SISSA

GRAVITY DUALS OF 2D SUSY GAUGE THEORIES

BASED ON:

- 0909.3106 with E. Conde and A.V. Ramallo (Santiago de Compostela)

[See also 0810.1053 with C. Núñez, P. Merlatti and A.V. Ramallo]

Daniel Areán
Milos, September 2009

OUTLINE

➢ INTRODUCTION. AdS/CFT and its generalisations

➢ GRAVITY DUAL OF 2d N=(1,1) from wrapped branes

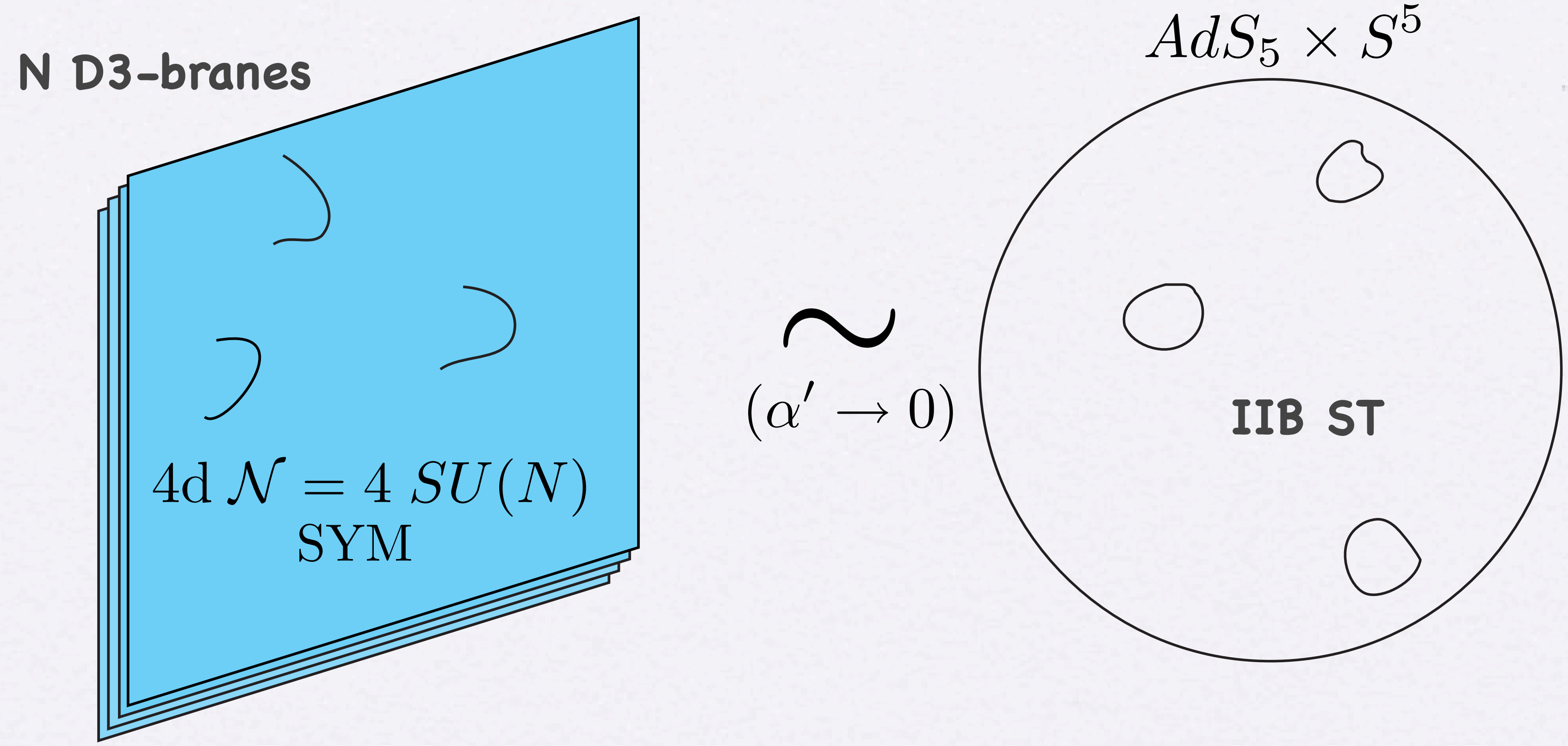
- Brane setup
- 10d SUGRA ansatz
- Gauged SUGRA approach (7d)
- Solution → Coulomb branch

➢ ADDING FLAVOR

- Flavor D5s
- Backreaction → smearing
- Flavored solution

➢ SUMMARY

AdS / CFT Correspondence



GENERALISE

- ★ d = 2
- ★ 2 (4) SUSYs
- ★ ~~Conformal~~
- ★ Add Flavor

$\longrightarrow$

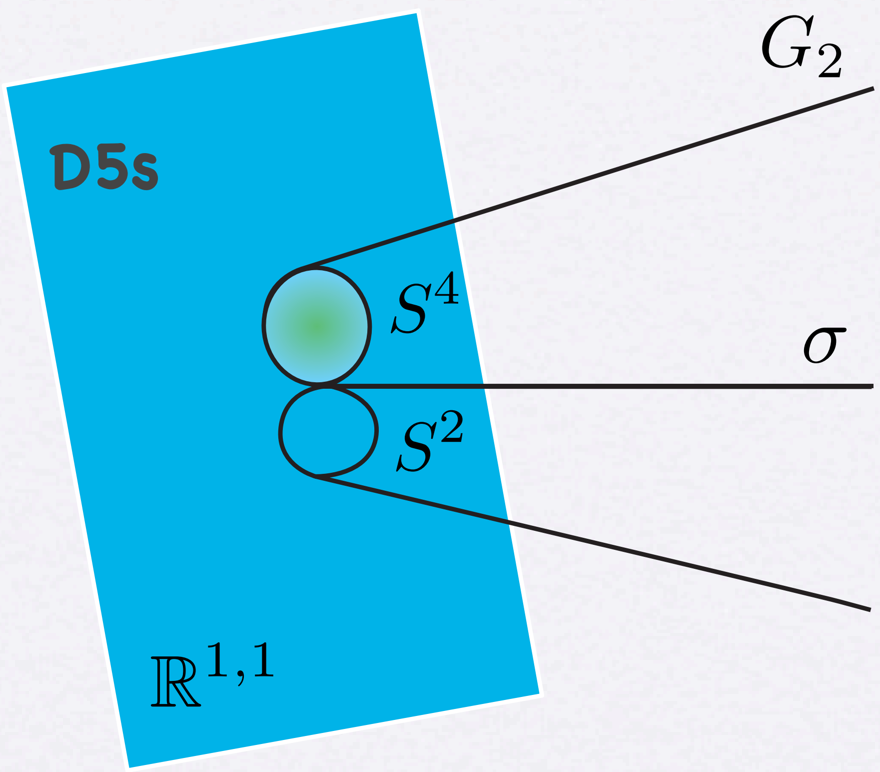
$$\boxed{2d \quad \begin{matrix} \mathcal{N}=(2,2) \\ \mathcal{N}=(1,1) \end{matrix} \quad \text{SYM} \ + \ N_f \ \text{flavors}}$$

★ USE WRAPPED BRANES

(4d: Maldacena & Núñez, Gauntlett et al, Bigazzi et al)
(3d: Chamseddine & Volkov, Maldacena & Nastase, Schvellinger & Tran, Gomis & Russo, Gauntlett et al)

DUAL TO N=(1,1) SYM FROM WRAPPED D5s

★ BRANE SETUP



$$\times \quad \mathbb{R}\ (\rho)$$

G_2 (over S^4 and N_3)

	$\mathbb{R}^{1,1}$		S^4				N_3			$\mathbb{R}$
D5	—	—	○	○	○	○	·	·	·	·

$N_3\ :\ (\sigma, \theta, \phi)$

DUAL TO N=(1,1) SYM FROM WRAPPED D5s

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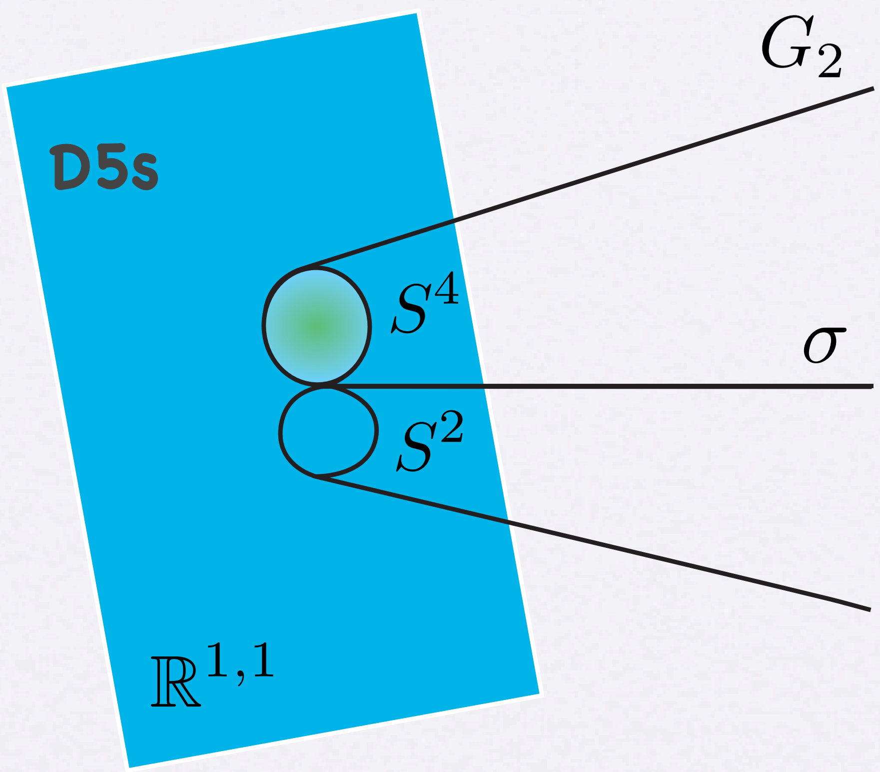


$\times \quad \mathbb{R}\ (\rho)$

G_2

	$\mathbb{R}^{1,1}$		S^4				N_3			$\mathbb{R}$
D5	—	—	○	○	○	○	·	·	·	·

$N_3 \ : \ (\sigma, \theta, \phi)$

- $G_2 \rightarrow$ 1/8 SUSY
- D5s (on a calibrated C_4) $\rightarrow$ 1/2 SUSY

$\longrightarrow$ **2 SUSYS**

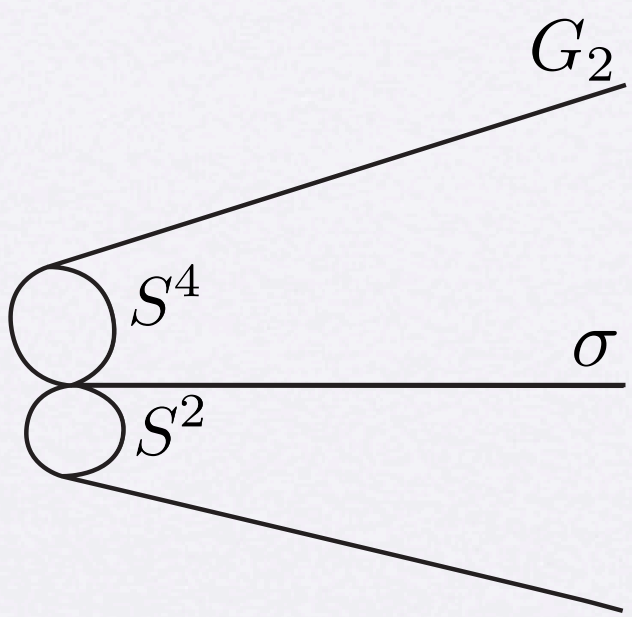
★ SUGRA ANSATZ

$\overbrace{\qquad\qquad\qquad\qquad}^{\mathrm{G}_2}$ ($S^4 \times N_3$)

	$\mathbb{R}^{1,1}$		S^4				N_3			$\mathbb{R}$
D5	$-$	$-$	$\bigcirc$	$\bigcirc$	$\bigcirc$	$\bigcirc$	$\cdot$	$\cdot$	$\cdot$	$\cdot$

$N_3\ :\ (\sigma,\theta,\phi)\qquad \mathbb{R}\ (\rho)$

◆ **(resolved)G_2 cone:** $ds_7^2 = \dfrac{(d\sigma)^2}{1-\frac{a^4}{\sigma^4}} + \dfrac{\sigma^2}{2}\, d\Omega_4^2 + \dfrac{\sigma^2}{4}\left(1-\dfrac{a^4}{\sigma^4}\right)\left[(E^1)^2+(E^2)^2\right]$ **(Bryant, Salamon) (Gibbons, Page, Pope)**



- S^4**:** $d\Omega_4^2 = \dfrac{4}{(1+\xi^2)^2}\left[d\xi^2 + \dfrac{\xi^2}{4}\left((\omega^1)^2+(\omega^2)^2+(\omega^3)^2\right)\right]$

- **fibered** S^2**:**

$$E^1 = d\theta + \frac{\xi^2}{1+\xi^2}\left(\sin\phi\,\omega^1 - \cos\phi\,\omega^2\right)$$

$$E^2 = \sin\theta\left(d\phi - \frac{\xi^2}{1+\xi^2}\,\omega^3\right) + \frac{\xi^2}{1+\xi^2}\cos\theta\left(\cos\phi\,\omega^1 + \sin\phi\,\omega^2\right)$$

G_2

	$\mathbb{R}^{1,1}$		S^4				N_3			$\mathbb{R}$
D5	$-$	$-$	$\bigcirc$	$\bigcirc$	$\bigcirc$	$\bigcirc$	$\cdot$	$\cdot$	$\cdot$	$\cdot$

$N_3 \,:\, (\sigma, \theta, \phi) \quad \mathbb{R}\,(\rho)$

- **10d metric** $ds^2 = e^{\Phi}\Big[\, dx_{1,1}^2 + \frac{z}{m^2}\, d\Omega_4^2 \Big] + \frac{e^{-\Phi}}{m^2 z^{\frac{4}{3}}}\Big[\, d\sigma^2 + \sigma^2 \big((E^1)^2 + (E^2)^2 \big) \Big] + \frac{e^{-\Phi}}{m^2}\,(d\rho)^2$

- **3-form** $F_3 = dC_2 \;, \quad C_2 = g_1 E^1 \wedge E^2 + g_2 \big(\mathcal{S}^{\xi} \wedge \mathcal{S}^3 + \mathcal{S}^1 \wedge \mathcal{S}^2 \big)$

$$\overbrace{\hphantom{S^4 \quad N_3}}^{\mathrm{G}_2}$$

	$\mathbb{R}^{1,1}$		S^4				N_3			$\mathbb{R}$
D5	$-$	$-$	$\bigcirc$	$\bigcirc$	$\bigcirc$	$\bigcirc$	$\cdot$	$\cdot$	$\cdot$	$\cdot$

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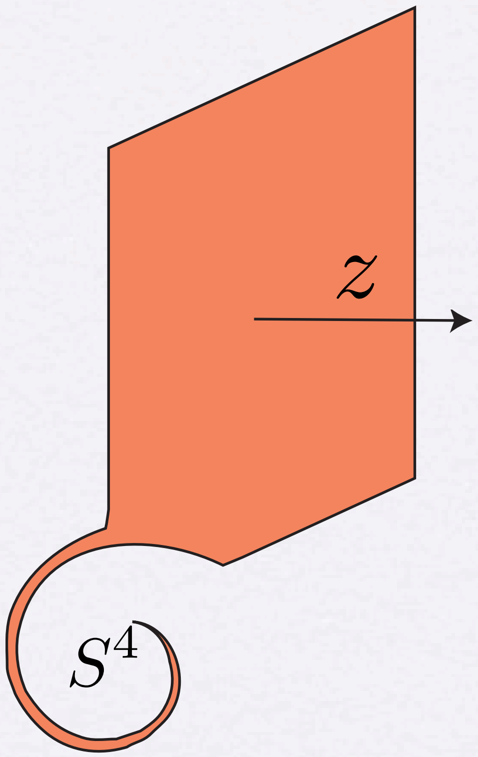
$\boxed{\text{SUSY}} \xrightarrow{\text{N=(1,1)}} \boxed{\text{BPSs}} \longrightarrow$

$z(\rho,\sigma)$	SIZE OF $\mathcal{C}_4$
$\Phi(\rho,\sigma)$	DILATON
$g_i(\rho,\sigma)$	3-FORM FLUX

- **BPSs are PDEs ☹, 7d Gauged SUGRA ➞ SOLUTION ☺**

★ **GAUGED SUGRA APPROACH** ⟶ **LINEAR DISTRIBUTION OF D5S**

- ◆ **Take 7d SO(4) Gauged SUGRA** ⟶ **Domain wall problem**

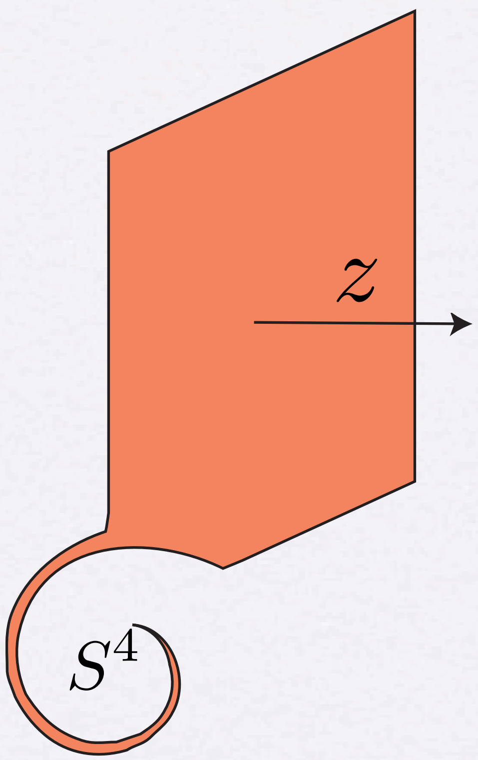


- **1d problem → BPSs easy** $\xrightarrow{\text{Uplift}}$ **10d solution in terms of** $\boxed{\text{c}}$

$$\left.\begin{array}{l} \rho \;\rightarrow\; \mathbb{R} \perp (\mathbb{R}^{1,1}, G_2) \\ \sigma \;\rightarrow\; G_2 \end{array}\right. \longleftrightarrow (z, \psi)$$

★ GAUGED SUGRA APPROACH ⟶ LINEAR DISTRIBUTION OF D5S

◆ Take 7d SO(4) Gauged SUGRA ⟶ Domain wall problem



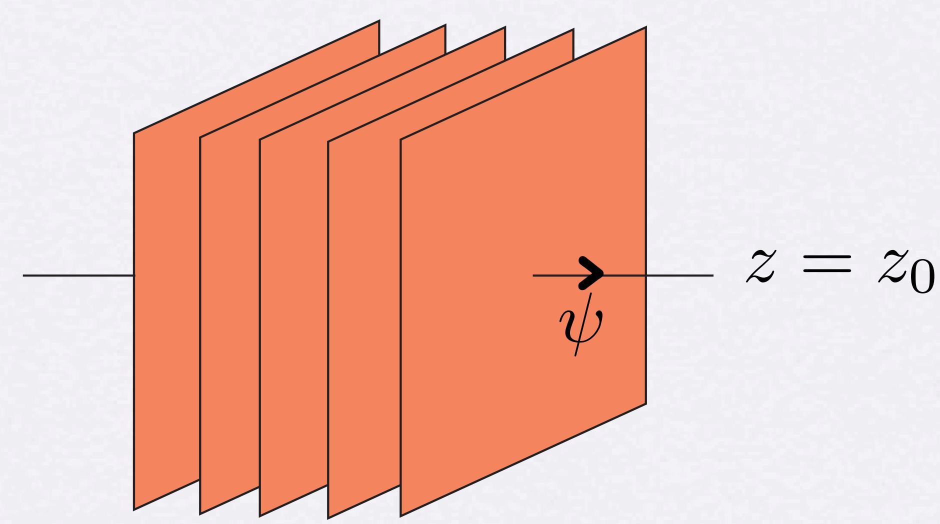
- 1d problem → BPSs easy $\xrightarrow{\text{Uplift}}$ 10d solution in terms of $\boxed{c}$

$$\begin{array}{l} \rho \to \mathbb{R} \perp (\mathbb{R}^{1,1}, G_2) \\ \sigma \to G_2 \end{array} \longleftrightarrow (z, \psi)$$

◆ UV (z→∞): $ds^2 \to$ $\boxed{\text{D5s along } \mathbb{R}^{1,1} \times S^4}$ [➡ Linear dilaton]

◆ IR (for c<-1):

- Singularity (good) at $z = z_0$
- Linear distribution (ψ)

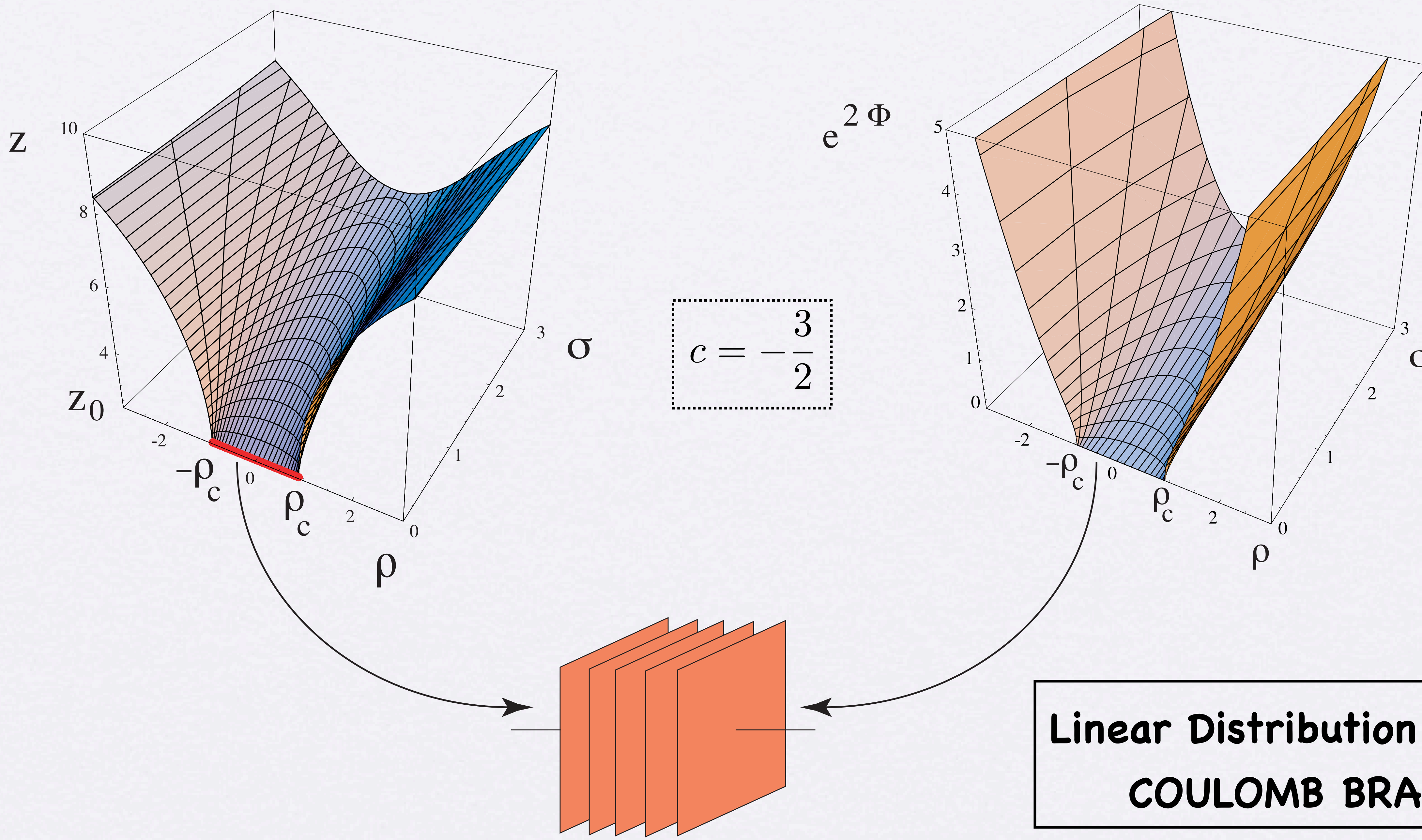


	$\mathbb{R}^{1,1}$		S^4				N_3			$\mathbb{R}$
			G_2							
D5	$-$	$-$	$\bigcirc$	$\bigcirc$	$\bigcirc$	$\bigcirc$	$\cdot$	$\cdot$	$\cdot$	$\cdot$

$N_3 \,:\, (\sigma, \theta, \phi) \qquad \mathbb{R}\,(\rho)$

- **Changing vbles.** $(z, \psi) \rightarrow (\rho, \sigma)$

➥ **Analytic (implicit) sol. for** $z(\rho, \sigma)$

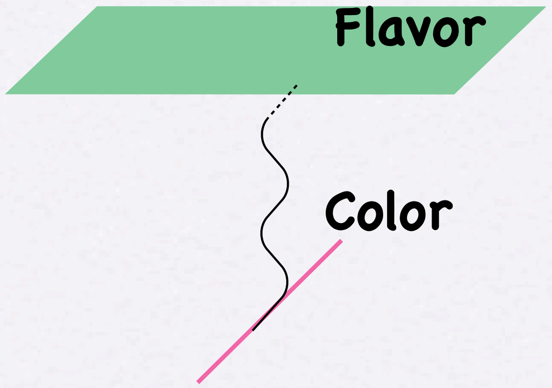


$$c = -\frac{3}{2}$$

$$(z = z_0, \psi) \rightarrow (|\rho| < \rho_c, \sigma = 0)$$

Linear Distribution of D5s
COULOMB BRANCH

★ ADDING FLAVOR



◆ Add an open string sector → FLAVOR BRANES

- Brane setup

Flavor D5s

- Non-compact $\mathcal{C}_4 \subset G_2$
- At fixed $\rho = \rho_Q$

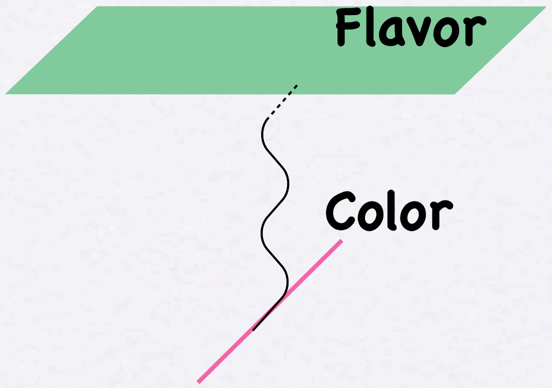
→

★ Global Sym: flavor

★ $m_Q \sim \rho_Q$

★ Same SUSY

★ ADDING FLAVOR



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⟶

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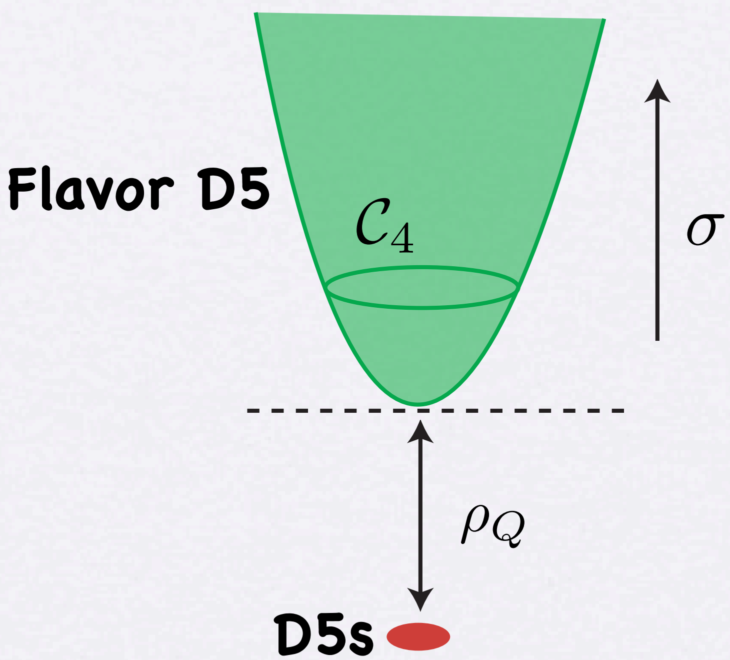
- Probe approximation $N_f \ll N_c\,,\ N_c \to \infty$

(Karch & Randall, Karch & Katz)

↕

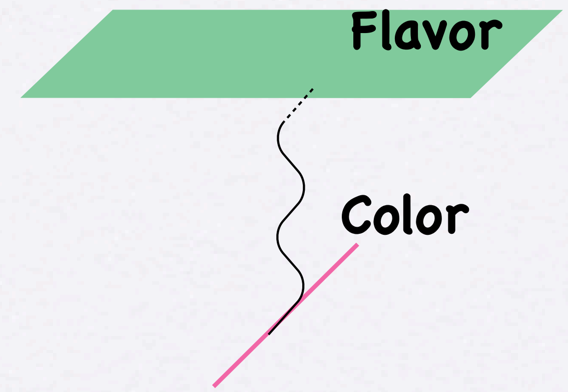
Quenched flavor in the large N_c limit.

⟶



★ ADDING FLAVOR

◆ Add an open string sector → FLAVOR BRANES



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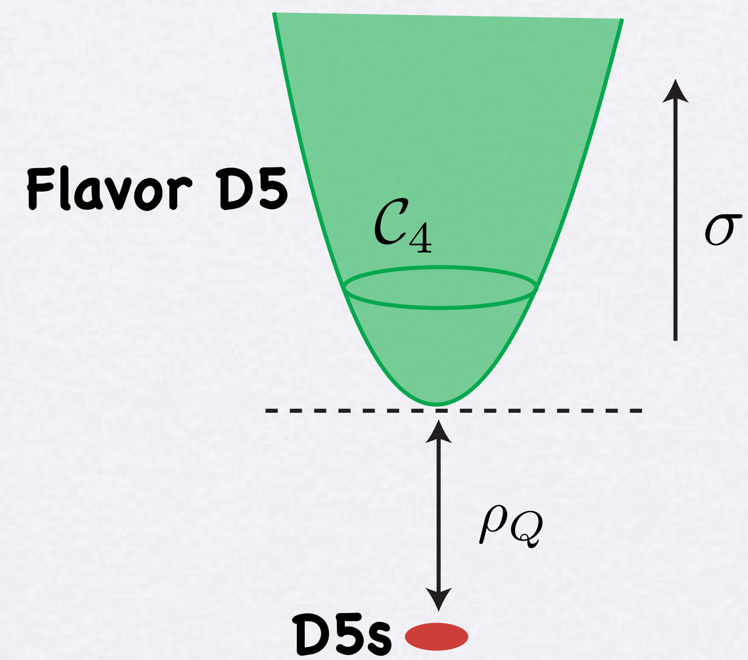


• Backreaction $N_f \sim N_c$

$N_f \, , \; N_c \to \infty$
N_f / N_c fixed

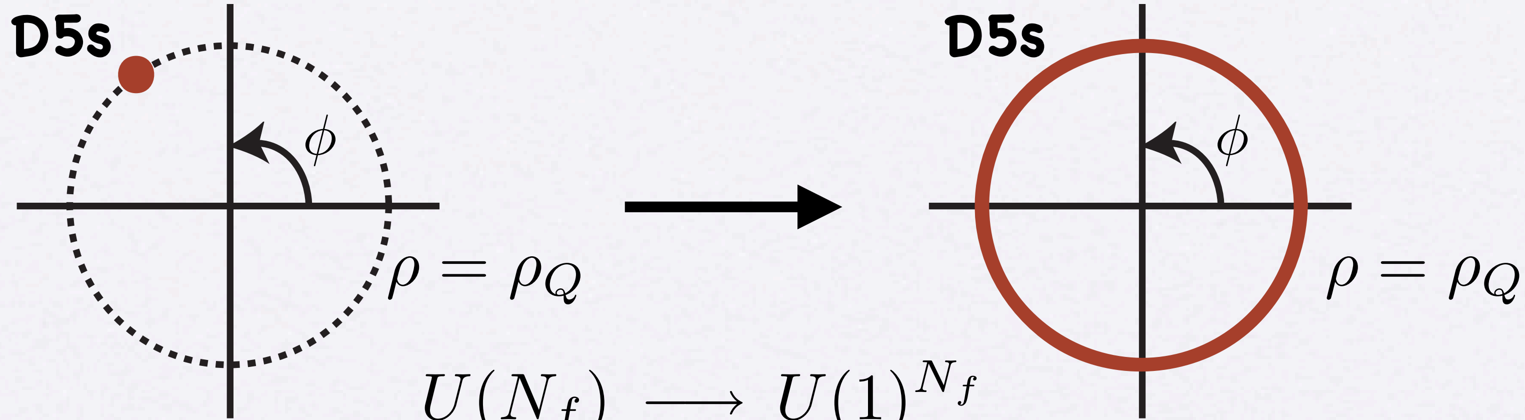
→ Veneziano limit

Quarks loops included

- **Computing the backreaction is difficult** $\boxed{S = S_{IIB} + S_{DBI}^{\text{flavor}} + S_{WZ}^{\text{flavor}}}$

➥ **Smearing**

(Bigazzi et al, Casero et al)



$$U(N_f) \longrightarrow U(1)^{N_f}$$

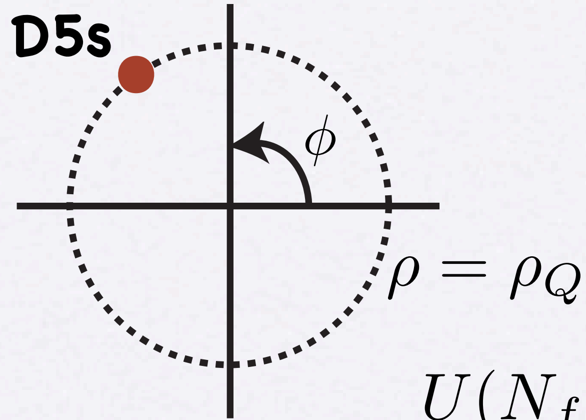
$$S_{WZ}^{flavor} = T_5 \sum^{N_f} \int_{\mathcal{M}_6^{(i)}} \hat{C}_6 \Longrightarrow -T_5 \int_{\mathcal{M}_{10}} \Omega \wedge C_6 \longrightarrow \boxed{dF_3 \; = \; 2\kappa_{10}^2 \, T_5 \, \Omega}$$

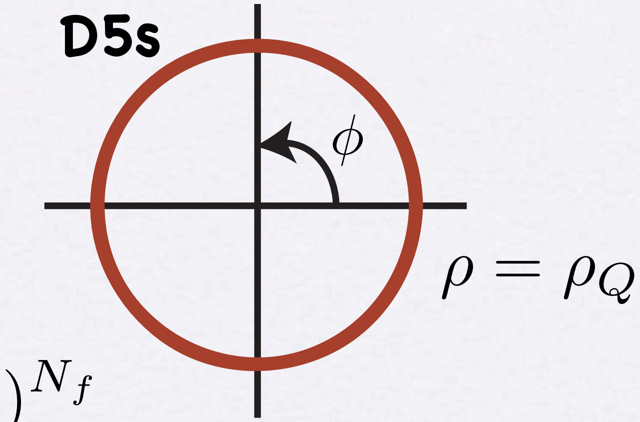
~~**Bianchi identity**~~

➥ **Ω + metric →** $\boxed{\textbf{Flavored BPSs}}$

- Computing the backreaction is difficult $\boxed{S = S_{IIB} + S_{DBI}^{\text{flavor}} + S_{WZ}^{\text{flavor}}}$

➥ **Smearing**
(Bigazzi et al, Casero et al)





$U(N_f) \longrightarrow U(1)^{N_f}$

$$S_{WZ}^{flavor} = T_5 \sum^{N_f} \int_{\mathcal{M}_6^{(i)}} \hat{C}_6 \implies -T_5 \int_{\mathcal{M}_{10}} \Omega \wedge C_6 \longrightarrow \boxed{dF_3 \;=\; 2\kappa_{10}^2\, T_5\, \Omega}$$ ~~Bianchi identity~~

➥ Ω + metric → $\boxed{\text{Flavored BPSs}}$

- D5 embeddings (κ-symmetry) ➔ Ω ,   this is hard!!

- D5-branes at $\rho = \rho_Q$
- Same SUSY (2)
- No new deformations of g_{ab}

⟶ $\boxed{\text{generic } \Omega}$ /

- Consistent BPSs (➔ EoM)
- Color ∩ Flavor = ∅

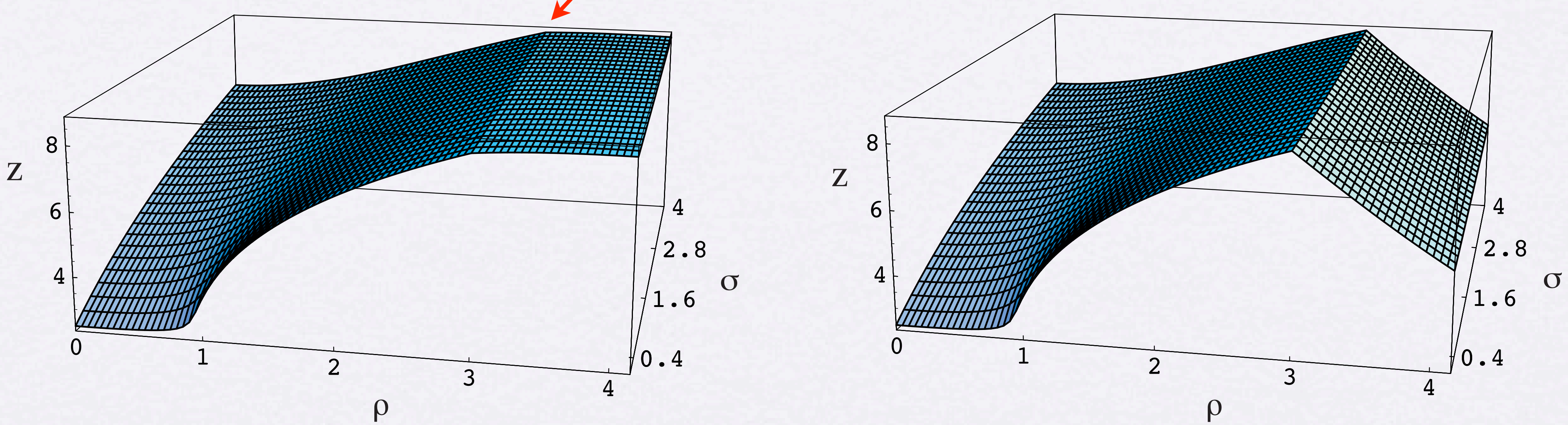
◆ **Particular charge distribution / homogeneous charge distribution along $\perp \mathbb{R}^3$**

- **Numerical solution with $z,\ \phi,\ g_i$ continuous at $\rho = \rho_Q$**

- **Coincides with the unflavored for $\rho < \rho_Q$**

$\rho = \rho_Q$

$x \equiv \dfrac{18\pi\, n_f}{N_c} = 0.2$ $\qquad\qquad$ $x = 1$



- **Flavor contributes as expected [$1/g_{YM}^2 \sim z^2(\rho, \sigma = 0)$]**

★ SUMMARY / TO TRY

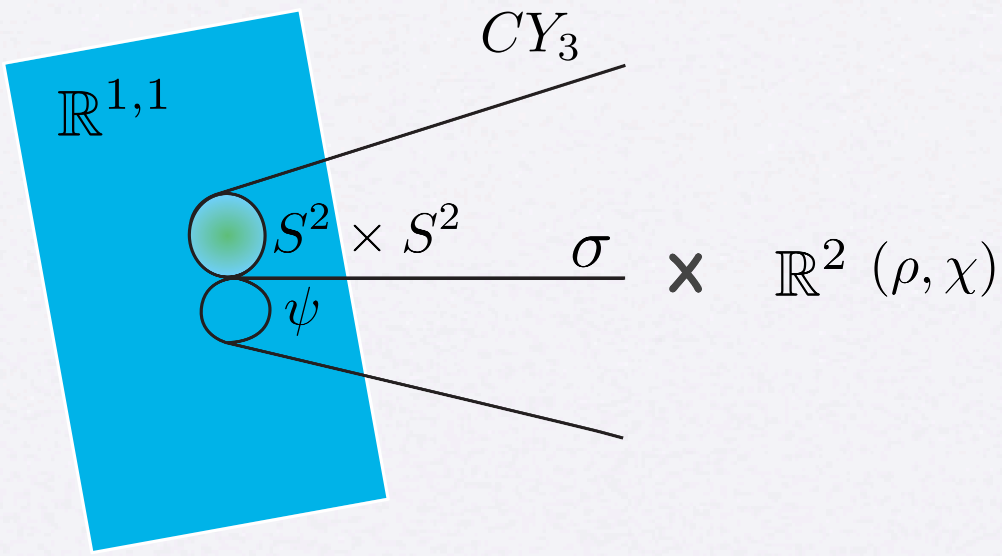
- Gravity duals of 2d N=(1,1) & (2,2) SUSY theories from wrapped D5s ✓
- Large number of flavors via backreacting flavor D5s ✓
- Explore the F.T. (a little) → color probe brane ✓ (E-r relation missing)
- Higgs branch → Color & flavor branes recombining
- Alternative setup → D3s on a 2-cycle of a CY3. Better UV.
- Non-singular background?
- Less SUSY → D5s on a 4-cycle of a Spin(7)

★ SUGRA DUALS OF 2D THEORIES WITH N=(2,2) SUSY

- D5s on a 4-cycle of a CY3 ~ 2d N = (2,2)

◆ CY3 → 1/4 SUSY
◆ D5s → 1/2 SUSY

⟶ $\boxed{\text{2 SUSYS}}$



▪ 10d Ansatz

▪ Metric → z (ρ,σ) & ϕ(ρ,σ)
▪ 3-form → g(ρ,σ)

⟶ $\boxed{\text{BPSs}}$

▪ Analyt. sol ✓
▪ EoM ✓

[7d Gauged SUGRA]

▪ Flavoring → D5s on a non-compact 4-cycle → Embeddings found

➥ Ω constructed → new BPSs → (Numeric) Flavored background