

# Kinetic Theory and Pure Gravity in AdS

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## Main Reference

Ramakrishnan Iyer and AM, "An AdS/CFT Connection between Boltzmann and Einstein," [arxiv:0907.1156[hep-th]]

# Preliminaries and Problems A

## Preliminaries

- Gauge/gravity duality at

- (a) strong coupling

- (b) large rank of the gauge group ( $N$ )

defines a “universal sector” of dynamics in gauge theories as dual of pure classical gravity in five dimensions. This is so because the theory of classical gravity always admits a consistent truncation to Einstein's equation in five dimensions with a negative cosmological constant.

The embedding of the universal sector in the full theory depends on the details of the theory but not the dynamics within the sector.

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- In this sector, all observables can be determined by the energy-momentum tensor *alone*. This is so because the metric which solves Einstein’s equation with negative cosmological constant is uniquely determined by the boundary stress tensor (Balasubramanian-Krauss stress tensor) which is identified with the energy-momentum tensor of the gauge theory.

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- “Universal sector” is constituted by a range of phenomena such as decoherence, local relaxation and hydrodynamics.
- (a) State in the field theory  $\leftrightarrow$  Solution in gravity with a smooth final horizon
- (b) Temperature of the final equilibrium  $\leftrightarrow$  Final temperature of the horizon
- Regularity/irregularity in the five dimensional solution of Einstein’s solution implies regularity/irregularity in the full solution of gravity as the lift to the full solution is trivial (without involving any warping). The transport coefficients in hydrodynamics can be systematically determined by the regularity of the future horizon.

# Preliminaries and Problems B

## Problems

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- A field-theoretic understanding of how all observables get determined by the energy-momentum tensor alone.
- To solve for the condition on the energy-momentum tensor which gives solutions of Einstein's equation with smooth future horizons.
- A precise way to decode phenomena in the gauge theory from the metric.

# Our Results

- We show that the relativistic semiclassical Boltzmann equation has “conservative solutions” which could be determined by the energy-momentum tensor alone. We can justify our study of Boltzmann equation at weak coupling because previous work of Arnold, Yaffe and others have demonstrated that an effective Boltzmann equation is as good as perturbative gauge theory to study, for example, transport phenomena in high temperature QCD.

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- We argue that these *conservative* solutions exist also in the exact microscopic theory.
- We naturally identify the *conservative solutions* with the *universal sector* at strong coupling and large  $N$ .
- We find the right method of extrapolating the *conservative* condition at weak coupling to *regularity* condition in gravity at strong coupling.

# Conservative Solutions (1/4)

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- The energy momentum tensor can always be parametrised by (a) the five hydrodynamic variables ( $\rho$ ,  $u_i$ ,  $p$ ) and (b) the shear stress tensor ( $p_{ij}$ ) in a comoving locally inertial frame. It can be shown that these ten variables are related to the first few velocity moments of the one particle phase-space distribution function  $f(\mathbf{x}, \xi)$  as below:

$$\rho = \int f(\mathbf{x}, \xi) d\xi, \quad u_i = \frac{1}{\rho} \int f(\mathbf{x}, \xi) \xi_i d\xi, \quad p = \frac{1}{3} \int f(\mathbf{x}, \xi) (\xi - \mathbf{u})^2 d\xi \quad (1)$$

$$p_{ij} = \int f(\mathbf{x}, \xi) (\xi_i - u_i)(\xi_j - u_j) d\xi$$

A local temperature ( $T$ ) can be defined by using the equation of state for ideal gas,  $p/\rho = RT$ , locally.



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- Let  $f^{(n)}$ , a tensor of rank  $n$  be the  $n$ -th velocity moment of  $f(\mathbf{x}, \xi)$  so that  $f^{(n)} = \int \mathbf{c}^n f d\xi$ , where  $c_i = \xi_i - u_i$ . At equilibrium all these  $f^{(n)}$ 's vanish. However in conservative solutions these do not vanish and in fact can be very large. These  $f^{(n)}$ 's are determined functionally in terms of the ten independent variables of the energy-momentum tensor.

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$$S_i = \frac{15pR}{2B^{(2)}} \frac{\partial T}{\partial x_i} + \frac{3}{2B^{(2)}} (2RT \frac{\partial p_{ir}}{\partial x_r} + 7Rp_{ir} \frac{\partial T}{\partial x_r} - \frac{2p_{ir}}{\rho} \frac{\partial p}{\partial x_r}) + \dots \quad (2)$$

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where  $B^{(2)}$  is a specific function of the molecular mass, radius, local density and temperature and can be determined from the Boltzmann equation. All the higher moments similarly can be systematically determined for such solutions in unique functional forms of the ten independent variables. These functional forms have systematic expansions in two parameters, the derivative expansion parameter which is (typical scale of variation/mean free path) and amplitude expansion parameter (typical value of non-hydrodynamic shear stress/hydrostatic pressure). Only spatial derivatives and no time derivative appear in the functional forms of  $f^{(n)}$ .

# Conservative Solutions (2/4)

- Since all the velocity moments of  $f$  are unique local functions of the ten variables and their spatial derivatives, it follows that  $f$  is also uniquely determined by the ten variables. Once  $f$  is determined, any observable can also be determined through  $f$ .

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- The ten variables satisfy the following equations of motion closed amongst themselves

$$\begin{aligned}
 & \frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_r}(\rho u_r) = 0 \tag{3} \\
 & \frac{\partial u_i}{\partial t} + u_r \frac{\partial u_i}{\partial x_r} + \frac{1}{\rho} \frac{\partial (\rho \delta_{ir} + p_{ir})}{\partial x_r} = 0 \\
 & \frac{\partial p}{\partial t} + \frac{\partial}{\partial x_r}(u_r p) + \frac{2}{3}(\rho \delta_{ir} + p_{ir}) \frac{\partial u_i}{\partial x_r} + \frac{1}{3} \frac{\partial S_r}{\partial x_r} = 0 \\
 & \frac{\partial p_{ij}}{\partial t} + \frac{\partial}{\partial x_r}(u_r p_{ij}) + \frac{\partial S_{ijr}}{\partial x_r} - \frac{1}{3} \delta_{ij} \frac{\partial S_r}{\partial x_r} \\
 & \quad + \frac{\partial u_j}{\partial x_r} p_{ir} + \frac{\partial u_i}{\partial x_r} p_{jr} - \frac{2}{3} \delta_{ij} p_{rs} \frac{\partial u_r}{\partial x_s} \\
 & \quad + p \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3} \delta_{ij} \frac{\partial u_r}{\partial x_r} \right) = B^{(2)}(\rho, T) p_{ij} \\
 & \quad + \sum_{\rho, q=0; \rho \geq q; (\rho, q) \neq (2,0)}^{\infty} B_{ij\nu\rho}^{(2;\rho,q)}(\rho, T) f_\nu^{(\rho)} f_\rho^{(q)}
 \end{aligned}$$

Above all the higher moments, including  $S_i$ , has been determined in terms of the hydrodynamic variables, the shear stress tensor and their *spatial* derivatives. Since spatial derivatives of arbitrary orders are present in these functional forms, we need analytic data as initial conditions for these equations

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- Any solution of the above equations of motion of the ten variables can be *uniquely* lifted to a full solution of the Boltzmann equation for  $f$  through the functional forms for  $f^{(n)}$  already determined.

# Conservative Solutions (3/4)

- There are two special kinds of *conservative* solutions
  - (a) *normal* or purely-hydrodynamic solutions [Enskog(1917), Burnett(1935), Chapman(1939)] where  $f$  is determined as functional of the five hydrodynamic variables and their spatial derivatives only
  - (b) homogenous non-hydrodynamic solutions where all hydrodynamic variables are constants and the shear stress tensor (therefore all the higher moments) is a function of time only, describing dynamics in only velocity space and hence relaxation.

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- The normal solutions can be found by noting that the equation for  $p_{ij}$  has a special algebraic solution given in terms of hydrodynamic variables only. This solution is unique. Upto two derivatives this solution is as below:

$$\begin{aligned}
 p_{ij} = & \eta \sigma_{ij} + \lambda_1 \frac{\eta^2}{\rho} (\partial_i u_j) \sigma_{ij} + \lambda_2 \frac{\eta^2}{\rho} \left( \frac{D}{Dt} \sigma_{ij} - 2(\sigma_{ik} \sigma_{kj} - \frac{1}{3} \delta_{ij} \sigma_{lm} \sigma_{lm}) \right) \\
 & + \lambda_3 \frac{\eta^2}{\rho T} (\partial_i \partial_j T - \frac{1}{3} \delta_{ij} \square T) + \lambda_4 \frac{\eta^2}{\rho \rho T} (\partial_i \rho \partial_j T + \partial_j \rho \partial_i T - \frac{2}{3} \delta_{ij} \partial_l \rho \partial_l T) \\
 & + \lambda_5 \frac{\eta^2}{\rho \rho T} (\partial_i T \partial_j T - \frac{1}{3} \delta_{ij} \partial_l T \partial_l T) + \dots
 \end{aligned} \tag{4}$$

where  $\sigma_{ij} = \partial_i u_j + \partial_j u_i - (2/3) \delta_{ij} \partial_l u_l$ ,  $\eta = (\rho/B^2)$  and the  $\lambda$ 's which are pure numbers can be determined from the Boltzmann equation. Note all time-derivatives can be replaced by spatial derivatives through hydrodynamic equations of motion. This matches with the second order expression for  $p_{ij}$  for normal solutions [Chapman and Cowling, Chapter 15]



# Conservative Solutions (4/4)

- Interestingly, the homogenous non-hydrodynamic solutions has singularities. For instance  $f_{ijkl}^{(4)} = (2B^{(2)}\delta_{(klmn)(ijtu)} - B_{(klmn)(ijtu)}^{(4,4,0)})^{-1} B_{(ijtu)(pqrs)}^{(4,2,2)} p_{pq} p_{rs} + \dots$  and this becomes indeterminate when  $(2B^{(2)}\delta_{(klmn)(ijtu)} - B_{(klmn)(ijtu)}^{(4,4,0)})$  regarded as an  $81 \times 81$  matrix fails to be invertible. Such singularities also appear in normal solutions in kinetic theory of liquids (to be discussed later) and has the interpretation of local nucleation of solid phase and so here the singularities probably signal local condensation of the liquid phase.

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- Any generic solution of the Boltzmann Equation at sufficiently late times is approximated by an appropriate conservative solution. Since the maximum of the propagation speeds of the linear modes increases as more and more moments are included [Boillat, Muller], we can argue that, at a sufficiently late time, the part of the higher moments functionally independent of the hydrodynamic variables and the shear stress tensor becomes irrelevant, so that the dynamics is well approximated by an appropriate conservative solution. Thus ten variables suffice to capture systematically a whole range of phenomena which includes hydrodynamics and relaxation.

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- It can be shown that the relativistic semiclassical Boltzmann equation has conservative solutions as well.

# Multi-Component Systems

In order to generalize conservative solutions of Boltzmann equation to relativistic gauge theories we also need to understand how to construct such solutions for multi-component systems.

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- In  $\mathcal{N} = 4$  SYM theory, all the particles form a multiplet whose internal degrees of freedom are spin and  $(SO(6)_R)$  charge along with the color indices. From the point of view of gravity, since in the universal sector we have pure gravity on the dual side, not only local and global charges and currents, but also the higher multipole moments of these charge distributions are absent at the boundary. So, the most natural reflection of this on the conservative solutions is that there is equipartition at every point in phase space over the internal, i.e the spin, charge and color degrees of freedom. Then we can easily construct an effective single component Boltzmann equation by summing over interactions in all spin, charge and color channels.

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- For other conformal gauge theories with gravity duals, we may also do the same even though all particles do not form a multiplet. This is possible because of mass degeneracy.

# Do Conservative Solutions exist in the Exact Microscopic Theory?

- The untruncated BBGKY hierarchy for non-relativistic systems is equivalent to the exact microscopic theory. Normal/purely hydrodynamic solutions have been constructed for the untruncated hierarchy [Born and Green (1949)]. These also exist if semiclassical corrections are included.

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- The viscosity, for instance receives corrections as in  $\eta = \frac{1}{15} \int \nu(r) \phi'(r) r^3 dr$  + the gaseous part ( $N'_2 = \nu(\mathbf{r} \cdot \mathbf{b}(\mathbf{v}) \cdot \mathbf{r} + \mathbf{r} \cdot \mathbf{b}'(\mathbf{v}) \cdot \mathbf{r})$ ), and increases with temperature unlike gases.



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- It is certainly plausible that the conservative solutions of the untruncated BBGKY hierarchy also exist. We are investigating this currently.

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- The viscosity, for instance receives corrections as in  $\eta = \frac{1}{15} \int \nu(r) \phi'(r) r^3 dr$  + the gaseous part ( $N'_2 = \nu(\mathbf{r} \cdot \mathbf{b}(\mathbf{v}) \cdot \mathbf{r} + \mathbf{r} \cdot \mathbf{b}'(\mathbf{v}) \cdot \mathbf{r})$ ), and increases with temperature unlike gases.
- It is certainly plausible that the conservative solutions of the untruncated BBGKY hierarchy also exist. We are investigating this currently.
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- So, it is likely that normal and conservative solutions exist in the exact relativistic quantum gauge theories like QCD such that a generic state at sufficient late times can be approximated by an appropriate conservative solution.
- The higher order transport coefficients could be exactly defined (at least implicitly) if we can construct normal solutions of the exact relativistic quantum gauge theories. We are investigating this currently as well.

# The Israel-Stewart-Muller Formalism and Gauge/Gravity duality

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- Interestingly, Ilya Prigogine also made an attempt to rewrite exact microscopic Hamiltonian dynamics in a “proto-thermodynamic” language. So our approach also conforms with his vision. In fact, it could be the first instance, where his vision could be concretely formulated and understood.

# A Proposal for the Regularity Condition in Gravity (1/2)

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- Now the “conservative” condition on the energy-momentum tensor becomes the “regularity” condition in gravity such that the dual solutions have smooth future horizons. So, on top of the conservation equation  $\partial^\mu [(\pi T)^4 (4u_\mu u_\nu + \eta_{\mu\nu}) + \pi_{\mu\nu}] = 0$  which gives us the forced Euler equation with  $\pi_{\mu\nu}$  as an independent variable; the regularity condition must involve five independent equations which tells us how any analytic initial data on  $\pi_{\mu\nu}$  evolves with time.

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- When  $\pi_{\mu\nu}$  is given in terms of hydrodynamic variables only, we will have “normal” solutions of the microscopic theory and the gravity duals at strong coupling could be easily identified with the “tubewise black brane solutions” found by Bhattachaya, et al. In a radial tube from every point at the boundary these solutions can be parametrised by the local hydrodynamic variables which from the gravity viewpoint are the Goldstone-like fields corresponding to boost and scale invariance, the maximally commuting broken symmetries present in the asymptotic geometry.

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- We propose the regularity condition as the most general equation for  $\pi_{\mu\nu}$  which can reproduce the correct purely hydrodynamic energy-momentum tensor known exactly upto second order in derivatives as a special solution.

# A Proposal for the Regularity Condition in Gravity (2/2)

Therefore, our regularity condition for pure gravity in  $AdS_5$  is:

$$\begin{aligned}
 (1 - \lambda_3) \left[ (u \cdot \partial) \pi^{\mu\nu} + \frac{4}{3} \pi^{\mu\nu} (\partial \cdot u) - \left( \pi^{\mu\beta} u^\nu + \pi^{\nu\beta} u^\mu \right) (u \cdot \partial) u_\beta \right] & \quad (5) \\
 = -\frac{2\pi T}{(2 - \ln 2)} \left[ \pi^{\mu\nu} + 2(\pi T)^3 \sigma^{\mu\nu} \right. \\
 - \lambda_3 (2 - \ln 2) (\pi T)^2 \left( (u \cdot \partial) \sigma^{\mu\nu} + \frac{1}{3} \sigma^{\mu\nu} (\partial \cdot u) - \left( u^\nu \sigma^{\mu\beta} + u^\mu \sigma^{\nu\beta} \right) (u \cdot \partial) u_\beta \right) \\
 - \lambda_4 (\ln 2) (\pi T)^2 (\sigma^{\alpha\mu} \omega_{\alpha\nu} + \sigma^{\alpha\mu} \omega_{\alpha\nu}) \\
 - 2\lambda_1 (\pi T)^2 \left( \sigma^{\alpha\mu} \sigma_{\alpha\nu} - \frac{1}{3} P^{\mu\nu} \sigma^{\alpha\beta} \sigma_{\alpha\beta} \right) \\
 \left. - (1 - \lambda_4) \frac{\ln 2}{(2 - \ln 2)} (\pi^\mu_{\alpha} \omega^{\alpha\nu} + \pi^\nu_{\alpha} \omega^{\alpha\mu}) \right] \\
 - \frac{2\lambda_2}{(2 - \ln 2)} \left[ \frac{1}{2} (\pi^{\mu\alpha} \sigma^\nu_{\alpha} + \pi^{\nu\alpha} \sigma^\mu_{\alpha}) - \frac{1}{3} P^{\mu\nu} \pi^{\alpha\beta} \sigma_{\alpha\beta} \right] \\
 + \frac{1 - \lambda_1 - \lambda_2}{(2 - \ln 2) (\pi T)^3} \left( \pi^{\mu\alpha} \pi^\nu_{\alpha} - \frac{1}{3} P^{\mu\nu} \pi^{\alpha\beta} \pi_{\alpha\beta} \right) + \\
 O\left(\pi^3, \pi \partial \pi, \partial^2 \pi, \pi^2 \partial u, \pi \partial^2 u, \partial^2 \pi, \partial^3 u, (\partial u)(\partial^2 u), (\partial u)^3\right)
 \end{aligned}$$

where the  $O(\pi^3, \pi \partial \pi, \dots)$  term indicates that the corrections to our proposal can include terms of the structures displayed or those with more derivatives or containing more powers of  $\pi_{\mu\nu}$  or both only. Also, the four  $\lambda_i$ 's ( $i = 1, 2, 3, 4$ ) are pure numbers.

Also,  $P^{\mu\nu}$  is the projection tensor in the plane orthogonal to the four-velocity  $u^\mu$ ,  $\sigma_{\mu\nu}$  is the hydrodynamic strain rate and  $\omega^{\mu\nu}$  is the hydrodynamic vorticity tensor

$$\begin{aligned}
 P^{\mu\nu} &= u^\mu u^\nu + \eta^{\mu\nu} & (6) \\
 \sigma^{\mu\nu} &= \frac{1}{2} P^{\mu\alpha} P^{\nu\beta} (\partial_\alpha u_\beta + \partial_\beta u_\alpha) - \frac{1}{3} P^{\mu\nu} (\partial \cdot u) \\
 \omega^{\mu\nu} &= \frac{1}{2} P^{\mu\alpha} P^{\nu\beta} (\partial_\alpha u_\beta - \partial_\beta u_\alpha)
 \end{aligned}$$

# Simple Predictions and Consistency Checks

- A simple prediction of our proposal is that there are at most three branches of linearized fluctuations constitute the universal sector. By the logic of our proposal, two of the three branches consists of the hydrodynamic sound and shear branches and in fact these branches are exactly the same as in the quasinormal mode spectrum obtained by solving the linearized fluctuations of the black brane with infalling boundary condition at the horizon. The third branch contains the mode  $\omega = -i\tau_\pi^{-1}, \mathbf{k} = 0$  with  $\tau_\pi = \frac{(1-\lambda_3)(2-\ln 2)}{2\pi T}$ . This mode at weak coupling was associated with relaxation or local equilibration in the quasiparticle-velocity space, so we will call this branch as the relaxation branch. Such a branch is not present in the quasinormal mode spectrum which also contains an infinite tower of different kinds of fluctuations.



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- Even if the quasinormal modes give regular linear perturbations, they need not survive in the full non-linear theory. On the other hand, our work in progress indicates that  $\lambda_3 = 1$  and the third mode is also not present as solution to our equations at the linear level, but non-hydrodynamic solutions do arise as solutions of the full non-linear equations.

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- It is also more plausible that the universal sector should contain only finite number of branches which could be blind to the particle content and other microscopic details of the theory.
- Since our proposal applies to the whole range of phenomena in the universal sector, one can do various consistency tests by looking at different configurations, for example homogeneous non-hydrodynamic configurations and combinations of hydrodynamic and non-hydrodynamic fluctuations systematically in two expansion parameters and determine the  $\lambda$ 's independently.

# Issues in Irreversibility, etc

## How does gravity see loss of quantum coherence?

To understand this, we may go back to the conservative solutions, but now construct them in quantum kinetic theories. It will be interesting if we can study the universal part of the phenomenology of pure to mixed state transition and at least some aspects of decoherence in general in terms of only ten variables parametrising the energy-momentum tensor.

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## How do we connect to experiment?

Our proposal implies that universal phenomena at strong coupling consists of the dynamics of three branches in the spectrum, namely the two hydrodynamic branches and the relaxation branch. It will be important to understand how we can connect this observation with actual experiments. The spectrum in the case of cold atoms tuned at Feshback resonance which is independent of all possible dimensionless parameters may give us support for our proposal.

Thank You

