First Annual School of the EU Network "Universnet" The Origin of the Universe
Sept. 24-29, 2007, Mytilene, Greece

Thermodynamic Constraint on Primordial Black Hole Formation

Hyun Kyu Lee
Hanyang University

- A finite size object with mass M changes the curvature of the space: Schwarzschild metric for a spherically symmetric case.
- Black hole has additional features:
  1. Event Horizon: $R_{BH} = 2M$
  2. Entropy: $S_{BH} = 4\pi M^2$

     Temperature: $T_{BH} \equiv \frac{1}{(dS/dM)} = \frac{1}{8\pi M}$

→ Thermodynamic identity

$$dM = T\,dS \quad (\hbar = c = G = k = 1)$$

- First law of thermodynamics
- Hawking radiation with temperture T
- Can the second law of thermodynamics,

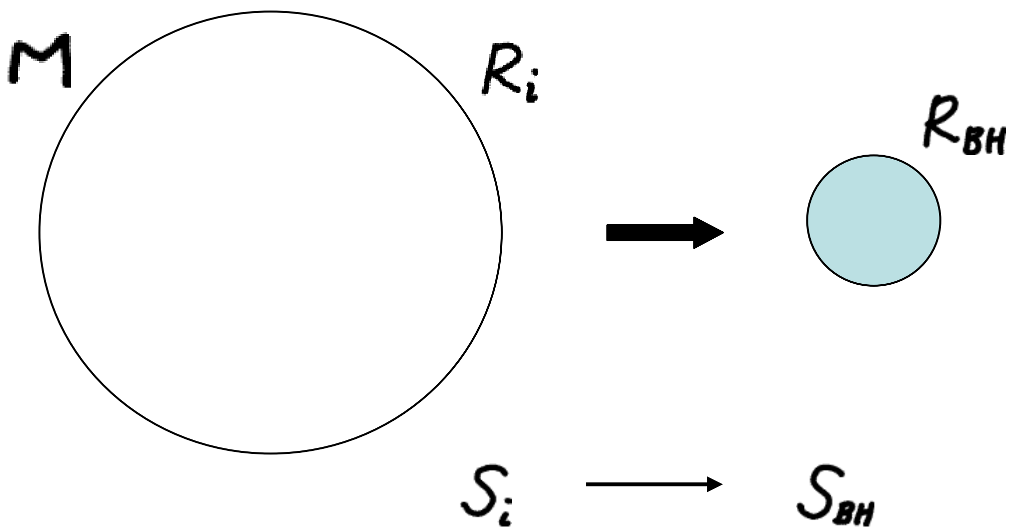
$$\Delta S = S_f - S_i \geq 0,$$

provide any constraints on the physical processes, which involve black holes ?

"Formation of primordial black hole in early Universe"

- PBH is supposed to be formed when a overdense region, $\rho > \bar{\rho}$, collapses into a black hole.
- Simple minded approach:

  Spherical collapse (isolated) of radiation



Entropy of progenitor depends on the equation of state

- Example: neutron-star-like matter

progenitor: $M_i \propto m_N N$

$S_i \propto N \rightarrow S_i = f M_i / m_N$

black hole: $M_{BH} = M_i$

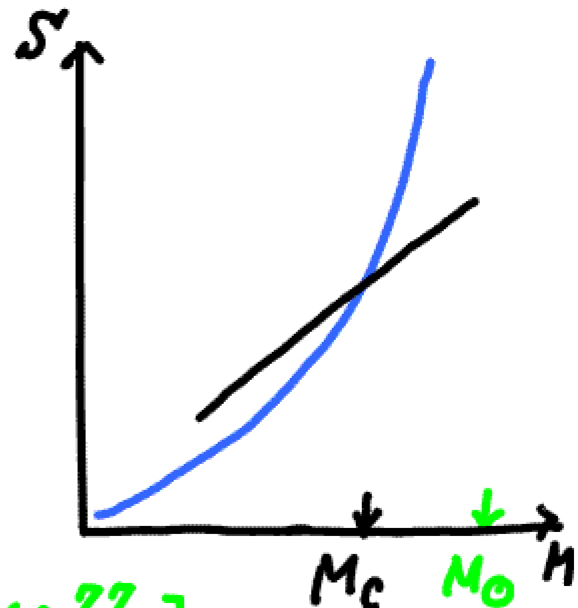
$S_{BH} = 4\pi (M_i/m_{pl})^2$

2nd law of thermodynamics

$\Rightarrow M_i \gtrsim M_c$

$M_c \sim m_N \left(\frac{m_{pl}}{m_N}\right)^2 \sim 10^{13} g$

[for $M_i = M_\odot$, $S_i \sim 10^{54}$ & $S_{BH} \sim 10^{77}$]



Progenitor in early Universe

- Size of over-dense region: Ri

$$r_{hor} = \frac{0.30}{\sqrt{g_*}}(\frac{m_{pl}c^2}{k_B T})^2 l_{pl}, \;\; l_{pl} = \hbar / m_{pl}\; c$$

- energy and entropy density:
  [radiation dominated, T >> m]

$$\rho = \frac{\pi^2}{30} g_* T^4, \;\;\; s = \frac{2\pi^2}{45} g_* T^3$$

- Parameters: $g_*, T\ (m_{pl})$ $\quad g_*^{SM} = 106.75$

$$S_{hor} = \frac{0.050}{\sqrt{g_*}}\left(\frac{m_{pl}}{T}\right)^3,$$

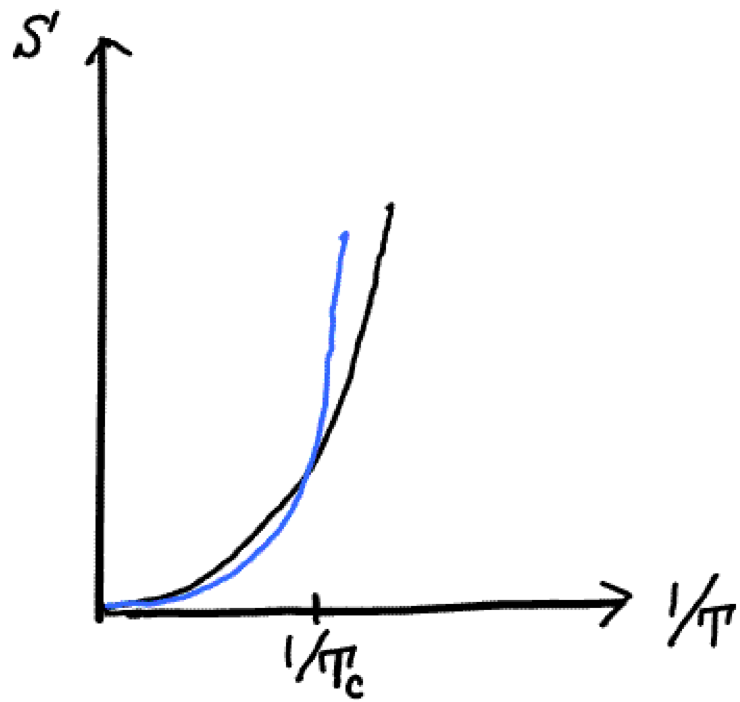
$$M_{hor} = \frac{0.038}{\sqrt{g_*}}\left(\frac{m_{pl}}{T}\right)^2 m_{pl}.$$

$$S_{pbh} = \frac{0.018}{g_*}\left(\frac{m_{pl}c^2}{k_B T}\right)^4$$

$$S_{pbh} \gtrsim S_{hor}$$

$$\Rightarrow T \lesssim T_c$$

$$T_c = \frac{0.36}{\sqrt{g_*}} m_{pl}$$



$$M_{pbh} = M_{hor} \gtrsim 0.29\sqrt{g_*}\, m_{pl}$$

HKL, PRD 66, 063001(02)

- Classical vs Quantum ?

  Compton wavelength of PBH at Tc:

$$\lambda_{pbh} = \frac{1}{2.4\sqrt{g_*}} \lambda_{hor}$$

$$\lambda_{pbh} < \lambda_{hor} \rightsquigarrow \text{classical}$$

- new of freedom, over density: $g_* \to \tilde{g}_*$

$$M_{pbh} > 3\kappa\, m_{pl}, \quad \kappa = \sqrt{\frac{\tilde{g}}{g_*}}$$

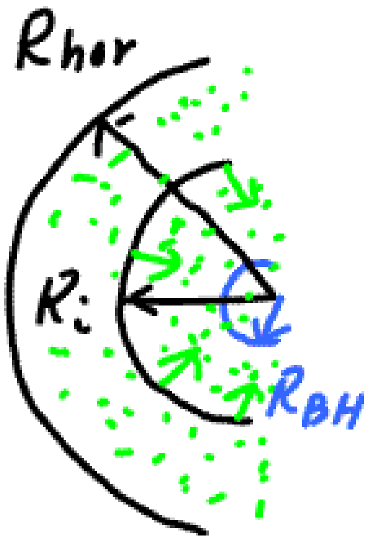
$$T_c < \frac{1.2\times10^{-2}}{\kappa} m_{pl}$$

$$\lambda_{pbh} < \frac{1.4\times10^{-2}}{\kappa} \lambda_{hor}$$

- Sub-horizon PBH formation

J.Chrisholm, PRD 74, 043512(06),

T.Harada, PRD 74,084004(06)



-initial size, Ri < horizon

-no heat production

$$M_{pbh} \Rightarrow f M_{hor} \quad (f < 1)$$

$$M_{pbh} \gtrsim \frac{4}{27}\sqrt{\frac{g_*}{5\pi}}\, m_{pl}$$

→ minimum mass of PBH ~ $O(m_{pl})$

* $\lambda_{pbh} \sim \lambda_{hor}$:

Quantum effect may be important.

Summary

- 2^{nd} law of thermodynamics on spherical collapse of a overdense region constrains PBH formation only for T < Tc ($\sim 10^{-2}\, m_{pl}$).
- Sub-horizon collapse also leads to minimum mass for PBH.
- Possible variations are due to the overdensity, additional degree of freedom, which gives lower Tc and higher minimum PBH mass.
- Underlying dynamics for the thermodynamical constraint to be investigated: Quantum Gravity?