

# Wilson Line Inflation

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Based on:

Gen.Rel.Grav.39:1203-1234,2007

# Inflation

- Horizon, Flatness, Monopole problems, ...

$$\tau = \int_0^t a(t)^{-1} dt$$

$$|\Omega - 1| = \frac{|k|}{a^2 H^2}$$

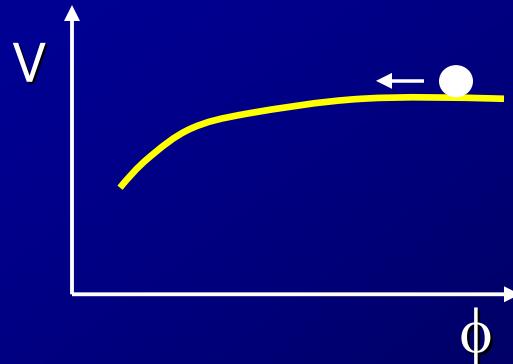
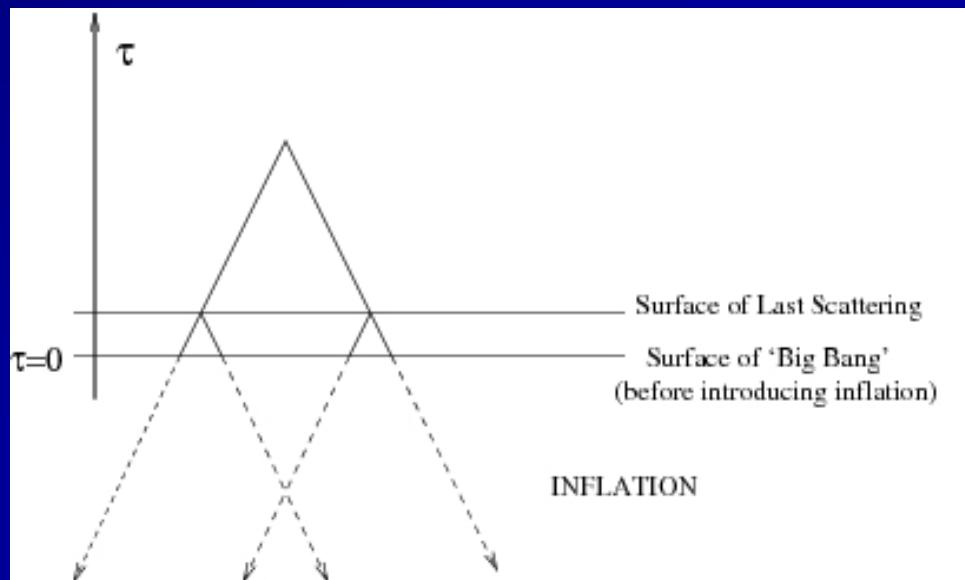
$$\pi_2(G / SM) \supset \pi_1(U(1)) \neq I$$

- Exponential expansion with  $H \sim \text{const}$

Comoving horizon decreases

Expansion dilutes curvature and monopoles

Slowly rolling down a flat



# Stringy Inflation

- String Theory Moduli as inflatons
- Break SUSY to lift potential
- Inflaton candidates
  - Closed String: dilaton, Kahler, cx structure
  - Open String: Brane positions, tachyon, Wilson lines
- Models: Racetrack, Kahler (Kahler)  
DDbar, Branes @ angles, D3-D7, DBI (brane separation)  
Warped tachyonic (open string tachyon)

# SUGRA $\eta$ problem

- Potential is of the form:  $V = V_0 + V_{\text{int}}(\varphi)$

where typically

$$V_0 = \frac{X}{r^\alpha}$$

- Sugra variable is  $T$ , where:  $2r = T + T^* + \varphi \varphi^*$

- After stabilisation:  $V_0 = V_0(\varphi)$

- Fine tuning!

(KKLMMT 03, BDKMS 07)

# Wilson Lines

- Consider gauge field:  $F = dA, \quad A \rightarrow A + d\chi$
- Wilson Line:  $U_\gamma = P \exp \oint_\gamma A$
- If  $\gamma$  contractible, then:  $U_\gamma = P \exp \int_C F, \quad \gamma = \partial C$   
and  $F = 0 \Rightarrow U = 1$
- However, if  $\pi_1(C) \neq I$  one can have:  $F = 0, \quad U \neq 1$
- Abuse of terminology  $U_\gamma \leftrightarrow A$
- In particular:  $A \rightarrow A + \lambda, \quad \lambda = const$

# Wilson Line Inflation: Setup

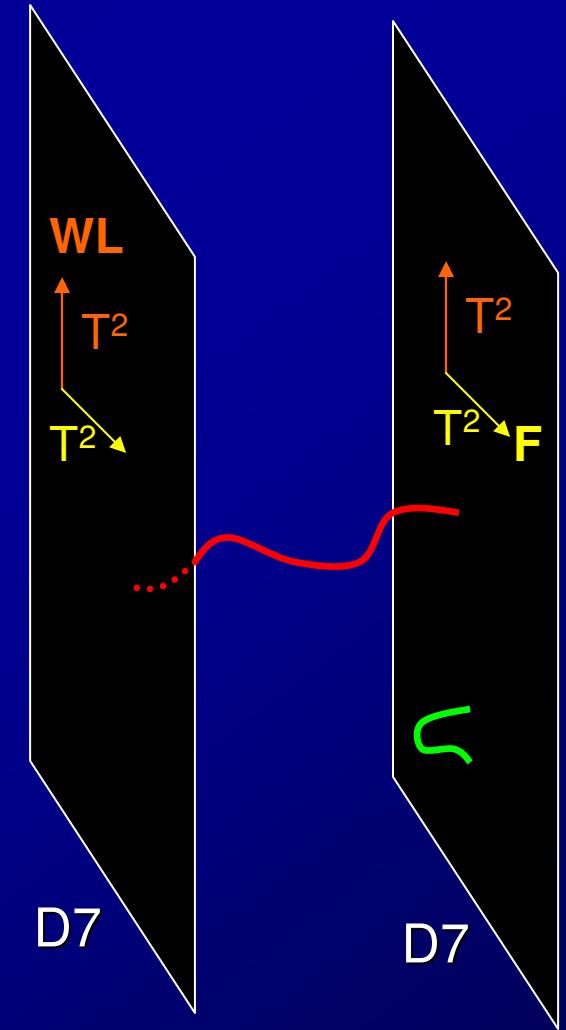
- Parallel D7 branes wrapping  
a 4-torus  $T^4 = T^2 \times T^2$
- Brane1: Turn on a **WL** in one  $T^2$

$$A = \frac{\lambda}{2\pi R_1} dx^1$$

- Brane2: Turn on **F** in the other  $T^2$

$$F = \frac{m}{2\pi R_3 R_4} dx^3 \wedge dx^4$$

- Open Strings: BCs, mode expansion,  
Virasoro operators, **MASS**



# Wilson Line Inflation: Potential

- BCs lead to twisted mode expansions (cf Branes @ angles)

- Mass op:

$$\alpha' M^2 = \sum_{i=1}^2 \frac{(y_i + 2\pi w_i)^2 \tilde{R}_i^2}{4\pi^2 \alpha'} + \frac{(\lambda + 2\pi n)^2 \alpha'}{4\pi^2 R_1^2} + N_\nu + \nu(\theta - 1)$$

- Interaction Energy given by Coleman-Weinberg formula:

$$V_{\text{int}} = 2 \int \frac{d^4 k}{(2\pi)^4} \int_0^\infty \frac{dt}{t} Tr \exp[-2\pi \alpha' t(k^2 + M^2)]$$

- Leads to:

$$V_{\text{int}}(\lambda, y) = -\frac{\sin^2(\theta/2) \tan(\theta/2)}{8\pi^3 \alpha'^2 \|Y, \Lambda\|^2} \quad (\text{cf G-BRZ, 2003})$$

where  $\|Y, \Lambda\|^2 = \sum_{i=1}^2 \frac{y_i^2 \tilde{R}_i^2}{\alpha'} + \frac{\lambda^2 \alpha'}{R_1^2} \equiv Y^2 + \Lambda^2$

# Wilson Line Inflation: Results

- Total potential:

$$V = V_0 + V_{\text{int}}(\lambda) = \frac{\text{Vol}(T^4)}{8\pi^3 \alpha'^4 g_s} \frac{\tan^2(\theta)}{4} - \frac{\sin^2(\theta/2) \tan(\theta/2)}{8\pi^3 \alpha'^2 (Y^2 + \Lambda^2)}$$

- Slow Roll

Parameters:

$$\varepsilon = 4(8\pi)^2 g_s \frac{\tilde{R}_1 \tilde{R}_2 \alpha'^4}{R_1^4 R_2^2 R_3^2 R_4^2} \frac{\sin^4(\theta/2) \tan^2(\theta/2)}{\tan^4(\theta)} \frac{\lambda^2}{\|Y, \Lambda\|^8}$$

$$\eta = (8\pi)^2 \frac{\tilde{R}_1 \tilde{R}_2 \alpha'}{R_1 R_2 R_3 R_4} \frac{\sin^2(\theta/2) \tan(\theta/2)}{\tan^2(\theta)} \frac{1}{\|Y, \Lambda\|^4} \left( 1 - \frac{4\alpha' \lambda^2}{R_1^2 \|Y, \Lambda\|^2} \right)$$

- Two cases

$\Lambda \ll Y, V_{\text{int}}(\phi) \sim \phi^2, \varepsilon, \eta +ve \quad \varepsilon \approx 10^{-13}, \eta \approx 3 \times 10^{-3}$

$\Lambda \gg Y, -V_{\text{int}}(\phi) \sim \phi^{-2}, \eta -ve \quad \varepsilon \approx 10^{-13}, \eta \approx -1.5 \times 10^{-2}$

- Hybrid-like exit:

$$M^{-2} = \sum_{i=1}^2 \frac{{y_i}^2 {\tilde{R}_i}^2}{4\pi^2 \alpha'^2} + \frac{\lambda}{4\pi^2 {R_1}^2} - \frac{|\theta|}{2\pi} \quad \text{strings}$$

- Fine tuning: 1/1000

# Realistic Compactifications

- Work in type IIB, KKLT-like setup
- Kahler moduli stabilised by non-pert W
- Turn on  $G_3 = F_3 + \phi H_3$  to stabilise brane positions  
(GMM, 2005)
- WLs appear in Kahler potential as positions
$$K = -3\log(T + T^* - \varphi\varphi^*)$$
- Results of previous setup apply to this case

# SUGRA $\eta$ problem

- Consider the IIB toroidal model ( $T^6 = T^2 \times T^2 \times T^2$ )
- Superpotential stabilisation fixes  $\text{Re } T_1, \text{Re } T_2, \text{Re } T_3$

where:

$$A_1 = \sqrt{\frac{2 e^\phi \text{Re } T_3}{\text{Re } T_1} \left( \text{Re } T_2 - \frac{1}{2} |\varphi|^2 \right)}$$
$$A_2 = \sqrt{\frac{2 e^\phi \text{Re } T_1 \text{Re } T_3}{\text{Re } T_2 - \frac{1}{2} |\varphi|^2}}$$
$$A_3 = \sqrt{\frac{2 e^\phi \text{Re } T_1}{\text{Re } T_3} \left( \text{Re } T_2 - \frac{1}{2} |\varphi|^2 \right)}$$

- Compute  $V_0$  in the 4D Einstein frame:

$$V_0 = \frac{(2\pi)^{-3} m^2 \alpha'^{-2} g_s^3}{4 A_1^2 A_2^3 A_3}$$

- Independent of  $\phi$  !

# Summary

- Model of inflation using WLs as inflatons
- Similar to Branes @ angles (T-duality)
- But: provides more tuning parameters
- Predictions: small  $\varepsilon$  (no gravitational waves)
  - HZ or slightly red scalar spectrum
  - Cosmic strings with  $G\mu < 10^{-7}$
- No SUGRA  $\eta$  problem!
- Heterotic?