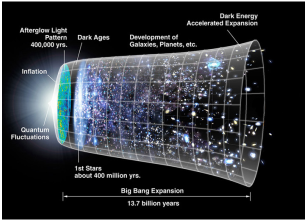
Resolution of Cosmic Singularities via String Theory

Johannes M. Oberreuter[1]
with Jan Pieter van der Schaar[1] and Koenraad Schalm[2]

[1]Institute for Theoretical Physics, University of Amsterdam
[2]Instituut-Lorentz for Theoretical Physics, University of Leiden

September 22, 2009

The Problem



Cartoon of the expanding universe (from the LAMDA archive, WMAP)

- The universe is expanding (observation)
- The universe has a beginning (Big Bang)
- The Big Bang is a singularity:
  General Relativity breaks down b/c length scales are small
- Quantum Gravity relevant at small length scales:
  must resolve the singularity
- Consistent evolution through the singularity

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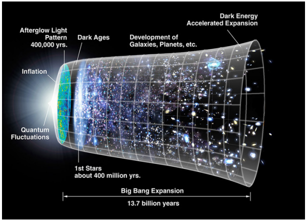

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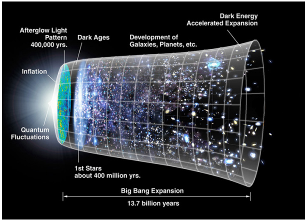

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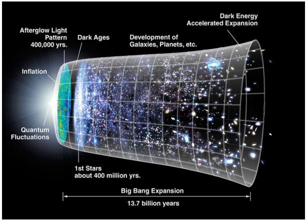

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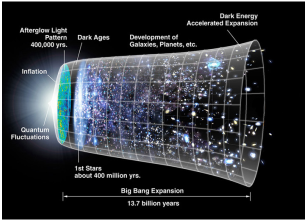

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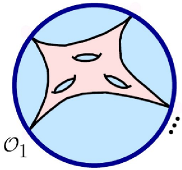

$$= \quad \langle \mathcal{O}_1(x_1) \cdots \rangle$$

- Sources of gauge invariant operators
  $\leftrightarrow$ boundary data of gravitational fields
- Boundary conditions for the bulk scalars needed:
  - $\phi(r) = \frac{\alpha \ln r}{r^2} + \frac{\beta}{r^2}$ for $r \to \infty$
  - Preserving full AdS symmetry: $\alpha = 0$
  - Breaking some symmetries: $\alpha = -\frac{\delta W(\beta)}{\delta \beta}$
  - Designer gravity: W arbitrary, real, smooth
    (invariant under global time translations)

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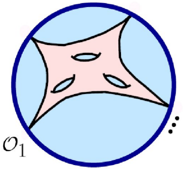

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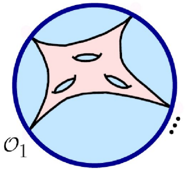

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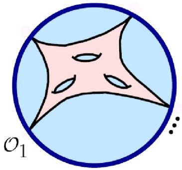

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Modification at the Boundary

- Take $\alpha = f\beta$
- For $f = 0$: Dual is $\mathcal{N} = 4$ SYM theory
- Bulk scalar couples to $\mathcal{O} \sim \text{tr}\left[\Phi_1^2 - \sum_{i=2}^{5} \Phi_i^2\right]$
- α: source for $\mathcal{O}$
  β: expectation value of $\mathcal{O}$
- $\alpha(\beta) \neq 0 \leftrightarrow$ multi-trace interaction $W(\mathcal{O})$
- Here: double-trace interaction
  $S = S_0 - W(\mathcal{O}) = S_0 + \frac{f}{2} \int \mathcal{O}^2$

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AdS-Cosmology

Cosmology

is a time dependent gravity background with a (cosmological) singularity at the beginning: the Big Bang.

In our setup

- spacetime must asymptote to AdS for using duality
- evolution is reversed, singularity at the end: Big Crunch
- metric degenerates in finite time

$$ds^2 = -dt^2 + a^2(t) d\sigma_4 \, , \quad a(t) \stackrel{t \to t_0}{\longrightarrow} 0$$

- Take Coleman-De Luccia instanton

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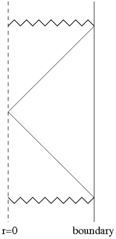
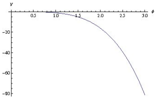


Figure: Cosmic Singularity in AdS space



Expectation: singularity removed in dual field theory.

This gauge theory

- has a negative potential unbounded from below

$$V(\phi) = -f\phi^4$$

- The theory has a tachyon

$\Rightarrow$ We need a turnaround of the potential to make the tachyon condensate.

T. Hertog and G. T. Horowitz 2004, 2005

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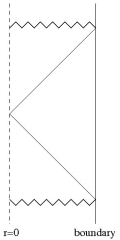

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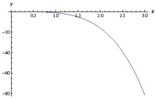

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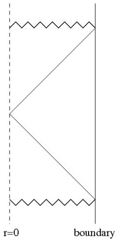

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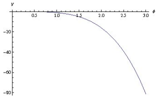

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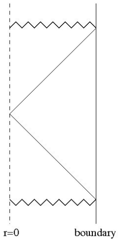

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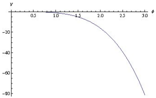

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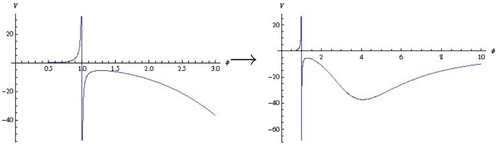
Expectation and Disapointment

- Generically, quantum effects are expected to turn around an unbounded potential.



- Coleman-Weinberg prescription: time evolution.

$$f \longrightarrow f(\mu) \longrightarrow f(\phi)$$

S. R. Coleman and E. J. Weinberg 1973

- Effective Potential is
  - one-loop exact
  - still unbounded:

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B. Craps, T. Hertog and N. Turok, 2007

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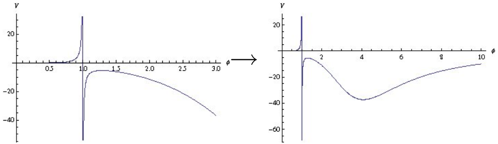

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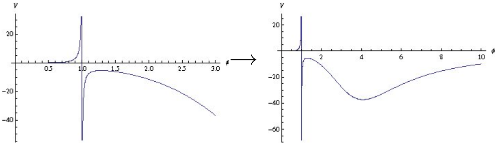

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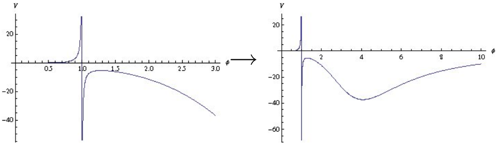

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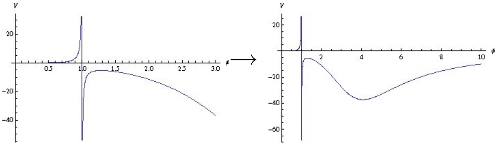

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Large-N Gauge Theories

- The *dual Quantum Field Theory* :
  gauge theory organised in double expansion in
  - the 't Hooft coupling $g^2_{\mathrm{YM}}N$ and
  - the inverse of the number of gauge degrees of freedom $\frac{1}{N}$
- Correlators look schematically as

$$\langle \mathcal{O}_1 \dots \rangle \sim \left((g^2N) + (g^2N)^2 + \dots\right)\left(1 + \frac{1}{N} + \frac{1}{N^2} + \dots\right) .$$

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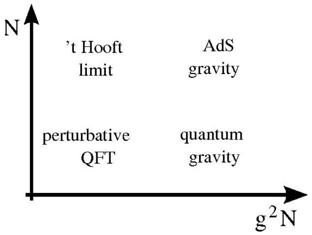

Oposite regimes of perturbation theory are related:

$$g_{\rm YM}^2 N \leftrightarrow \frac{R^4_{\rm curvature}}{l_s^4} \quad , \quad \frac{1}{N} \leftrightarrow g_{\rm string}$$

$\Rightarrow$ We need to include $\frac{1}{N}$ corrections to turn around the potential:

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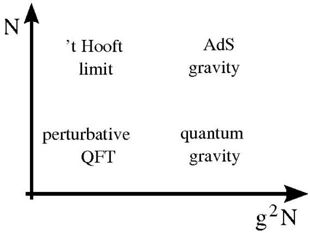

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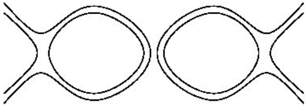
Proof of Principle

Corrected potential comes from a linear term in the β-function:

$$\beta(f) \equiv \mu \frac{\partial f}{\partial \mu} \sim \quad -f^2 \quad \underbrace{-(g^2 N)^2 \frac{f}{N}}_{\text{absent at 1-loop order}} \quad .$$

The corresponding diagram is



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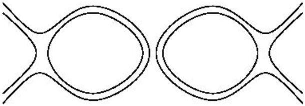

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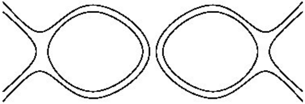

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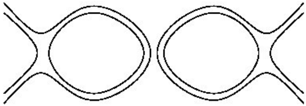
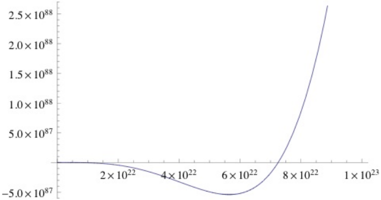


- Both couplings renormalized by $1/N$ effects
- β-functions are coupled and have non-trivial RG-flow

Preliminary results

$$V_{\text{eff.}} = - f_{\text{ren.}}\big|_{\mu \to \phi}\, \phi^4$$



But only for scalar theory at one loop level

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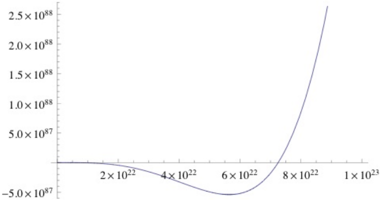

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Summary and Outlook

- More general boundary conditions admit time-dependent backgrounds.
- By means of the Gauge/Gravity duality cosmic singularities are expected to be resolved in a dual gauge theory picture.
- The large-N limit is not sufficient: field theory has no groundstate.
- $1/N$-corrections become important at large curvatures (hence close the the singularity) and turn around the potential.
- The formerly singular region remains bounded in size: Black Hole rather than Big Crunch?
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