Abundance of Thermal WIMPs in Non-standard Cosmological Scenarios

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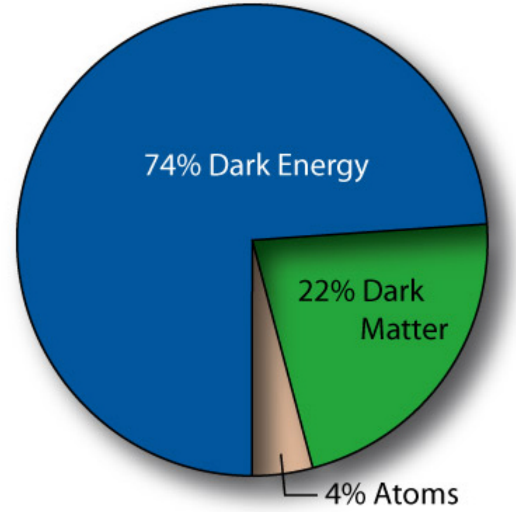
Refs:

- PRD73 (2006) 123502 [hep-ph/0603165]
- arXiv:0704.1590 [hep-ph], to appear in PRD

1. Motivation



[http://map.gsfc.nasa.gov]

- Observations of
  - cosmic microwave background
  - structure of the universe
  - etc.

➡ Non-baryonic dark matter: $0.8 < \Omega_{\rm DM} h^2 < 0.12$

- Weakly interacting massive particles (WIMPs) χ are good candidates for cold dark matter (CDM)

The predicted thermal relic abundance naturally explains the observed dark matter abundance: $\Omega_{\chi,{\rm standard}} h^2 \sim 0.1$

- Neutralino (LSP); $1^{\rm st}$ KK mode of the B boson (LKP); etc.

Investigation of early universe using CDM abundance

- The relic abundance of thermal WIMPs is determined by the Boltzmann equation: $\dot{n}_\chi + 3Hn_\chi = -\langle\sigma_{\rm eff} v\rangle(n_\chi^2 - n_{\chi,{\rm eq}}^2)$

  (and the reheat temperature: T_R)

- The (effective) cross section $\sigma_{\rm eff}$ can be (hopefully) determined from collider and DM detection experiments

⇒ **We can test the standard CDM scenario and investigate conditions of very early universe: $T_R, H, \cdots$**

- Standard scenario or non-standard scenarios?

  [Scherrer; Salati; Rosati; Profumo, Ullio; Pallis; Maisero, Pietroni, Rosati; ...]

Outline

This work

- We provide an approximate analytic treatment that is applicable to low-reheat-temperature scenarios
- Based on the assumption that dark matter is made of thermal WIMPs,
  - we derive the lower bound on the reheating temperature
  - we constrain possible modifications of the Hubble parameter

c.f. Cosmic p^+, γ ⇒ Bounds on pre-BBN expansion

[Schelke, Catena, Fornengo, Masiero, Pietroni PRD74 (2006); Donato, Fornengo, Schelke, JCAP0703 (2007)]

1. Motivation
2. Standard calculation of WIMP relic abundance
3. Low-temperature scenario
4. Constraints on the very early universe from WIMP dark matter
5. Summary

2. Standard calculation of the WIMP relic abundance

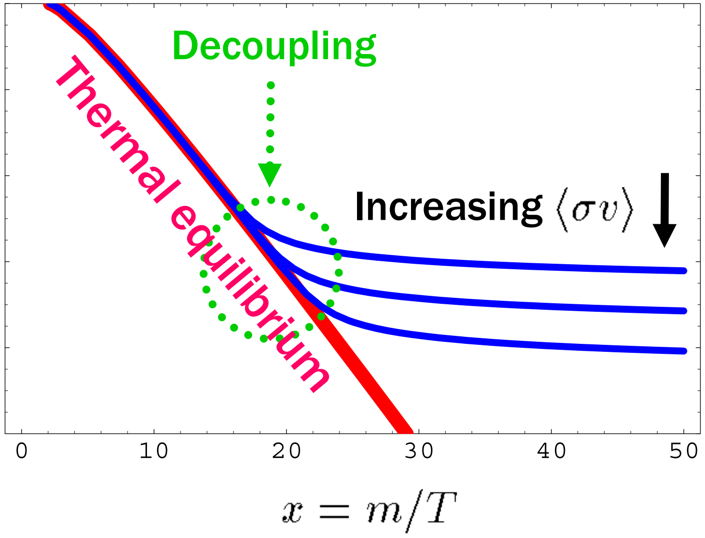
[Scherrer, Turner, PRD33(1986); Griest, Seckel, PRD43(1991); ...]

- Conventional assumptions:
  - $\chi = \bar{\chi}$, single production of χ is forbidden
  - Thermal equilibrium was maintained:
    $T_R(\text{Reheat temp}) \geq T_F(\text{Freezeout temp})$
- For adiabatic expansion the Boltzmann eq. is

$$\frac{\mathrm{d}Y_\chi}{\mathrm{d}x} = -\frac{\langle \sigma v \rangle s}{Hx}(Y_\chi^2 - Y_{\chi,\mathrm{eq}}^2),$$

$$Y_{\chi(,\mathrm{eq})} = \frac{n_{\chi(,\mathrm{eq})}}{s}, x = \frac{m_\chi}{T}$$

Co-moving number density



- χ decoupled when they were non-relativistic in RD epoch:

$$\langle \sigma v \rangle = a + 6b/x + \mathcal{O}(1/x^2), \quad n_{\chi,\mathrm{eq}} = g_\chi \, (m_\chi T/2\pi)^{3/2} \mathrm{e}^{-m_\chi/T}$$

$$\Omega_{\chi,\mathrm{standard}} h^2 \simeq 0.1 \times \left(\frac{a + 3b/x_F}{10^{-9}\ \mathrm{GeV}^{-2}} \right)^{-1} \left(\frac{x_F}{22} \right) \left(\frac{g_*}{90} \right)^{-1/2} \sim \Omega_{\mathrm{DM}} h^2$$

3. Low-temperature scenario

- Assumptions: $T_R \leq T_F;\ Y_\chi(x_0) = 0\,,\ \ x_0 = m_\chi / T_R$
- Zeroth order approximation: For $T_R \ll T_F$, χ annihilation is negligible:

$$\frac{dY_0}{dx} = 0.028\, g_\chi^2 g_*^{-3/2} m_\chi M_{\rm Pl} \mathrm{e}^{-2x} x \left(a + \frac{6b}{x} \right)$$

The solution Y_0 is proportional to the cross section:

- First order approximation:
  - Add a correction term describing annihilation to Y_0 : $Y_1 = Y_0 + \delta\ (\delta < 0)$
  - As long as $|\delta| \ll Y_0$, the evolution equation for δ is

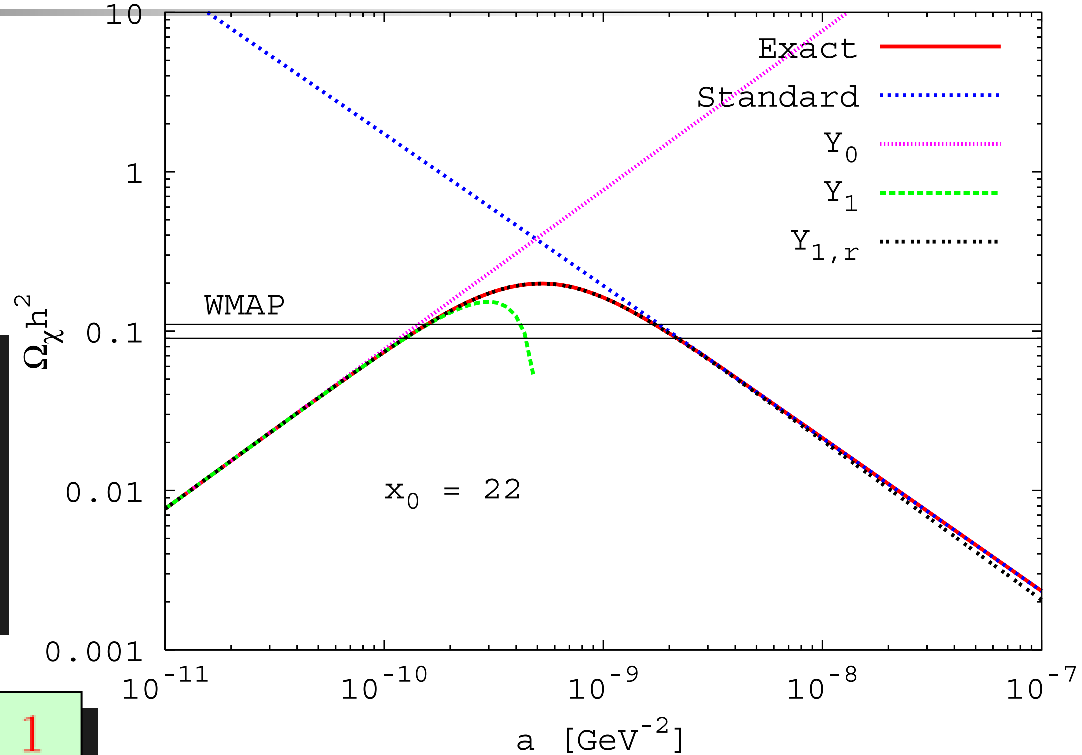
$$\frac{d\delta}{dx} = -1.3\, \sqrt{g_*} m_\chi M_{\rm Pl} \left(a + \frac{6b}{x} \right) \frac{Y_0(x)^2}{x^2}$$

An analytic expression for δ can be obtained and is proportional to σ^3

Resummed ansatz

- We can obtain an excellent analytic formula for $T_R \leq T_F$ through resummation:

$$Y(x) = Y_0 + \delta = Y_0 \left(1 + \frac{\delta}{Y_0}\right) \simeq \frac{Y_0}{1 - \delta/Y_0} \equiv Y_{1,r}$$



- For $|\delta| \gg Y_0$, $Y_{1,r}(x) \simeq -\frac{Y_0^2}{\delta} \propto \frac{1}{\sigma}$

$x_0 \rightarrow x_F$ **Standard formula**

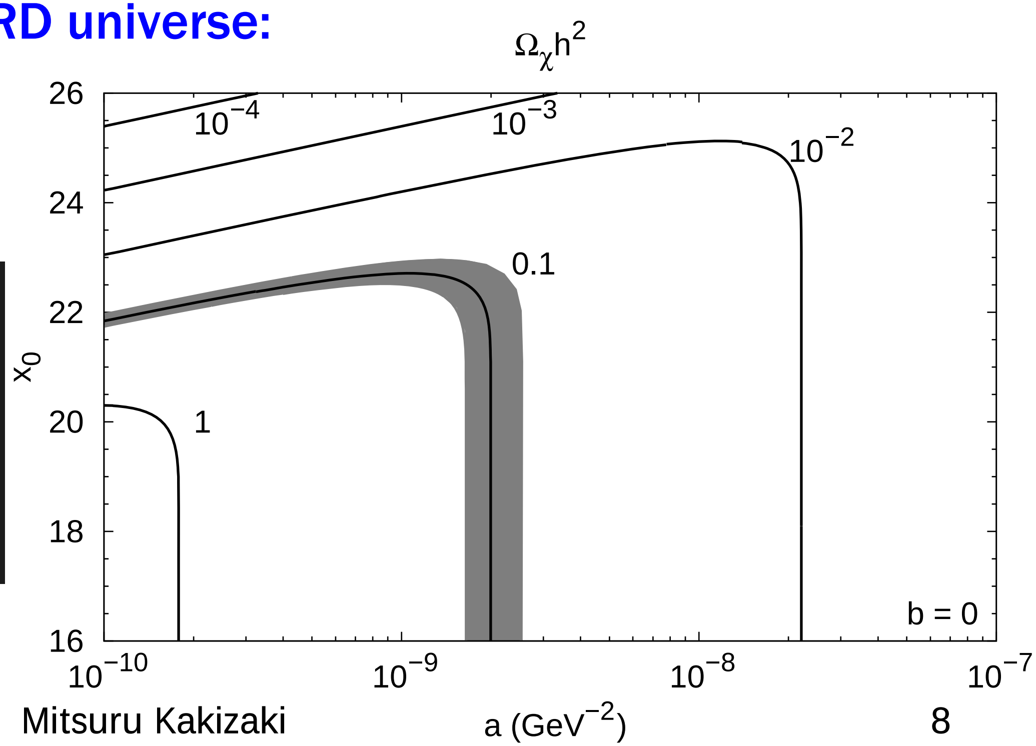
At late times, $Y_{1,r}(x \rightarrow \infty) = \dfrac{x_0}{1.3\sqrt{g_*}\, m_\chi M_{\rm Pl}(a + 3b/x_0)}$

4. Constraints on the very early universe from WIMP DM

- **Out-of-equilibrium case:** $\sigma \nearrow \Rightarrow \Omega h^2 \nearrow$; $T_0 = m_\chi / x_0 \nearrow \Rightarrow \Omega h^2 \nearrow$
  Equilibrium case: $\sigma \nearrow \Rightarrow \Omega h^2 \searrow$; $\Omega_\chi h^2$ is independent of T_R

- **Thermal relic abundance in the RD universe:**

$0.8 < \Omega_{\rm DM} h^2 < 0.12$



Requirement that $\Omega_\chi h^2 \simeq 0.1$

⇓

Lower bound on the reheat temperature: $T_R > m_\chi / 23$

Modified expansion rate

- Various cosmological models predict a non-standard early expansion
  ⇒ Predicted WIMP relic abundances are also changed
- Idea:

Once we know σ as well as $\Omega_{\rm DM} h^2$,
we can constrain the expansion rate at around WIMP decoupling
within the framework of thermal WIMP production

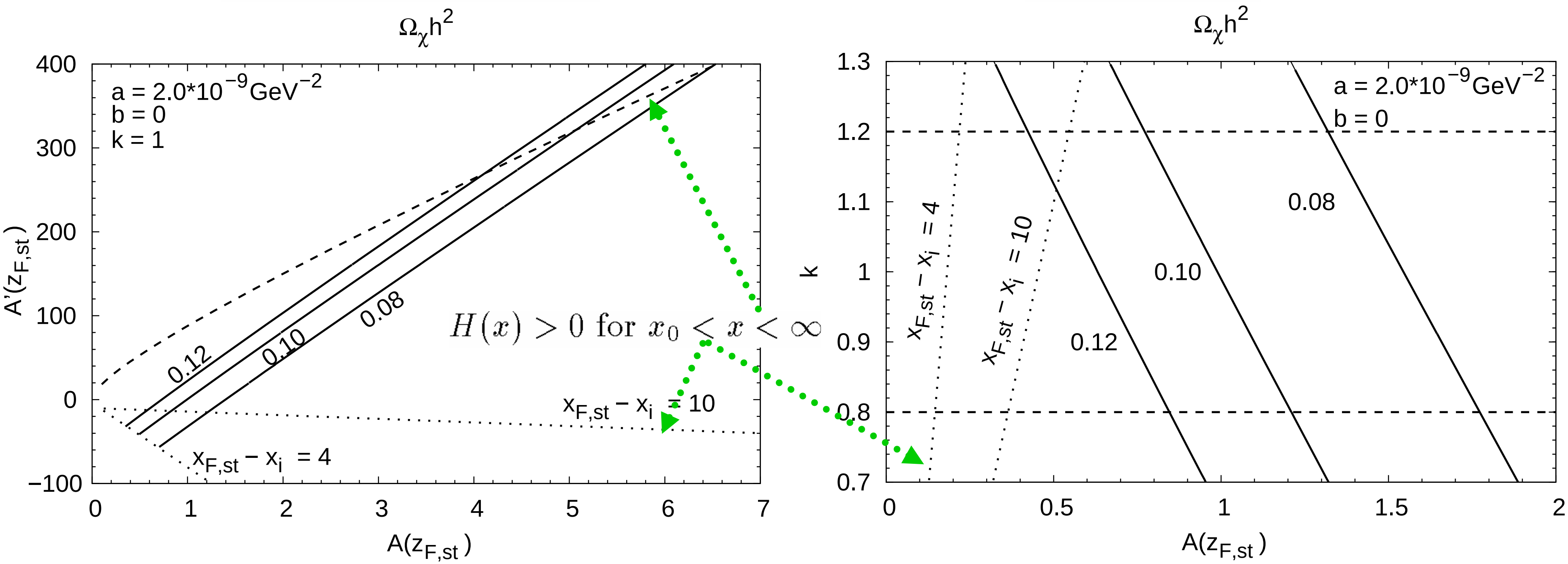
- Parametrization: $A(z) = H_{\rm st}(z)/H(z)\,, \quad z \equiv T/m_\chi = 1/x$
- We need to know $A(z)$ only for $z_{\rm BBN} = 10^{-5} - 10^{-4} \leq z \leq z_F \sim 1/20 \ll \mathcal{O}(1)$
  ⇒ Taylor expansion of $A(z)$ in powers of $(z - z_{F,\rm st})$:

$$A(z) = A(z_{F,\rm st}) + (z - z_{F,\rm st})A'(z_{F,\rm st}) + \frac{1}{2}(z - z_{F,\rm st})^2 A''(z_{F,\rm st})$$

subject to the BBN limit: $0.8 \leq k \equiv A(z \to z_{\rm BBN}) \leq 1.2$

Constraints on modifications of the Hubble parameter

- **Case for** $k \equiv A(z \to z_{\rm BBN}) = 1$
- **Case for** $A''(z_{F,\rm st}) = 0$



x_i : **Maximal temperature where our expansion is valid**

$\Omega_\chi h^2$ depends on all $H(T_{\rm BBN} < T < T_F)$ ⇨ Weaker constraints on $H(T_F)$

5. Summary

- Using the CDM relic density we can examine very early universe at around $T \sim m_\chi/20 \sim \mathcal{O}(10)$ GeV (well before BBN $T_{\rm BBN} \sim \mathcal{O}(1)$ MeV)

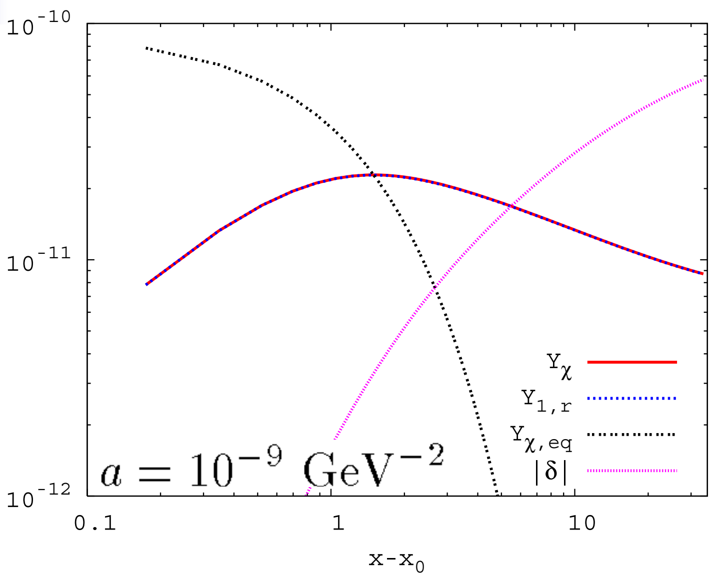
- We find an approximate analytic formula that is valid for all $T_R \leq T_F$

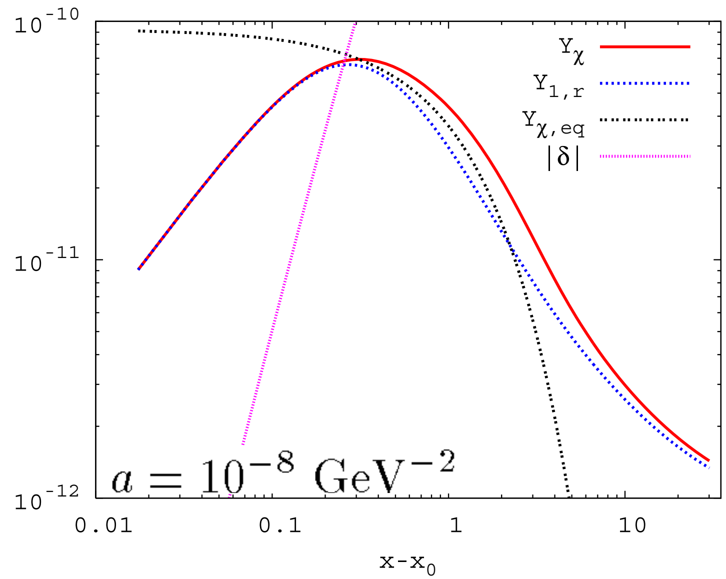
- By requiring $\Omega_{\chi,\rm thermal} h^2 = \Omega_{\rm DM} h^2$,
  we found the lower bound on the reheat temperature: $T_R > m_\chi/23$

- The sensitivity of $\Omega_{\chi,\rm thermal} h^2$ on $H(T_F)$ is weak
  because $\Omega_{\chi,\rm thermal} h^2$ depends on all $H(T_{\rm BBN} < T < T_F)$

Backup slides

Evolution of solutions

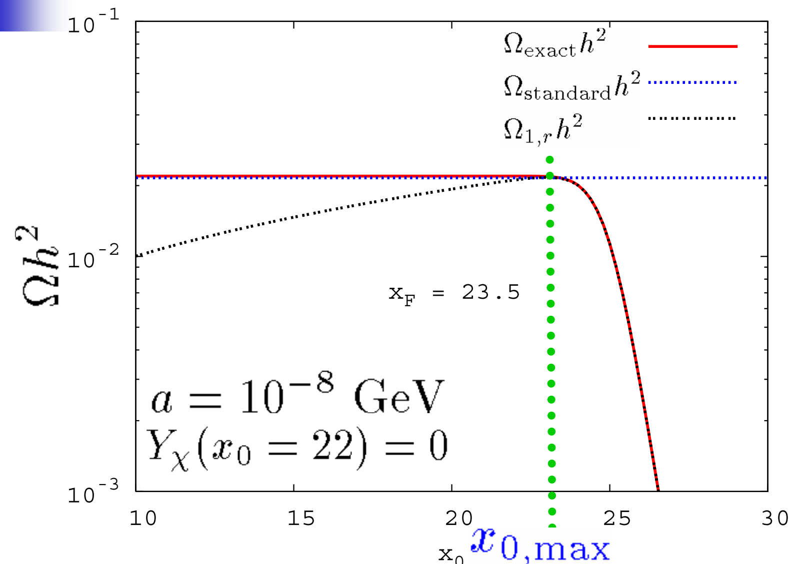


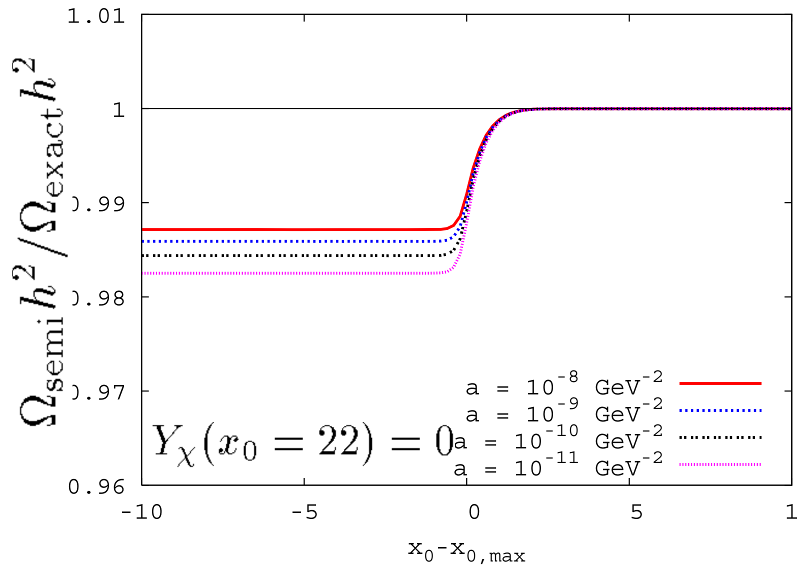


Y_χ : Exact result, $Y_{1,r}$: Re-summed ansatz, $b = 0, \ Y_\chi(x_0 = 22) = 0$

- The re-summed ansatz $Y_{1,r}$ describes the full temperature dependence of the abundance when equilibrium is not reached
- For larger cross section the deviation becomes sizable for $x - x_0 \sim 1$, but the deviation becomes smaller for $x \gg x_0$

Semi-analytic solution





- $Y_{1,r}(x_0, x \to \infty)$ $(\propto \Omega_{1,r} h^2)$ has a maximum (left)
- New semi-analytic solution can be constructed: $\Omega_{\rm semi} h^2$ (right)

For $x_0 > x_{0,\max}$, use $Y_{1,r}(x_0)$; for $x_0 < x_{0,\max}$, use $Y_{1,r}(x_{0,\max})$

The semi-analytic solution $\Omega_{\rm semi} h^2$ reproduces the correct final relic density $\Omega_{\rm exact} h^2$ to an accuracy of a few percent