Correlators of operators with a large R-charge

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arxiv:0905.2273
JHEP

Outline

- Background
- Schur polynomials
- Near 1/4-BPS sector
- New expansion parameters

Background

- AdS/CFT
- N=4 SYM dual to IIB strings on asymptotically $AdS_5 x S^5$
- Build composite scalar operators out of complex scalars
- Large R-charge in field theory corresponds to large energy in gravity
- Gravitational back-reaction gives rise to new geometries

Schur polynomial basis

$$\chi_R(Z) = \frac{1}{n!} \sum_{\sigma \in S_n} \chi_R(\sigma) Z^{i_1}_{i_{\sigma(1)}} \cdots Z^{i_n}_{i_{\sigma(n)}}$$

- R is a rep of S_n, a Young diagram with n boxes
- Schurs exactly diagonalize two-point function

$$\langle \chi_R(Z^\dagger) \chi_S(Z) \rangle = \delta_{RS} f_R$$

- f_R is product of weights

$$f_{\yng(2,1)} = N(N+1)(N-1)$$

Schur polynomial basis

- Product rule (from character nature)

$$\chi_R(Z)\chi_S(Z) = \sum_T g_{RST}\chi_T(Z)$$

- Hooks

$$\text{hooks}_{\yng(2,1)} = 1 \cdot 1 \cdot 3$$

Schur polynomial basis

- Arose as a basis for describing giant graviton systems
- Natural to describe open string excitations of giant gravitons
- Class of 1/2 BPS supergravity solutions parametrized by Young tableaux - LLM backgrounds
- Simple to study gauge theory dual to these backgrounds

Restricted Schur polynomials

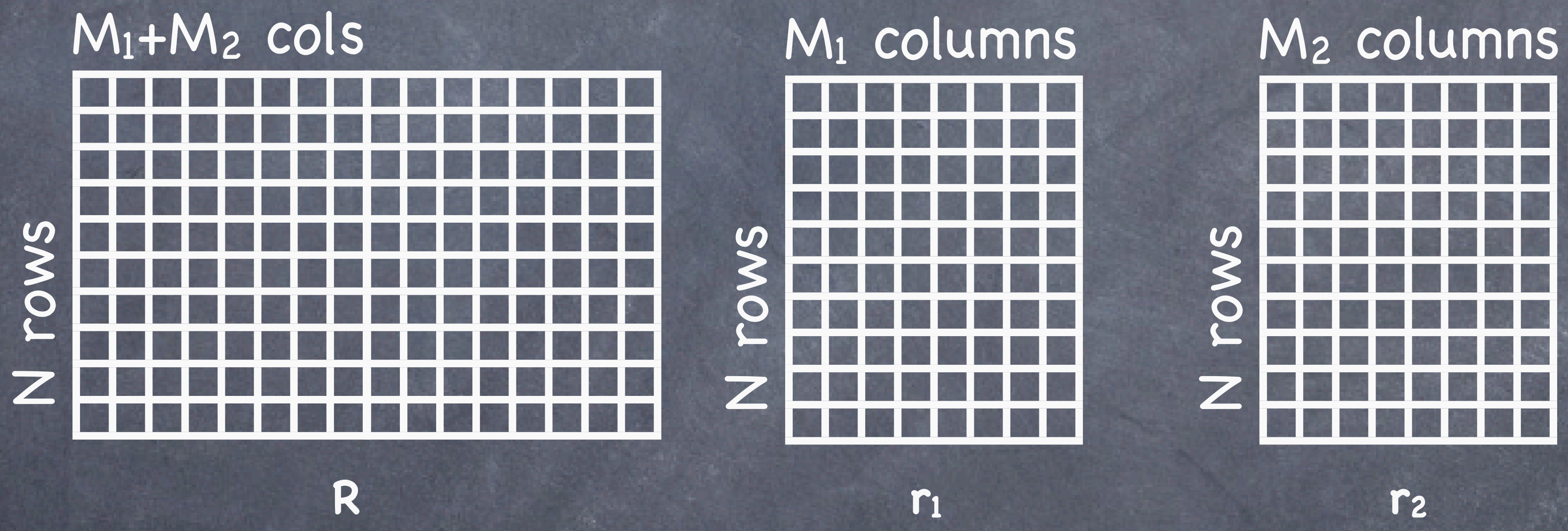
$$\chi_{R,(r_n,r_m)}(Z,Y) =$$

$$\frac{1}{n!m!}\sum_{\sigma\in S_{n+m}}\chi_{(r_n,r_m)}(\sigma)Z^{i_1}_{i_{\sigma(1)}}\cdots Z^{i_n}_{i_{\sigma(n)}}Y^{i_{n+1}}_{i_{\sigma(n+1)}}\cdots Y^{i_{n+m}}_{i_{\sigma(n+m)}}$$

- R is a rep of S_{n+m}, a Young diagram with n+m boxes
- $r_n \times r_m$ is a rep of $S_n \times S_m$ subduced from the rep R of S_{n+m}
- Satisfy analogous product rule

2-charge backgrounds

- Restricted Schur polynomials of NM_1 Z fields and NM_2 Y fields



- For this case, the product rule is

$$\chi_{R,(r_1,r_2)}(Z,Y) = \frac{\text{hooks}_R}{\text{hooks}_{r_1}\ \text{hooks}_{r_2}} \chi_{r_1}(Z)\chi_{r_2}(Y)$$

2-charge backgrounds

- It is interesting to consider the anomalous dimension of the 2-charge background
- At leading order, and 1-loop in λ we have

$$\Delta = N(M_1 + M_2) + 4\lambda M_1 M_2\,, \qquad \lambda = N g_{YM}^2$$

Near 1/4-BPS operators

- In arxiv:0801.4457, charged near-extremal black holes in AdS were considered with charges $J_1 = NM_1 \sim O(N^{3/2})$ and $J_2 = NM_2 \sim O(N^{3/2})$
- These charges allowed a near-horizon decoupling limit in the gravity theory
- Size of field theory object's gravity dual

$$R = \sqrt{\frac{J}{N}} R_{\mathrm{AdS}} \qquad R^2_{\mathrm{AdS}} = \sqrt{g^2_{YM} N \alpha'}$$

- For $J = O(N^{3/2})$, size of object R diverges when measured in terms of R_{AdS}

Near 1/4-BPS operators

- On the field theory side, the 2-charge backgrounds with $M_1 = O(\sqrt{N})$ and $M_2 = O(\sqrt{N})$ have the same quantum numbers
- With these quantum numbers and λ large but fixed we have a BMN-like sector

$$\Delta - J_1 - J_2 = 4\lambda M_1 M_2 \sim N$$

$$\eta \equiv \frac{\Delta - J_1 - J_2}{J_1 + J_2} \sim N^{-\frac{1}{2}} \to 0$$

- That the anomalous dimension is proportional to $M_1 M_2$ suggests it arises from open string sectors stretching between the two stacks of giants

Near 1/4-BPS operators

- We study the effective 't Hooft coupling for this $J_i = O(N^{3/2})$ operator.
- 't Hooft observed that matrix model perturbation expansion looks like

$$\sum_{g=0}^{\infty} N^{2-2g} f_g(\lambda)$$

- We will look for a structure like this

Near 1/4-BPS operators

- Set $M_1 = M_2 = M$ and compute (using usual two-point Schur correlators)

$$I_2 = \langle \chi_{R,(r_1,r_2)}(Z,Y)^\dagger \chi_{R,(r_1,r_2)}(Z,Y)\rangle = \left(\frac{G_2(N+M+1)}{G_2(N+1)G_2(M+1)}\right)^2$$

- $G_2(n+1)$ is the Barnes function

$$G_2(z+1) = \Gamma(z)G_2(z)\,, \qquad G_2(n+1) = \prod_{k=1}^{n-1} k!$$

- which has the asymptotic expansion

$$\log G_2(N+1) = \frac{N^2}{2}\log(N) - \frac{1}{12}\log(N) - \frac{3}{4}N^2 + \frac{N}{2}\log(2\pi) + \zeta'(-1) + \sum_{g=2}^{\infty}\frac{B_{2g}}{2g(2g-2)N^{2g-2}}$$

Near 1/4-BPS operators

- If we now set $N = \alpha M^2$ with $\alpha = O(1)$ compared to N, we can expand I_2
- Expansion is not unique, but generically has two parts, $I_2 = F_{non\text{-}pert} F_{pert}$
- F_{pert} admits an expansion in 1/M
- This suggests that we identify the genus counting parameter $g_2 = 1/M \sim 1/\sqrt{N}$
- So we anticipate an effective 't Hooft coupling

$$\tilde{\lambda} = \frac{g_{YM}^2}{g_2} = g_{YM}^2 M = \frac{1}{\sqrt{\alpha}} g_{YM}^2 \sqrt{N}$$

Near 1/4-BPS operators

- Express anomalous dimension in terms of effective 't Hooft coupling

$$\Delta = 2NM\left(1+2\tilde{\lambda}\right) = 2\alpha M^3\left(1+2\tilde{\lambda}\right)$$

- Looks like polynomial in $\tilde{\lambda}$ times some power of M. Compared to generic form, $\tilde{\lambda}$ looks like the correct 't Hooft coupling.
- Can take large N and λ limit while keeping $\tilde{\lambda}$ arbitrary
- Able to do perturbative field theory in this sector and still compare with gravity

Near 1/4-BPS operators

- The genus counting parameter $g_2 = 1/M$ and effective 't Hooft coupling $\tilde{\lambda} = g_{YM}^2 M$ are parameters we'd expect for U(M) gauge theory.
- Natural since we have order M giant gravitons whose low-energy worldvolume theory will have a U(M) gauge group.
- Suggests that the near-horizon dynamics of the charged AdS black holes in this sector are captured by the dynamics of open strings on a bound state of intersecting giants.

Near 1/4-BPS operators

- Note though that these particular backgrounds are too simple to describe black hole microstates. Large entropy - need triangular Young tableaux, which are harder to calculate with.
- But they should still exhibit the effective weak coupling, even when the original field theory is strongly coupled.

Summary

- Found a class of nearly 1/4-BPS operators with anomalous dimension suppressed by quantum numbers, protected in large N limit.
- In this sector, there is a new 't Hooft coupling. Can use perturbative field theory and still compare to gravity by taking large N and λ.
- Parameters match those expected for low energy worldvolume theory on intersecting branes comprising the background.

Anomalous dimension

- Derivation uses a trick of expressing the problem in terms of restricted Schurs with open strings attached
- When contracting all fields except the attached open string words we will use

$$\left\langle \chi^{(1)}_{R,R'}(Z,W)\chi^{(1)}_{R,R'}(Z^\dagger,W^\dagger)\right\rangle = A\left\langle \mathrm{Tr}\,(WW^\dagger)\right\rangle + B\left\langle \mathrm{Tr}\,(W)\mathrm{Tr}\,(W^\dagger)\right\rangle$$

Anomalous dimension

- Allowed index structure for open string word correlators is

$$\langle W^i_j (W^\dagger)^k_l \rangle = \delta^i_l \delta^k_j F_0 + \delta^i_j \delta^k_l F_1$$

- F_0 and F_1 are correlators obtained by different ways of tracing the string together
- Here we use dummy strings labels only, so don't worry about F_0 and F_1

Anomalous dimension

- Plug in to find

$$\left\langle \chi^{(1)}_{R,R'}(Z,W)\chi^{(1)}_{R,R'}(Z^\dagger,W^\dagger)\right\rangle = A(N^2F_0+NF_1)+B(N^2F_1+NF_0)$$

- By comparing to the known formula

$$\left\langle \chi^{(1)}_{R,R'}(Z,W)\chi^{(1)}_{R,R'}(Z^\dagger,W^\dagger)\right\rangle = \frac{\mathrm{hooks}_R}{\mathrm{hooks}_{R'}} f_R F_0 + c_{RR'} f_R F_1$$

- We can find

$$A = \left(\frac{\mathrm{hooks}_R}{\mathrm{hooks}_{R'}}N^2 - c_{RR'}N\right)\frac{f_R}{N^4-N^2}$$

$$B = \left(N^2 c_{RR'} - N\frac{\mathrm{hooks}_R}{\mathrm{hooks}_{R'}}\right)\frac{f_R}{N^4-N^2}$$

Anomalous dimension

- We compute the normalized correlation function

$$\langle \chi_{r_1}(Z^\dagger)\chi_{r_2}(Y^\dagger)\chi_{r_1}(Z)\chi_{r_2}(Y)\rangle$$

- D-term, self energy and gluon exchange cancel, leaving only F-term

$$I_1 = \langle \chi_{r_1}(Z^\dagger)\chi_{r_2}(Y^\dagger)\chi_{r_1}(Z)\chi_{r_2}(Y)\mathrm{Tr}\left([Z,Y][Z^\dagger,Y^\dagger]\right)\rangle$$

- Normal ordering means we can compute

$$\left\langle \mathrm{Tr}\left([\frac{\partial}{\partial Z},\frac{\partial}{\partial Y}][\frac{\partial}{\partial Z^\dagger},\frac{\partial}{\partial Y^\dagger}]\right)\chi_{r_1}(Z^\dagger)\chi_{r_2}(Y^\dagger)\chi_{r_1}(Z)\chi_{r_2}(Y)\right\rangle$$

Anomalous dimension

- This can be rewritten with dummy open string labels

$$\left\langle \mathrm{Tr}\,([\frac{\partial}{\partial W},\frac{\partial}{\partial V}][\frac{\partial}{\partial W^\dagger},\frac{\partial}{\partial V^\dagger}])\chi^{(1)}_{r_1,r_1'}(Z^\dagger,W^\dagger)\chi^{(1)}_{r_2,r_2'}(Y^\dagger,V^\dagger)\chi^{(1)}_{r_1,r_1'}(Z,W)\chi^{(1)}_{r_2,r_2'}(Y,V)\right\rangle$$

- And then using the formula for contracting everything but open strings we get

$$\mathrm{Tr}\,([\frac{\partial}{\partial W},\frac{\partial}{\partial V}][\frac{\partial}{\partial W^\dagger},\frac{\partial}{\partial V^\dagger}])\left(A_1\mathrm{Tr}\,(WW^\dagger)+B_1\mathrm{Tr}\,(W)\mathrm{Tr}\,(W^\dagger)\right)\left(A_2\mathrm{Tr}\,(VV^\dagger)+B_2\mathrm{Tr}\,(V)\mathrm{Tr}\,(V^\dagger)\right)$$

- With

$$A_1=\frac{M_1}{N}f_{r_1},\quad A_2=\frac{M_2}{N}f_{r_2},\quad B_i=0$$

Anomalous dimension

- We easily obtain

$$\frac{I_1}{f_{r_1} f_{r_2}} = -2NM_1M_2 + 2\frac{M_1M_2}{N}$$

- Which is related to the 1-loop anomalous dimension by $-2g_{YM}^2$ and so, at leading order

$$\Delta = N(M_1 + M_2) + 4\lambda M_1 M_2 \,, \qquad \lambda = N g_{YM}^2$$

- Note there are no assumptions about M_i