5th Aegean Summer School
From Gravity to Thermal Gauge Theories:
The AdS/CFT Correspondence
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Topics on bi-gravity

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I. General Aspects of Bi-Gravity

The bi-gravity action

$$I = I_{EH}(g_{\mu\nu}) + I_{EH}(f_{\mu\nu}) + I_{int}$$

Physical metric Metric "matter" Interaction

The interaction term I_{int} breaks half of the diffeomorphism symmetry. It will be assumed to be of the form,

$$\int dx(-g)^m(-f)^{1/2-m}V\left(f^{\mu\nu}g_{\mu\nu}, f^{\mu\nu}g_{\nu\rho}f^{\rho\sigma}g_{\sigma\mu}, \ldots\right)$$

This is a family including a particular case originally introduced by Isham, Salam, and Strathdee in 1971, which is the one we shall use (except for the addition of cosmological constants for each sector and the requirement of m=0)

$$I = \frac{1}{16\pi G}\int\left\{\sqrt{-g}(R-2\Lambda)+\sqrt{-f}(K-2\lambda)+\frac{1}{\ell^2}\sqrt{-f}\left[-f^{\alpha\beta}g_{\alpha\beta}+\kappa\left((f^{\alpha\beta}g_{\alpha\beta})^2-f^{\alpha\beta}g_{\beta\gamma}f^{\gamma\delta}g_{\delta\alpha}\right)\right]\right\}$$

This particular form has some interesting properties:

$\Lambda, \lambda, l, \kappa$
Free parameters

- Pauli-Fierz at linearized level (1 massive, 1 massless graviton)
- $\kappa = 0$ is equivalent to Eddington-Born-Infeld Gravity – containing dark matter. (Bañados, 2008)
- The measure has been fixed to give rise to dark matter
- It gives rise to dark energy, even in the absence of comological terms (Damour, Kogan, and Papazoglou, 2002)
- Danger: there is an extra degree of freedom, which may be a source of trouble (Boulware and Deser 1972)). But see Damour and Kogan, 2002 for comments on this point.

II. The Vacuum and Dark Energy

The Vacuum Solution

The vacuum (most symmetric solution) of this theory is given by de Sitter in both sectors

$$ds_g^2 = -dt^2 + a(t)^2 d\vec{x}^2, \quad ds_f^2 = X^2(-dt^2 + a(t)^2 d\vec{x}^2)$$

$$a = e^{\frac{H}{\ell}t}$$

$$\begin{aligned} X^2 &= \frac{1-(\Lambda l^2 - 6\kappa)}{1-\lambda l^2} \\ H^2 &= \frac{1-(\Lambda l^2 - 6\kappa)\lambda l^2}{3(1-\lambda l^2)} \end{aligned}$$

The second metric is a source of dark energy!

A symmetry breaking vacuum

$$ds_g^2 = -dt^2 + a(t)^2 d\vec{x}^2, \quad ds_f^2 = X^2(-dt^2 + A^2 a(t)^2 d\vec{x}^2)$$

$$a = e^{\frac{H}{\ell} t}$$

This metric is also de Sitter. However, both metrics have different sets of Killing vectors, (Except for the common translational Killing vectors)

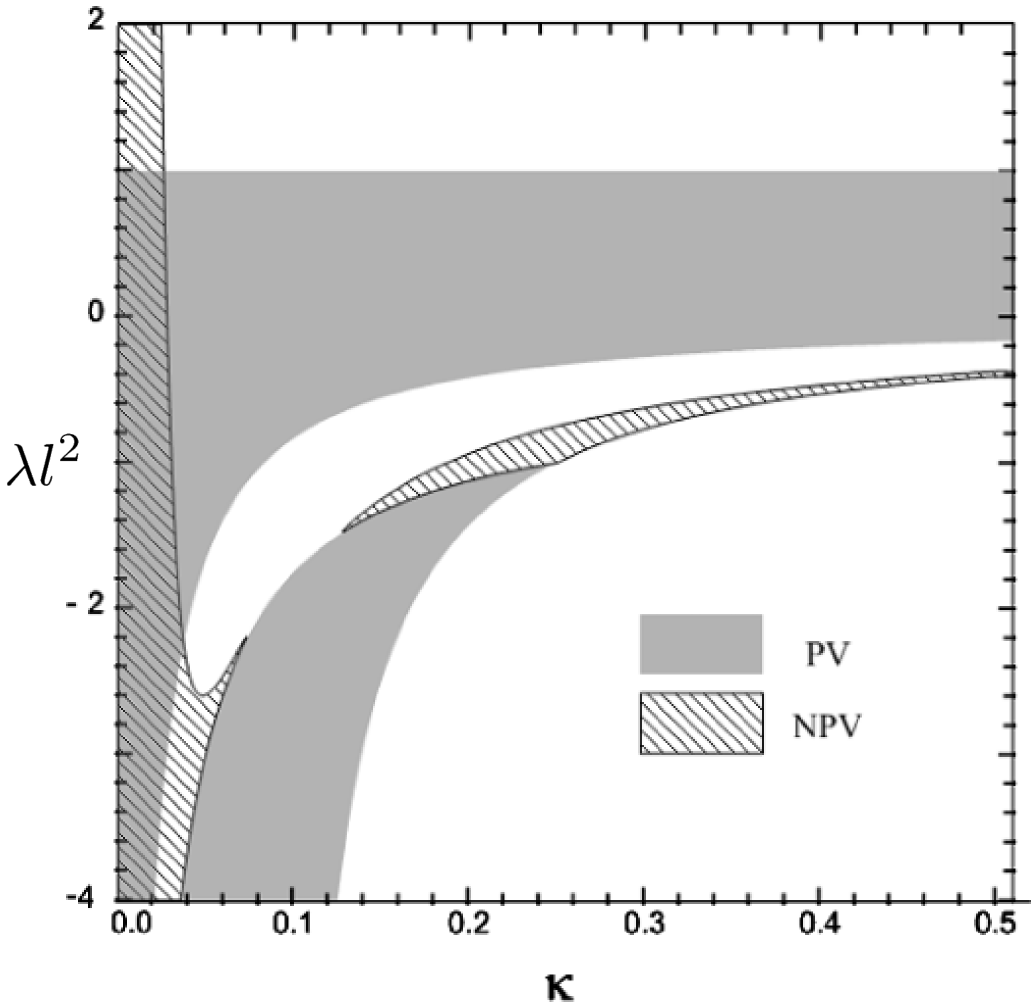
In general there are three solutions of this sort, given by the solutions of

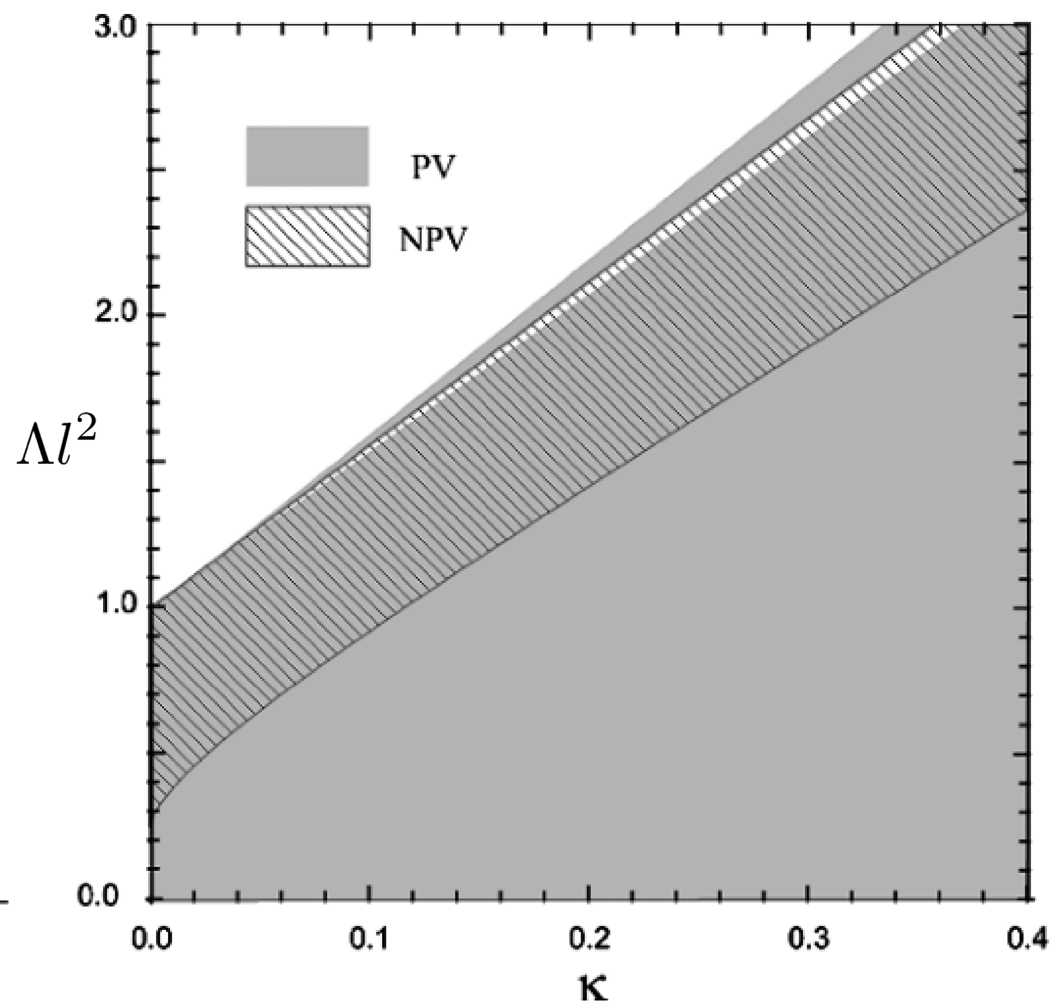
$$\begin{aligned} \kappa &= \frac{X^2 A^2}{4} \\ \Lambda l^2 &= 3H^2 + \frac{X^2 A^3}{2} \\ \lambda l^2 &= -\frac{3}{4X^2A^2} + \frac{3H^2}{X^2} - \frac{1}{4X^2} \end{aligned}$$

Blas, Deffayet and Garriga, 2006

Stability of vacuum

We study now the stability of the vacuum under spherically symmetric perturbations

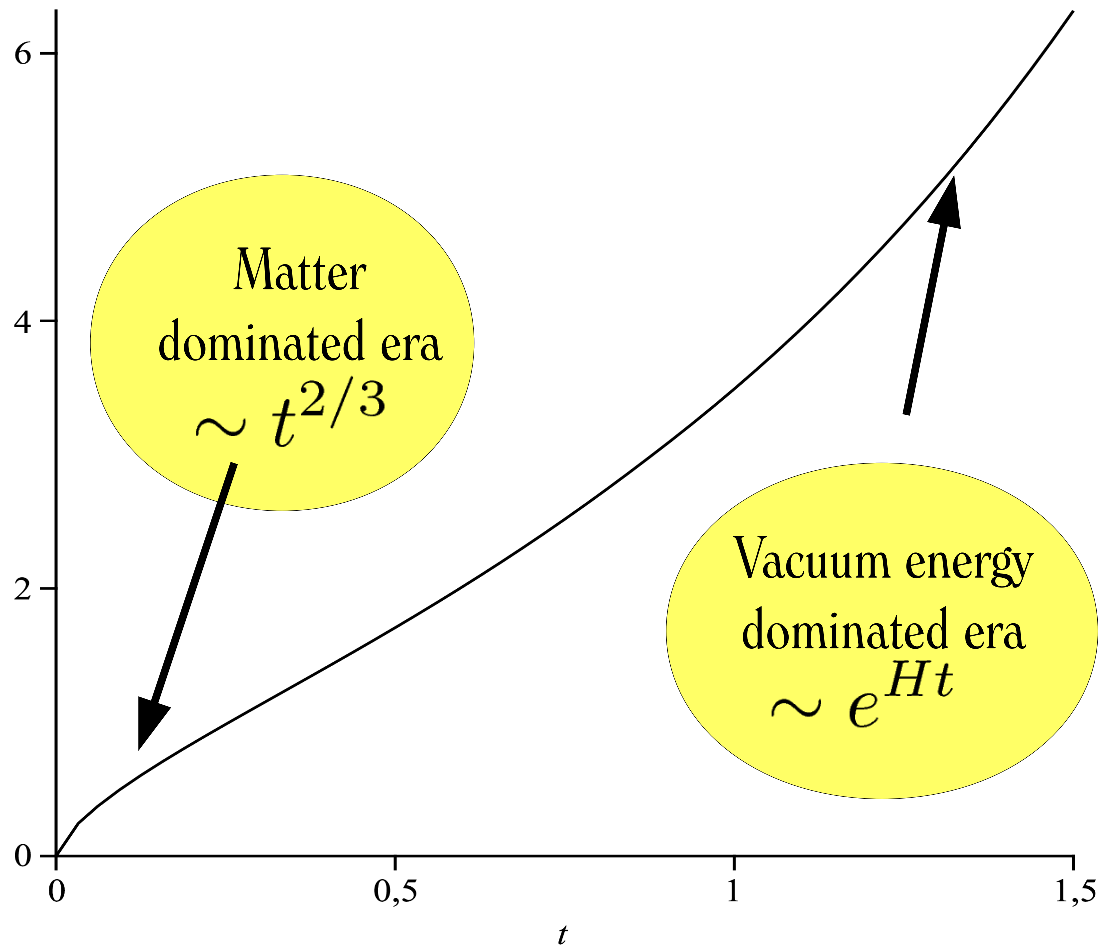




III. Dark Matter

The Darkest Matter

We may now try some more complex situations. This we must do numerically.

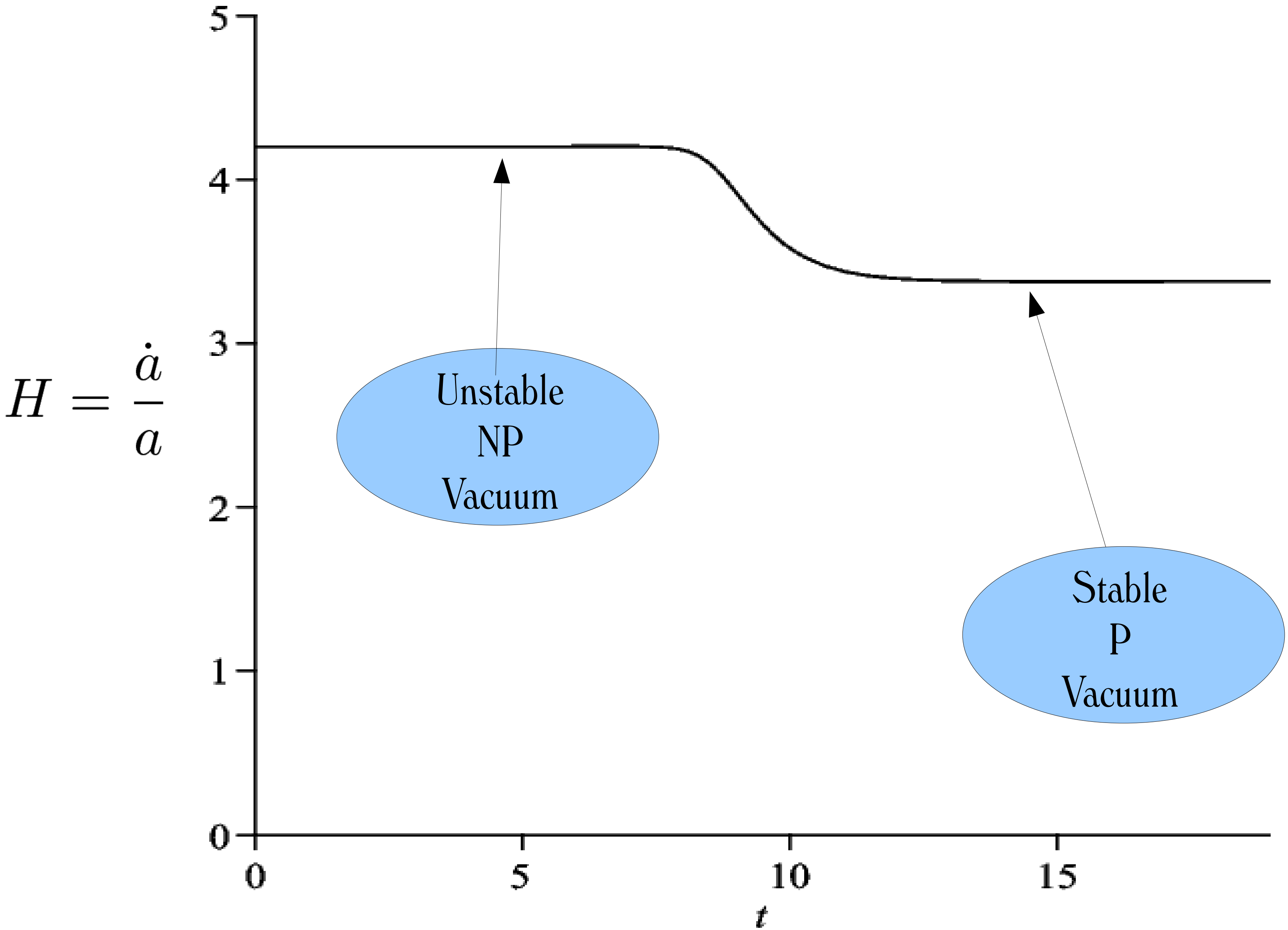


Scale factor $a(t)$.

In the EBI case, $\kappa=0$,the cosmological implications of such a model was studied in detail (Bañados, Ferreira, and Skordis, 2009). For that case, however, the inclusion of an external cosmological constant seem unavoidable.

IV. An inflationary toy

“Top-hill” inflation-like behavior



V. Black holes and thermodynamics

Bi-Black Hole Thermodynamics

Spherical solutions: Isham and Storey (1978)

$$ds_g^2 = -J(r)dt^2 + J(r)^{-1}dr^2 + r^2 d\Omega^2$$
$$ds_f^2 = -C(r)dt^2 + A(r)dr^2 + 2H(r)dtdr + B(r)d\Omega^2$$

$$J(r) = 1 - \left(\Lambda - \frac{8\kappa^2}{l^2\Delta}\right)\frac{r^2}{3} - \frac{2M}{r}$$
$$B(r) = 4\kappa r^2$$
$$H(r) = \sqrt{\Delta^2 - A(r)C(r)}$$
$$C(r) = \frac{\Delta^2}{4\kappa}\left[1 - \left(\frac{4\kappa^2}{\Delta^2 l^2} + \frac{3}{4l^2} + 4\lambda\kappa\right)\frac{r^2}{3} - \frac{2m}{r}\right]$$
$$A(r) = \frac{3l^2 r\Delta}{\kappa(\Delta l^2(\Lambda r^3 - 3r + 6M) - 8\kappa^2 r^3)^2}\left[\left(\frac{3}{16} + l^2\left(\lambda\kappa - \frac{\Lambda}{4}\right)\right)r^3\right.$$
$$\left. - \frac{3l^2(M-m)}{2}\right)\Delta^3 + 2\Delta^2\kappa^2 r^3 - 4\kappa^2\Delta\left(\left(\Lambda l^2 - \frac{1}{4}\right)r^3 - 3rl^2 + 6l^2 M\right) + 32\kappa^4 r^3\Big]$$

- ➔3 parameters, M, m and Δ
- ➔Both f and g represent de Sitter or anti-de Sitter black holes.
- ➔Horizons at different values of r

It is quite complicated to understand how to proceed…let's study a simpler case

Consider the simplest, **Euclidean**, bi-gravity, that is, no interaction (there is interaction anyway, through the thermal bath)

$$I[g_{\mu\nu}, f_{\mu\nu}] = I_{EH}[g_{\mu\nu}] + I_{EH}[f_{\mu\nu}]$$

The most general solution with spherical symmetry is

$$\boxed{\begin{aligned} ds_g^2 &= f^2 dt^2 + f^{-2} dr^2 + r^2 d\Omega^2 \\ ds_f^2 &= \lambda^2 h^2 dt^2 + h^{-2} dr^2 + r^2 d\Omega^2, \end{aligned}}$$

$$\begin{aligned} f^2 &= 1 - 2M/r \\ h^2 &= 1 - 2m/r \end{aligned}$$

Note that λ cannot be removed. It is a physical quantity

Compute the Gibbs free energy by evaluating the Euclidean action for fixed values of β and λ

$$I_E = \beta G = \beta M + \beta\lambda m - \frac{1}{4}A - \frac{1}{4}a,$$

Area of g-horizon

Area of f-horizon

One identifies

- Internal energy $U=M$
- Inverse temperature β
- Chemical Potential λ corresponding to the intensive parameter $N=-m$
- Entropy $S = \dfrac{A+a}{4}$

The temperature $1/\beta$ and the Potential λ may be determined either By

- Requiring regularity at the horizon
- Minimizing the free energy for M,m

We get

$$\boxed{\lambda = \frac{m}{M} \qquad \beta = 8\pi M}$$

The interacting case goes exactly en the same way. There are no derivatives in the interaction and therefore no further boundary terms for the action. It is just more involved.

The parameter Δ corresponds to the chemical potential associated to the second mass.

Note that even if the diffeomorphism symmetry has bee broken, we can define charges corresponding to 2, independent, timelike asymptotical Killing vectors.

Thanks