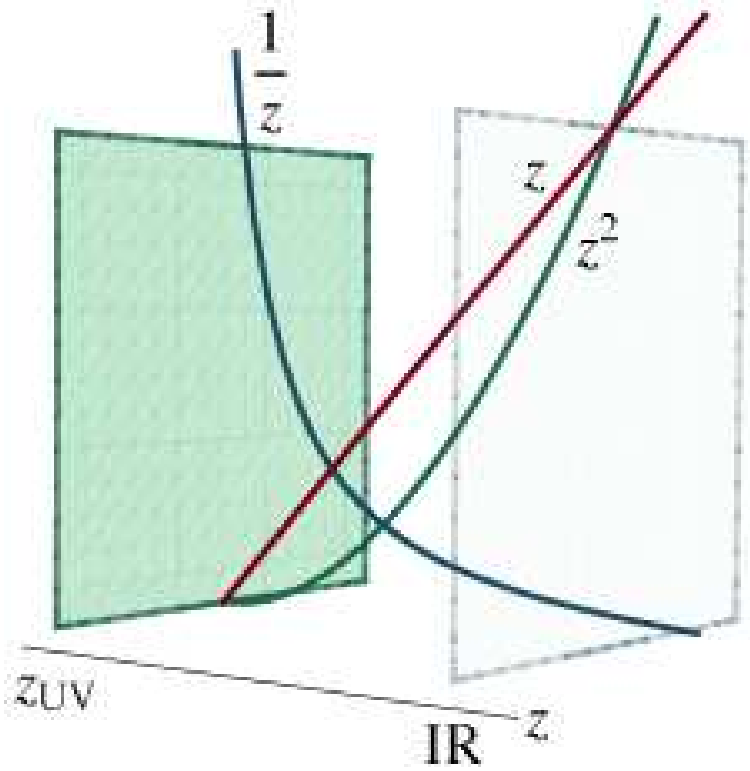
Akamatsu+Hatsuda+Hirano, 2008

The "soft wall"

♠ The asymptotic spectrum can be fixed by introducing a non-dynamical dilaton profile $\Phi \sim r^2$ (soft wall)

Karch+Katz+Son+Stephanov

• It is not a solution of equations of motion: the metric is still AdS: Neither $g_{\mu\nu}$ nor Φ solves the equations of motion.



The relevant "defects"

• $B_{\mu\nu} \rightarrow$ Fundamental string (F_1). This is the YM (glue) string: fundamental tension $\ell_s^2 \sim \mathcal{O}(1)$

• Its dual $\tilde{B}_\mu \rightarrow NS_0$: Tension is $\mathcal{O}(N_c^2)$. It is an effective magnetic baryon vertex binding N_c magnetic quarks.

• $C_5 \rightarrow D_4$: Space filling flavor branes. They must be introduced in pairs: $D_4 + \bar{D}_4$ for charge neutrality/tadpole cancelation $\rightarrow$ gauge anomaly cancelation in QCD.

• $C_4 \rightarrow D_3$ branes generating the gauge symmetry.

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• $C_3 \to D_2$ branes : domain walls separating different oblique confinement vacua (where $\theta_{k+1} = \theta_k + 2\pi$). Its tension is $\mathcal{O}(N_c)$

• $C_2 \to D_1$ branes: These are the magnetic strings:
(strings attached to magnetic quarks) with tension $\mathcal{O}(N_c)$

• $C_1 \to D_0$ branes. These are the baryon vertices: they bind N_c quarks, and their tension is $\mathcal{O}(N_c)$.
Its instantonic source when we add flavor is the (solitonic) baryon in the string theory.

• $C_0 \to D_{-1}$ branes: These are the Yang-Mills instantons.

The string effective action

- as $N_c \to \infty$, only string tree-level is dominant.

- Relevant field for the vacuum solution: $g_{\mu\nu}, a, \phi, F_5$.

- The vev of $F_5 \sim N_c\ \epsilon_5$. It appears always in the combination $e^{2\phi}F_5^2 \sim \lambda^2$, with $\lambda \sim N_c\ e^{\phi}$ All higher derivative corrections $(e^{2\phi}F_5^2)^n$ are $\mathcal{O}(1)$.
A non-trivial potential for the dilaton will be generated already at string tree-level.

- This is not the case for all other RR fields: in particular for the axion as $a \sim \mathcal{O}(1)$

$$(\partial a)^2 \sim \mathcal{O}(1) \quad , \quad e^{2\phi}(\partial a)^4 = \frac{\lambda^2}{N_c^2}(\partial a)^4 \sim \mathcal{O}\left(N_c^{-2}\right)$$

Therefore to leading order $\mathcal{O}(N_c^2)$ we can neglect the axion.

The UV regime

- In the far UV, the space should asymptote to AdS_5.

- The 't Hooft coupling should behave as $(r \to 0)$

$$\lambda \sim \frac{1}{\log(r\Lambda)} + \cdots \quad \to \quad 0 \quad , \quad r \sim \frac{1}{E}$$

- The effective action to leading order in N_c is

$$S_{eff} \sim \int d^5x\sqrt{g}\; e^{-2\phi}\; \Big(F(R\;,\;\xi\;) + \;4(\partial\phi)^2\;\Big) \quad , \quad \xi \equiv -e^{2\phi}\frac{F_5^2}{5!}$$

- For weak background fields

$$F = \frac{2}{3}\frac{\delta c}{\ell_s^2} + R + \frac{1}{2}\xi + \mathcal{O}(R^2, R\xi, \xi^2) \quad , \quad \delta c = 10 - 5 = 5$$

The equation for the four form is

$$\nabla^\mu \Big(F_\xi\; F_{\mu\nu\rho\sigma\tau}\Big) = 0 \quad , \quad F_\xi\; F_{\mu\nu\rho\sigma\tau} = \frac{N_c}{\ell_{AdS}}\frac{\epsilon_{\mu\nu\rho\sigma\tau}}{\sqrt{g}} \to \xi\; F_\xi(\xi, R)^2 = \frac{\lambda^2}{\ell_{AdS}^2}$$

We may use the alternative action where the 4-form is "integrated-out"

$$S_{\text{tree}} = M^3 N_c^2 \int d^5x\sqrt{g}\ \frac{1}{\lambda^2}\left[4\frac{\partial\lambda^2}{\lambda^2} + F(R,\xi) - 2\xi F_\xi(R,\xi)\right] \quad , \quad \xi\ F_\xi^2 = \frac{\lambda^2}{\ell_{AdS}^2}$$

To continue further we must solve $\xi\ F_\xi^2 = \frac{\lambda^2}{\ell_{AdS}^2}$. There are several possibilities:

(a) $\xi \to 0$ as $\lambda \to 0$ (turns out to be inconsistent with equations of motion).

(b) $\xi \to \xi_*(R)$ as $\lambda \to 0$.

$$F \simeq c_0(R) + \frac{c_1(R)}{2}(\xi - \xi_*(R))^2 + \mathcal{O}\left[(\xi - \xi_*(R))^3\right]$$

$$\xi \equiv \xi_*(R) + \delta\xi \simeq \xi_*(R) - \frac{\lambda}{c_1(R)\ \ell_{AdS}\ \sqrt{\xi_*(R)}} + \mathcal{O}(\lambda^2)$$

The gravitational equation implies that for AdS to be the leading solution (at $\lambda = 0$) we must have

$$c_0(R_*) = 0 \quad , \quad \left.\frac{\partial c_0(R)}{\partial R}\right|_{R=R_*} = 0$$

F is therefore zero to next order and the first non-trivial contribution is at quadratic order

$$F(R,\xi) = \frac{\lambda^2}{2c_1(R_*)\ \ell_{AdS}^2\ \xi_*(R_*)} + \frac{1}{2}\left.\frac{\partial^2 c_0(R)}{\partial R^2}\right|_{R=R_*}(R-R_*)^2 + \cdots$$

Solving the equations we find the one-loop β-function coefficients as

$$b_0 = \frac{\ell_{AdS}\sqrt{\xi_*(R_*)}}{16}$$

and the correction subleading correction to the AdS_5 metric

$$e^A = \frac{\ell}{r}\left[1 + \frac{w}{\log(\Lambda r)} + \cdots\right] \quad , \quad \delta R = \frac{40w}{\ell^2 \log(\Lambda r)} + \cdots$$

$$w = \frac{-5 + \frac{\frac{\delta\xi_*}{\delta R}(R*)}{\xi_*(R_*)}R_*}{c_0''(R_*)}\ \frac{\xi_*(R_*)}{80R_*}$$

- This turns out to be a regular expansion of the solution in powers of

$$\frac{P_n(\log\log(r\Lambda))}{(log(r\Lambda))^n}$$

- Effectively this can be rearranged as a "perturbative" expansion in $\lambda(r)$.

In the case of running coupling, the radial coordinate can be substituted by $\lambda(r)$.

- Using λ as a radial coordinate the solution for the metric can be written

$$E \equiv e^A = \frac{\ell}{r(\lambda)}\left[1 + c_1\lambda + c_2\lambda^2 + \cdots\right] = \ell\,(e^{-\frac{b_0}{\lambda}})\left[1 + c_1'\lambda + c_2'\lambda^2 + \cdots\right] \quad , \quad \lambda \to 0$$

The axion

Similar arguments lead to an action of the form

$$S = N_c^2 \; S_{g,\phi} + S_{axion} + \cdots$$

$$S_{axion} \sim \int d^5x \sqrt{g} \; G(R,\lambda) \; (\partial a)^2$$

- Higher powers of $(\partial a)^2$ are subleading in N_c.

- We may therefore find the solution using the solution of the metric-dilaton system.

UV conclusions

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Conclusion 1: The asymptotic AdS_5 is stringy, but the rest of the geometry is "perturbative around the asymptotics". We cannot however do computations even if we know the structure.

Conclusion 2: It has been a mystery how can one get free field theory at the boundary. This is automatic here since all non-trivial connected correlators are proportional to positive powers of λ that vanishes in the UV.

The IR regime: general ex[ectations

• Here the situation is more obscure. The constraints/input will be: confinement, discreteness of the spectrum and mass gap.

• We do expect that $\lambda \to \infty$ (or becomes large) at the IR bottom.

• Intuition from N=4 and other 10d strongly coupled theories suggests that in this regime there should be an (approximate) two-derivative description of the physics.

• The simplest solution with this property is the linear dilaton solution with

$$\lambda \sim e^{Qr} \quad , \quad V(\lambda) \sim \delta c = 10 - D \quad \to \quad \text{constant} \quad , \quad R = 0$$

• Self-consistency of this assumption implies that the string frame curvature should vanish in the IR.

• This property persists with potentials $V(\lambda) \sim (\log \lambda)^P$. Moreover all such cases have confinement, a mass gap and a discrete spectrum (except the P=0 case).

• At the IR bottom (in the string frame) the scale factor vanishes, and 5D space becomes (asymptotically) flat.

The axion background

- The axion solution can be interpreted as a "running" θ-angle

- This is in accordance with the absence of UV divergences (all correlators $\langle Tr[F \wedge F]^n \rangle$ are UV finite), and Seiberg-Witten type solutions.

- The axion action is down by $1/N_c^2$

$$S_{axion} = -\frac{M_p^3}{2} \int d^5x \sqrt{g}\ Z(\lambda)\ (\partial a)^2$$

$$\lim_{\lambda \to 0} Z(\lambda) = Z_0 \left[1 + c_1 \lambda + c_2 \lambda^2 + \cdots\right] \quad , \quad \lim_{\lambda \to \infty} Z(\lambda) = c_a \lambda^d + \cdots \quad , \quad d = 4$$

- The equation of motion is

$$\ddot{a} + \left(3\dot{A} + \frac{\dot{Z}(\lambda)}{Z(\lambda)}\right) \dot{a} = 0 \quad \rightarrow \quad \dot{a} = \frac{C\ e^{-3A}}{Z(\lambda)}$$

- The full solution is

$$a(r) = \theta_{UV} + 2\pi k + C \int_0^r dr \frac{e^{-3A}}{Z(\lambda)} \quad , \quad C = \langle Tr[F \wedge F] \rangle$$

- $a(r)$ is a running effective θ-angle. Its running is non-perturbative,

$$a(r) \sim r^4 \sim e^{-\frac{4}{b_0 \lambda}}$$

- The vacuum energy is

$$E(\theta_{UV}) = -\frac{M^3}{2} \int d^5x \sqrt{g}\ Z(\lambda)\ (\partial a)^2 = -\frac{M^3}{2} C a(r) \Big|_{r=0}^{r=r_0}$$

- Consistency requires to impose that $a(r_0) = 0$. This determines C and

$$E(\theta_{UV}) = \frac{M^3}{2} \text{Min}_k \frac{(\theta_{UV} + 2\pi k)^2}{\int_0^{r_0} \frac{dr}{e^{3A} Z(\lambda)}}$$

$$\frac{a(r)}{\theta_{UV} + 2\pi k} = \frac{\int_r^{r_0} \frac{dr}{e^{3A} Z(\lambda)}}{\int_0^{r_0} \frac{dr}{e^{3A} Z(\lambda)}}$$

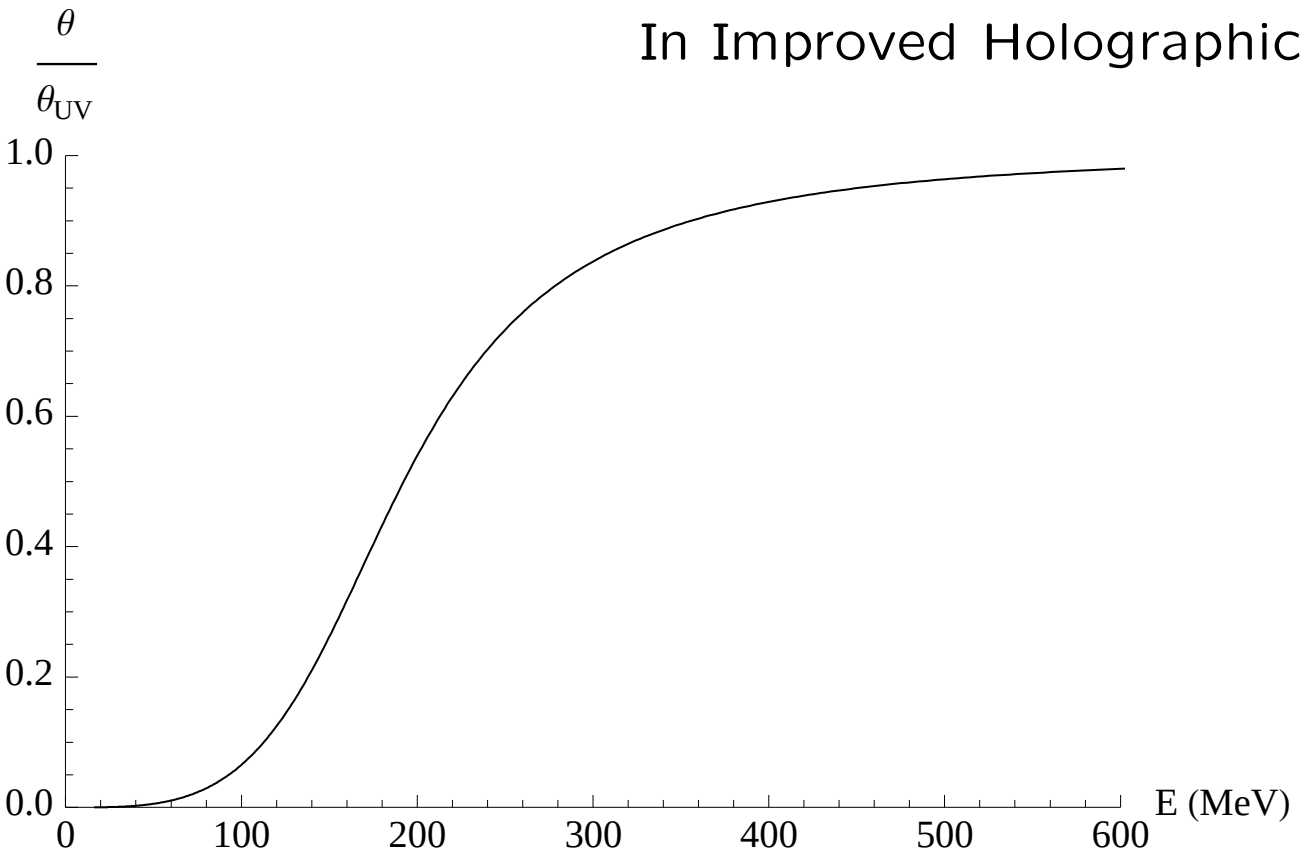
- The topological susceptibility is given by

$$E(\theta) = \frac{1}{2} \chi\ \theta^2 + \mathcal{O}(\theta^4) \quad , \quad \chi = \frac{M^3}{\int_0^{r_0} \frac{dr}{e^{3A} Z(\lambda)}}$$

• The effective θ-angle "runs" also in the D4 model for QCD, and also vanishes in the IR

$$\theta(U) = \theta(1 - U_0^3/U^3)$$

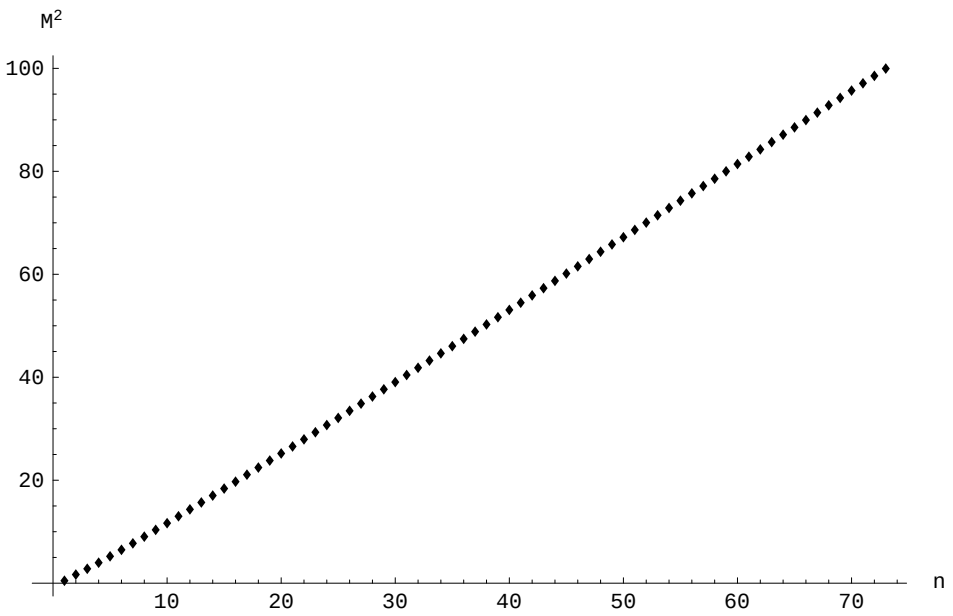
• In Improved Holographic QCD:



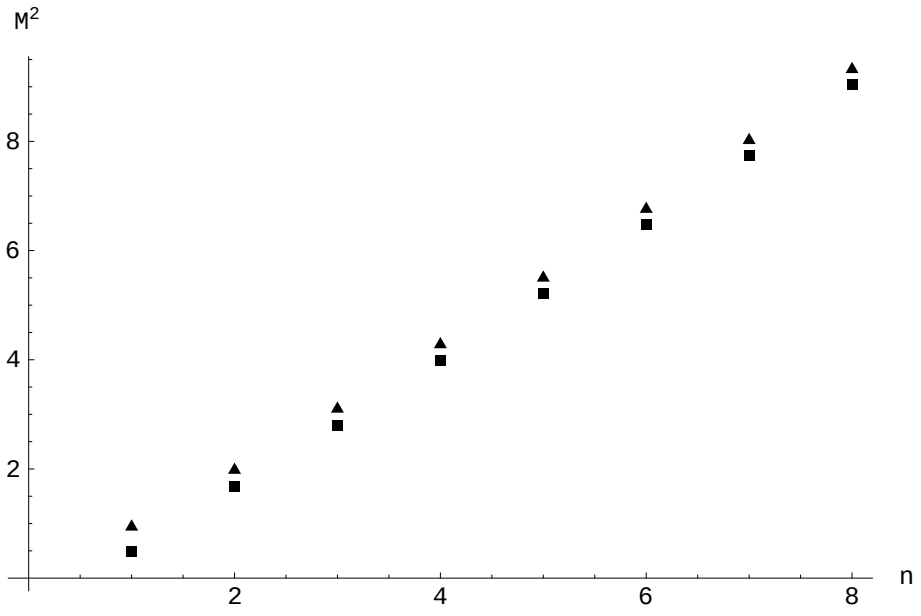
We have taken: $Z(\lambda) = Z_0(1 + c_a\lambda^4) \simeq 133(1 + 0.26\lambda^4)$

RETURN

Linearity of the glueball spectrum



(a)

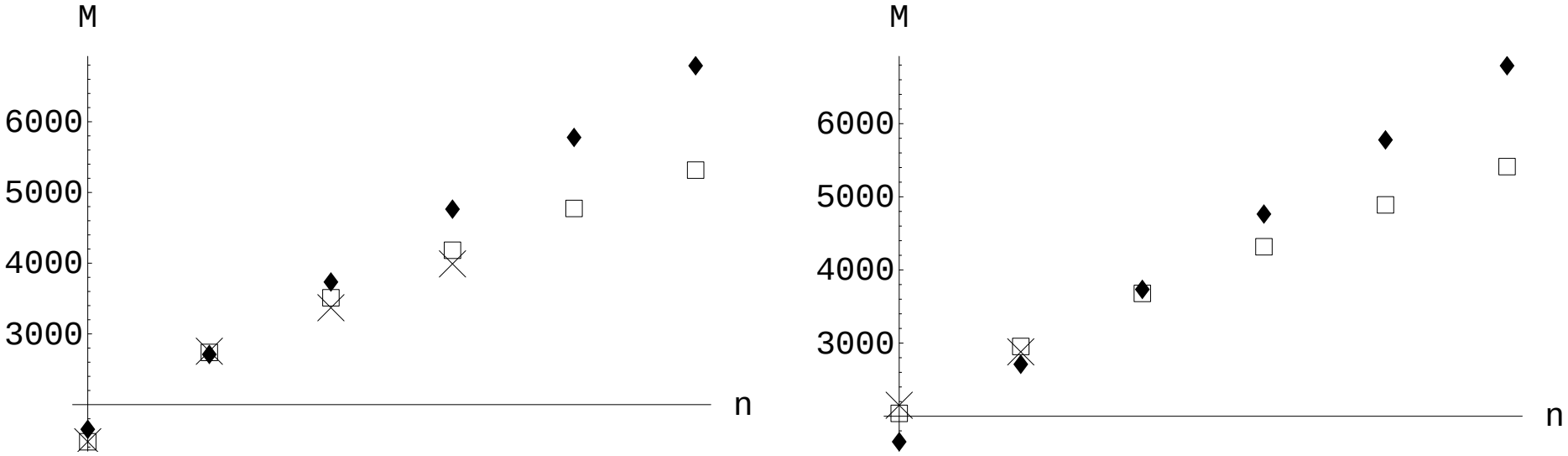


(b)

(a) Linear pattern in the spectrum for the first 40 0^{++} glueball states. M^2 is shown units of $0.015\ell^{-2}$.

(b) The first 8 0^{++} (squares) and the 2^{++} (triangles) glueballs. These spectra are obtained in the background I with $b_0 = 4.2, \lambda_0 = 0.05$.

Comparison with lattice data (Meyer)



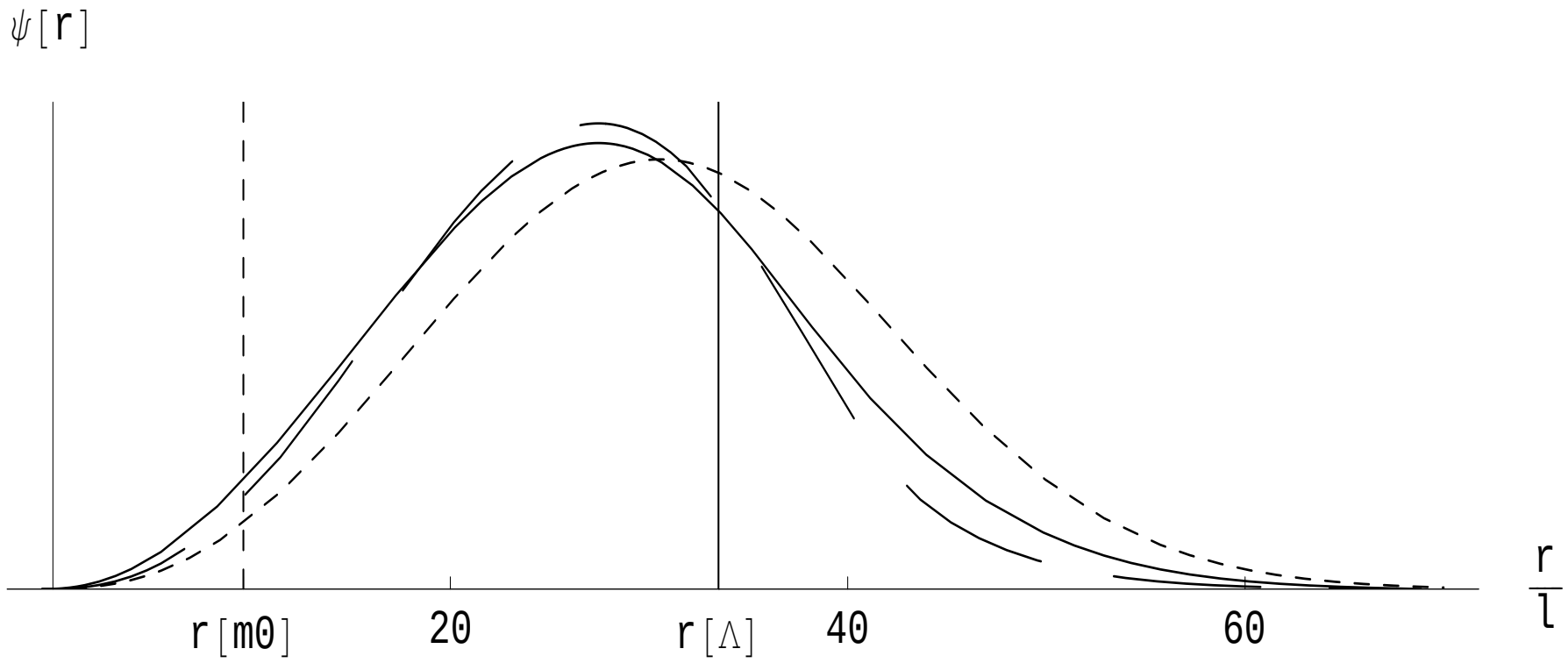
Comparison of glueball spectra from our model with $b_0 = 4.2, \lambda_0 = 0.05$ (boxes), with the lattice QCD data from Ref. I (crosses) and the AdS/QCD computation (diamonds), for (a) 0^{++} glueballs; (b) 2^{++} glueballs. The masses are in MeV, and the scale is normalized to match the lowest 0^{++} state from Ref. I.

The fit to glueball lattice data

J^{PC}	Ref I (MeV)	Our model (MeV)	Mismatch	$N_c \to \infty$	Mismatch
0^{++}	**1475 (4%)**	**1475**	0	**1475**	0
2^{++}	2150 (5%)	2055	4%	2153 (10%)	5%
0^{-+}	**2250 (4%)**	**2243**	0		
0^{++*}	**2755 (4%)**	**2753**	0	2814 (12%)	2%
2^{++*}	2880 (5%)	2991	4%		
0^{-+*}	3370 (4%)	3288	2%		
0^{++**}	3370 (4%)	3561	5%		
0^{++***}	3990 (5%)	4253	6%		

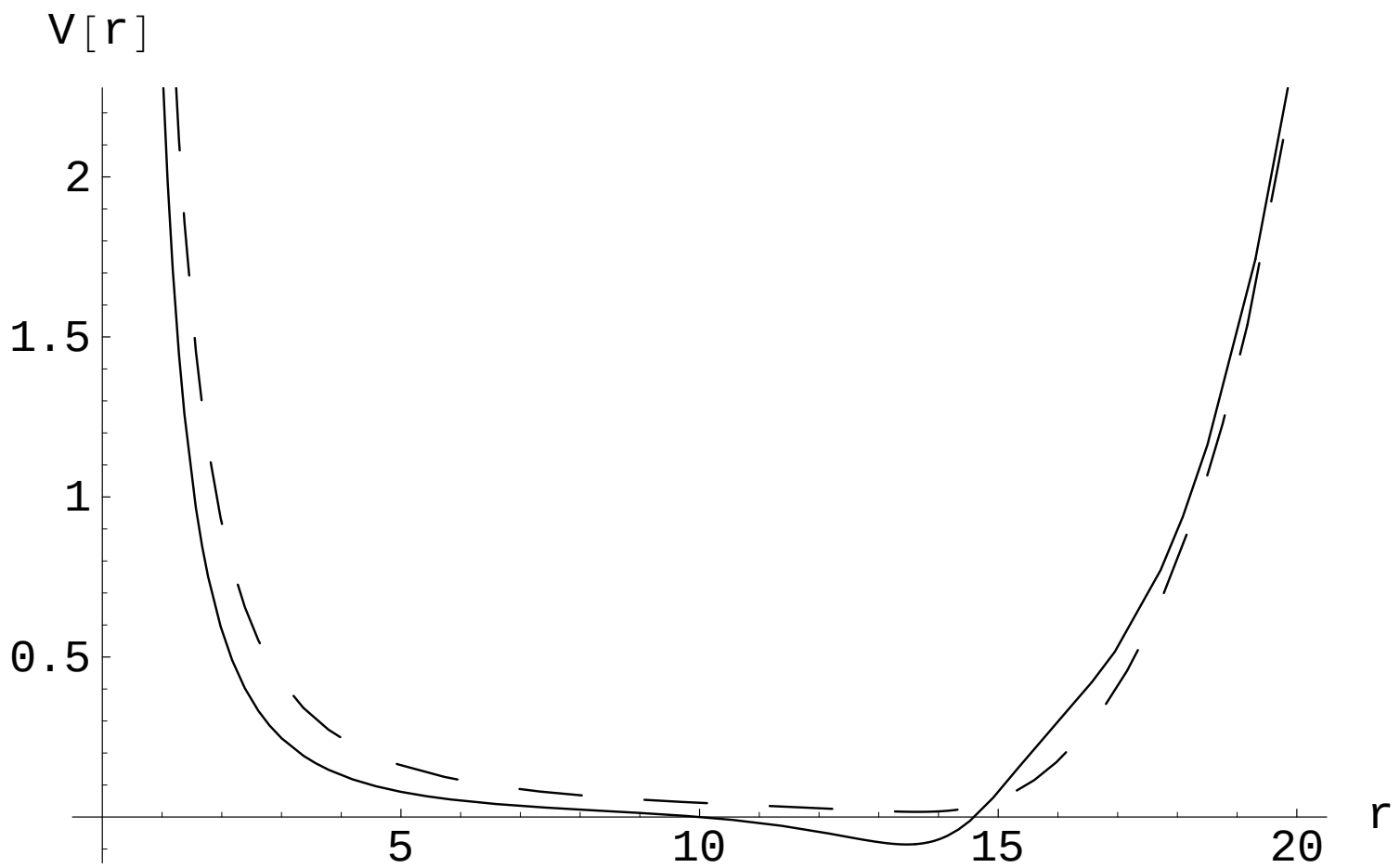
Comparison between the glueball spectra in Ref. I and in our model. The states we use as input in our fit are marked in red. The parenthesis in the lattice data indicate the percent accuracy.

The glueball wavefunctions



Normalized wave-function profiles for the ground states of the 0^{++} (solid line) ,0^{-+} (dashed line), and 2^{++} (dotted line) towers, as a function of the radial conformal coordinate. The vertical lines represent the position corresponding to $E = m_{0^{++}}$ and $E = \Lambda_p$.

Comparison of scalar and tensor potential



Effective Schrödinger potentials for scalar (solid line) and tensor (dashed line) glueballs. The units are chosen such that $\ell = 0.5$.

The tachyon DBI action

- The flavor action is the $D_4 - \bar{D}_4$ action: $S[T, A^L, A^R] = S_{DBI} + S_{WZ}$

$$S_{DBI} = \int dr d^4x \, \frac{N_c}{\lambda} \, \mathbf{Str} \left[V(T) \left(\sqrt{-\det\left(g_{\mu\nu} + D_{\{\mu} T^\dagger D_{\nu\}} T + F^L_{\mu\nu}\right)} + \right.\right.$$

$$\left.\left. + \sqrt{-\det\left(g_{\mu\nu} + D_{\{\mu} T^\dagger D_{\nu\}} T + F^R_{\mu\nu}\right)} \right) \right]$$

$$D_\mu T \equiv \partial_\mu T - i T A^L_\mu + i A^R_\mu T \quad , \quad D_\mu T^\dagger \equiv \partial_\mu T^\dagger - i A^L_\mu T^\dagger + i T^\dagger A^R_\mu$$

transforming covariantly under flavor gauge transformations

$$T \to V_R T V_L^\dagger \quad , \quad A^L \to V_L (A^L - i V_L^\dagger d V_L) V_L^\dagger \quad , \quad A^R \to V_R (A^R - i V_R^\dagger d V_R) V_R^\dagger$$

- For the vacuum structure and spectrum $Str = Tr$.

The tachyon WZ action

- The WZ action is given by

Kennedy+Wilkins, Kraus+Larsen, Takayanagi+Terashima+Uesugi

$$S_{WZ} = T_4 \int_{M_5} C \wedge \mathrm{Str}\ \exp\left[i2\pi\alpha' \mathcal{F}\right]$$

- M_5 is the world-volume of the D4-$\overline{\text{D4}}$ branes that coincides with the full space-time.

- C is a formal sum of the RR potentials $C = \sum_n (-i)^{\frac{5-n}{2}} C_n$,

- $\mathcal{F}$ is the curvature of a superconnection $\mathcal{A}$:

$$i\mathcal{A} = \begin{pmatrix} iA_L & T^\dagger \\ T & iA_R \end{pmatrix}, \qquad i\mathcal{F} = \begin{pmatrix} iF_L - T^\dagger T & DT^\dagger \\ DT & iF_R - TT^\dagger \end{pmatrix}$$

$$\mathcal{F} = d\mathcal{A} - i\mathcal{A} \wedge \mathcal{A} \quad , \quad d\mathcal{F} - i\mathcal{A} \wedge \mathcal{F} + i\mathcal{F} \wedge \mathcal{A} = 0$$

- Under (flavor) gauge transformation it transforms homogeneously

$$\mathcal{F} \to \begin{pmatrix} V_L & 0 \\ 0 & V_R \end{pmatrix} \mathcal{F} \begin{pmatrix} V_L^\dagger & 0 \\ 0 & V_R^\dagger \end{pmatrix}$$

- Expanding:

$$S_{WZ} = T_4 \int C_5 \wedge Z_0 + C_3 \wedge Z_2 + C_1 \wedge Z_4 + C_{-1} \wedge Z_6$$

where Z_{2n} are appropriate forms coming from the expansion of the exponential of the superconnection.

- $Z_0 = 0$, signaling the global cancelation of 4-brane charge, which is equivalent to the cancelation of the gauge anomaly in QCD.

$$Z_2 = d\Omega_1 \quad , \quad \Omega_1 = iSTr(V(T^\dagger T))Tr(A_L - A_R) - \log\det(T) d(StrV(T^\dagger T))$$

Casero+Kiritsis+Paredes

- This term provides the Stuckelberg mixing between $Tr[A_\mu^L - A_\mu^R]$ and the QCD axion that is dual to C_3. Dualizing the full action we obtain

$$S_{CP-odd} = \frac{M^3}{2N_c^2} \int d^5x \sqrt{g} Z(\lambda) \left(\partial a + i\Omega_1\right)^2$$

$$= \frac{M^3}{2}\int d^5x\sqrt{g}Z(\lambda)\left(\partial_\mu a + \zeta\partial_\mu V(\tau) - \sqrt{\frac{N_f}{2}}V(\tau)A_\mu^A\right)^2$$

$$\zeta = \Im \log\det T \quad , \quad A_L - A_R \equiv \frac{1}{2N_f}A^A\mathbf{I} + (A_L^a - A_R^a)\lambda^a$$

• This term is invariant under the $U(1)_A$ transformations, reflecting the QCD $U(1)_A$ anomaly.

$$\zeta \to \zeta + \epsilon \quad , \quad A_\mu^A \to A_\mu^A - \sqrt{\frac{2}{N_f}}\partial_\mu\epsilon \quad , \quad a \to a - N_f\epsilon V(\tau)$$

• This is responsible for the mixing between the QCD axion and the η' $\to$ we have two scalars a, ζ and an (axial) vector, A_μ^A. Then an appropriate linear combination of the two scalars will become the 0^{-+} glueball field while the other will be the η'. The transverse (5d) vector will provide the tower of $U(1)_A$ vector mesons.

• The term $C_1 \times Z_4 \sim V\ C_1\left[F_L\wedge F_L + F_R\wedge F_R\right] + \cdots$ couples the flavor instanton density to the baryon vertex.

• Using $Z_6 = d\Omega_5$ we may rewrite the last term as

$$\int F_0\wedge\Omega_5 \quad , \quad F_0 = dC_{-1}$$

$F_0 \sim N_c$ is nothing else but the dual of the five-form field strength. This term then provides the correct Chern-Simons form that reproduces the flavor anomalies of QCD. It contains the tachyon non-trivially

Casero+Kiritsis+Paredes

• To proceed further and analyze the vacuum solution we set $T = \tau$ 1 and set the vectors to zero. Then the DBI action collapses to

$$S[\tau, A_M] = N_c N_f \int dr d^4x \; e^{-\Phi} V(\tau) \sqrt{-\det\left(g_{\mu\nu} + \partial_\mu \tau \partial_\nu \tau\right)}$$

We assume the following tachyon potential, motivated/calculated in studies of tachyon condensation:

$$V(\tau) = V_0 e^{-\frac{\mu^2}{2}\tau^2}$$

where μ has dimension of mass. It is fixed by the requirement that τ has the correct bulk mass to couple to the quark bilinear operator on the boundary.

Tachyon dynamics

- In the vacuum the gauge fields vanish and $T \sim 1$. Only DBI survives

$$S[\tau] = T_{D_4} \int dr d^4x \, \frac{e^{4A_s(r)}}{\lambda} \, V(\tau) \, \sqrt{e^{2A_s(r)} + \dot{\tau}(r)^2} \quad , \quad V(\tau) = e^{-\frac{\mu^2}{2}\tau^2}$$

- We obtain the nonlinear field equation:

$$\ddot{\tau} + \left(3\dot{A}_S - \frac{\dot{\lambda}}{\lambda}\right)\dot{\tau} + e^{2A_S}\mu^2\tau + e^{-2A_S}\left[4\dot{A}_S - \frac{\dot{\lambda}}{\lambda}\right]\dot{\tau}^3 + \mu^2\tau\,\dot{\tau}^2 = 0.$$

- In the UV we expect

$$\tau = m_q \, r + \sigma \, r^3 + \cdots \quad , \quad \mu^2\ell^2 = 3$$

- We expect that the tachyon must diverge before or at $r = r_0$. We find that indeed it does at the (dilaton) singularity. For the $r_0 = \infty$

backgrounds

$$\tau \sim \exp\left[\frac{2}{a}\,\frac{R}{\ell^2}\,r\right] \qquad \text{as} \qquad r \to \infty$$

• Generically the solutions have spurious singularities: $\tau(r_*)$ stays finite but its derivatives diverges because:

$$\tau \sim \tau_* + \gamma\sqrt{r_* - r}.$$

The condition that they are absent determines σ as a function of m_q.

• The easiest spectrum to analyze is that of vector mesons. We find ($r_0 = \infty$)

$$\Lambda_{glueballs} = \frac{1}{R}, \qquad \Lambda_{mesons} = \frac{3}{\ell}\left(\frac{\alpha\ell^2}{2R^2}\right)^{(\alpha-1)/2} \propto \frac{1}{R}\left(\frac{\ell}{R}\right)^{\alpha-2}.$$

This suggests that $\alpha = 2$ preferred also from the glue sector (linear trajectories).

Organizing the vacuum solutions

- The β-function can be mapped uniquely to the dilaton potential $V(\lambda)$.

- A useful variable is the phase variable

$$X \equiv \frac{\lambda'}{3\lambda A'} = \frac{\beta(\lambda)}{3\lambda}$$

- We can introduce a (pseudo)superpotential

$$V(\lambda) = \left(\frac{4}{3}\right)^3 \left[W^2 - \left(\frac{3}{4}\right)^2 \left(\frac{\partial W}{\partial \Phi}\right)^2\right]$$

and write the equations in a first order form:

$$A' = -\frac{4}{9}W \quad , \quad \Phi' = \frac{dW}{d\Phi}$$

$$\beta(\lambda) = -\frac{9}{4}\lambda \frac{d\log W}{d\log \lambda}$$

♠ The equations have three integration constants: (two for Φ and one for A) One is fixed by $\lambda \to 0$ in the UV. The other is Λ. The one in A is the choice of energy scale.

The IR regime

For any asymptotically AdS_5 solution ($e^A \sim \frac{\ell}{r}$):

- The scale factor $e^{A(r)}$ is monotonically decreasing

Girardelo+Petrini+Porrati+Zaffaroni
Freedman+Gubser+Pilch+Warner

- Moreover, there are only three possible, mutually exclusive IR asymptotics:

♠ *there is another asymptotic AdS_5 region, at $r \to \infty$, where* $\exp A(r) \sim \ell'/r$, *and $\ell' \leq \ell$* (equality holds if and only if the space is exactly AdS_5 everywhere);

♠ there is a curvature singularity at some finite value of the radial coordinate, $r = r_0$;

♠ **there is a curvature singularity at $r \to \infty$, where the scale factor vanishes and the space-time shrinks to zero size.**

Wilson-Loops and confinement

• Calculation of the static quark potential using the vev of the Wilson loop calculated via an F-string worldsheet.

Rey+Yee, Maldacena

$$T\ E(L) = S_{minimal}(X)$$

We calculate

$$L = 2\int_0^{r_0} dr \frac{1}{\sqrt{e^{4A_S(r)-4A_S(r_0)}-1}}.$$

It diverges when e^{A_s} has a minimum (at $r = r_*$). Then

$$E(L) \sim T_f\ e^{2A_S(r_*)}\ L$$

• Confinement $\rightarrow$ $A_s(r_*)$ is finite. This is a more general condition that considered before as A_S `is not monotonic in general`.

• Effective string tension

$$T_{\text{string}} = T_f\ e^{2A_S(r_*)}$$

Comments on confining backgrounds

• For all confining backgrounds with $Q = 4/3,\ 0 < P < 1$ although the space-time is singular in the Einstein frame, the string frame geometry is asymptotically flat for large r. Therefore only λ grows indefinitely.

• String world-sheets do not probe the strong coupling region, at least classically. The string stays away from the strong coupling region.

• Therefore: singular confining backgrounds have generically the property that the singularity is *repulsive*, i.e. only highly excited states can probe it. This will also be reflected in the analysis of the particle spectrum (to be presented later)

• The confining backgrounds must also screen magnetic color charges. This can be checked by calculating 't Hooft loops using D_1 probes:

♠ All confining backgrounds with $r_0 = \infty$ and most at finite r_0 screen properly

♠ In particular "hard-wall" AdS/QCD confines also the magnetic quarks.

Particle Spectra: generalities

- Linearized equation:

$$\ddot{\xi} + 2\dot{B}\dot{\xi} + \Box_4 \xi = 0 \quad , \quad \xi(r,x) = \xi(r)\xi^{(4)}(x), \qquad \Box\xi^{(4)}(x) = m^2\xi^{(4)}(x)$$

- Can be mapped to Schrodinger problem

$$-\frac{d^2}{dr^2}\psi + V(r)\psi = m^2\psi \quad , \quad V(r) = \frac{d^2B}{dr^2} + \left(\frac{dB}{dr}\right)^2 \quad , \quad \xi(r) = e^{-B(r)}\psi(r)$$

- Mass gap and discrete spectrum visible from the asymptotics of the potential.

- Large n asymptotics of masses obtained from WKB

$$n\pi = \int_{r_1}^{r_2} \sqrt{m^2 - V(r)}\ dr$$

- Spectrum depends only on initial condition for λ ($\sim \Lambda_{QCD}$) and an overall energy scale (e^A) that must be fixed.

- scalar glueballs

$$B(r) = \frac{3}{2}A(r) + \frac{1}{2}\log\frac{\beta(\lambda)^2}{9\lambda^2}$$

- tensor glueballs

$$B(r) = \frac{3}{2}A(r)$$

- pseudo-scalar glueballs

$$B(r) = \frac{3}{2}A(r) + \frac{1}{2}\log Z(\lambda)$$

- Universality of asymptotics

$$\frac{m^2_{n\to\infty}(0^{++})}{m^2_{n\to\infty}(2^{++})} \to 1 \quad , \quad \frac{m^2_{n\to\infty}(0^{+-})}{m^2_{n\to\infty}(0^{++})} = \frac{1}{4}(d-2)^2$$

predicts $d = 4$ via

$$\frac{m^2}{2\pi\sigma_a} = 2n + J + c,$$

General criterion for confinement

• `the geometric version`:
A geometry that shrinks to zero size in the IR is dual to a confining 4D theory if and only if the Einstein metric in conformal coordinates vanishes as (or faster than) e^{-Cr} as $r \to \infty$, for some $C > 0$.

• It is understood here that a metric vanishing at finite $r = r_0$ also satisfies the above condition.

♠ `the superpotential`
A 5D background is dual to a confining theory if the superpotential grows as (or faster than)

$$W \sim (\log \lambda)^{P/2} \lambda^{2/3} \quad \text{as} \quad \lambda \to \infty \quad , \quad P \geq 0$$

♠ `the` β`-function` A 5D background is dual to a confining theory if and only if

$$\lim_{\lambda \to \infty} \left(\frac{\beta(\lambda)}{3\lambda} + \frac{1}{2} \right) \log \lambda = K, \qquad -\infty \leq K \leq 0$$

(No explicit reference to any coordinate system) Linear trajectories correspond to $K = -\frac{3}{16}$

Classification of confining superpotentials

Classification of confining superpotentials $W(\lambda)$ as $\lambda \to \infty$ in IR:

$$W(\lambda) \sim (\log \lambda)^{\frac{P}{2}} \lambda^Q \quad , \quad \lambda \sim E^{-\frac{9}{4}Q} \left(\log \frac{1}{E}\right)^{\frac{P}{2Q}} , \qquad E \to 0.$$

- $Q > 2/3$ or $Q = 2/3$ and $P > 1$ leads to confinement and a singularity at finite $r = r_0$.

$$e^A(r) \sim \begin{cases} (r_0 - r)^{\frac{4}{9Q^2-4}} & Q > \frac{2}{3} \\ \exp\left[-\frac{C}{(r_0-r)^{1/(P-1)}}\right] & Q = \frac{2}{3} \end{cases}$$

- $Q = 2/3$, and $0 \leq P < 1$ leads to confinement and a singularity at $r = \infty$ The scale factor e^A vanishes there as

$$e^A(r) \sim \exp[-Cr^{1/(1-P)}].$$

- $Q = 2/3, P = 1$ leads to confinement but the singularity may be at a finite or infinite value of r depending on subleading asymptotics of the superpotential.

♠ If $Q < 2\sqrt{2}/3$, no *ad hoc* boundary conditions are needed to determine the glueball spectrum → One-to-one correspondence with the β-function This is unlike standard AdS/QCD and other approaches.

- when $Q > 2\sqrt{2}/3$, the spectrum is not well defined without extra boundary conditions in the IR because both solutions to the mass eigenvalue equation are IR normalizable.

Confining β-functions

A 5D background is dual to a confining theory if and only if

$$\lim_{\lambda\to\infty}\left(\frac{\beta(\lambda)}{3\lambda}+\frac{1}{2}\right)\log\lambda=K,\qquad -\infty\leq K\leq 0$$

(No explicit reference to any coordinate system). Linear trajectories correspond to $K=-\frac{3}{16}$

- We can determine the geometry if we specify K:
- $K=-\infty$: the scale factor goes to zero at some finite r_0, not faster than a power-law.
- $-\infty<K<-3/8$: the scale factor goes to zero at some finite r_0 faster than any power-law.
- $-3/8<K<0$: the scale factor goes to zero as $r\to\infty$ faster than $e^{-Cr^{1+\epsilon}}$ for some $\epsilon>0$.
- $K=0$: the scale factor goes to zero as $r\to\infty$ as e^{-Cr} (or faster), but slower than $e^{-Cr^{1+\epsilon}}$ for any $\epsilon>0$.

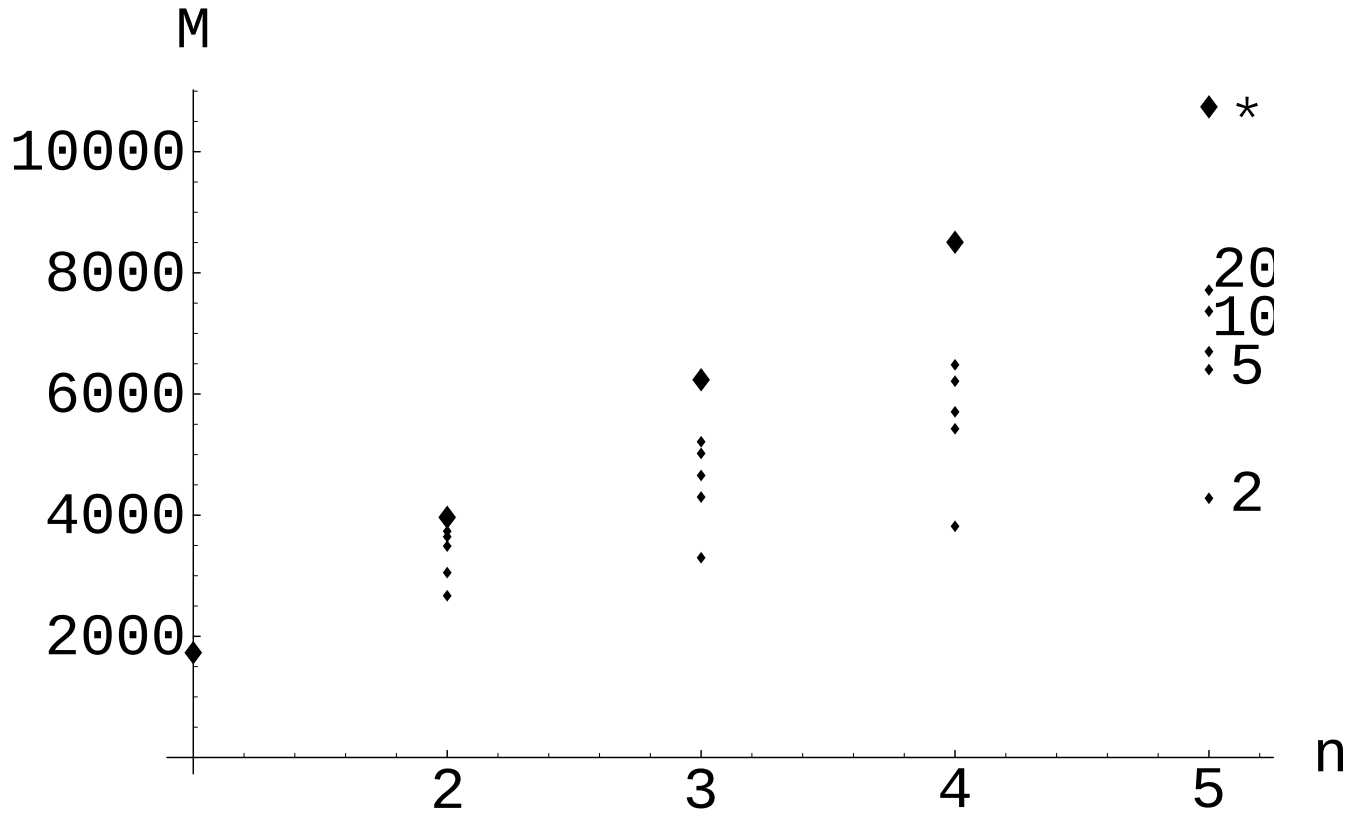
The borderline case, $K=-3/8$, is certainly confining (by continuity), but whether or not the singularity is at finite r depends on the subleading terms.

The lattice glueball data

J^{++}	Ref. I $(m/\sqrt{\sigma})$	Ref. I (MeV)	Ref. II (mr_0)	Ref. II (MeV)	$N_c \to \infty (m/\sqrt{\sigma})$
0	$3.347(68)$	$1475(30)(65)$	$4.16(11)(4)$	$1710(50)(80)$	$3.37(15)$
0^*	$6.26(16)$	$2755(70)(120)$	$6.50(44)(7)$	$2670(180)(130)$	$6.43(50)$
0^{**}	$7.65(23)$	$3370(100)(150)$	NA	NA	NA
0^{***}	$9.06(49)$	$3990(210)(180)$	NA	NA	NA
2	$4.916(91)$	$2150(30)(100)$	$5.83(5)(6)$	$2390(30)(120)$	$4.93(30)$
2^*	$6.48(22)$	$2880(100)(130)$	NA	NA	NA
R_{20}	$1.46(5)$	$1.46(5)$	$1.40(5)$	$1.40(5)$	$1.46(11)$
R_{00}	$1.87(8)$	$1.87(8)$	$1.56(15)$	$1.56(15)$	$1.90(17)$

Available lattice data for the scalar and the tensor glueballs. Ref. I =H. B. Meyer, [arXiv:hep-lat/0508002]. and Ref. II = C. J. Morningstar and M. J. Peardon, [arXiv:hep-lat/9901004] + Y. Chen *et al.*, [arXiv:hep-lat/0510074]. The first error corresponds to the statistical error from the the continuum extrapolation. The second error in Ref.I is due to the uncertainty in the string tension $\sqrt{\sigma}$. (Note that this does not affect the mass ratios). The second error in the Ref. II is the estimated uncertainty from the anisotropy. In the last column we present the available large N_c estimates according to B. Lucini and M. Teper, [arXiv:hep-lat/0103027]. The parenthesis in this column shows the total possible error followed by the estimations in the same reference.

α-dependence of scalar spectrum



The 0^{++} spectra for varying values of α that are shown at the right end of the plot. The symbol * denotes the AdS/QCD result.

$B_2 - C_2$ mixing

- B_2 and C_2 are typically massless.
- In the presence of C_4 flux, this is not the case:

$$S = -M^3 \int d^5x \sqrt{g} \left[\frac{e^{-2\phi}}{2 \cdot 3!} H_3^2 + \frac{1}{2 \cdot 3!} F_3^2 + \frac{1}{2 \cdot 5!} F_5^2 \right] \ , \ F_3 = dC_2 \ , \ H_3 = dB_2 \ , \ F_5 = dC_4 - C_2 \wedge H_3$$

The equations of motion that stem from this action are*

$$\nabla^\mu (e^{-2\phi} H_{3,\mu\nu\rho}) + \frac{1}{4} F_{5,\nu\rho\alpha\beta\gamma} F_3{}^{\alpha\beta\gamma} = 0 \quad , \quad \nabla^\mu F_{3,\mu\nu\rho} + \frac{1}{4} F_{5,\nu\rho\alpha\beta\gamma} H_3{}^{\alpha\beta\gamma} = 0$$

$$\nabla^\mu F_{5,\mu\nu\rho\sigma\tau} = 0 \quad \rightarrow \quad F_{5,\mu\nu\rho\sigma\tau} = \frac{\epsilon_{\mu\nu\rho\sigma\tau}}{\sqrt{g}} \frac{2N_c}{3\ell_s}$$

Substituting

$$\nabla^\mu (e^{-2\phi} H_{3,\mu\nu\rho}) + \frac{N_c}{6\ell_s} \frac{\epsilon_{\nu\rho\alpha\beta\gamma}}{\sqrt{g}} F_3{}^{\alpha\beta\gamma} = 0 \quad , \quad \nabla^\mu F_{3,\mu\nu\rho} + \frac{N_c}{6\ell_s} \frac{\epsilon_{\nu\rho\alpha\beta\gamma}}{\sqrt{g}} H_3{}^{\alpha\beta\gamma} = 0$$

We finally decouple the equations:

$$\nabla^\mu \left[\nabla^\nu (e^{-2\phi} H_{3,\mu\rho\sigma} + \text{cyclic} \right] + \frac{N_c^2}{12 \cdot 5! \ell_s^2} H_{3,\nu\rho\sigma} = 0$$

and a similar one for F_3. This equation has uniform N_c scaling for $e^\phi \sim \frac{\lambda}{N_c}$

- Both B_2 and C_2 combine to a massive two-tensor, that is dual to the $C-odd$ non-conserved operator $Tr[F_{[\mu a} F^{ab} F_{b\nu]} + \frac{1}{4} F_{ab} F^{ab} F_{\mu\nu}]$ with UV dimension 6.

RETURN

$D_0 - F_1$ charges

We may dualize $C_2 \to C_1$

$$(F_3)_{\mu\nu\rho} = \frac{\epsilon_{\mu\nu\rho\sigma\tau}}{2\sqrt{g}} \left(F^{\sigma\tau} + \frac{N_c}{\ell_s} B^{\sigma\tau} \right) \quad , \quad F = dC_1$$

The equations become

$$\nabla^\mu \left(e^{-2\phi} H_{\mu\nu\rho} \right) + \left(\frac{N_c}{2\ell_s} \right)^2 B_{\nu\rho} + \frac{N_c}{4\ell_s} F_{\nu\rho} = 0 \quad , \quad \nabla^\sigma \left(F_{\sigma\tau} + \frac{N_c}{\ell_s} B_{\sigma\tau} \right) = 0$$

and stem from a Stuckelberg-type action

$$S = -M^3 \int d^5x \sqrt{g} \left[\frac{e^{-2\phi}}{2 \cdot 3!} H_3^2 + \frac{1}{4} \left(F_{\mu\nu} + \frac{N_c}{\ell_s} B_{\mu\nu} \right)^2 + \frac{2N_c^2}{9\ell_s^2} \right]$$

Under B_2 gauge transformations C_1 transforms

$$\delta B_2 = d\Lambda_1 \quad , \quad \delta C_1 = -\frac{N_c}{\ell_s} \Lambda_1$$

- This implies that N_c units of fundamental string charge can cancel one unit of C_1 charge.

RETURN

$D_1 - NS_0$ charges

We now dualize $B_2 \to \tilde{B}_1$

$$e^{-2\phi}(H_3)_{\mu\nu\rho} = \frac{\epsilon_{\mu\nu\rho\sigma\tau}}{2\sqrt{g}}\left(\tilde{F}^{\sigma\tau} + \frac{N_c}{\ell_s}C^{\sigma\tau}\right) \quad , \quad \tilde{F} = d\tilde{B}_1$$

The equations become

$$\nabla^\mu\left((F_3)_{\mu\nu\rho}\right) + e^{2\phi}\left(\frac{N_c}{2\ell_s}\right)^2 C_{\nu\rho} + e^{2\phi}\frac{N_c}{4\ell_s}\tilde{F}_{\nu\rho} = 0 \quad , \quad \nabla^\sigma\left[e^{2\phi}\left(F_{\sigma\tau} + \frac{N_c}{\ell_s}B_{\sigma\tau}\right)\right] = 0$$

and stem from a Stuckelberg-type action

$$S = -M^3\int d^5x\sqrt{g}\left[\frac{1}{2\cdot 3!}F_3^2 + \frac{e^{2\phi}}{4}\left(\tilde{F}_{\mu\nu} + \frac{N_c}{\ell_s}C_{\mu\nu}\right)^2 + \frac{2N_c^2}{9\ell_s^2}\right]$$

Under C_2 gauge transformations C_1 transforms

$$\delta C_2 = d\Lambda_1 \quad , \quad \delta\tilde{B}_1 = -\frac{N_c}{\ell_s}\Lambda_1$$

• This implies that N_c units of fundamental D-string charge can cancel one unit of $\tilde{B}_1$ charge.

RETURN

Bosonic string or superstring? II

• Consider the axion a dual to $Tr[F \wedge F]$. We can show that it must come from a RR sector.

In large-N$_c$ YM, the proper scaling of couplings is obtained from

$$\mathcal{L}_{YM} = N_c \; Tr\left[\frac{1}{\lambda}F^2 + \frac{\theta}{N_c}F \wedge F\right] \quad , \quad \zeta \equiv \frac{\theta}{N_c} \sim \mathcal{O}(1)$$

It can be shown

$$E_{YM}(\theta) = N_c^2 \; E_{YM}(\zeta) = N_c^2 \; E_{YM}(-\zeta) \simeq C_0 \; N_c^2 + C_1\theta^2 + C_2\frac{\theta^4}{N_c^2} + \cdots \quad \textit{Witten}$$

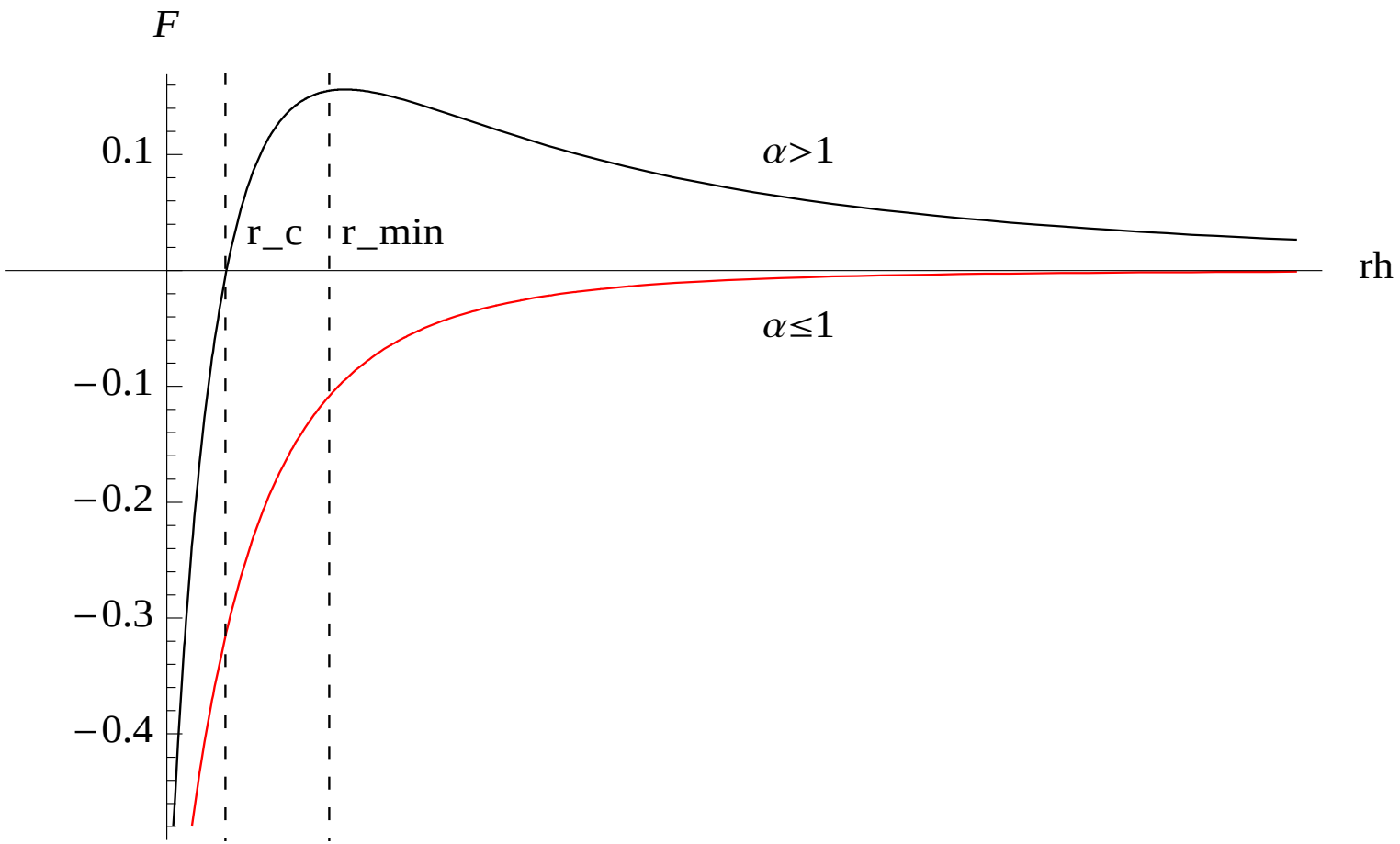
In the string theory action

$$S \sim \int e^{-2\phi}\,[R + \cdots] + (\partial a)^2 + e^{2\phi}(\partial a)^4 + \cdots \quad , \quad e^{\phi} \sim g_{YM}^2 \quad , \quad \lambda \sim N_c e^{\phi}$$

$$\sim \int \frac{N_c^2}{\lambda^2}\,[R + \cdots] + (\partial a)^2 + \frac{\lambda^2}{N_c^2}(\partial a)^4 + \cdots \quad , \quad a = \theta[1 + \cdots]$$

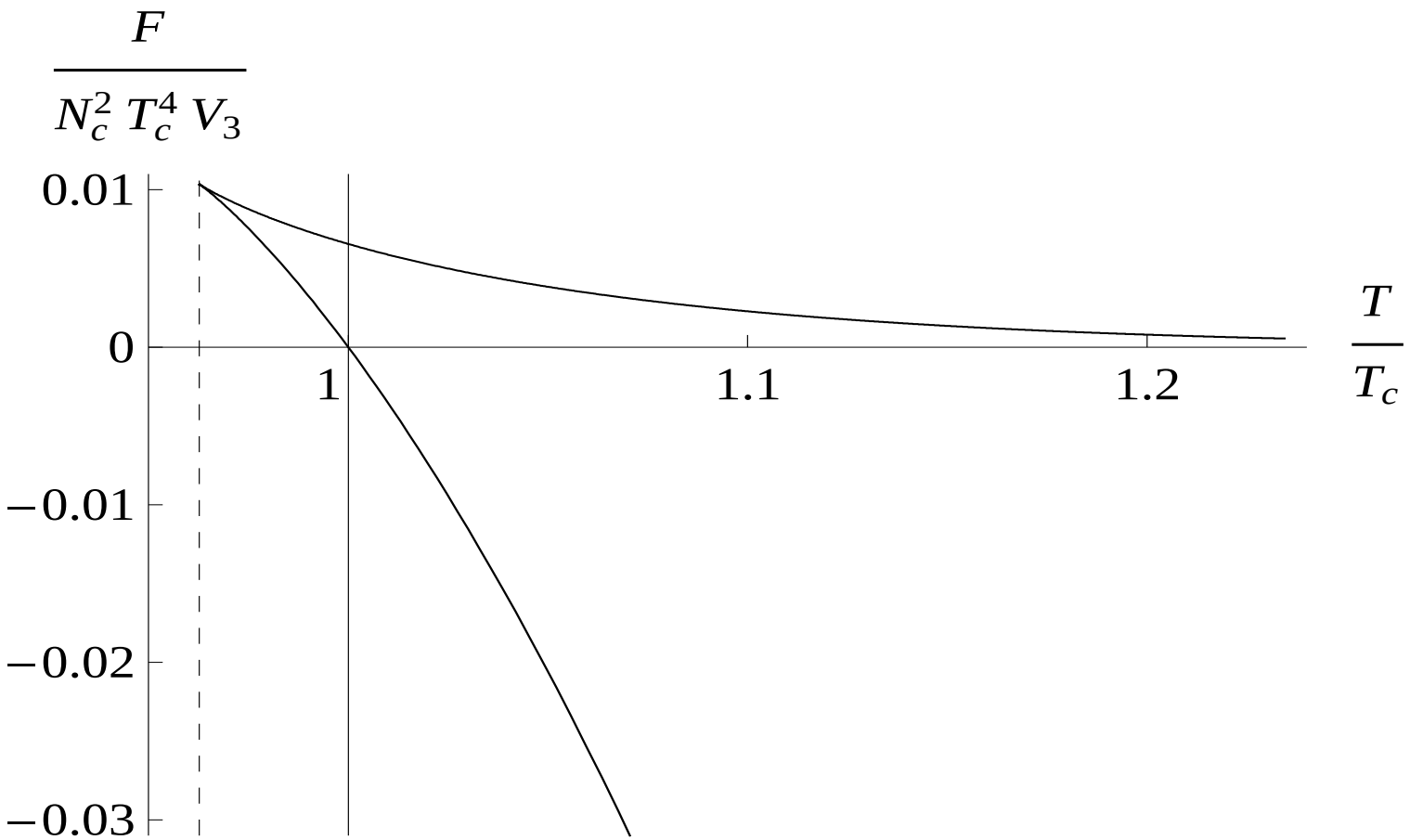
RETURN

Free energy versus horizon position



We plot the relation $\mathcal{F}(r_h)$ for various potentials parameterized by a. $a = 1$ is the critical value below which there is no first order phase transition .

The transition in the free energy



Finite-T Confining Theories

• There is a minimal temperature T_{min} for the existence of Black-hole solutions

• When $T < T_{\text{min}}$ only the "thermal vacuum solution" exists: it describes the confined phase at small temperatures.

• For $T > T_{\text{min}}$ there are two black-hole solutions with the same temperature but different horizon positions. One is a "large" BH the other is "small".

• When $T > T_{\text{min}}$ three competing solutions exist. The large BH has the lowest free energy for $T > T_c > T_{min}$. It describes the deconfined "Gluon-Glass" phase.

The conformal anomaly in flat space

- In YM we have the following anomaly equation in flat space:

$$T^{\mu}_{\mu} = \frac{\beta(\lambda_t)}{4\lambda_t^2} Tr[F^2],$$

- Defining the pressure p and energy density ρ,

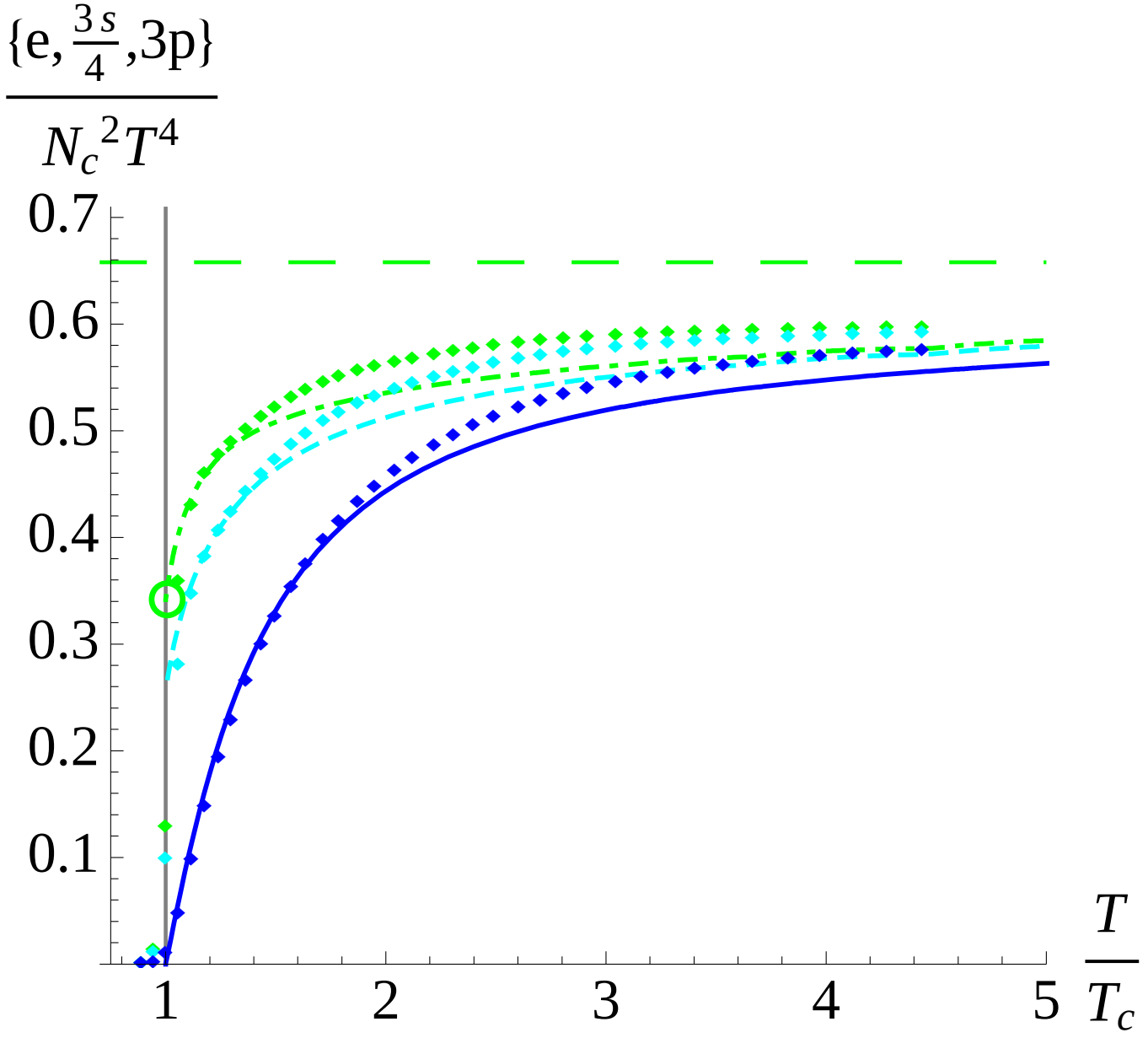
$$p = -\frac{\mathcal{F}}{V_3}, \qquad \rho = \frac{\mathcal{F} + TS}{V_3},$$

the trace is

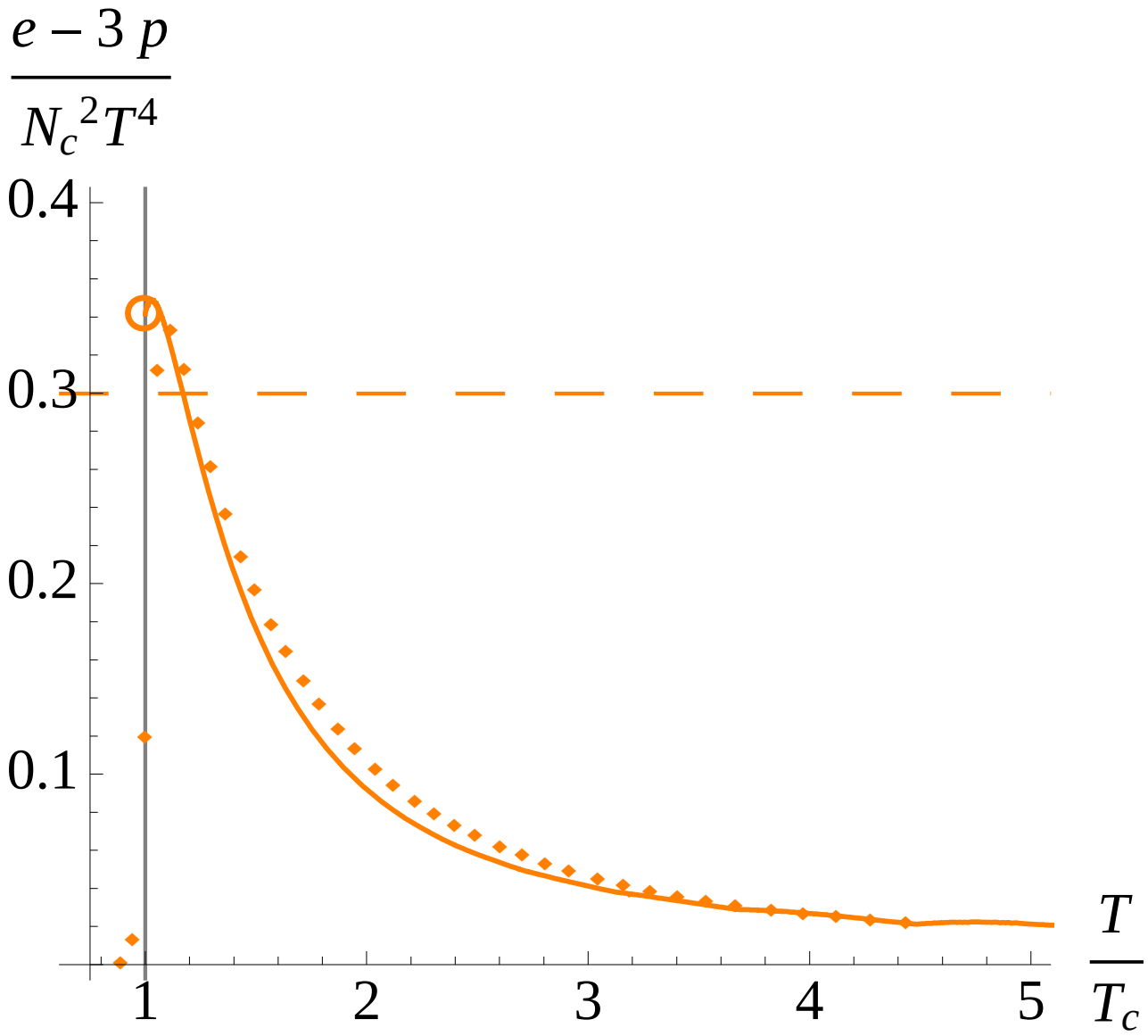
$$\langle T^{\mu}_{\mu} \rangle_R = \rho - 3p = 60 M_p^3 N_c^2 \mathcal{G}(T) = \frac{\beta(\lambda_t)}{4\lambda_t^2} (\langle Tr[F^2] \rangle_T - \langle Tr[F^2] \rangle_o),$$

- The left hand side is the trace of the renormalized thermal stress tensor, $\langle T^{\mu}_{\mu} \rangle_R = \langle T^{\mu}_{\mu} \rangle - \langle T^{\mu}_{\mu} \rangle_o$, and it is proportional to $\mathcal{G} \sim \langle Tr[F^2] \rangle$,

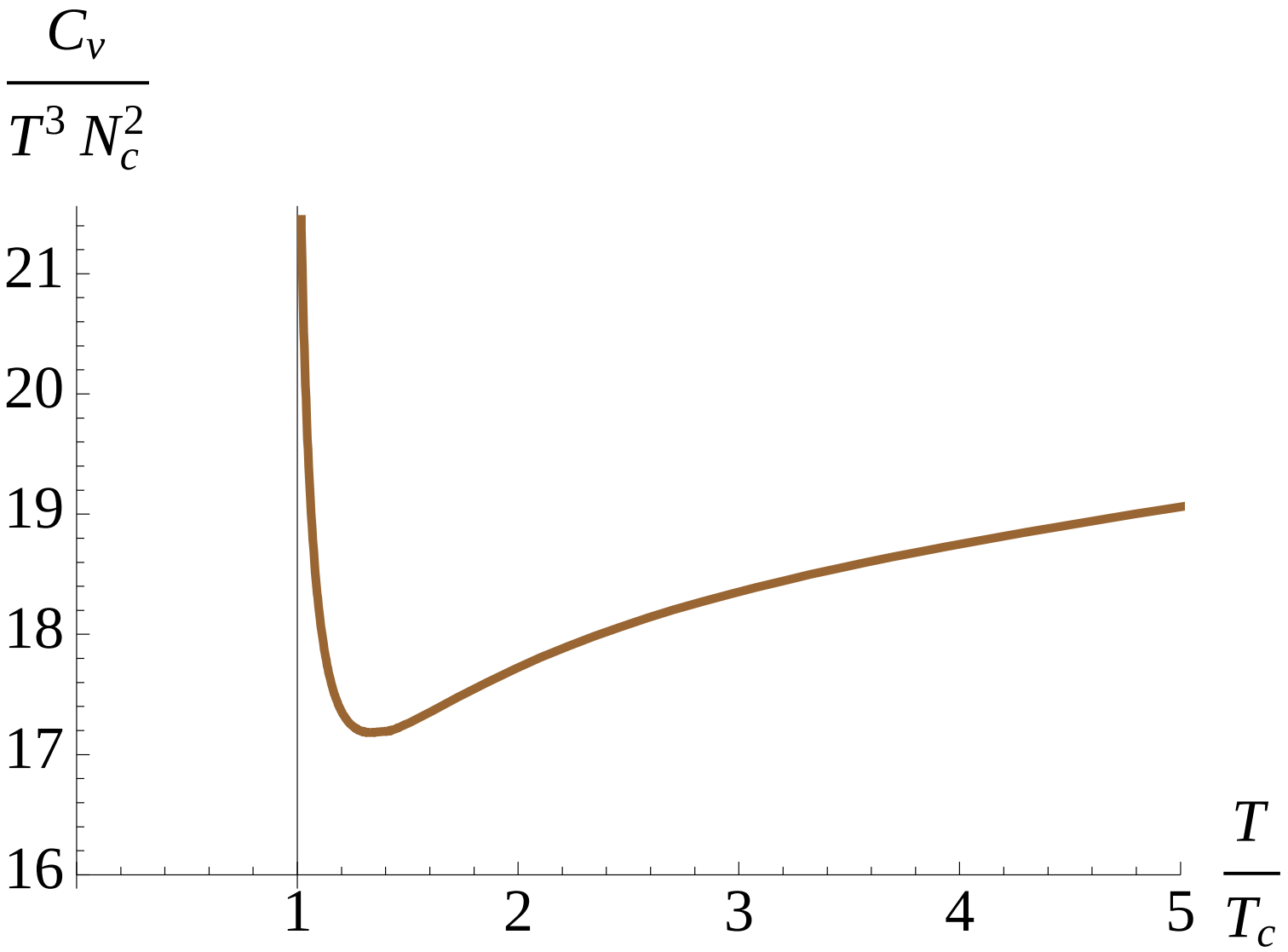
Thermodynamic variables



Equation of state



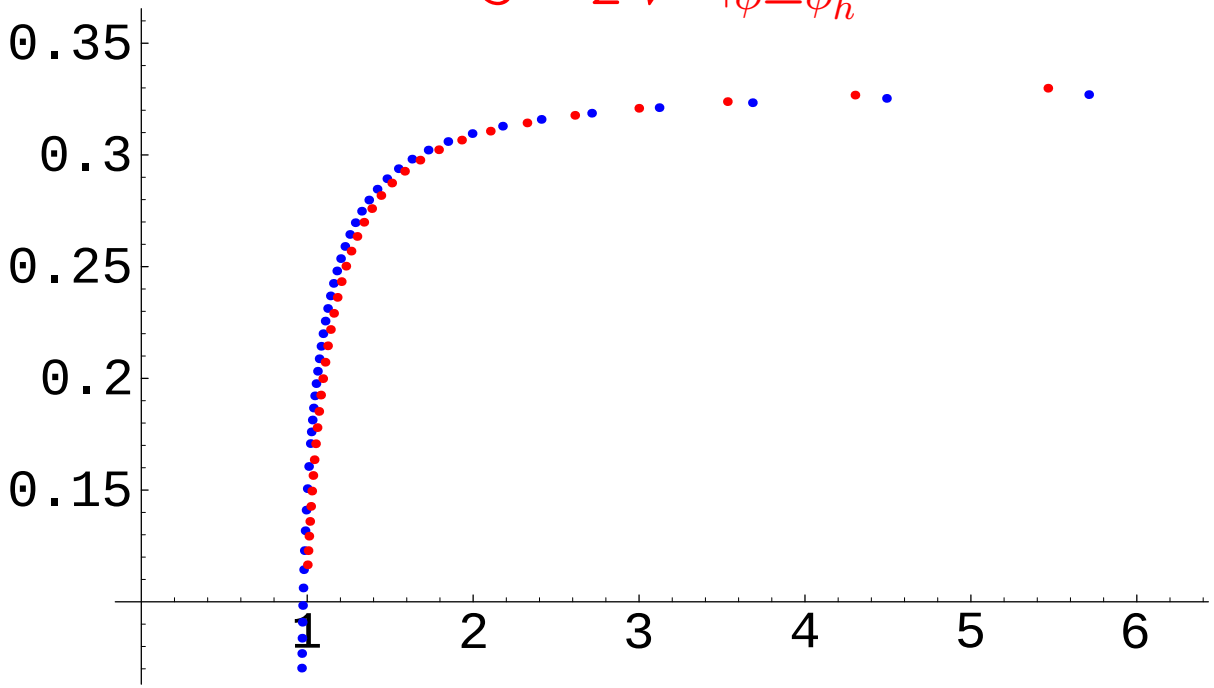
The specific heat



Comparing to Gubser+Nelore's formula

• Gubser+Nelore proposed the following approximate formula for the speed of sound

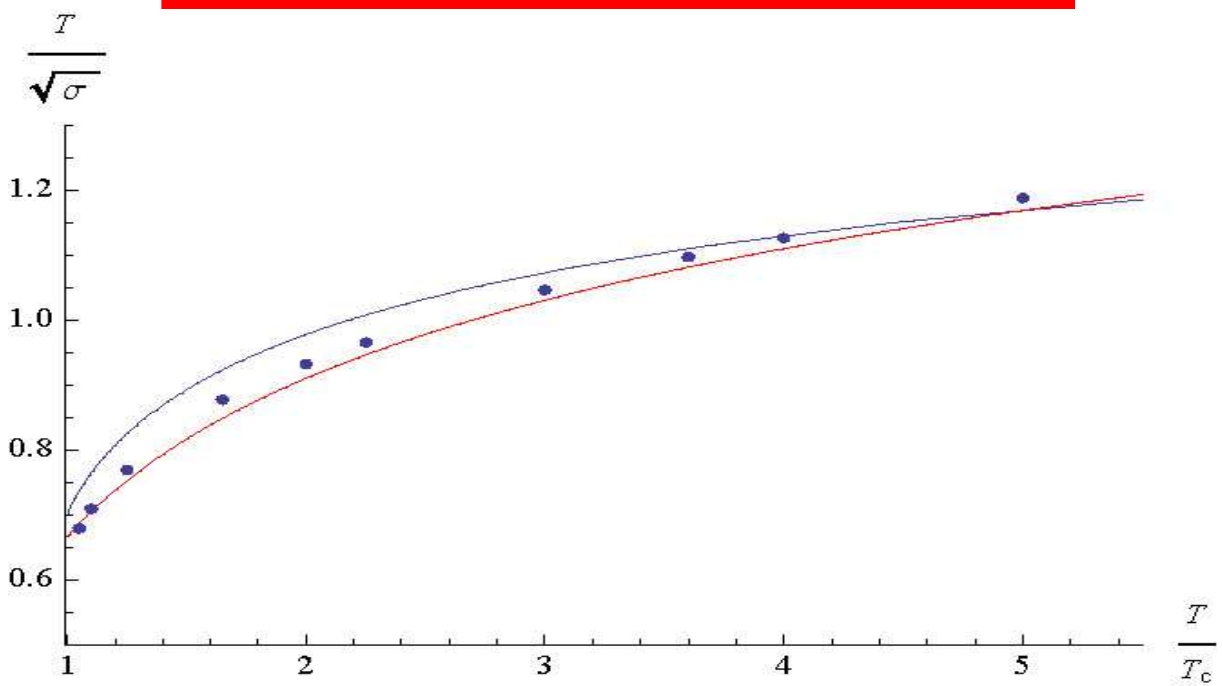
$$c_s^2 \simeq \frac{1}{3} - \frac{1}{2}\frac{V'^2}{V^2}\Big|_{\phi=\phi_h}$$



Gursoy (unpublished) 2009

• Red curve=numerical calculation, Blue curve=Gubser's adiabatic/approximate formula.

Spatial string tension



G. Boyd et al. 1996

- The blue line is the spatial string tension as calculated in Improved hQCD, with no additional fits.

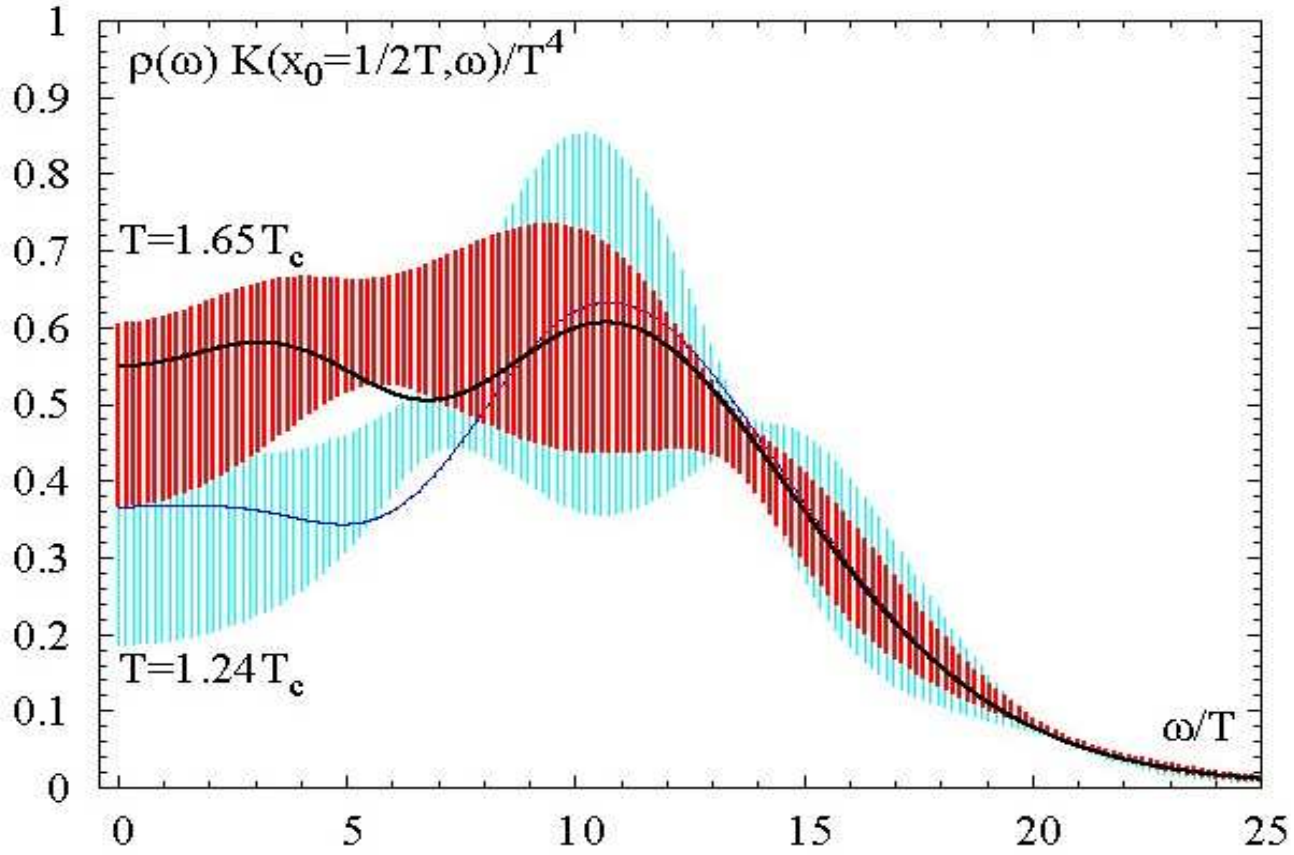
Nitti (unpublished) 2009

- The red line is a semi-phenomenological fit using

$$\frac{T}{\sqrt{\sigma_s}} = 0.51 \left[\log\frac{\pi T}{T_c} + \frac{51}{121}\log\left(2\log\frac{\pi T}{T_c}\right)\right]^{\frac{2}{3}}$$

Alanen+Kajantie+Suur-Uski, 2009

Shear Viscosity bounds from lattice

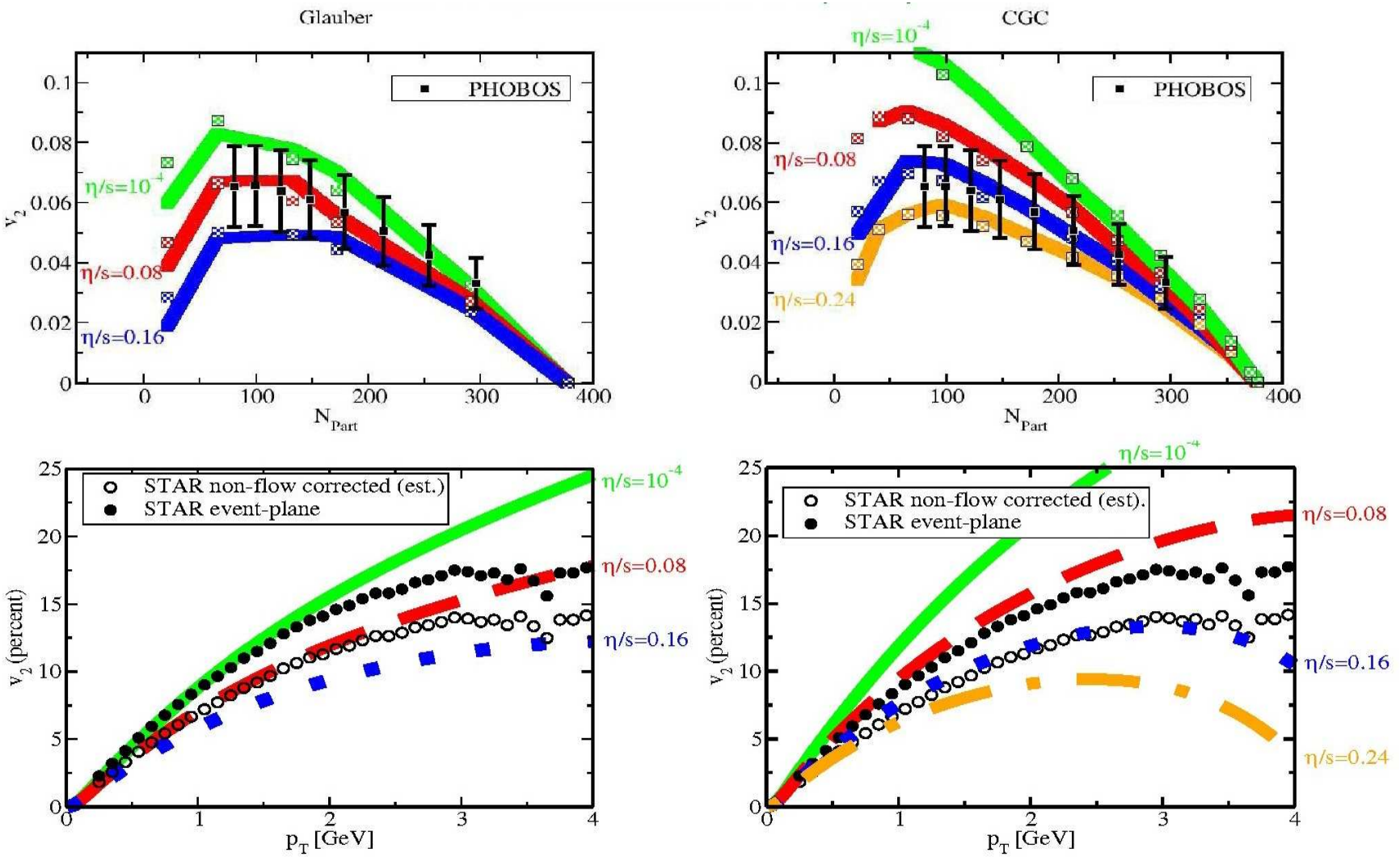


H. Meyer 2007

$$4\pi\frac{\eta}{s} = \begin{cases} 1.68(42), & T = 1.65\ T_c, \\ 1.28(70), & T = 1.24\ T_c. \end{cases}$$

shear viscosity data

- V_2 is the elliptic flow coefficient



Luzum+Romatchke 2008

The Bjorken Relation

- Consider that after the collision of the nuclear pancakes a lot of particles are produced at $t = \tau$. These are confined in a slice of longitudinal width dz and transverse area A.

- The longitudinal velocities have a spread $dv_L = \frac{dz}{\tau}$.

- Near the middle region $v_L \to 0$

$$\frac{dy}{dv_L} = \frac{d}{dv_L}\left[\frac{1}{2}\log\frac{1+v_L}{1-v_L}\right] = \frac{1}{1-v_L^2} \simeq 1$$

- We may now write

$$dN = dv_L\ \frac{dN}{dv_L} \simeq \frac{dz}{\tau}\ \frac{dN}{dy} \quad \to \quad \frac{dN}{dz} \simeq \frac{1}{\tau}\ \frac{dN}{dy}$$

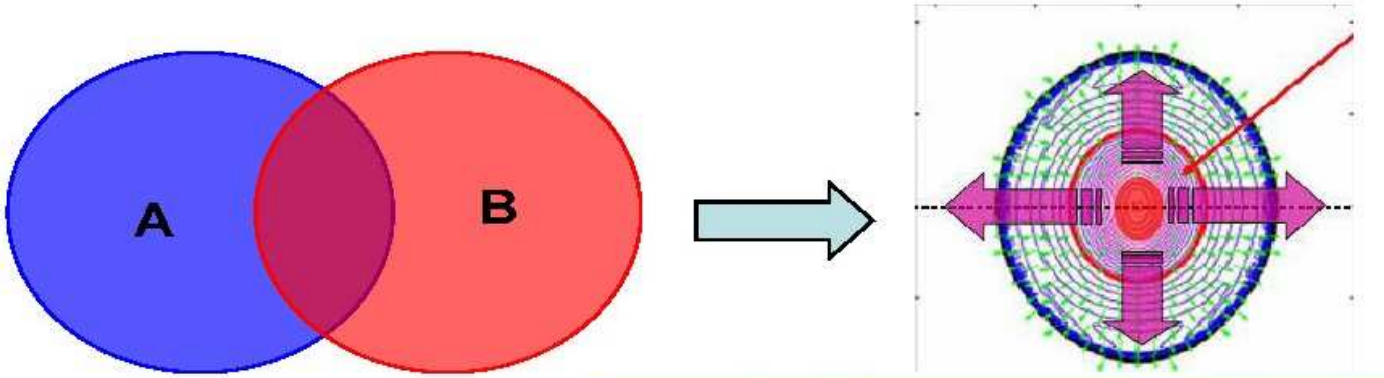
- If $\langle E_T \rangle \simeq \langle m_T \rangle$ is the average energy per particle then the energy density in this area at $t = \tau$ is given by the Bjorken formula:

$$\langle \epsilon(\tau) \rangle \simeq \frac{dN\langle m_T \rangle}{dz\ A} = \frac{1}{\tau}\frac{dN}{dy}\frac{\langle m_T \rangle}{A} = \frac{1}{\tau\ A}\frac{dE_T^{\text{total}}}{dy}$$

- It is valid if (1) τ can be defined meaningfully (2) The crossing time $\ll \tau$.

RETURN

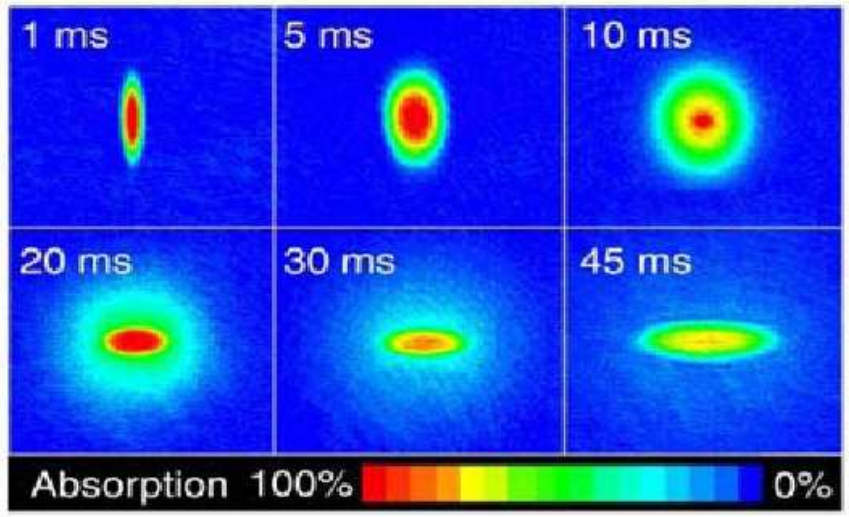
Elliptic flow in ultracold gases



Na_{23}

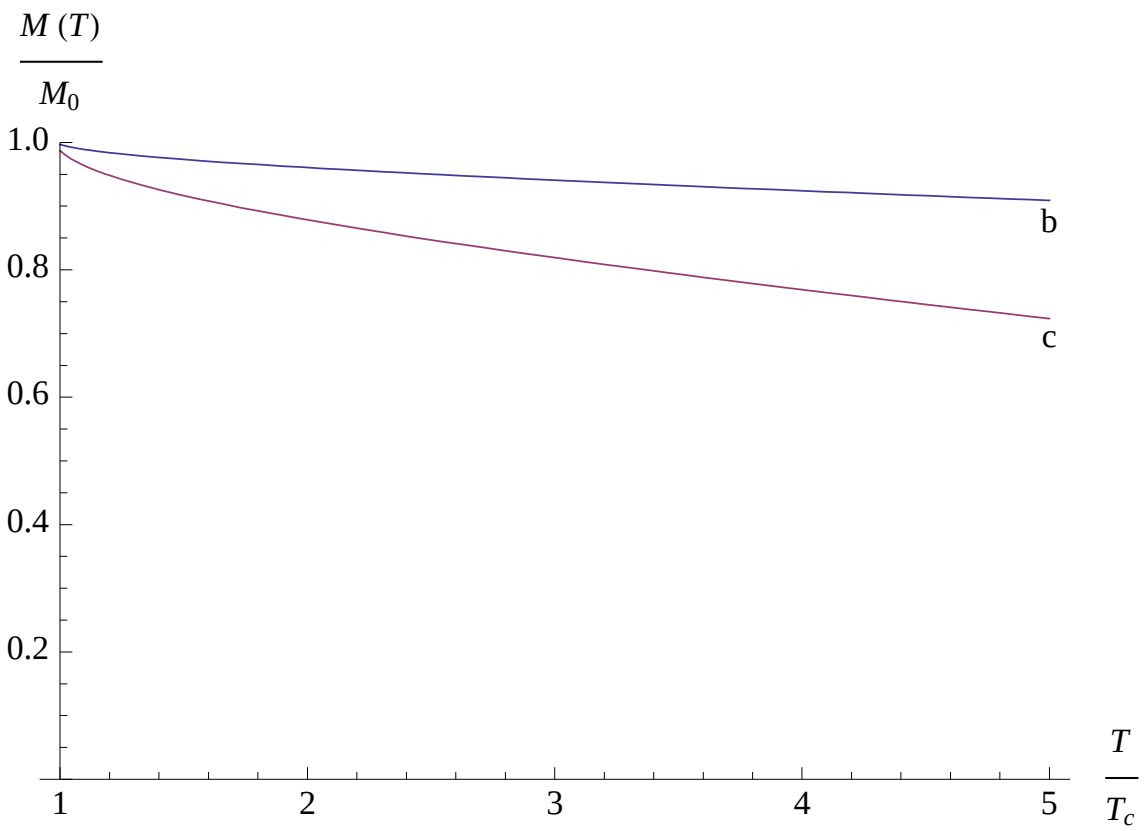
Bose condensate

Ketterle group, MIT



RETURN

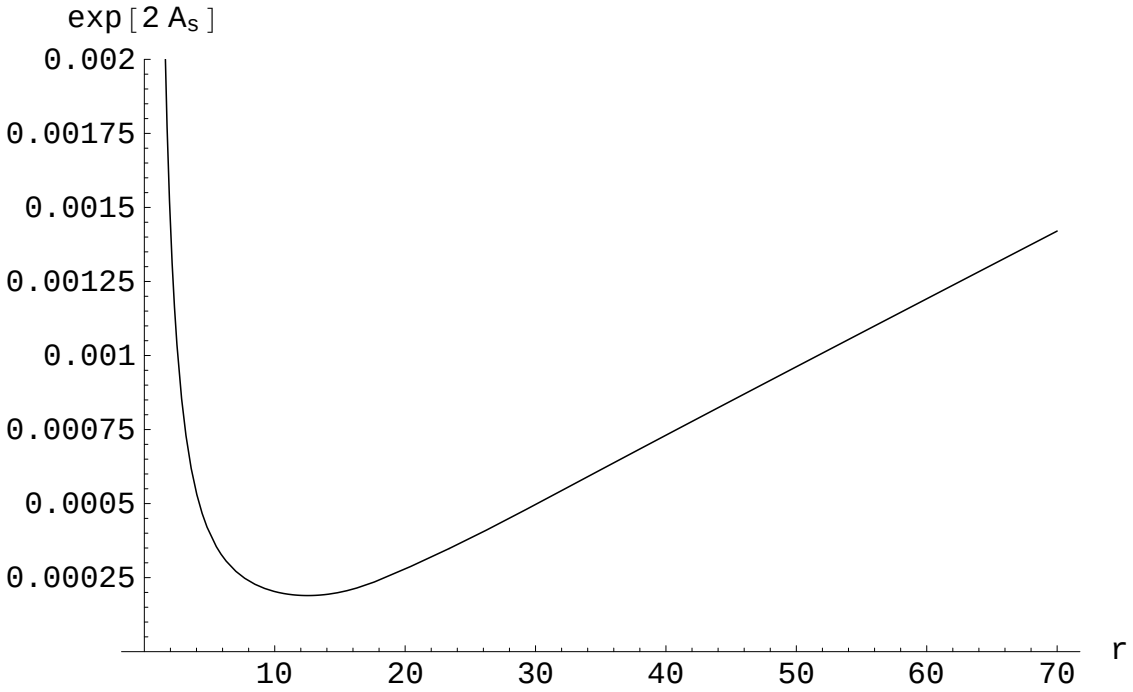
The thermal mass



- The mass is defined via a straight string hanging in the bulk
- It is qualitatively in agreement with lattice calculation of the position of the quarkonium resonance shift at finite temperature.

Datta+Karsch+Petreczky+Wetzorke 2004

The string frame scale-factor



$$E(L) \sim \frac{e^{2A_S(r_*)}}{4\pi\ell_s^2}\, L \quad , \quad \sigma_{\text{string}} = \frac{e^{2A_S(r_*)}}{4\pi\ell_s^2}$$

- Note that the singularity is at $r = \infty$.

RETURN

Diffusion times in different schemes

T_{QGP}, MeV	τ_{charm} (fm/c) (direct)	τ_{charm} (fm/c) (energy)	τ_{charm} (fm/c) (entropy)
220	-	3.96	3.64
250	5.67	3.14	2.96
280	4.27	2.56	2.47
310	3.45	2.12	2.08
340	2.88	1.80	1.78
370	2.45	1.54	1.53
400	2.11	1.33	1.34

The diffusion times for the charm quark are shown for different temperatures, in the three different schemes. Diffusion times have been evaluated with a quark initial momentum fixed at $p \approx 10\ GeV$.

Gursoy+Kiritsis+Michalogiorgakis+Nitti, 2009

bosonic string or superstring?

• The string theory must have no on-shell fermionic states at all because there are no gauge invariant fermionic operators in pure YM. (even in the presence of quarks and modulo baryons that are expected to be solitonic).

♠ We do expect a superstring however since there should be RR fields.

♠ A RR field we expect to have is the RR 4-form, as it is necessary to "seed" the D_3 branes responsible for the gauge group.

• It is non-propagating in 5D

• We will see later however that it is responsible for the non-trivial IR structure of the gauge theory vacuum.

• The most solid indication: There is a direct argument that the axion, dual to the instanton density $F \wedge F$ must be a RR field (as in $\mathcal{N} = 4$).

• Therefore the string theory must be a 5d-superstring theory resembling the II-0 class.

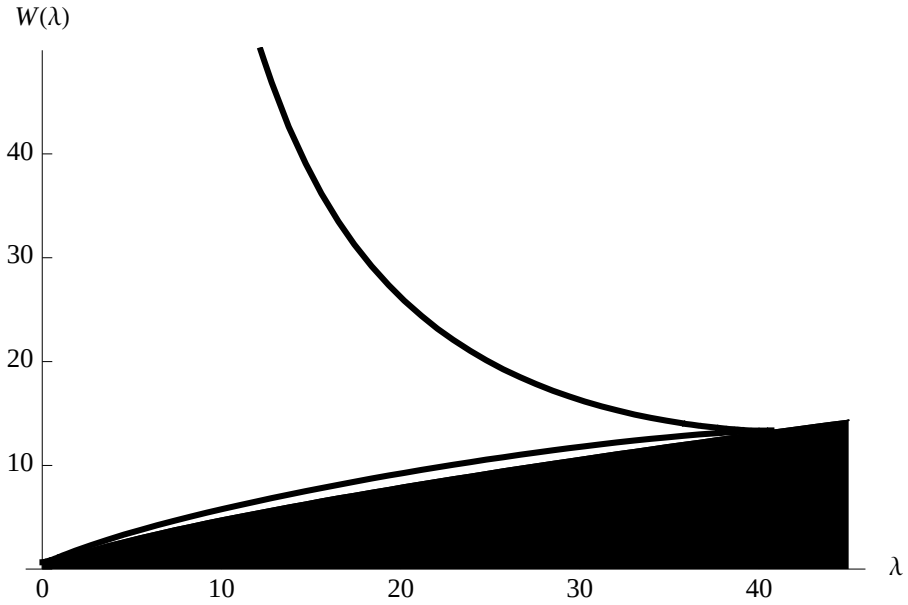
The good the bad and the ugly

- The "Bad" type. This is a one parameter family of solutions with

$$W(\lambda) \sim \lambda^{\frac{4}{3}}$$

It has a bad IR singularity.

♠ The "Ugly" type. This is a one parameter family of solutions. In such solutions there are two branches but they never reach the IR $\lambda \to \infty$. Instead λ goes back to zero



Detailed plan of the presentation

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- RHIC collision 11 minutes
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- The minimal string theory spectrum 34 minutes
- (many) holographic approaches to QCD 37 minutes
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FINITE TEMPERATURE

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- The UV regime 144 minutes
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- The IR regime: general expectations 151 minutes
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- Linearity of the glueball spectrum 161 minutes
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- Comparison of scalar and tensor potential 168 minutes
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- Confining β-functions 213 minutes
- The lattice glueball data 214 minutes
- α-dependence of scalar spectrum 215 minutes
- $B_2 - C_2$ mixing 218 minutes
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- Bosonic string or superstring? II 224 minutes
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