A novel world–sheet functional approach to Liouville strings & Implications to Cosmology

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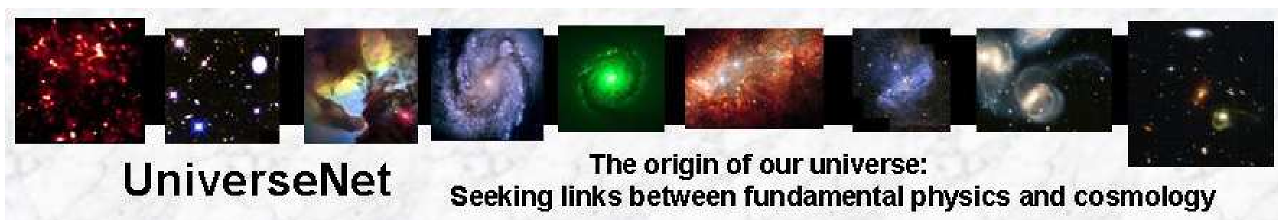


Work with J. Alexandre and N. Mavromatos

OUTLINE

- 1st quantized String theory as a conformal theory on the world – sheet

- Motivation for Liouville string theory : restoration of lost conformal invariance in non – equilibrium cosmologies (eg. strings on colliding brane-worlds, etc)

- A new approach to Liouville field theory, through an exact functional method

Motivation : (I) Free Bosonic String Theory

Free bosonic string propagating in *D* – dimensional flat space-time

$$S^* = -\frac{1}{4\pi\alpha'}\int d^2\sigma \sqrt{\gamma}\gamma^{ab}\eta_{\mu\nu}\partial_a X^\mu \partial_b X^\nu$$

$$\sigma^a = (\tau, \sigma) \; , \; X^\mu = \left(X^0, X^1, \ldots X^{D-1}\right)$$

This is a conformal theory on the 2 – dim world – sheet.

Central charge : $c_{tot} = c_X + c_g = D - 26$

Conformal invariance ⇒ $D = 26$ → *Critical Bosonic String*

Motivation : (II) Strings in Background fields

Now consider a deformed world – sheet action :

$$S = S^* + g^i \int d^2\sigma V_i$$

g^i : couplings

V_i : vertex operators

Conformal invariance ⟺ **vanishing of Weyl anomaly coefficients** $\hat{\beta}^i = 0$

⤶ **equations of motion for background fields in target – space**

⟺ **effective target – space action**

Motivation : (II) Strings in Background fields

e.g. for Graviton and Dilaton backgrounds :

World – sheet action (2 – dim) :

$$S = \frac{1}{4\pi\alpha'} \int d^2\sigma \sqrt{\gamma} \left[\gamma^{ab} G_{\mu\nu}(X) \partial_a X^\mu \partial_b X^\nu + \alpha' R^{(2)} \Phi(X) \right]$$

$$\hat{\beta}^G_{\mu\nu} = \alpha' R_{\mu\nu} + 2\alpha' \nabla_\mu \nabla_\nu \Phi + \mathcal{O}\left(\alpha'^2\right)$$

$$\hat{\beta}^\Phi = \frac{D-26}{6} - \frac{\alpha'}{2} \nabla^2 \Phi + \alpha' \nabla_a \Phi \nabla^a \Phi + \mathcal{O}\left(\alpha'^2\right)$$

Conformal invariance : $\hat{\beta}^G_{\mu\nu} = \hat{\beta}^\Phi = 0$ **look like equations of motion!**

Effective action in *D* – dim target space (in σ – model frame) :

$$S_{eff} \propto \int d^D x \sqrt{-G} e^{-2\Phi} \left[-\frac{2(D-26)}{3\alpha'} + R^{(D)} + 4\partial_\mu \Phi \partial^\mu \Phi \right] + \mathcal{O}\left(\alpha'\right)$$

Motivation : (II) Strings in Background fields

$$\left(\hat{\beta}^i = \beta^i + \delta g^i \,, \quad \beta^i = \frac{d g_R^i}{d \ln \mu} \right.$$

δg^i : change under general coordinate diffeomorphisms in target space

μ : world – sheet RG scale

g_R^i : renormalized coupling $\left. \right)$

more general, vacuum energy

$$\frac{D-26}{6} \to Q^2$$

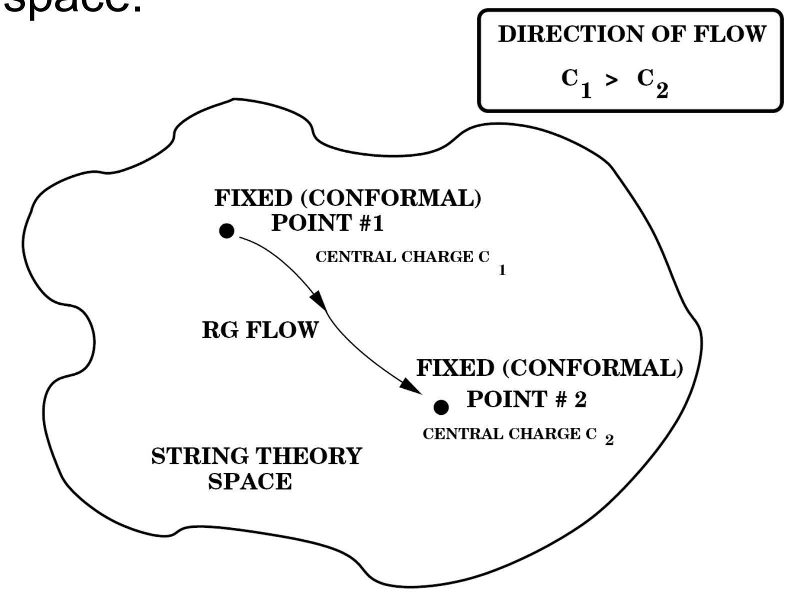
Motivation : (III) Non - critical Strings

"**String theory space**" : space of coupling constants, $\{g^i\}$

(Zamolodchikov : metric space, $\mathcal{G}_{ij} = z^2 \bar{z}^2 < V_i(z,\bar{z}) V_j(0,0) >_g$ …)

fixed points : conformal invariance conditions are satisfied $\hat{\beta}^i = 0$

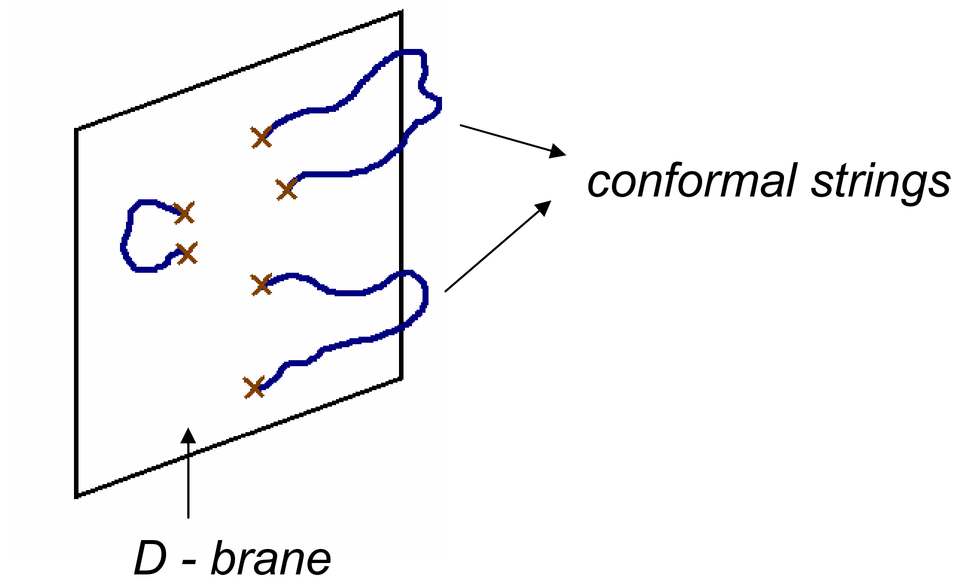
Non – equilibrium processes may result in **non – critical** (non – conformal) string configurations, i.e. configurations which lie **away from fixed points** in the string theory space.



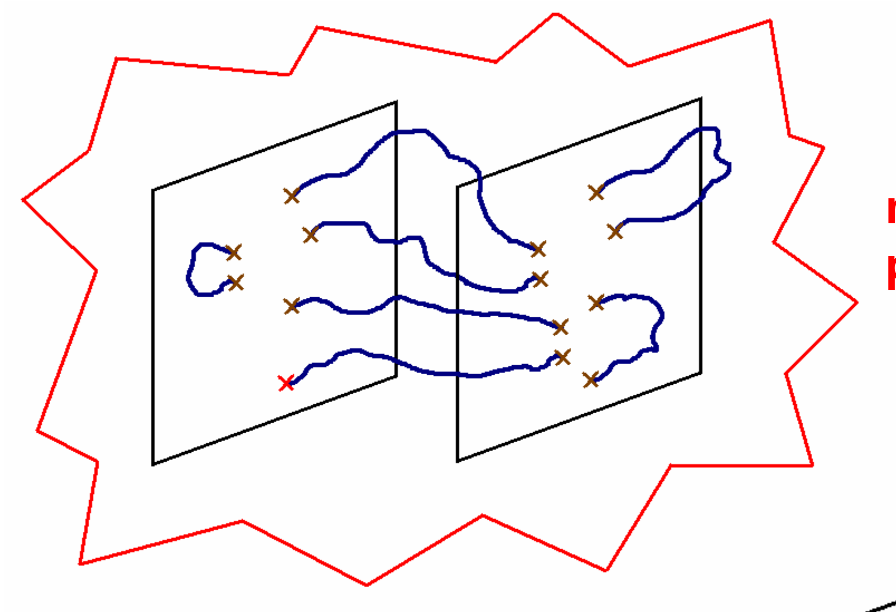
Motivation : (III) Non - critical Strings

Example of such a non – equilibrium procedure :

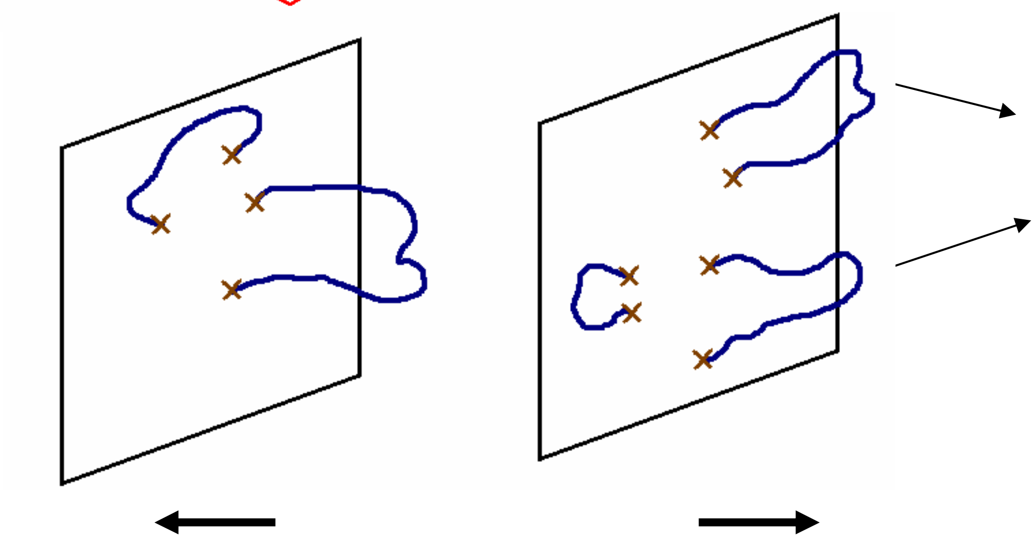
Brane collision (early universe ?)



Motivation : (III) Non - critical Strings



non – smooth process !!



non – conformal strings !

Liouville string theory

We are away from the critical theory : $\hat{\beta}^i \neq 0$

How do we restore conformal invariance ?

→ **Liouville – dressing procedure**

Couple our model with an extra world – sheet quantum field $\phi(\tau, \sigma)$ with action:

$$S_L = \frac{1}{8\pi} \int d^2\sigma \sqrt{\gamma} \left(-\text{sign}\left(Q^2\right) \gamma^{ab} \partial_a \phi \partial_b \phi + Q R^{(2)} \phi + 4\pi \mu e^{2b\phi} \right)$$

Every non – conformal operator, V_i, of conformal dimension h_i becomes "**Liouville – dressed**" :

$$V_i^L(\phi, X^\mu) \equiv e^{\alpha_i \phi(\tau,\sigma)} V_i(X^\mu)$$

Conformal, as long as :

$$\alpha_i(\alpha_i + Q) = \Delta_i \equiv h_i - 2$$

Q : "charge at infinity"

Q^2 : central charge deficit

Liouville string theory

generalized conformal invariance conditions : $\ddot{g}^i + Q\dot{g}^i = -\hat{\beta}^i + \mathcal{O}\left(\dot{g}^2\right)$

$$\dot{g}^i \equiv \frac{\partial g^i}{\partial \phi_0} \;, \quad \hat{\beta}^i = d\text{-dim beta functions}$$

In **supercritical (central charge surplus,** $Q^2 > 0$ **)** models : ϕ_0 can be identified with **target time**, and conformal invariance can be restored in one dimension higher

⇒ **Need to study Liouville field theory on 2 - dim world - sheet**

Non-perturbative approach to Liouville theory

Novel non-perturbative functional field theory method :

in α'

In **early universe cosmology**, higher order **curvature** $\left(R^2,\ R^4,\ \ldots\right)$ terms could be relevant.

These are terms of order α' in the effective action.

➡ Our method should be non – perturbative in α'

Non-perturbative approach to Liouville theory

Novel non-perturbative functional field theory method :

Apparent technical similarities with usual (Wilsonian) renormalization, but essential differences :

- Fixed world – sheet cutoff

Fixed world – sheet area, A (Distler – Kawai)

$$Z(A) = \int \mathcal{D}\phi \mathcal{D}X e^{-S} \delta\left(\int d^2\sigma e^{\alpha\phi}\sqrt{\gamma} - A\right)$$

- Running bare parameter(s)

Liouville field theory : Q^2 = vacuum energy

Non-perturbative approach to Liouville theory

Liouville field theory on 2 – dim flat world – sheet

Bare action :

$$S = \int d^2\xi \left\{ \frac{Q^2}{2} \partial_a \phi \partial^a \phi + V_Q\left(\phi\right) \right\}$$

Path integral quantization leads to the **effective action**, Γ, whose evolution equation with respect to the bare parameter, Q^2 , is :

$$\dot{\Gamma} \equiv \frac{\partial \Gamma}{\partial Q^2} = \int d^2\xi \frac{1}{2} \partial_a \phi \partial^a \phi + \frac{1}{2} \mathrm{Tr} \left[\frac{\partial}{\partial \xi^a} \frac{\partial}{\partial \zeta_a} \left(\frac{\delta^2 \Gamma}{\delta \phi_\xi \delta \phi_\zeta} \right)^{-1} \right]$$

Exact evolution equation, independent of any loop expansion

Non-perturbative approach to Liouville theory

We assume the following functional dependence of the effective action :

$$\Gamma[\phi] = \int d^2\xi \left\{ \frac{Z_Q(\phi)}{2} \partial_a \phi \partial^a \phi + V_Q(\phi) \right\}$$

Alexandre, Ellis, Mavromatos, JHEP ***0703*** *(2007) 060*

$$Z_Q = \text{independent of } \phi$$

$$\dot{Z}_Q = 1 \qquad Z_Q = Q^2$$

$$\dot{V} = \frac{\Lambda^2}{8\pi Z} - \frac{V''}{8\pi Z^2} \ln\left(1 + \frac{Z\Lambda^2}{V''}\right)$$

$$\dot{Z} = 1 + \frac{5\,(Z')^2}{8\pi Z^3} \ln\left(1 + \frac{Z\Lambda^2}{V''}\right) + \frac{7}{24\pi}\frac{Z'}{Z^2}\frac{V^{(3)}}{V''} - \frac{47}{48\pi}\frac{(Z')^2}{Z^3}$$

Non-perturbative approach to Liouville theory

Solution :

$$Z_Q = Q^2$$

↔ **no quantum corrections for Z_Q ,** ***consistent with conformal invariance***

- In the limit $Q^2 >> 1$:

$$V(\phi) = \mu^2 P_Q(\phi) e^{\phi} \simeq \mu^2 \left(1 + \frac{\ln(Q^2)}{8\pi Q^2}\right) \exp\left\{\left(1 - \frac{1}{8\pi Q^2}\right)\phi\right\}$$

- In the limit $Q^2 \to 0$:

$$V(\phi) = \mu^2 P_Q(\phi) e^{\phi} \simeq \frac{\Lambda^2}{8\pi} \left|\ln(Q^2)\right| \exp\left\{\frac{\phi}{|\ln(Q^2)|}\right\} \simeq \frac{\Lambda^2}{8\pi} \left|\ln(Q^2)\right|$$

Jackiw

Conclusions & Outlook

- We tested a novel functional method in the context of Liouville strings, motivated by non – critical stringy Q – cosmologies

- We found no wave function renormalization in the Liouville model, as expected from the general conformal invariance restoring properties of Liouville mode.

Outlook :

Application to violent cosmic phenomena such as colliding brane – worlds, D–particle space – time foam (capture of strings by D0 – branes as source of non – conformal invariance) etc.