Non-adiabatic perturbations in DBI cosmology

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Based on hep-th/0792666 with Gregory, Zavala, Easson, Mota
and work in progress

Outline

- Motivations: why DBI cosmology is interesting?
  - Single field case: acceleration via brane D-cceleration.
  - Two field case: more possibilities for the brane trajectories.
- Gaussian cosmological perturbations
  - Adiabatic modes: crucial dependence on the sound speed.
  - Non-adiabatic modes: how do they seed curvature perturbations?
- Conclusions

Introduction

Why DBI cosmology is interesting?

- Possibility to obtain inflation also with generic potentials (no slow roll conditions).
- The idea is to use the non-standard form of DBI kinetic tems.

Then ten d metric is

$$ds^2_{10} = h^{-1/2}(\eta)\, g_{\mu\nu} dx^\mu dx^\nu + h^{1/2}(\eta)\, g_{mn} dy^m dy^n \quad ; \quad h(\eta) = \frac{\lambda}{\eta^4}$$

The four dimensional effective action is (here $\phi = \sqrt{T_3}\eta$)

$$\frac{M_{Pl}^2}{2} \int d^4x \sqrt{-g}\, R -$$

$$- \; g_s^{-1} \int d^4x \sqrt{-g} \left[h^{-1} \sqrt{1 - h\, g_{mn}\, \dot{\phi}^m \dot{\phi}^n} - h^{-1} + V(\phi^m) \right].$$

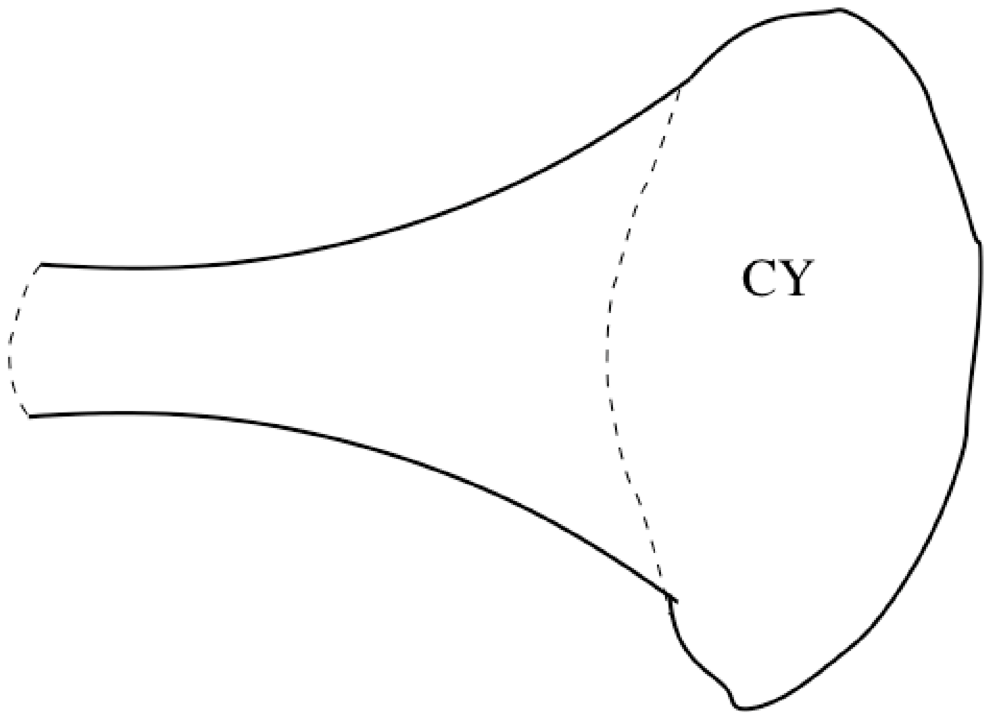
The fields that drive inflation have then non-canonical kinetic terms

$$\frac{1}{h}\left[\sqrt{1 - h\, g_{mn}\, \dot{\phi}^m \dot{\phi}^n} - 1\right] = \frac{1}{2} g_{mn}\, \dot{\phi}^m \dot{\phi}^n + \text{corrections}$$

- Implies a velocity limit: the speed of the brane is less than $1/h$.
- At the tip of the throat, where h becomes large, the brane must move very slowly.

$$\Downarrow$$

The potential term dominates and inflation can occurr!



The homogeneous solution

Silverstein and Tong, in the single field case, found an interesting inflationary trajectory

- They took a potential $V = m^2\phi^2$
- The EOMs for the system admit the following solution

$$H = \frac{1}{\epsilon t} + \dots\,, \quad \phi = \frac{\sqrt{\lambda}}{t} - \frac{\sqrt{\lambda}}{6\,t^5} + \dots\,, \quad a(t) = a_0 t^{\frac{1}{\epsilon}} + \dots$$

$$m^2 = \frac{9\lambda}{4} - 1\,, \quad \epsilon = \frac{2}{3\lambda}$$

- In order to have inflation, m must be large: a steep potential!
- Example of power-like inflation, with variable speed of sound

$$c_s = \frac{1}{\gamma} \quad , \quad \gamma = \frac{1}{\sqrt{1 - hv^2}}$$

Gaussian perturbations

The previous system looks promising for obtaining enough inflation

$$N = \frac{1}{\epsilon} \log \frac{\phi_{in}}{\phi_{fin}}$$

What about the behavior of cosmological perturbations?

[Garriga, Mukhanov]

- Consider perturbations for the inflaton field ϕ, and the metric.
- Define gauge invariant curvature perturbations ξ and ζ:

$$\dot{\xi} = \frac{a\,\dot{\phi}^2}{c_S\,H^2}\,\zeta\,,$$
$$\dot{\zeta} = \frac{H^2\,c_S^3}{\dot{\phi}^2\,a^3}\,(\Delta\xi)$$

- Define $v = a\zeta$, expand in Fourier modes $v = v_k e^{ikx}$. The equation to solve is

$$\frac{d^2 v_k}{d\tau^2} + \left(c_S^2 k^2 - \frac{\nu^2 - \frac{1}{4}}{\tau^2}\right) v_k = 0 \quad , \quad \nu^2 = \frac{2+5\epsilon}{(-1+\epsilon)^2} + \frac{1}{4}$$

- The power spectrum for scalar perturbations is nearly scale invariant:

$$n_s - 1 \simeq \mathcal{O}(\epsilon^2)$$

- Non-gaussianities instead are non-negligible. They are induced by the time-varying speed of sound. They impose an upper bound on γ, that translates on

$$\epsilon \geq \frac{1}{10}\left(\frac{M_p^2 g_s}{\phi^2}\right)$$

Question

What about considering the dynamics along the other coordinates?

Motion along the angular coordinates

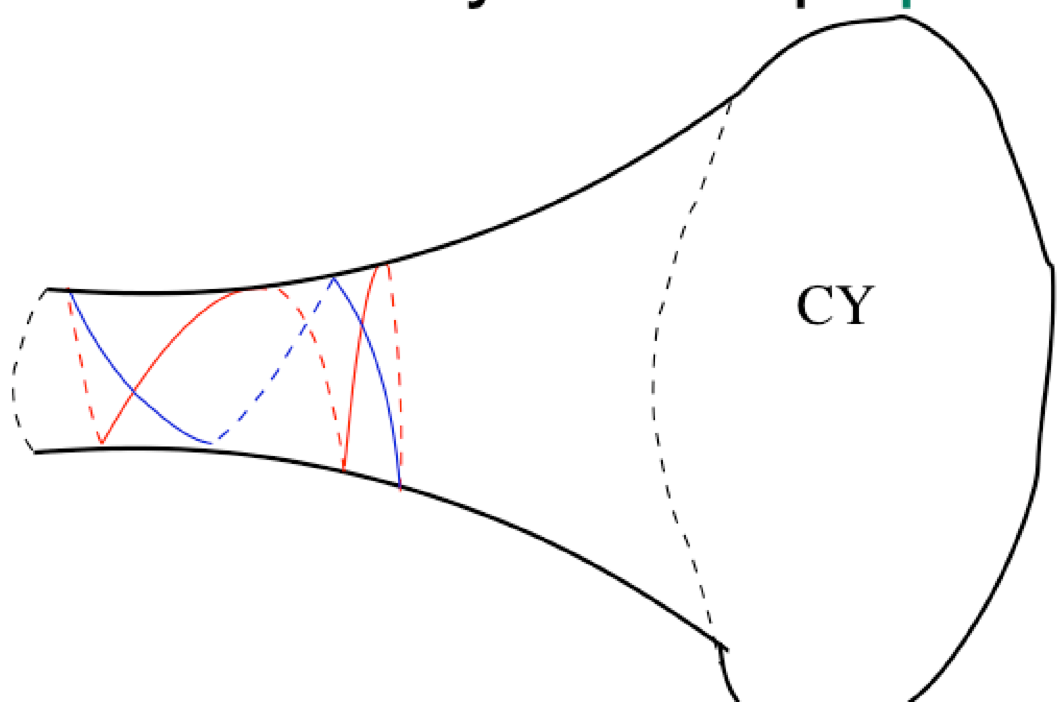
- A couple of observations
  - A trajectory where the brane moves also along the angular directions is more generic!
  - One may hope to modify the behavior of perturbations, in an useful way.

- Suppose we allow for motion on the brane along one angular direction, and angular momentum is conserved.

$$v^2 = \dot{\phi}^2 + \phi^2\dot{\theta}^2 \quad \Rightarrow \quad \frac{d}{dt}\left[a^3\phi^2\gamma\dot{\theta}\right] = 0 \quad \Rightarrow \quad \ell = a^3\phi^2\gamma\dot{\theta}$$

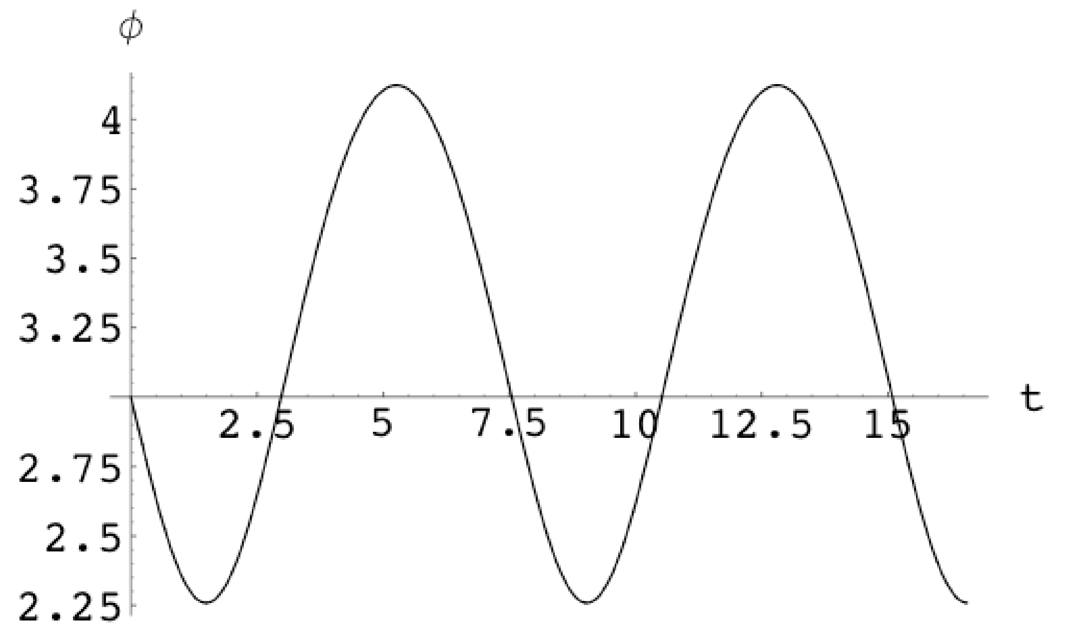
Angular momentum
is conserved

- The brane trajectories may develop qualitatively new features.

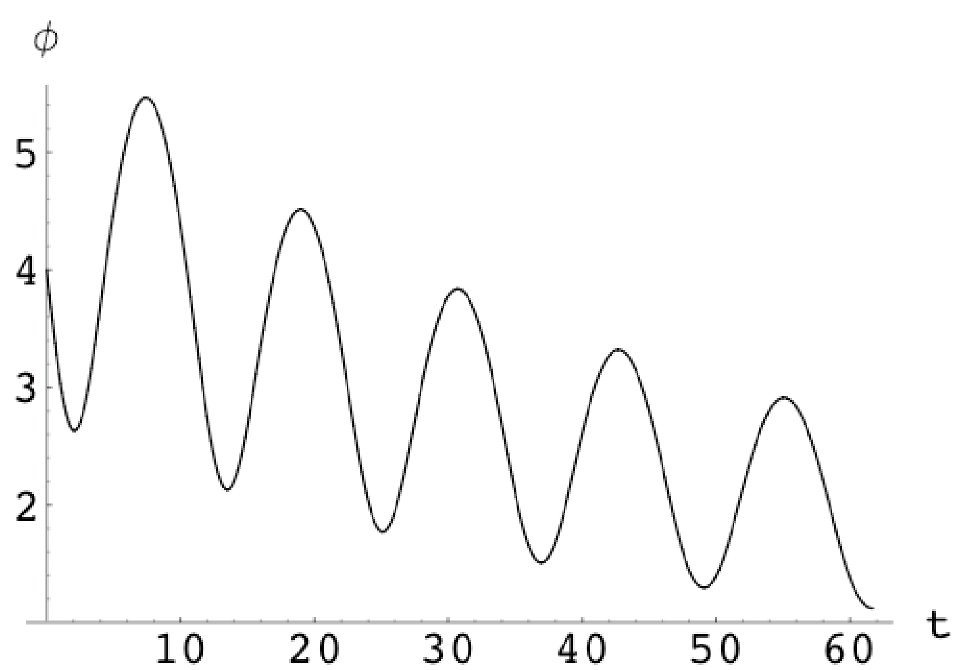


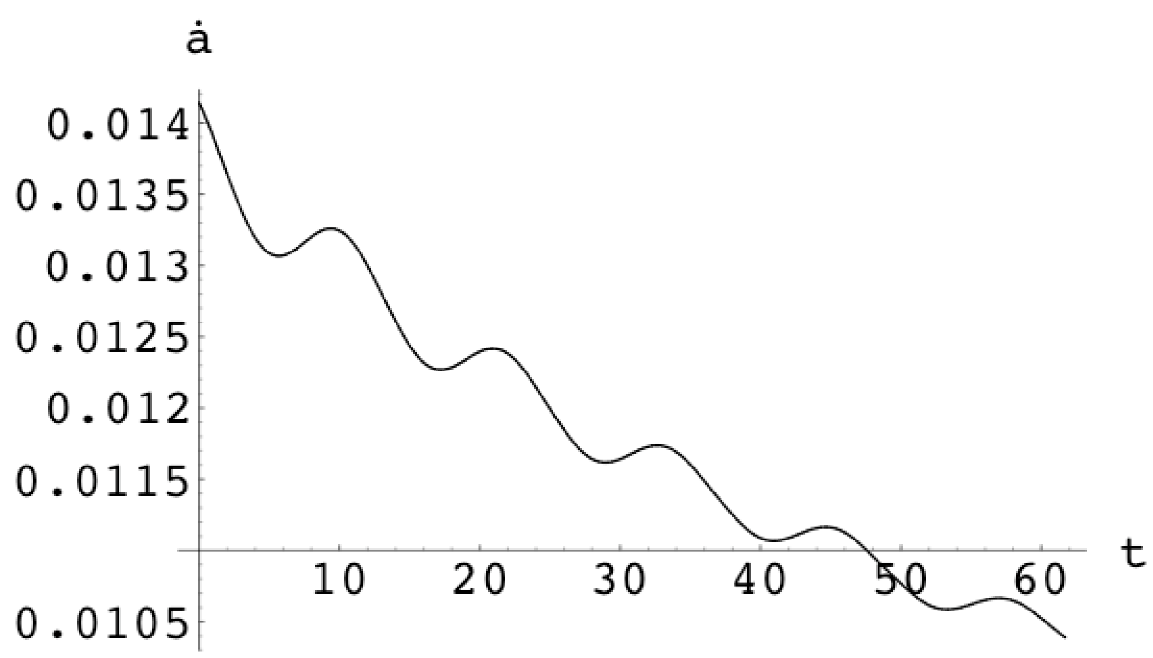
- Examples of possible cosmologies

Brane trajectory
with no gravity



Brane position and scale factor with gravity switched on .





- The system has at least two nice features
  - Induced cosmology gets short kicks of acceleration at the turning points.
  - Between the brane bounces, cosmology can undergo longer periods of inflation.

    $\Rightarrow$ The γ factor does not necessarily become large.

What happens at the cosmological perturbations?

Cosmological perturbations

- Since the brane moves along different directions, various fields can contribute to the evolution of perturbations .

- At the homogeneous level, it is convenient to define the angle

$$\cos\alpha = \frac{\dot{\phi}}{\sqrt{2X}} \quad , \quad \sin\alpha = \frac{\phi\dot{\theta}}{\sqrt{2X}}$$

And redefine the coordinates introducing an averaged trajectory field σ,

$$d\,\sigma \;=\; \cos\alpha\, d\phi + \phi \sin\alpha\, d\theta$$

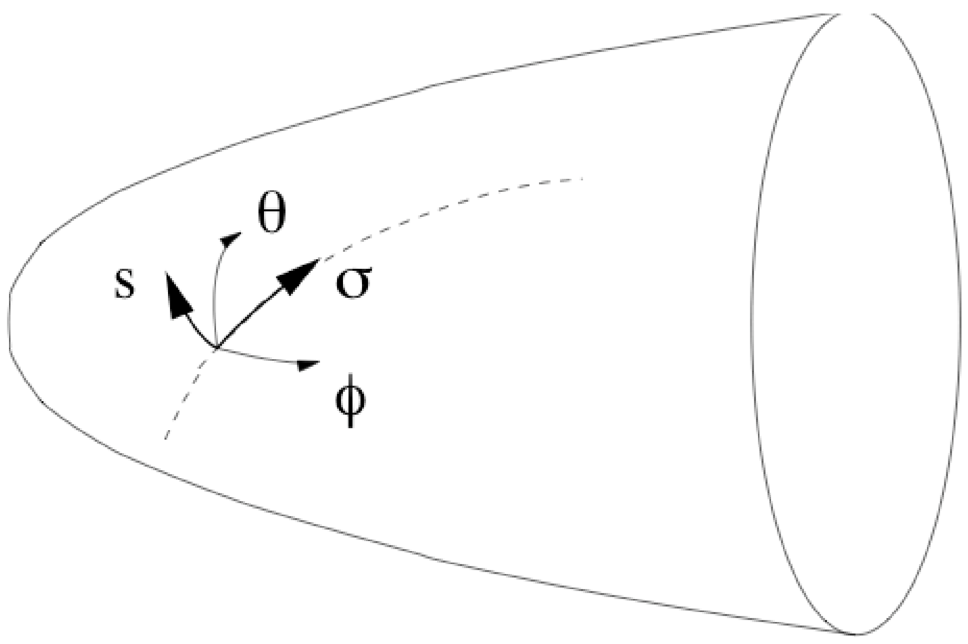
and an orthogonal, entropy field s:

$$d\,s \;=\; \phi\cos\alpha\, d\theta - \sin\alpha\, d\phi$$

Then one has

$$\dot{\sigma}^2 \;=\; v^2 \quad , \quad \dot{s} \;=\; 0$$

$$\sin\alpha \;=\; \frac{\ell}{a^3\,\phi\,\gamma\,\dot{\sigma}}$$



- The perturbations are consequently defined by

$$ds^2 = -(1+2\Phi)\, dt^2 + (1-2\Phi)\, a^2(t)\gamma_{ij}dx^i dx^j$$

$$\frac{\delta\sigma}{\dot\sigma} = \cos^2\alpha \left(\frac{\delta\phi}{\dot\phi}\right) + \sin^2\alpha \left(\frac{\delta\theta}{\dot\theta}\right) \quad , \quad \frac{\delta s}{\dot\sigma\, \sin\alpha\cos\alpha} = \frac{\delta\theta}{\dot\theta} - \frac{\delta\phi}{\dot\phi}$$

or, in a gauge invariant way,

$$\Phi a \equiv 4\pi G H \xi \quad , \quad \frac{\delta\sigma}{\dot\sigma} \equiv \frac{\zeta}{H} - \left(\frac{4\pi G}{a}\right)\xi\,.$$

where ξ corresponds to a (re-weighted) Newtonian potential while ζ is the curvature perturbation.

Another useful gauge-invariant quantity is $\delta\sigma_\Phi \equiv \frac{\dot\sigma}{H}\,\zeta$

Equations of motion for the perturbations

The equations of motion for the cosmological perturbations result

$$\dot{\xi} \;=\; \frac{a\,(E+P)}{H^2}\zeta\;,$$

$$\begin{aligned}(E+P)\,\frac{\dot{\zeta}}{H} \;&=\; \frac{H\,c_S^2}{a^3}\,(\Delta\xi) \;-\; \tan\alpha\,\left(\dot{P}-c_S^2\dot{E}\right)\,\frac{\delta s}{\dot{\sigma}}\\ &=\; \frac{H\,c_S^2}{a^3}\,(\Delta\xi) \;-\; \sin\alpha\,\left(V_{,\phi}\,(1+c_S^2)+\frac{h_{,\phi}}{h^2}\,(1-c_S)^2\right)\,\delta s\end{aligned}$$

Important properties

- The coupling between curvature and entropy modes vanishes for $\alpha \to 0$
- It does not vanish even when the potential V is zero.
  - It is sensitive to the curved background: h is non trivial.
  - The speed of sound is $c_S \neq 1$.

It is convenient to pass to second order equations

$$\delta\ddot{\sigma}_\Phi + \left(3H + 3\frac{\dot{\gamma}}{\gamma}\right)\delta\dot{\sigma}_\Phi + \left(U_{\sigma_\Phi} + c_s^2\frac{k^2}{a^2}\right)\delta\sigma_\Phi = -\left(\frac{H\,\dot{\sigma}c_S^2}{a^3\,(E+P)}\right)\left[\frac{a^3\tan\alpha}{Hc_S^2}\left(\dot{P} - c_S^2\dot{E}\right)\frac{\delta s}{\dot{\sigma}}\right]^{\cdot}$$

where

$$U_{\sigma_\Phi} \equiv \frac{\dot{\sigma}H^2c_S^2}{a^3\,(E+P)}\left[\left(\frac{\dot{H}}{H} - \frac{\ddot{\sigma}}{\dot{\sigma}}\right)\frac{a^3(E+P)}{\dot{\sigma}H^2c_S^2}\right]^{\cdot}$$

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and

$$\delta\ddot{s} \;+\; \left(3H + \frac{\dot{\gamma}}{\gamma}\right)\delta\dot{s} \;+\; \left(U_s + \frac{k^2}{a^2}\right)\delta s \;=\; -\,\frac{k^2}{a^2}\,\frac{\dot{\sigma}\;\tan\alpha\,H}{a\,(E+P)^2}\,\left(\dot{P} - c_S^2\dot{E}\right)\,\xi$$

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where

$$U_s = \tan\alpha\left[3H\tan\alpha\left(\frac{\ddot{\sigma}}{\dot{\sigma}} - \frac{\dot{\alpha}}{\tan\alpha} + 3Hc_S^2 - \cos\alpha\frac{\dot{\sigma}}{\phi}\right) + \dot{\sigma}\left(\frac{\dot{\alpha}}{\dot{\sigma}} - \frac{\cos\alpha}{\phi\,\tan\alpha}\right)^{\cdot}\right]$$

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Important properties

- Entropy perturbations evolve independently of curvature perturbations at large scales: $k^2/a^2 \ll 1$.
- Different perturbations propagate with different speeds. Important effects for the evolution of the perturbations.

It is convenient to pass to second order equations

$$\delta\ddot{\sigma}_\Phi+\left(3H+3\frac{\dot{\gamma}}{\gamma}\right)\delta\dot{\sigma}_\Phi+\left(U_{\sigma_\Phi}+c_s^2\,\frac{k^2}{a^2}\right)\delta\sigma_\Phi\;=\;-\left(\frac{H\,\dot{\sigma}c_S^2}{a^3\,(E+P)}\right)\left[\frac{a^3\tan\alpha}{Hc_S^2}\left(\dot{P}-c_S^2\dot{E}\right)\,\frac{\delta s}{\dot{\sigma}}\right]^{\bullet}$$

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Does it have effects for inflation?

- If the angular moment is conserver, α decays fast: $\sin\alpha \;=\; \frac{l\,c_S}{a^3\phi\dot{\sigma}}$
  - Angular motion doesn't affect inflation: [Branonium, by Burgess et al.]
- We must consider more general trajectories.

Conclusions

- We have explored features of DBI cosmology in multifield case
  - Motivated by brane motion in a warped throat
- Angular motion affects cosmological trajectories
  - Expanding universes with short kicks of acceleration.
  - To have inflation, need to renounce to conserved l.
- Outlined the formalism to treat cosmological perturbations
  - Non-standard features of non-adiabatic vs adiabatic modes
  - Question: do these observations affect non-gaussian features?