Hawking radiation at strong coupling

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Hawking radiation at strong coupling

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- Veronika Hubeny, Don Marolf, MR: to appear

Outline

- Motivation
- CFTs on black hole backgrounds
- Black funnels & black droplets
- Examples of black funnels
- Novel dynamics of plasma in black hole bgs
- Summary and discussion

Motivation

Motivation

How do black holes radiate strongly coupled matter?

- Hawking radiation is conventionally discussed for perturbative fields.
- Quantify differences when matter is strongly coupled?
- Precise nature of the quantum stress tensor due to the radiation (matter)?
- How does the nature of the radiation change if the matter can be in distinct phases?
- Dynamical behaviour during collapse: how does the field theory state evolve. Relatedly, Unruh vacuum.

Motivation

Brane-world black holes

- Does one have a complete understanding of the potential set of brane-world black holes in induced gravity models?
- Are there new solutions apart from localized black holes and black strings?
- Why don't large localized black holes seem to exist?

Emparan, Fabbri, Kaloper

Fitzpatrick, Randall, Wiseman

- Induced gravity models will be treated as AdS/CFT with a UV cut-off.

CFTs on black hole backgrounds

CFTs on black hole backgrounds

Perturbative fields

- Consider a conformally coupled scalar field in a black hole background such as Schwarzschild.

$$ds_\partial^2 = \gamma_{\mu\nu}\, dx^\mu\, dx^\nu = -{}^d\mathfrak{f}(r)\, dt^2 + \frac{dr^2}{{}^d\mathfrak{f}(r)} + r^2\, d\Omega_{d-2}^2 \,.$$

$${}^d\mathfrak{f}_{\rm Schw}(r) = 1 - \frac{r_+^{d-3}}{r^{d-3}} \,.$$

- One can also consider the Schwarzschild-AdS black hole:

$${}^d\mathfrak{f}_{\rm SAdS}(r) = \frac{r^2}{\ell_d^2} + 1 - \frac{r_+^{d-3}}{r^{d-3}} \left(1 + \frac{r_+^2}{\ell_d^2}\right).$$

CFTs on black hole backgrounds

Perturbative fields: Results

- At large distances $r \gg r_+$ one has thermal radiation at

$$T_{\rm local} = \frac{1}{\sqrt{^d\mathfrak{f}(r)}}\, T_{\rm BH}$$

- On the black hole horizon the quantum stress tensor evaluates to:

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$$T_\mu^{\ \nu} = \frac{2\,\pi^4}{15}\, T_{\rm BH}^4\, {\rm diag}\,\{3,3,1,1\}$$

- Note that vacuum polarization renders local energy density negative at the horizon.

CFTs on black hole backgrounds

Beyond perturbation theory: Expectations

- Even when the matter fields are strongly coupled one should obtain a stress tensor that is regular on the horizon and is thermal asymptotically.
- The interpolation between the two regimes will be sensitive the details of the system.
- Also take into account the phase structure of the QFT.

- To understand these issues we are going to embed the problem within the AdS/CFT correspondence, which is the cleanest framework to discuss these issues.

The AdS/CFT story:
black funnels and droplets

The AdS/CFT story

QFT + Gravity $\rightsquigarrow$ Classical gravity

- Consider a situation in the AdS/CFT framework where the boundary field theory is on a non-dynamical black hole background $\mathcal{B}_d$ with metric $\gamma_{\mu\nu}$.
- Dynamics of these field theories is governed at strong coupling by ``asymptotically locally AdS'' geometries which asymptote to $\mathcal{B}_d$.
- This involves finding bulk spacetimes $\mathcal{M}_{d+1}$ which have as their conformal boundary the preferred manifold $\mathcal{B}_d$.

- One has to therefore find solutions to Einstein's equations with a negative cosmological constant which has the correct boundary conditions.

The AdS/CFT story

Known solutions for boundary black holes

- Consider the case where the boundary is the asymptotically flat Schwarzschild black hole.
- One solution which satisfies the field equations is the AdS black string

$$ds^2 = \frac{1}{z^2}\, ds^2_{\partial,\mathrm{Schw}} + \frac{dz^2}{z^2}\ .$$

- This solution is singular on the Poincare horizon (due to the black hole singularity which extends along z).
- Ignoring this issue we can ask what the induced stress tensor is on the boundary.
- The result is simply:

$$T_\mu{}^\nu = 0$$

Schwarzschild boundary conundrums:

- The vanishing stress tensor for the AdS black string is at odds with expectations of thermal Hawking radiation.
- Anticipate strong vacuum polarization effects due to the strongly interacting matter.
- Rather curious that the conventional Hawking thermal spectrum is not reproduced.
- A similar result can be derived for the boundary Schwarzschild-AdS black hole.
- Note that this result in particular implies that if we make the boundary gravity dynamical then it would see no back-reaction from the strongly coupled matter sector!

Phase structure of the CFT

Resolution: Confinement vs. Deconfinement

- The vanishing stress tensor for the AdS black string seems to suggest that the dual field theory is actually in the confined phase, i.e.,

$$0 \sim \mathcal{O}\left(1\right)$$

- Typically in the AdS/CFT context, the boundary field theories on Minkowski space doesn't undergo a confinement-deconfinement transition.
- At any non-zero temperature, the preferred phase is the deconfined phase with $\mathcal{O}\left(N^2\right)$ free energy.

- It is somewhat reassuring that the black string geometry actually is classically unstable, indicating that it is not a local minimum of the free energy.

Phase structure of the CFT

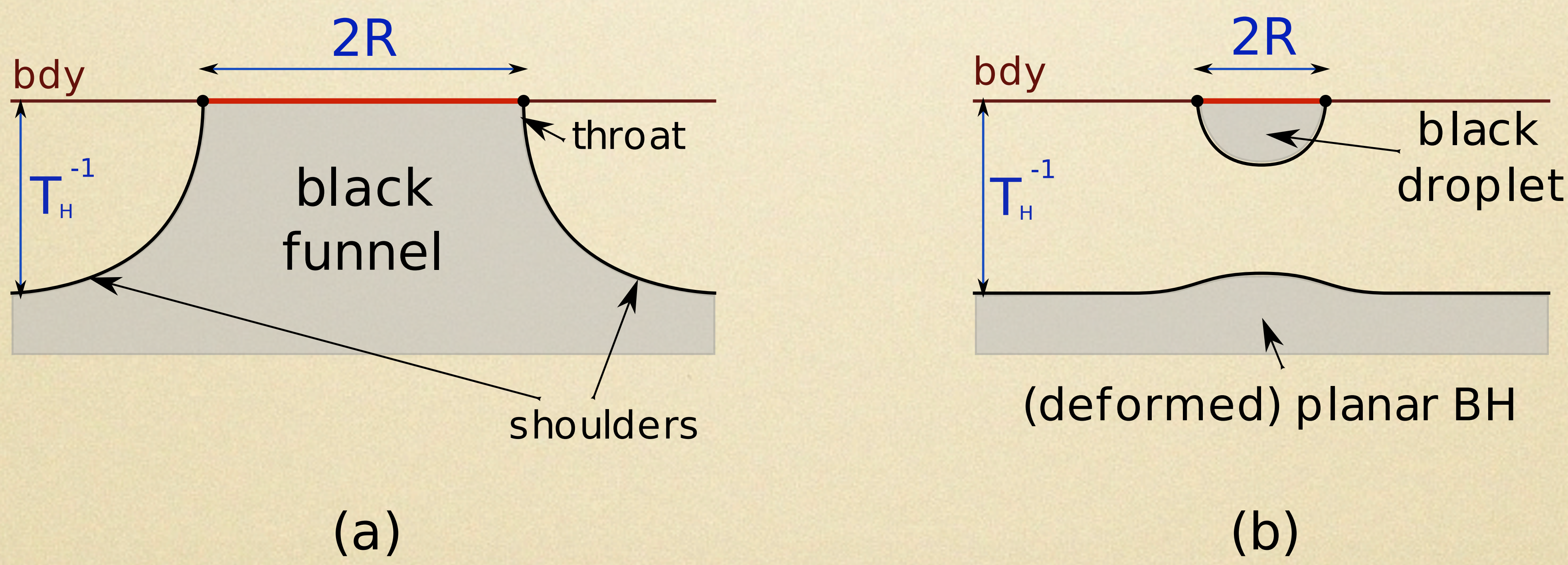
Whither the Deconfined phase?

- The stable phase at any finite temperature ought to be the deconfined phase.
- This phase has a thermal stress tensor asymptotically and should give rise to a strongly coupled version of the Page stress tensor.
- In AdS/CFT this should just amount to a multiplicative renormalization of the free field result (the famous 3/4).
- Question then remains: what is the bulk solution which encapsulates the physics of a deconfined field theory plasma in equilibrium with the thermal Hawking radiation of the boundary black hole?
- Relatedly, is there a unique candidate bulk geometry or can there be phase transitions?

Black Funnels, Black Droplets in AdS

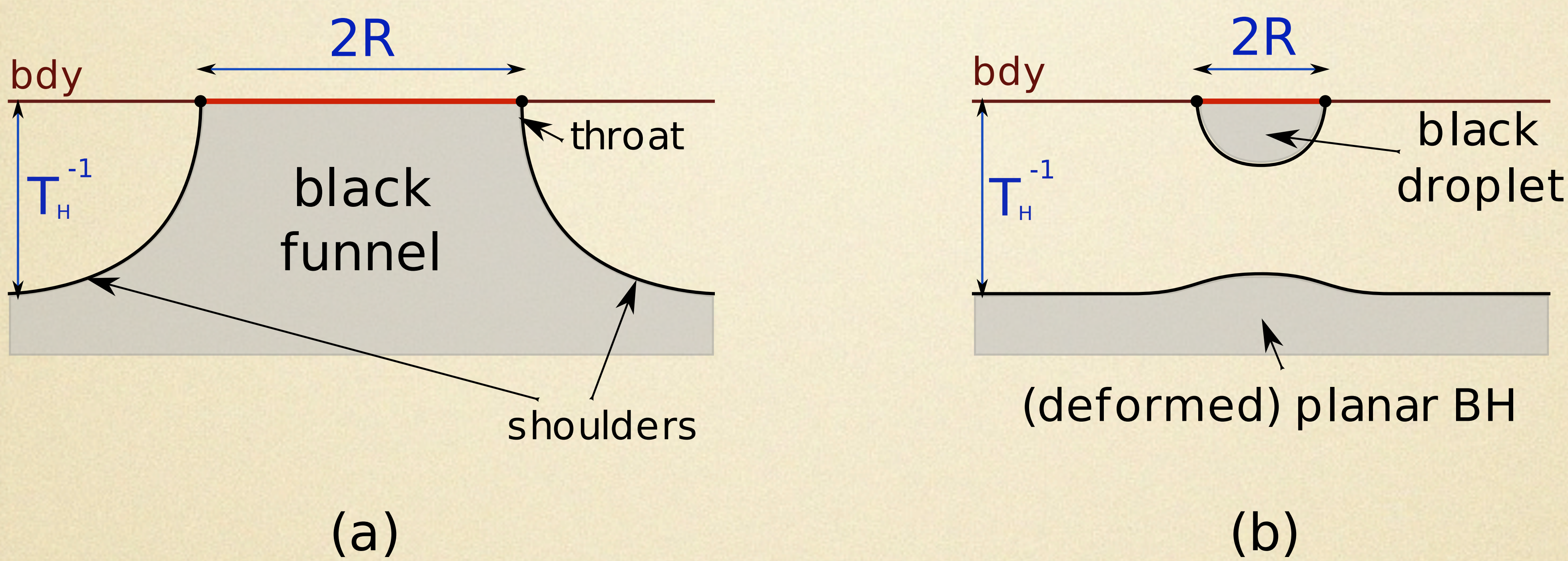
Motivating new classes of solutions

- The deconfined phase is dual to either
  - a Black Funnel geometry.
  - a Black Droplet together with a bulk black hole (disconnected horizons).



Black Funnels, Black Droplets in AdS

Funnels vs droplets



- Note that the boundary black hole can have two scales
  - characteristic size R.
  - Hawking temperature T_H.

- A-priori these can be unrelated and the dominant saddle point depends on $R\,T_H$.

Black Funnels & the phase structure of the CFT

A Conjecture: field theory on Schwarzschild bg

- The stable deconfined phase is dual to a Black Funnel geometry.
- Corresponds to the phase of the CFT in equilibrium with the thermal Hawking radiation.
- The unstable deconfined phase is dual to the AdS black string.

Extensions:

- Can be extended to situations where the field theory has a confining phase and to boundary metrics which have different asymptotics.

Examples

Black Funnels: Examples

Black funnels in AdS_3

- The simplest explicit example of black funnels can be obtained when the bulk is three dimensional.
- The non-dynamical boundary metric can be taken to the be the two dimensional black hole with metric

$$ds^2 = -\tanh^2 r\, dt^2 + dr^2$$

- Einstein's equations (with -ve cc) can be easily solved.
- Work in Fefferman-Graham coordinates for simplicity and take the metric ansatz to be

$$ds^2 = \frac{1}{z^2}\left(-f(r,z)\, dt^2 + g(r,z)\, dr^2 + dz^2\right)$$

Black Funnels: Examples

Black funnels in AdS_3

- The metric functions turn out to be:

$$\begin{aligned} f(r,z) &= \frac{1}{16}\left(4\,\tanh r + z^2\,\frac{1-2\,r_+\,\cosh^4 r}{\sinh r\,\cosh^3 r}\right)^2 \\ g(r,z) &= \left(1 + z^2\,\frac{2\,r_+\cosh^4 r - 1 - 4\,\sinh^2 r}{4\,\sinh^2 r\,\cosh^2 r}\right)^2 \end{aligned}$$

- Naively we have a one-parameter family of solutions, but the parameter r_+ is fixed by demanding regularity:

$$T_{\mathrm{bdy}} = \frac{1}{2\,\pi} \implies r_+ = \frac{1}{2}$$

Black Funnels: Examples

Black funnels in AdS_3

- There is a bulk event horizon located at

$$z_H(r) = \frac{2\,\cosh r}{\sqrt{\cosh^2 r + 1}}\,, \quad \& \;\; r = 0$$

Black Funnels: Examples

Black funnels/droplets in AdS_4

- One can also find black funnel like solutions within the AdS C-metric family.
- These turn out to correspond to boundary black holes which are asymptotically $\mathbf{R} \times \mathbf{H}^2$.
- Once again the expectations regarding the stress tensor are borne out by explicit computation.
- There are also black droplets within the C-metric family.
- Curiously, except for a degenerate case the solutions tend not to represent equilibrium states (temperatures don't match between the droplet and the planar black hole).
- There also exist funnels and droplets for spatially compact boundaries.

Novel dynamics of the deconfined plasma

Funnels vs droplets

Novel dynamics in the deconfined phase

- Black funnels represent deconfined plasma in equilibrium with the bdy black hole.
- Perturbations of this system relax as expected in the deconfined regime.

- Black droplets + planar bhs are more exotic.
- While the planar bh ensures that the system is in deconfined phase, perturbations of the system relax as if it were confined!
- This is because any change on one horizon has to be transferred to the other through bulk Hawking radiation which is a very slow process.

Funnels vs droplets

Black holes and finite size excitations

- Slow equilibration of droplets is curious from the field theory.
- Suggests that the intrinsic excitations of the field theory are very weakly coupled to the boundary black hole.
- Natural interpretation: field theory excitations have finite size (cf. size of hadrons in string theory).
- If $R_e \gg R$ then the field theory quanta just fly past the bdy black hole without much ado (modulo tidal effects).
- In the bulk one again sees this by the small distortion on the droplet caused by a gravitational perturbation deep inside the bulk.

Summary

Summary & Discussion

New exotic AdS black holes

- Holographic techniques allow us for the first time to investigate the Hawking radiation of strongly coupled quanta.
- Analysis of the dual gravitational solutions leads to a new class of solutions which we've dubbed black funnels.
- Explicit solutions exist when the boundary is 2 dimensional, or BTZ or an exotic hyperbolic black hole.
- Not only can such solutions be constructed in a limited class of examples, but also they confirm our intuition regarding the quantum stress tensor due to curved spacetime effects.

Summary & Discussion

Realistic Black funnels?

- ◉ Black funnels with Schwarzschild boundary should exist (should be doable numerically).
- ◉ Would be interesting to explore the explicit behaviour of these solutions and figure out the quantum stress tensor induced by strongly coupled $\mathcal{N} = 4$ SYM in this background.

There are important implications for:

- ✤ Plasma balls in confining theories
- ✤ Brane-world and induced gravity models.
- ✤ Curiosities for field theories on asymptotically AdS bhs.