ON WHAT SCALE SHOULD INFLATIONARY OBSERVABLES BE CONSTRAINED?

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Precision Cosmology

WMAP 3

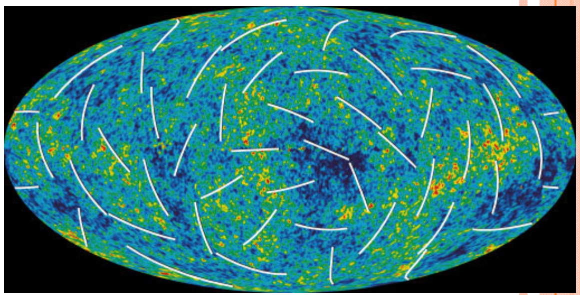
- Most sensitive all-sky map of the CMB made to date
- Initial conditions for cosmic structure formation that seeded the

Primordial Power Spectrum

Two challenges for cosmology:

• observationally, we wish to extract the spectrum's amplitude and scale dependence from data

• theoretically we seek to understand the origin of the perturbations



Analysis of CMB data

• *Restrict to single field inflationary models and ask*:

• Already possible to constrain the **shape of the inflaton potential?**

• Necessary to go **beyond Harrison-Zel'dovich** (scale invariant) ?

•From a model building point of view means :
inflaton potential is not flat
inflation is not driven by a pure cosmological constant.

•WMAP 3 data:
✓ **non-trivial shape** of potential
✓ non-vanishing second derivative of potential
✓ first derivative remains **unbounded from below**
(since no detection of primordial gravitational waves)

WMAP 3: Running of the scalar spectral index - implications for inflation

•Preference for running

$$\frac{dn_S}{d\ln k} = -0.05^{+0.028}_{-0.029}$$

•***Peiris and Easther (2006)***
Inflation cannot provide this ammount of running and sufficient number of e-folds (*potential too steep – inflation ends quickly*)

•Now: still some viable models but if constraints continue to tighten around central value these also ruled out

•***Ballesteros and Espinosa (2006)*** showed that even NRO in potential supressed by a high energy scale can flatten the potential give sufficient inflation with this amount of running.

Choice of Pivot scale

- Include running ⟹ n_S **changes with scale**
- One more degree of freedom ⟹ **uncertainty on n_S increases**.

•CosmoMc: specify scale k_* to obtain constraints:

- code takes observations of anisotropy at one angle and translates into another angle

•In principe this choice is *arbitrary*:

- changing the pivot scale means expanding about a different point in the potential
- inflationary observables $\{n_S, r\}$ will be given at another scale.

- **THIS SCALE HAS SO FAR BEEN CHOSEN BY HAND:**

Kurki-Suoni et al, 2004
Finelli et al, 2006
Liddle et al, 2006

Choice of Pivot Scale

But I will show that when

- running is included
- parameter space is multidimensional

⟹ **Choice is no longer arbitrary**

OK provided:

1. One specifies the full multi-dimensional parameter distribution

2. The model is internally self consistent

1.Multidimensional parameter distributions:

- N-dimensional parameter space, (normally 6-8)
- Presenting constraints:
  8D parameter space is **compressed** and projected onto a *1D* or *2D* plane of confid. limits, while **marginalizing** over the other parameters.
- Implies:
  - **loss of information** on the limits of other parameters
  - loss of information of **correlations** between parameters

So:

We **can't shift the pivot scale** anymore since we **lost information** on correlations between parameters

2. Self consistency

Power Spectra - Power Laws with different indices.

$$(n_S - 1 \neq n_T)$$

Amplitude of the tensor power spectrum is set by the **Consistency Equation**

$$2\frac{A_T^2}{A_S^2} \cong -n_T$$

dependent on scale!

- If we impose this at one scale it will not hold at other scales,

MEANS

- **Power spectra we obtain by imposing at one scale is different from the one we'll obtain by choosing another scale**

Correct way is impose ***Hierarchy of Consistency Equations***

MC and A. Liddle (2006)

- 2nd consistency equation ensures that the first holds on all scales!

$$\frac{dn_{\rm T}}{d\ln k} = n_{\rm T}\left[n_{\rm T} - (n_{\rm S} - 1)\right]$$

- If we include scalar running have to include tensor running

$$\begin{aligned} A^2_{\rm S}(k) &\propto (k/k_*)^{(n_{\rm S}-1)+(dn_{\rm S}/d\ln k)\ \ln k/k_*} \\ A^2_{\rm T}(k) &\propto (k/k_*)^{n_{\rm T}+(dn_{\rm T}/d\ln k)\ \ln k/k_*}\ , \end{aligned}$$

Running of the scalar spectral index

$$\frac{dn_S}{d\ln k}$$

Data analysis:

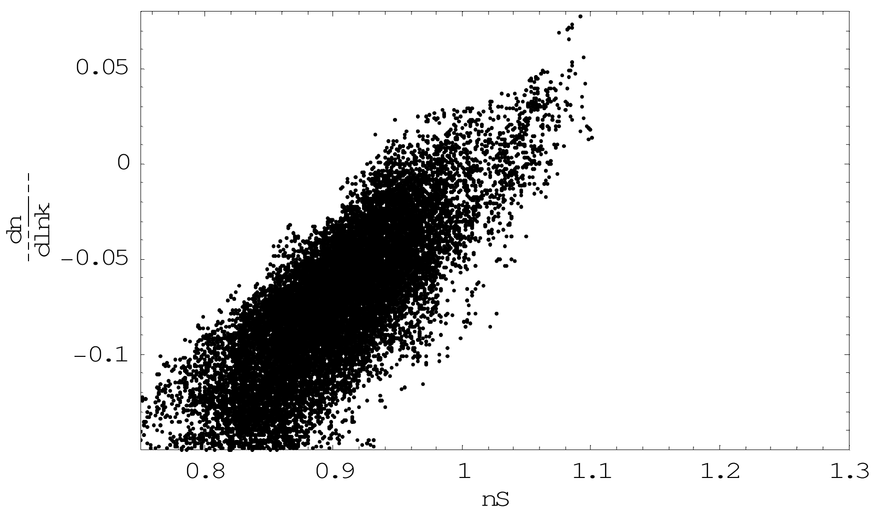
- One more degree of freedom → uncertainty on n_S increases.
- Copeland, Grivell, Liddle (1998)

uncertainty in n_S is recovered at scale where tilt and running **decorrelate.**

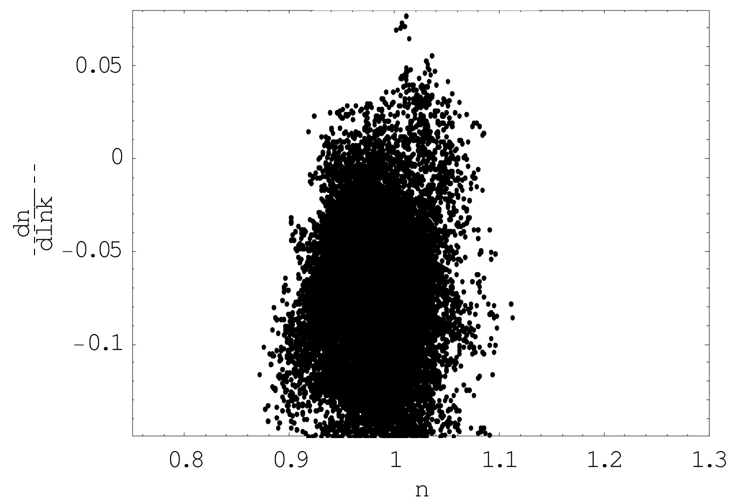
$$n_S(k) = n_S(k_*) + \frac{dn_S}{d\ln k}\ln\frac{k}{k_*}$$

Decorrelating the scalar tilt and its running:

k=0.05



k=0.017



$$n_S = A + B\, dn_S/d\ln k$$

$$k' = 0.017\,\mathrm{Mpc}^{-1}$$

$$B = -\ln k/k_*$$

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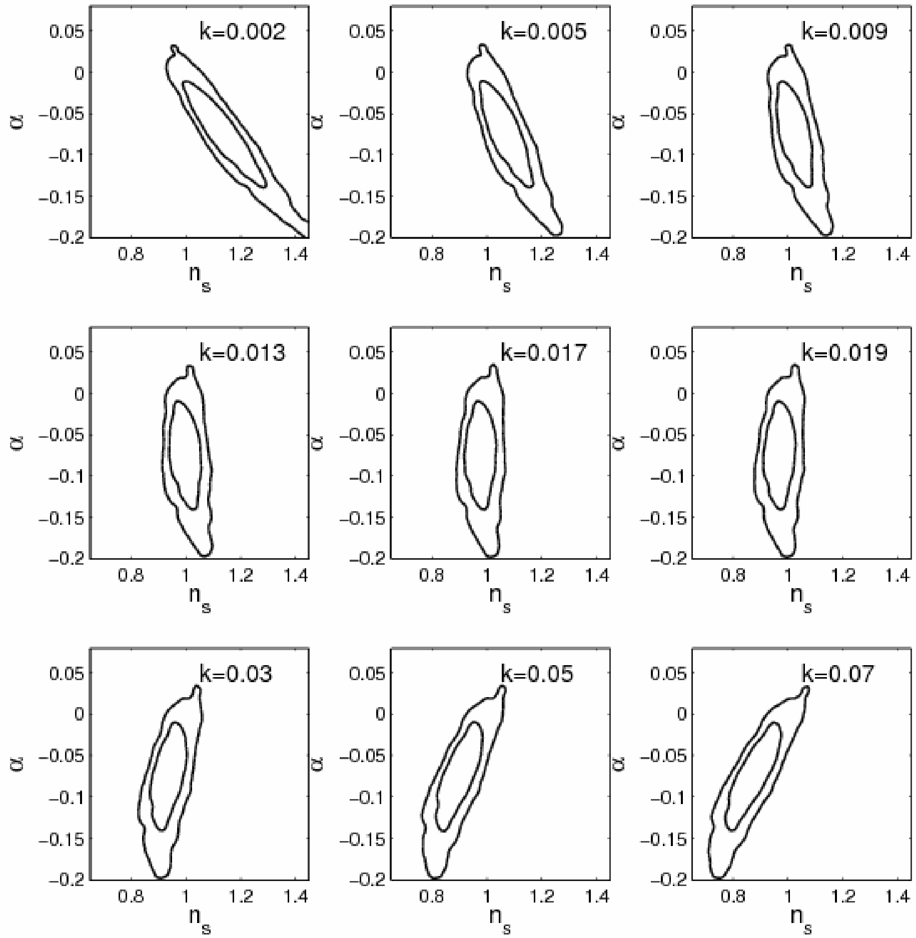
Now use same formalism for other scales…



- Unit Jacobian: area isn't altered
- However:

WMAP scale ***k=0.002***:
clearly angled contour for n_S compared with the pivot scale
k=0.017

WMAP scale is not allowing extracting most information out of data

WMAP: $\boldsymbol{0.9 < n_S < 1.5}$ **Pivot Scale:** $\boldsymbol{0.95 < n_S < 1.05}$

what happens for the $\{n_S\,, r\}$ plane ?

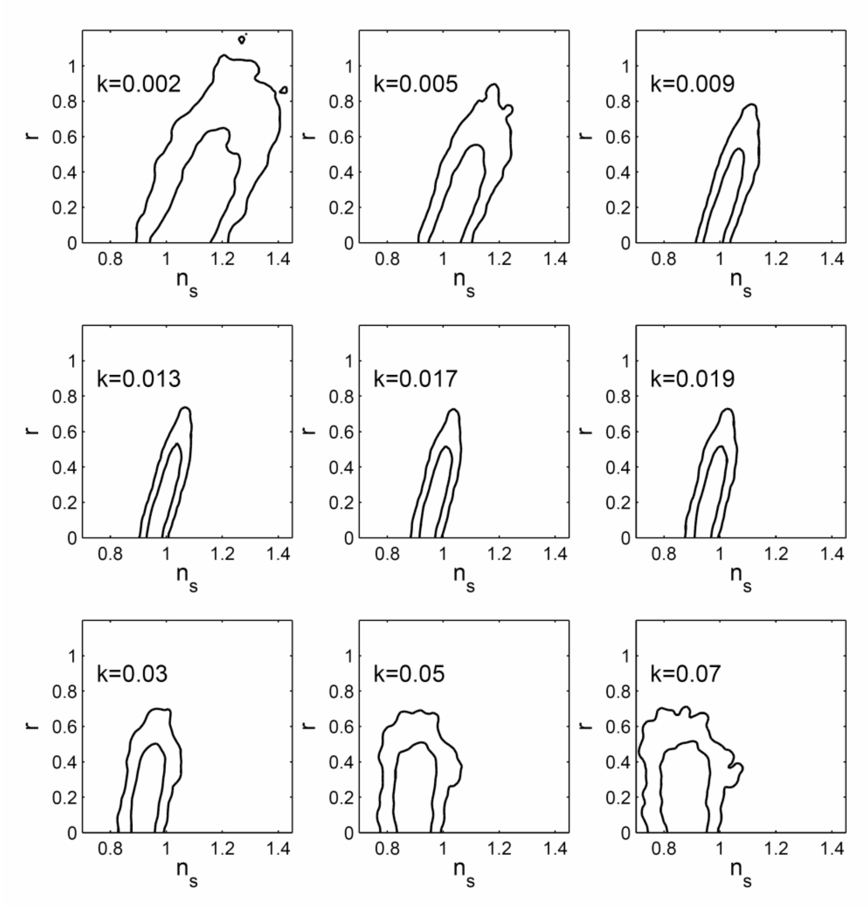
Tensor-to-scalar ratio: $r(k) \equiv 16A_T^2(k)/A_S^2(k)$

- Use same formalism for ***r***:
  - Expand the ***scalar*** and ***tensor*** amplitudes and substitute in ***r***

$$\frac{r(k)}{r(k_*)} = \frac{1 + n_T \ln\frac{k}{k_*} + \frac{1}{2}\left[n_T^2 + \frac{dn_T}{d\ln k}\right]\ln^2\frac{k}{k_*}}{1 + (n_S - 1)\ln\frac{k}{k_*} + \frac{1}{2}\left[(n_S - 1)^2 + \frac{dn_S}{d\ln k}\right]\ln^2\frac{k}{k_*}}$$

Now we can consider distribution at other scales…

2D distribution at other scales for $\{n_S , r\}$



- Now transformation **alters contour areas** as well as shape

- **Significant reduction in confidence contours** when different scales are considered

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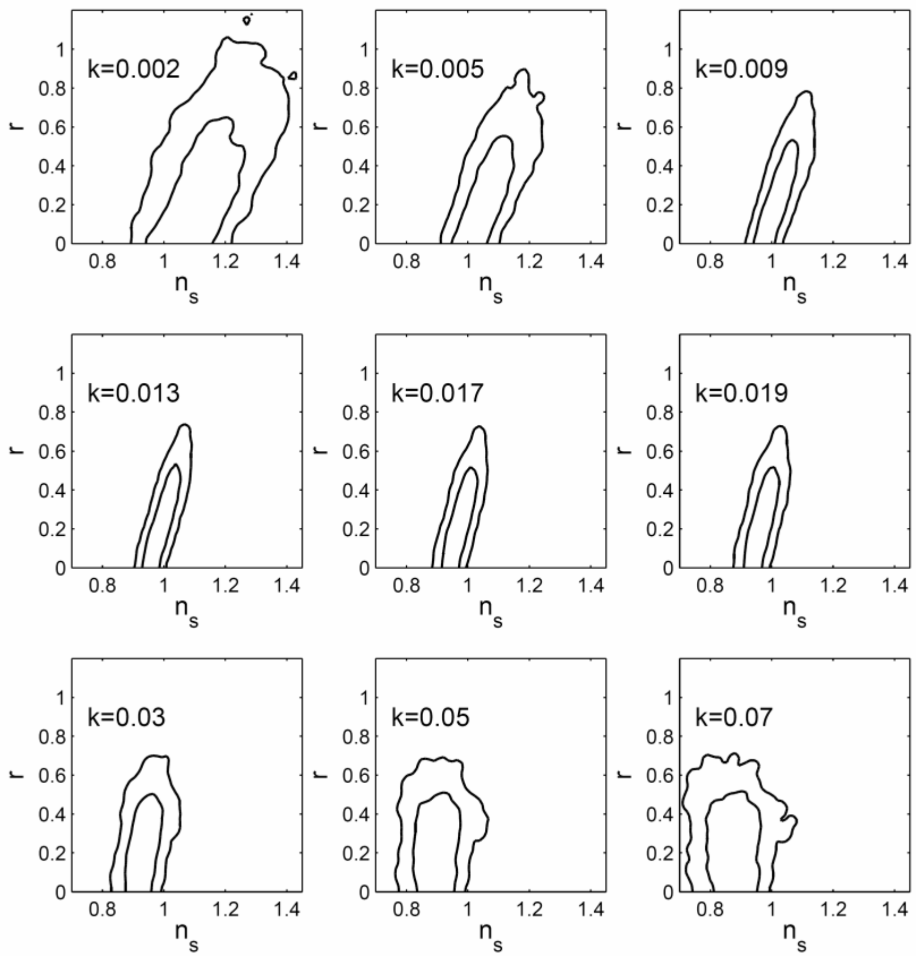
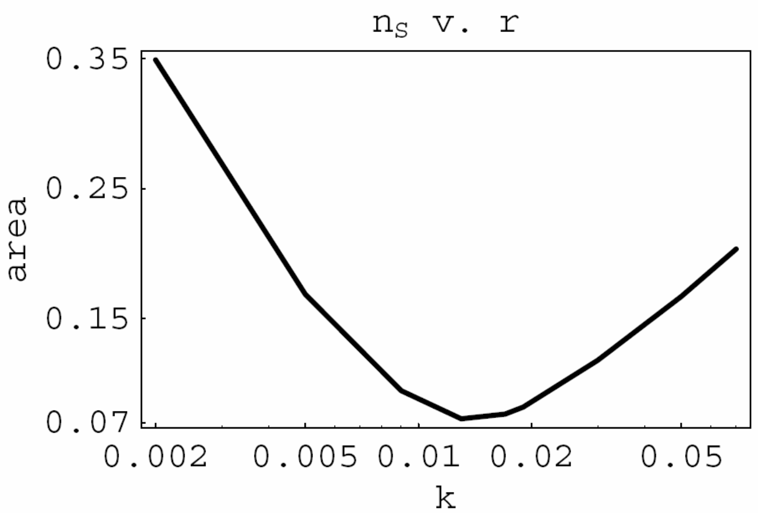




Reduction by a factor of 5 in parameter space!

Slow roll parameters

lowest order

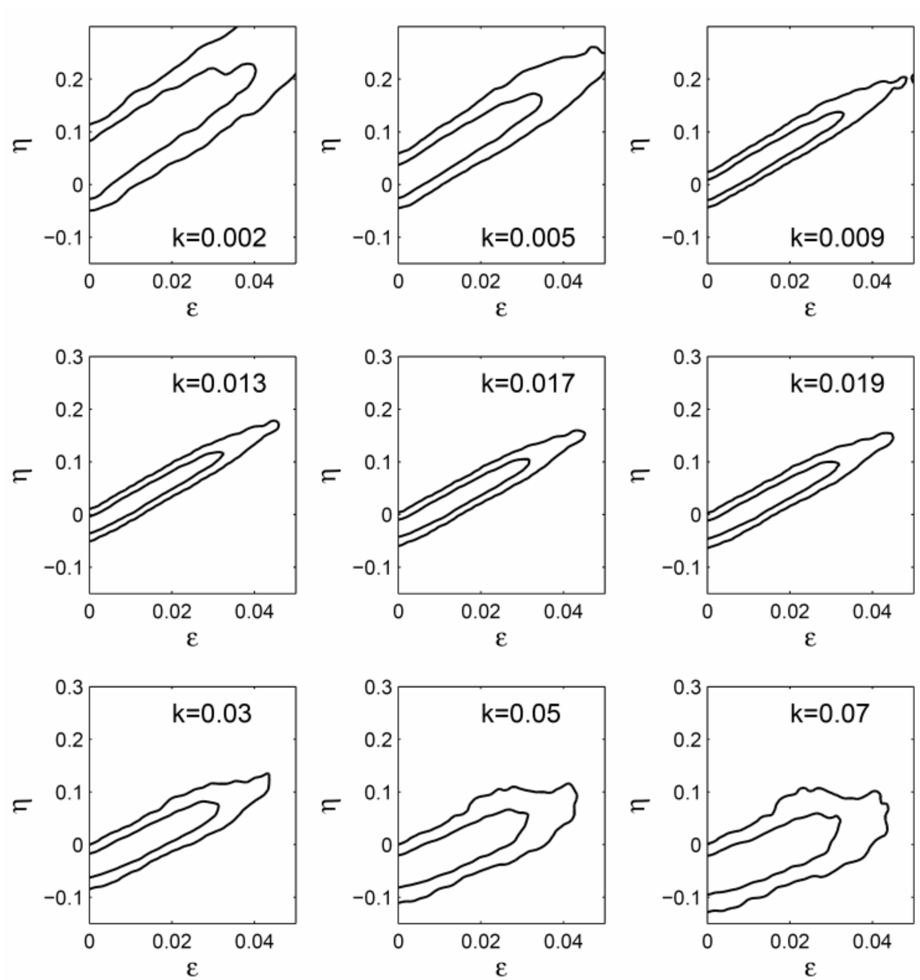
$$\epsilon = \frac{m_{\rm Pl}^2}{16\pi}\left(\frac{V'}{V}\right)^2 \quad ; \quad \eta = \frac{m_{\rm Pl}^2}{8\pi}\frac{V''}{V}$$

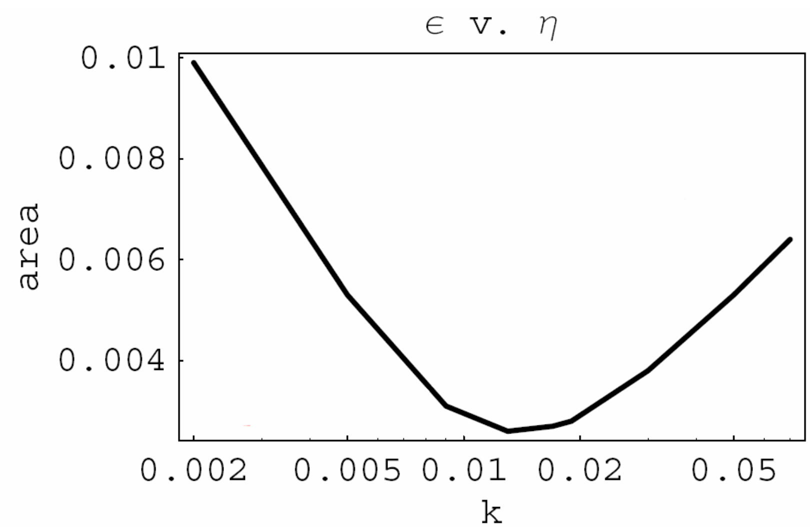
- Control the shape of the potential

  ε controls the slope

  η controls the curvature

Are these robust to a change in scale....?





- **Also strong variation with scale for ε, η**
  - Scale minimizing constraints is same as for n_S, r

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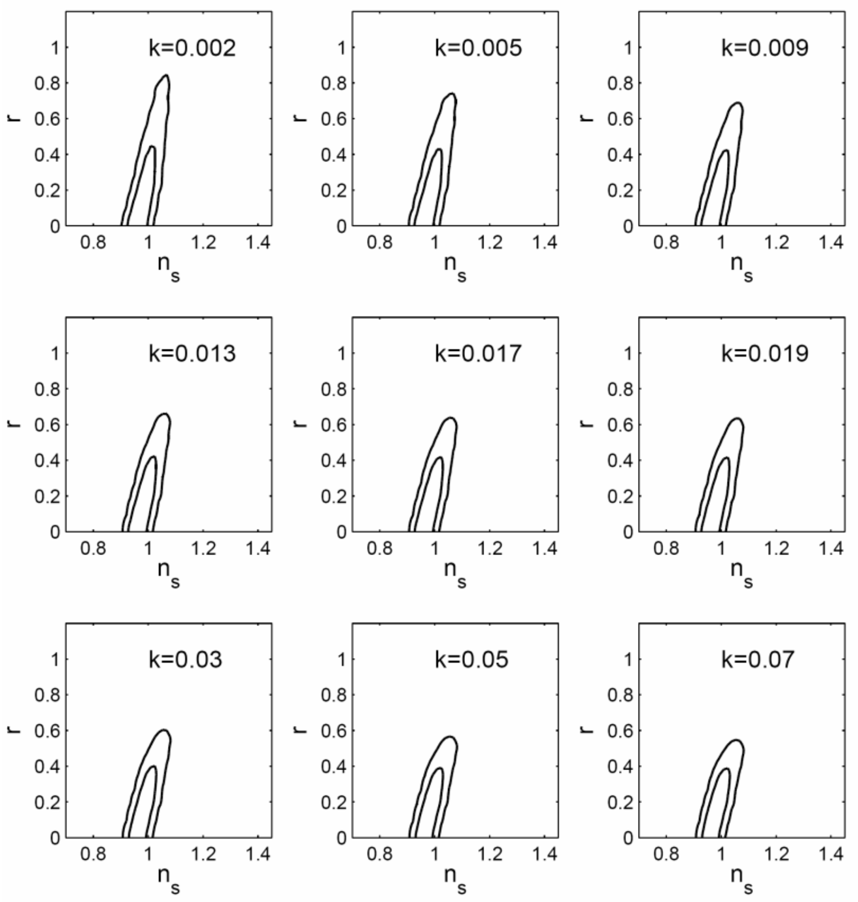
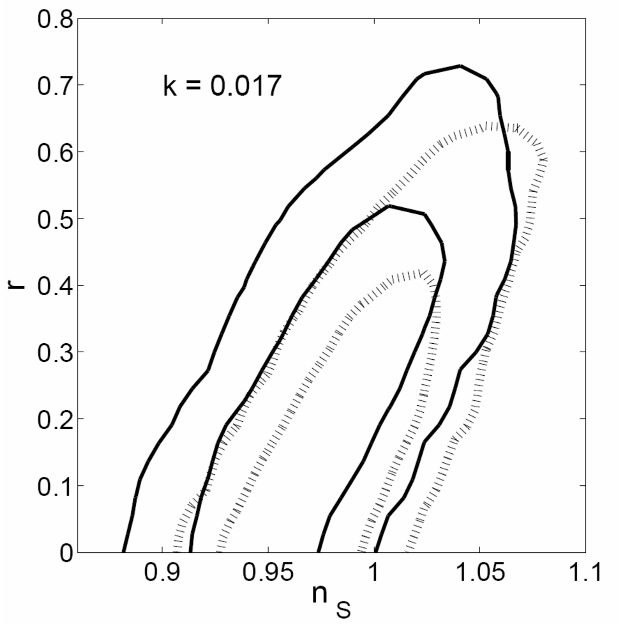
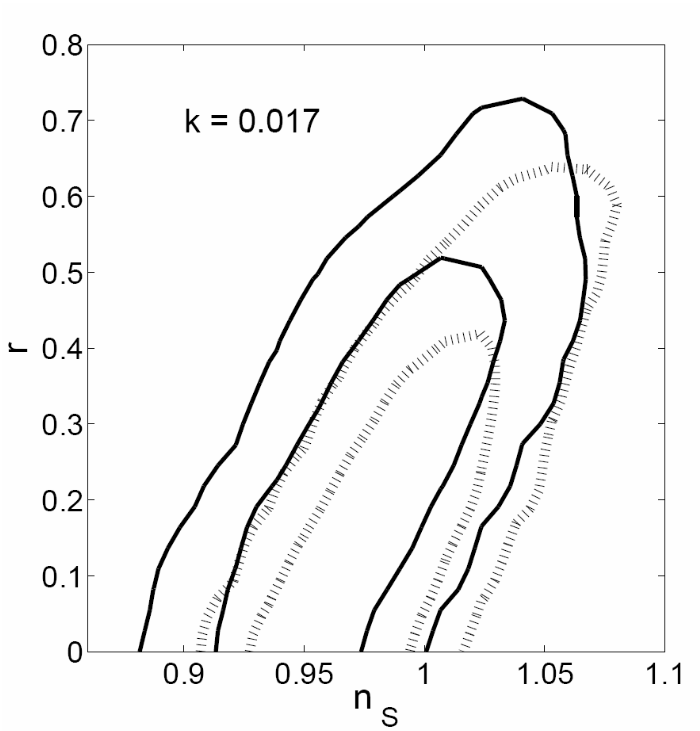
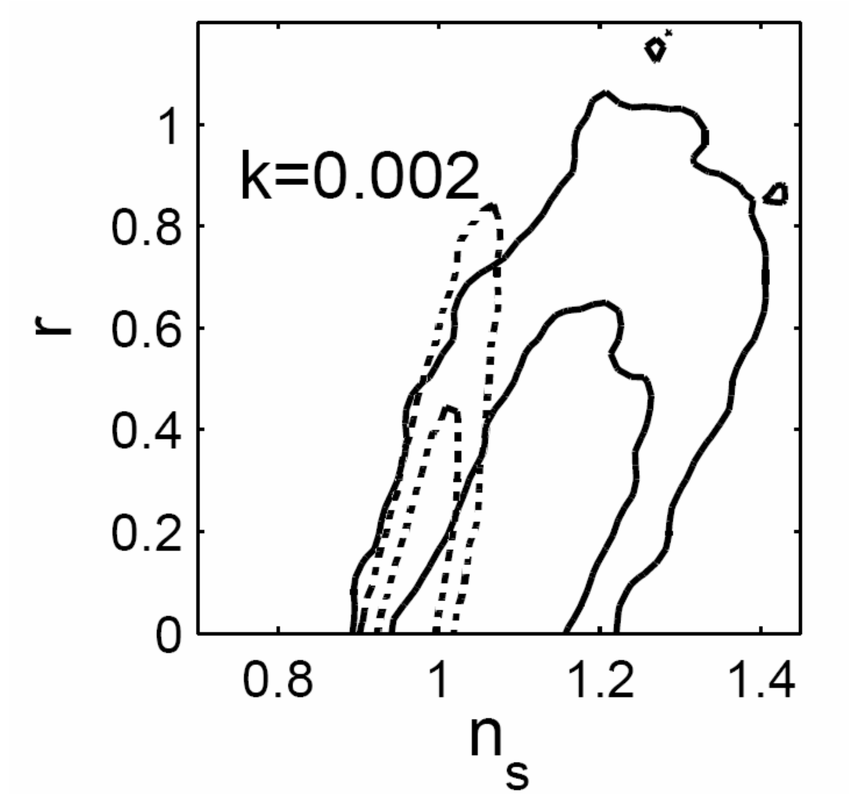
When no running is included





No change in constraints but <u>degradation of limits</u>





• Pivot scale $k=0.017$ Mpc^{-1}: Uncertainty increase of *20%*

• WMAP scale $k=0.002$ Mpc^{-1} : Uncertainty increase of *500%*

• WMAP scale gives huge deteoriation of constraints when running is included!

Conclusions

- Chosing a pivot scale is important when running is present

- Appropriate scale is that that decorrelates n_S and running

- At this scale ***k=0.017* Mpc^{-1},** n_S is **best determined**

- Marginalized coinstraints n_S, r, ε, η depend significantly on the choice of scale in the presence of running

- Same criterion can be used to define an optimal scale for any data set compilation

- At optimal scale constraints on n_S are only mildly degraded when including running **in contrast to WMAP scale.**

- Different scales for different observables?

Lowest Order

- The relation between the two descriptions is

$$\epsilon \simeq \frac{r}{16} \quad ; \quad \eta \simeq \frac{3}{16} r - \frac{1}{2}(1 - n_{\mathrm{S}})$$

WMAP 3 vs. WMAP 1

WMAP I:

- low values of C_l at small l
- glitches around the first peak and $l \sim 40$
- apparent evidence for a running of the sclar spectral indice when combining with small scale data

WMAP III:

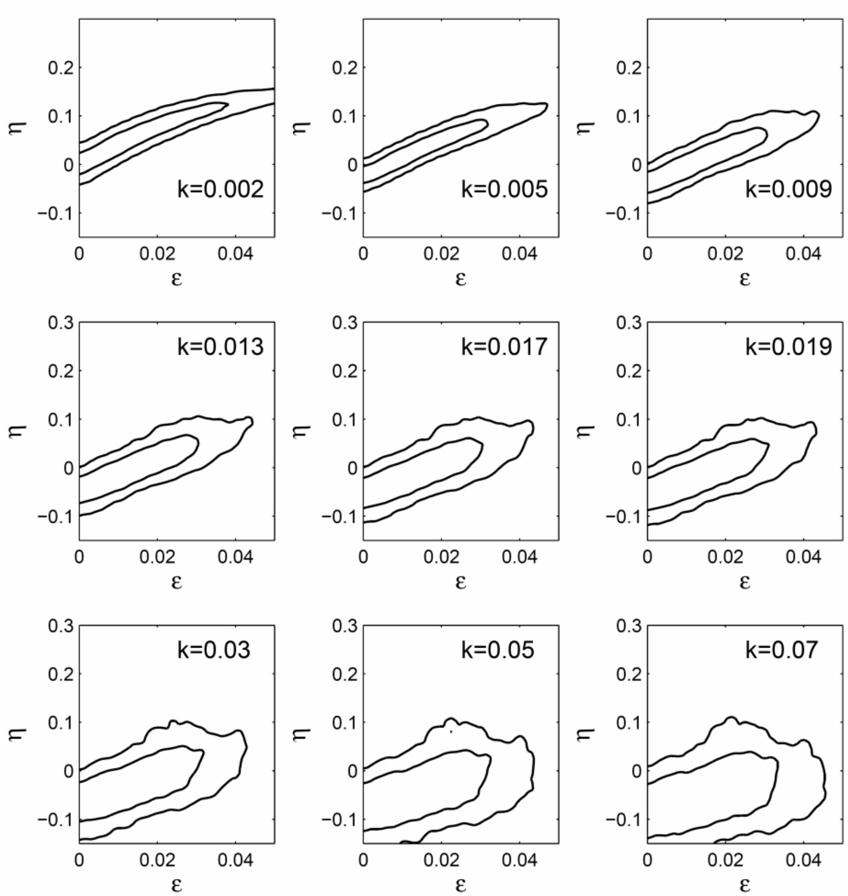
- quadrupole still ~ 2σ lower than lcdm but octupole moved closer to lcdm
- glitches around the first peak not seen but at $l \sim 40$ still present
- preference for running when viewed alone or in combination with other data

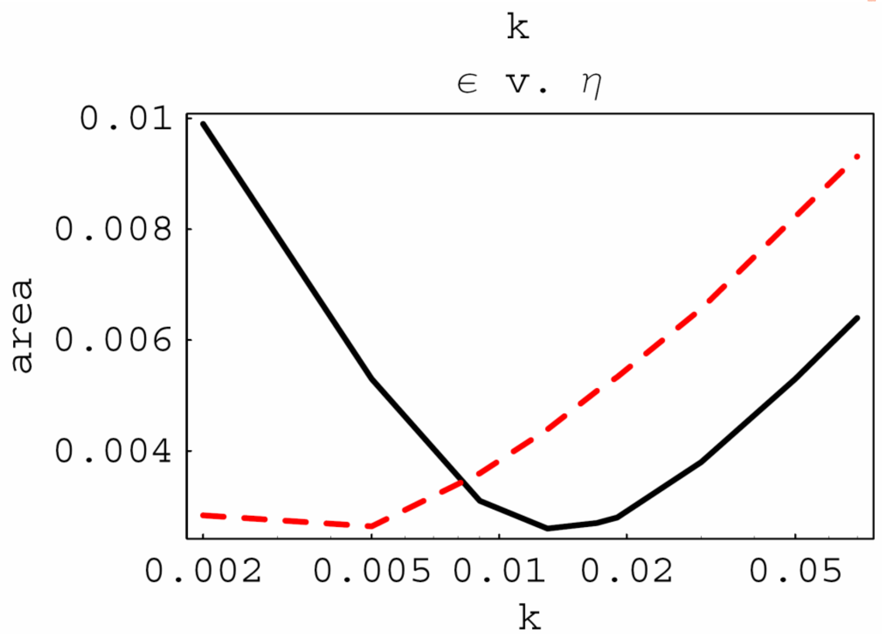
Slow roll parameters

next order

From the next order expressions for the potential *Lidsey et al (1995)*

$$
\begin{aligned}
\epsilon &= \frac{r}{16}\,\frac{1-0.85\,r/16+0.53(1-n_\mathrm{S})}{1+0.21\,r/16} \\
\eta &= \frac{1}{3}\,\frac{1}{1+0.21\,r/16}\left\{\frac{9}{16}r-\frac{3}{2}(1-n_\mathrm{S})\right. \\
&\quad +(36C+2)\left(\frac{r}{16}\right)^2-\frac{1}{4}(1-n_\mathrm{S})^2 \\
&\quad \left.-(12C-6)\frac{r}{16}(1-n_\mathrm{S})-\frac{1}{2}(3C-1)\frac{dn_\mathrm{S}}{d\ln k}\right\}
\end{aligned}
$$





- η to next order depends on the running so the ellipse widens considerably

- different minimal area occurs because of a cancellation of terms and is accidental

Other applications:

σ_8

- When constraining density perturbations using galaxy clusters commonly the parameter σ_8 is quoted (the amplitude of perturbations smoothed at $8\ h^{-1}$ Mpc)

- However the normalization is best determined at a somewhat smaller scale and marginalizing over quantites such as Ω_0 to quote constraints on σ_8 can increase the statistical uncertainty on the normalization