Thermodynamics of charged black holes with a nonlinear electrodynamics source

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Motivation

- A simple nonlinear electrodynamics model in higher dimensions
- a) To propose a conformally invariant EM lagrangian in higher dimensions
- b) To find interesting solutions, like BH spacetimes
- c) To introduce a new laboratory to explore different asymptotic behaviors of the metric and matter fields

Plan

- Black hole solutions
- Thermodynamical properties
- Future directions

In collaboration with Hernán A. González (PUC) and Mokhtar Hassaïne (Universidad de Talca)
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Action and field equations

- $$I[g_{\mu\nu}, A_\mu] = \int d^d x \sqrt{-g} \left[\frac{R}{2\kappa} - \alpha (F_{\mu\nu} F^{\mu\nu})^p \right]$$

  where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

- $$G_{\mu\nu} = \kappa T_{\mu\nu} = 4\alpha\kappa \left(p F_{\mu\rho} F_\nu{}^\rho F^{p-1} - \frac{1}{4} g_{\mu\nu} F^p \right)$$

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Special cases

- $p = 1$: Maxwell electrodynamics
- $p = d/4$: conformally invariant case

Spherically symmetric charged black holes

- Static and spherically symmetric metric with a radial electric field

$$ds^2 = -N^2(r)f^2(r)\,dt^2 + \frac{dr^2}{f^2(r)} + r^2 d\Omega^2_{d-2} \qquad A = A(r)dt$$

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$$b \equiv \frac{2(pd - 4p + 1)}{2p - 1} \qquad B \equiv -\frac{2\kappa\alpha(-1)^p C^{2p} 2^p (2p-1)^2}{(d-2)(d-2p-1)}$$

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Case $b = d - 3$

- $b = d - 3$ is equivalent to $p = (d-1)/2$, so d must be odd

$$f^2(r) = 1 - \frac{A}{r^{d-3}} + \kappa\alpha(-1)^p 2^{p+1} C^{d-1} \frac{\ln r}{r^{d-3}}$$

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Electric and spherically symmetric case

- Note that $F = -2(NF_{tr})^2 < 0$, then F^p is a real number only if p is an integer or a rational number with odd denominator. Hence, the exponent p is restricted to be an element of the following set

$$\tilde{\mathbb{Q}} = \left\{ \frac{n}{2m+1}, \quad (n,m) \in \mathbb{Z} \times \mathbb{Z} \right\}.$$

- Energy condition $T^{\hat{0}\hat{0}} = -\kappa\alpha(2p-1)F^p > 0$ implies

$$\mathrm{sgn}(\alpha) = \begin{cases} -(-1)^p & \text{for} \quad p > 1/2 \\ (-1)^p & \text{for} \quad p < 1/2 \end{cases}$$

Singularities

- For the general solution, the scalar curvature is

$$R = \frac{2\kappa\alpha(-2)^p C^{2p}(d-4p)}{(d-2)} \frac{1}{r^{\frac{2p(d-2)}{2p-1}}}$$

- There is a singularity at the origin $r = 0$ if $p > 1/2$ or $p \leq 0$
- There is a singularity at infinity $r = \infty$ if $0 < p < 1/2$
- We are interested in finding solutions without naked singularities, then the range $0 < p < 1/2$ is excluded.
- It is easy to show that this curvature regularity condition at ∞, in addition to the positive energy condition, ensure that the weak energy condition is satisfied.

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Cases

$$f^2(r) = 1 - \frac{A}{r^{d-3}} + \frac{B}{r^b}$$

Type	b	p	Remarks
I	$b > d-3$	$\frac{1}{2} < p < \frac{d-1}{2}$	Standard asymptotically flat case
II	$0 < b < d-3$	$p > \frac{d-1}{2}$ or $p < -\frac{1}{d-4}$	Electric term with relaxed falloff
III	$b \leq 0$	$-\frac{1}{d-4} \leq p < 0$	Asymptotically non flat case $b > -2$
Log	$b = d-3$	$p = \frac{d-1}{2}$ with d odd	Logarithmic case

Thermodynamics

- We consider the Euclidean approach. Using the saddle point approximation, the free energy for a thermodynamical ensemble is identified with the Euclidean action on shell, i. e.

$$\beta G = I_E(\text{on-shell})$$

  where β is the period of the euclidean time.
- Usually, the computation of I_E(on-shell) yields a trivial or a divergent result.

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- In order to regularize the action, we consider the hamiltonian version with a boundary term (Regge-Teitelboim). i.e.

$$I_E = \text{canonical euclidean action} + K$$

where K is a surface term.

- The canonical action is a linear combination of the constraints and it vanishes on-shell (The terms corresponding to "$p\dot{q}$" are supposed to vanished since the thermal equilibrium requires static or stationary fields.)
- Thus, $I_E(\text{on-shell}) = K(\text{on-shell})$ Now, how to fix K ? The surface term K is fixed by requiring that the action has an extremum

$$\delta I_E = \text{ eom } + \text{surface terms} + \delta K = 0$$

so $\delta K = -\text{surface terms}$

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- Here we are interested only in the static, spherically symmetric case with a radial electric field

$$ds^2 = N(r)^2 f(r)^2 d\tau^2 + \frac{dr^2}{f(r)^2} + r^2 d\Omega_{d-2}^2 \quad A = \phi(r) d\tau$$

$$\begin{aligned} I_E &= -\beta\,\Omega_{d-2} \int_{r_+}^{\infty} dr \left\{ \frac{(2p-1)\alpha N(-2)^{\frac{p}{2p-1}}}{r^{\frac{d-2}{2p-1}}} \left(\frac{\mathcal{P}}{4\alpha p} \right)^{\frac{2p}{2p-1}} \right. \\ &- \left. \frac{d-2}{2\kappa} N r^{d-2} \left[\frac{(f^2)'}{r} - \frac{d-3}{r^2}(1-f^2) \right] + \phi \mathcal{P}' \right\} + K \end{aligned}$$

where $\mathcal{P} \equiv 4\alpha p\, N_\infty\, r^{d-2} F^{p-1} F^{r\tau}$ is the rescaled canonical radial momentum.

Thermodynamics

- In what follows, we consider the formalism of the grand canonical ensemble, and hence we will consider the variation of the action keeping fixed the temperature and the electric potential, $\Phi = \phi(r_+)$.
  This implies that the variation of the surface term is given by

$$\delta K = \beta\, \Omega_{d-2} \left[-\frac{d-2}{2\kappa} N r^{d-3} \delta f^2 + \phi \delta \mathcal{P} \right]_{r_+}^{\infty} \equiv \delta K(\infty) - \delta K(r_+)$$

Thermodynamics

- For $b \neq d-3$, the variations of the fields at infinity are given by

$$\delta f^2|_\infty = -r^{-(d-3)}\delta A + r^{-b}\delta B, \quad \delta\mathcal{P}|_\infty = \delta\mathcal{P}_0,$$

and hence we have

$$\begin{aligned}\delta K(\infty) &= \beta\,\Omega_{d-2}\left[\frac{d-2}{2\kappa}\delta A\right.\\ &+ \left.\left(-\frac{d-2}{2\kappa}\delta B + \frac{2p-1}{d-2p-1}C\delta\mathcal{P}_0\right) r^{d-3-b}\right].\end{aligned}$$

For $b < d-3$, the contribution proportional to r^{d-3-b} may blow up at infinity but since the variation multiplying this term identically vanishes, the boundary variation at infinity is finite

$$\delta K(\infty) = \frac{d-2}{2\kappa}\beta\,\Omega_{d-2}\delta A \to K(\infty) = \frac{d-2}{2\kappa}\beta\,\Omega_{d-2}A$$

Thermodynamics

- In order to evaluate the variation of the metric function $f^2(r)$ at the horizon r_+, we use the fact that the solution satisfies $f^2(r_+) = 0$ together with the absence of conical singularities at the horizon $\beta = \frac{4\pi}{(f^2)'|_{r_+}}$:

$$\begin{aligned} \delta f^2|_{r_+} &= -(f^2)'|_{r_+}\delta r_+ = -\frac{4\pi}{\beta}\delta r_+, \\ \phi\,\delta\mathcal{P}|_{r_+} &= \phi(r_+)\delta\mathcal{P}_0. \end{aligned}$$

  The variation of the boundary term is easily integrated at the horizon yielding

$$K(r_+) = -\Phi\beta\mathcal{P}_0\Omega_{d-2} + \frac{2\pi}{\kappa}\Omega_{d-2}r_+^{d-2}.$$

Thermodynamics

- Finally, the on shell Euclidean action which reduces to the boundary term K reads

$$I_E = \beta\frac{d-2}{2\kappa}\Omega_{d-2}A - \beta\Phi\mathcal{P}_0\Omega_{d-2} - \frac{2\pi}{\kappa}\Omega_{d-2}r_+^{d-2}.$$

$$\begin{aligned} M &= \left(\frac{\partial I_E}{\partial\beta}\right)_\Phi - \frac{\Phi}{\beta}\left(\frac{\partial I_E}{\partial\Phi}\right)_\beta = \frac{(d-2)\Omega_{d-2}}{2\kappa}A, \\ Q &= -\frac{1}{\beta}\left(\frac{\partial I_E}{\partial\Phi}\right)_\beta = \Omega_{d-2}\mathcal{P}_0, \\ S &= \beta\left(\frac{\partial I_E}{\partial\beta}\right)_\Phi - I_E = \frac{2\pi}{\kappa}\Omega_{d-2}r_+^{d-2}. \end{aligned}$$

Thermodynamics

- Finally, the on shell Euclidean action which reduces to the boundary term K reads

$$I_E = \beta\frac{d-2}{2\kappa}\Omega_{d-2}A - \beta\Phi\mathcal{P}_0\Omega_{d-2} - \frac{2\pi}{\kappa}\Omega_{d-2}r_+^{d-2}.$$

$$\begin{aligned} M &= \left(\frac{\partial I_E}{\partial\beta}\right)_\Phi - \frac{\Phi}{\beta}\left(\frac{\partial I_E}{\partial\Phi}\right)_\beta = \frac{(d-2)\Omega_{d-2}}{2\kappa}A, \\ Q &= -\frac{1}{\beta}\left(\frac{\partial I_E}{\partial\Phi}\right)_\beta = \Omega_{d-2}\mathcal{P}_0, \\ S &= \beta\left(\frac{\partial I_E}{\partial\beta}\right)_\Phi - I_E = \frac{2\pi}{\kappa}\Omega_{d-2}r_+^{d-2}. \end{aligned}$$

Thermodynamics

Generalized Smarr formula

- We obtain a generalized Smarr formula given by

$$M = \frac{d-2}{d-3} ST + \frac{pd - 4p + 1}{p\,(d-3)} Q\Phi$$

- The same result can be obtained using a) Komar integrals and b) a Noether conserved current which is associated to a scale symmetry of the reduced action (see details in arXiv:0909.1365)

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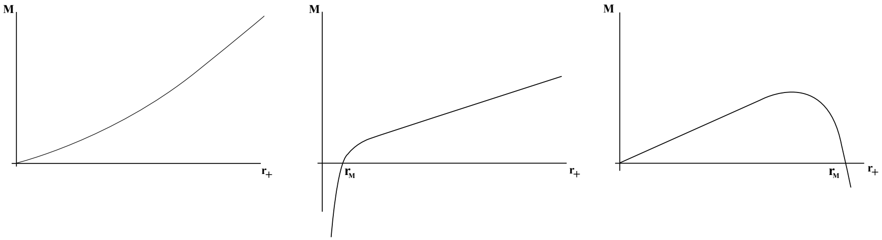
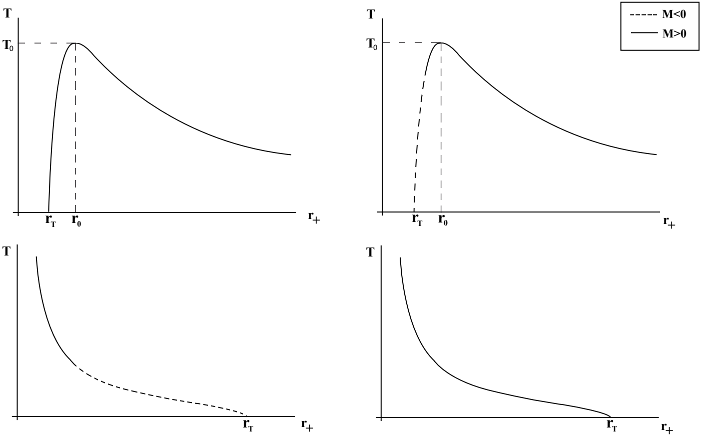
Thermodynamics



Mass of the black hole in terms of the event horizon radius r_+ at fixed electric potential Φ. The first graph corresponds to the solutions with $p \in \left(\frac{1}{2}, \frac{d-1}{2}\right)$, while the second one represents the solutions with $p > \frac{d-1}{2}$. The last graph is identified with the solutions with a negative exponent $p < 0$.

Temperature against the horizon r_+



Temperature against the horizon r_+ for different ranges of the exponent p at fixed electric potential Φ. At the top left, the graph corresponds to the range $p \in (\frac{1}{2}, \frac{d-1}{2})$, while the top right graph represents the solutions with $p > \frac{d-1}{2}$. The last graphs are respectively for $p < \frac{1}{4-d}$ and $p \in (\frac{1}{4-d}, 0)$.

Thermodynamics

Local stability

- Thermodynamics stability $\sim$ small fluctuations of the state functions around the equilibrium, and since the first order terms vanish, the stability is only a statement about the second order variations.
- Evaluate the sign of the heat capacity C_Φ at constant potential

$$C_\Phi \equiv T \left(\frac{\partial S}{\partial T} \right)_\Phi$$

as well as the sign of the electrical permittivity ϵ_T at constant temperature

$$\epsilon_T \equiv \left(\frac{\partial Q}{\partial \Phi} \right)_T .$$

Thermodynamics

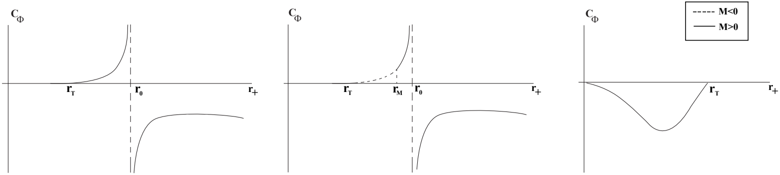


Figure: Specific heat at fixed potential with $p \in (\frac{1}{2}, \frac{d-1}{2})$, $p > \frac{d-1}{2}$ and $p < 0$.

Thermodynamics

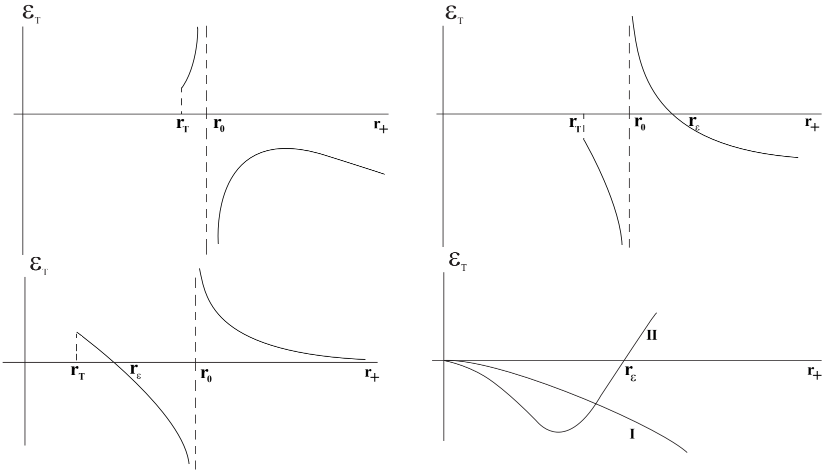


Figure: The electrical permittivity against r_+ for $p \in (\frac{1}{2}, 1)$, $p \in (1, \frac{d-1}{2})$ and $p > \frac{d-1}{2}$. For $p < 0$, there are two branches: the branch I corresponds to $p > \frac{1}{2(3-d)}$ while the branch II is relative to $p < \frac{1}{2(3-d)}$.

Global stability

- We now turn to the study of the global stability in order to determine whether our solutions are thermodynamically preferred over the Minkowski background.
- The Gibbs free energy $G = I_E/\beta$ is an appropriate state function to compare two solutions in the grand canonical ensemble.
- For example, it is well-known that in the standard Einstein-Maxwell theory, the Minkowski spacetime is always favored over the Reissner-Nordstrom solution since in this case the free energy of this latter is positive.

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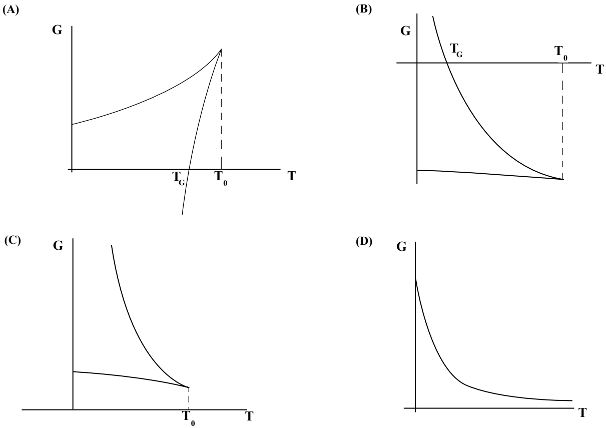

Figure: The Gibbs free energy in terms of temperature for $p \in (\frac{1}{2}, 1)$, $p \in (1, \frac{d-1}{2})$, $p > \frac{d-1}{2}$ and $p < 0$ at fixed electric potential Φ. A first-order phase transition can be observed only in the first graph at the temperature T_G.

Future directions

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- Rotating solutions

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