Neutrinos and cosmology

1) Baryogenesis

- ABC of cosmology
- Baryogenesis
- Baryogenesis in the SM

2) Neutrino masses

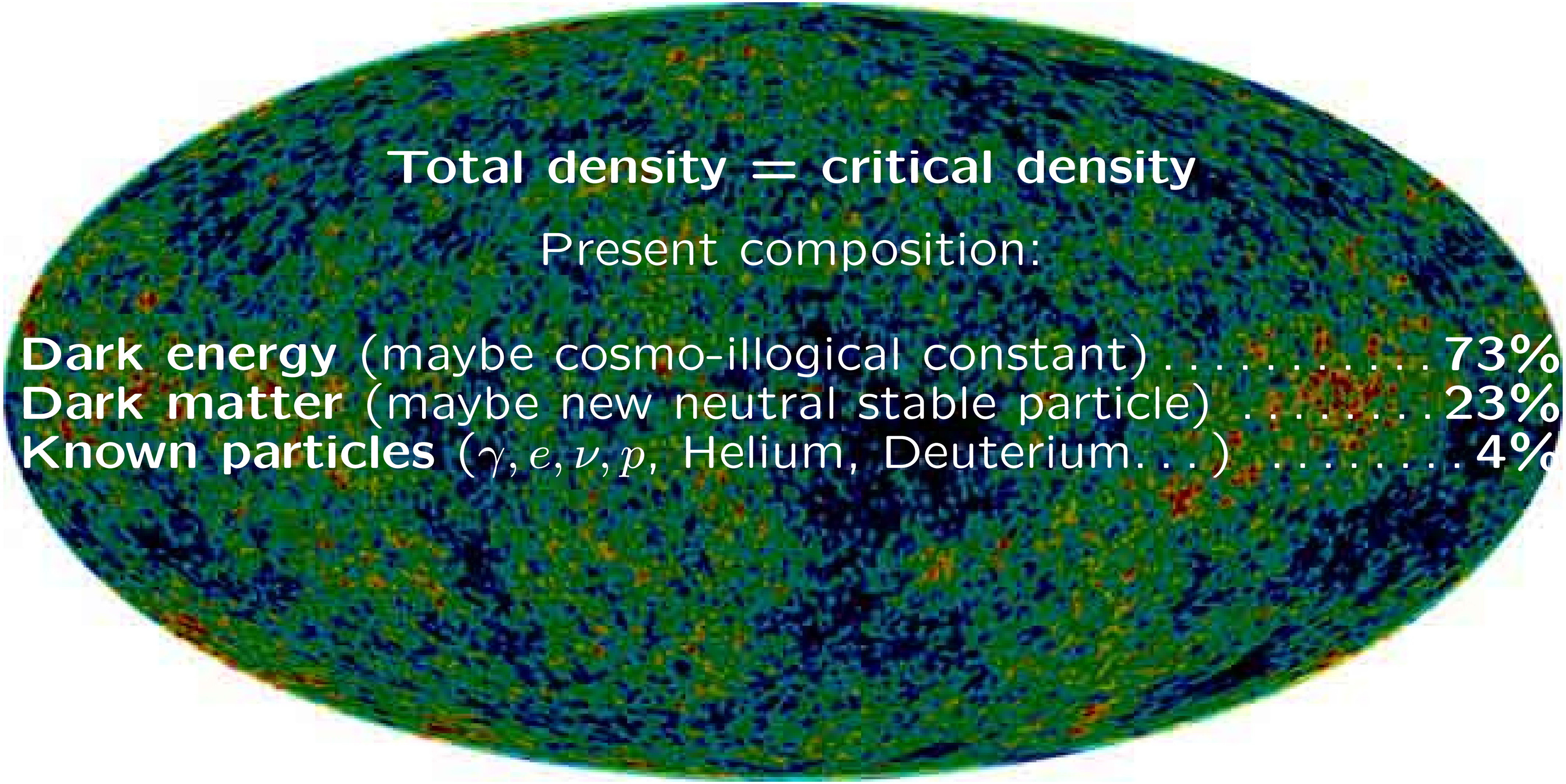
- Neutrino masses
- Leptogenesis: estimates

$1 + 2 = 3$) Leptogenesis

- Leptogenesis: precise computation
- Testing leptogenesis?

Alessandro Strumia, UniverseNet, Mytylene 2007.

Inventory



Total density = critical density

Present composition:

Dark energy (maybe cosmo-illogical constant) **73%**
Dark matter (maybe new neutral stable particle) **23%**
Known particles (γ, e, ν, p, Helium, Deuterium. . .) **4%**

Inflation explains $\rho = \rho_{\text{cr}}$. Big-bang explains $n_e = n_p$, $n_{^4\text{He}}/n_p \approx 0.25/4$, $n_{\text{D}}/n_p \approx 3\ 10^{-5}/2$, $n_{\nu_i} \stackrel{?}{=} n_{\bar{\nu}_i} \stackrel{?}{=} 3n_\gamma/22$,. . . We do not understand DM, n_B/n_γ.

Big bang: $H \sim T^2/M_{\rm Pl}$

Homogeneous $\rho(t)$ expands according to Newton acceleration

$$\ddot{R} = -\frac{GM(r<R)}{R^2} = -\frac{4\pi G\rho(t)}{3}R$$

Get 'energy constant' k assuming non-relativistic matter: $\rho(t) \propto 1/R^3(t)$:

$$\frac{d}{dt}\left[\frac{1}{2}\dot{R}^2 - \frac{4\pi}{3}G\rho R^2\right] = 0 \qquad H^2 \equiv \frac{\dot{R}^2}{R^2} = \frac{8\pi G}{3}\rho - \frac{k}{R^2}$$

Critical case $k=0$: needs $\rho = 3H^2/8\pi G \equiv \rho_{\rm cr}$ and expands for free. Valid for all ρ in general relativity, where k is curvature; inflation smoothes $k \to 0$.

Matter in thermal equilibrium at temperature $T \gg m$ has density

$$n_{\rm eq} \sim T^3 \qquad \rho_{\rm eq} \sim T^4$$

one particle with energy $\sim T$ per de-Broglie wavelength $\sim 1/T$.
Non relativistic particles are Boltzmann-suppressed: $n_{\rm eq} \sim e^{-m/T}(mT)^{3/2}$.

PS: in units $\hbar = c = 1$ $G = 1/M_{\rm Pl}^2$ with $M_{\rm Pl} \sim 10^{19}$ GeV.

Dark matter as thermal relic

What happens to a stable particle at $T < m$?

Scatterings try to give thermal equilibrium

$$n_{\rm DM} \propto \exp(-m/T).$$

But at $T \lesssim m$ they become too slow:

$$\Gamma \sim \langle n_{\rm DM}\sigma\rangle \lesssim H \sim T^2/M_{\rm Pl}$$

Out-of-equilibrium relic abundance:

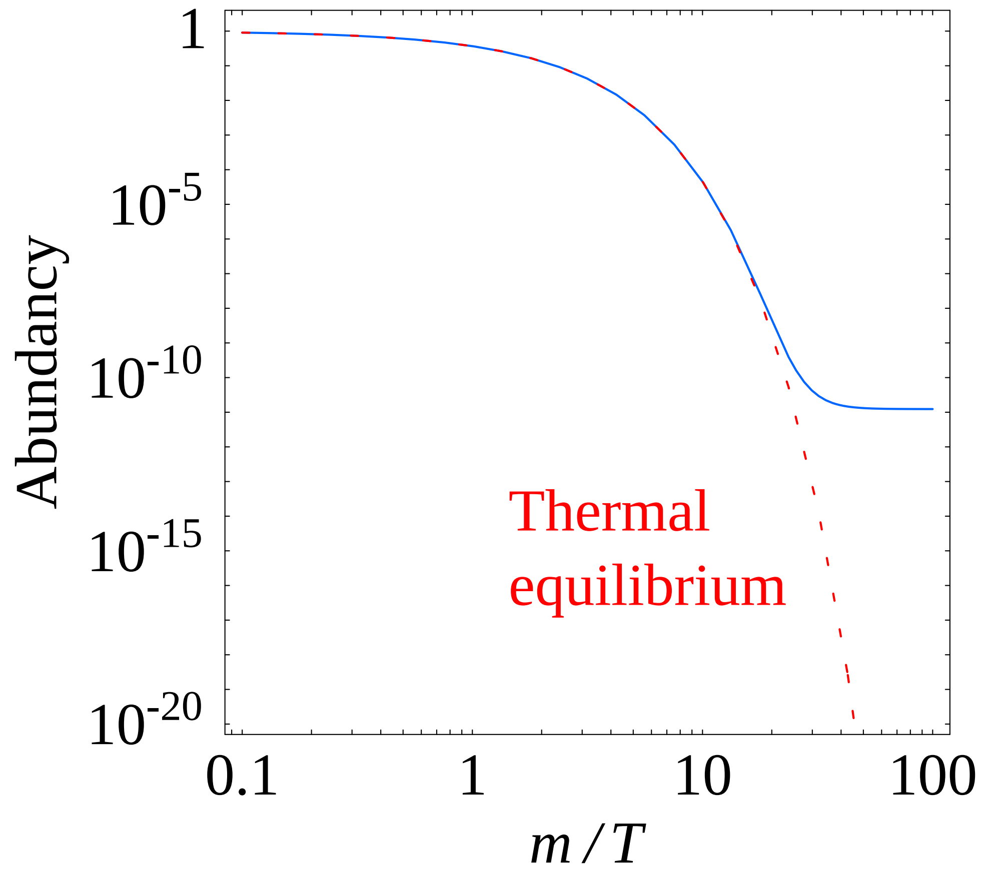
$$\frac{n_{\rm DM}}{n_\gamma} \sim \frac{T^2/M_{\rm Pl}\sigma}{T^3} \sim \frac{1}{M_{\rm Pl}\sigma m}$$

$$\frac{\rho_{\rm DM}}{\rho_\gamma} \sim \frac{m}{T_{\rm now}}\frac{n_{\rm DM}}{n_\gamma} \sim \frac{1}{M_{\rm Pl}\sigma T_{\rm now}}$$

Inserting $\rho_{\rm DM} \sim \rho_\gamma$ and $\sigma \sim g^2/m^2$ fixes

$$m/g \sim \sqrt{T_{\rm now} M_{\rm Pl}} \sim \text{TeV}$$

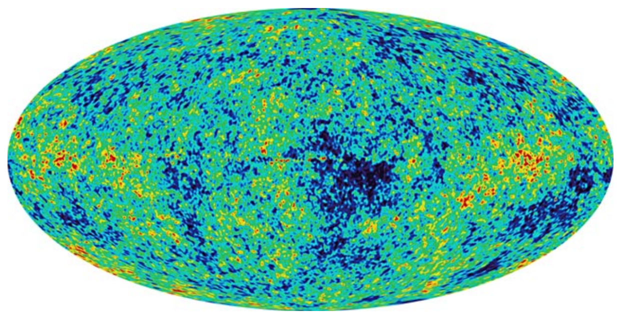
Testable: LHC + direct + indirect



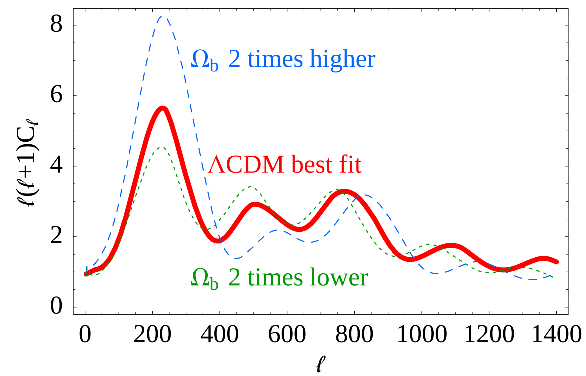
Measuring $n_B/n_\gamma = 6 \cdot 10^{-10}$

$T_{\rm now} \approx 3^\circ$K directly tells $n_\gamma \sim T^3_{\rm now} \approx 400/\,$cm^3. $n_B \sim 1/\,$m^3 follows from

(1) Anisotropies in the cosmic microwave background: $n_B/n_\gamma = (6.3 \pm 0.3)\ 10^{-10}$.



$\leftrightarrow$

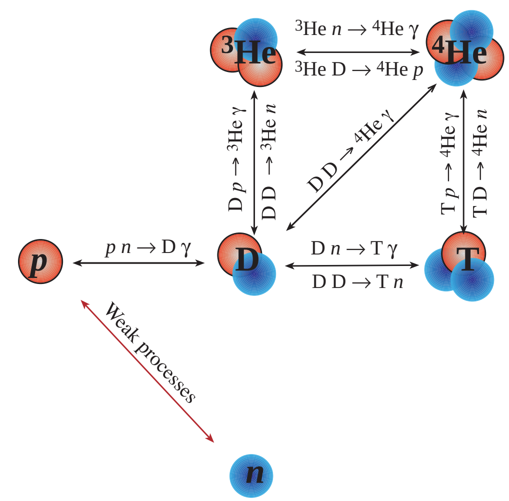


(2) Big Bang Nucleosynthesis: the D abundance implies

$$n_B/n_\gamma = (6.1 \pm 0.5) 10^{-10}$$

because many γ push in the $\leftarrow$ direction reactions like

proton neutron $\leftrightarrow$ Deuterium γ



(3) Less precise direct counts: only 10% of baryons are luminous (stars...).

(1) and (2) are indirect but different: their agreement makes the result trustable

Baryogenesis

Baryogenesis

$n_B/n_\gamma \sim 6\ 10^{-10}$ is a strange number, because means that when the universe cooled below $T \approx m_p$ we survived to nucleon/antinucleon annihilations as

$$1000000001\ \frac{\text{protons}}{\text{pico-m}^3} \qquad - \qquad 1000000000\ \frac{\text{anti-protons}}{\text{pico-m}^3}$$

(Proton freeze-out gives $n_p/n_\gamma = n_{\bar{p}}/n_\gamma \sim 1/M_{\text{Pl}}\sigma m_p \sim m_p/M_{\text{Pl}} \sim 10^{-18}$)

Might be the initial condition, but suspiciously small or large (in inflation).

Can a $p/\bar{p}$ asymmetry can be generated dynamically from nothing?

Yes, if 3 trivial Sacharov conditions are satisfied
(his big achivement was realizing that it is an interesting question).

1. Baryon number B is violated
2. C and CP are violated
   (otherwise p and $\bar{p}$ behave in the same way)
3. At some epoch the universe went out of equilibrium
   (CPT implies $m_p = m_{\bar{p}}$ so that in thermal equilibrium $n_p = n_{\bar{p}}$)

In the Standard Model

A lot of non trivial works showed that the SM does not satisfy the 3 conditions. The naïve answer seems no, yes, yes. The true answer is yes, no, no:

1. B is 'anomalous' and violated by thermal tunneling ('sphalerons' which conserve $B-L$) at rate faster than universe expansion if $100\,\text{GeV} \lesssim T \lesssim 10^{12}\,\text{GeV}$.

2. CP is violated, but not enough.
   CP violation vanishes if some quark were massless, and many are light.

3. No out-of-equilibrium.
   EW phase transition is smooth for $m_{\text{Higgs}} > 70\,\text{GeV}$, as demanded by data.

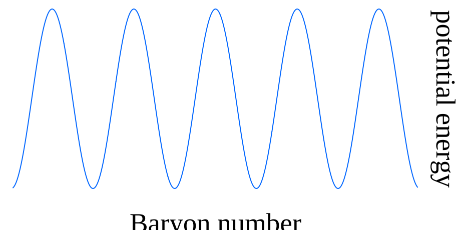
New physics is needed. Rules of the game changed: sphalerons requilibrate $n_p - n_{\bar{p}}$ unless $B - L$ is violated.

If something generates a Lepton asymmetry, sphalerons extend it into a Baryon asymmetry.

What sphalerons are?

Physics is AQFT (A = Advanced: no intuitive explanation): anomalies combined with $\mathrm{SU}(2)_L$ extended field configurations.

$$\partial_\mu J^B_\mu \sim n_F F^a_{\mu\nu}\tilde{F}^a_{\mu\nu} \sim$$



Quantum tunneling would give $\tau(p \to \pi^0 \bar{e}) \sim \dfrac{e^{4\pi/\alpha_2}}{m_p} \sim 10^{140}\,\mathrm{yr}$. But all $n_F = 3$ generations must be involved, like an $u_L u_L d_L e_L\ c_L c_L s_L \mu_L\ t_L t_L b_L \tau_L$ operator. So no p decay: only negligible tritium decay.

One observed effect due to analogous $\mathrm{SU}(3)_c$ effects: η' mass.

So don't doubt that sphalerons really exist, and this is all what one needs to know to understand leptogenesis quantitatively.

A few candidates

GUT baryogenesis. GUT baryogenesis after preheating. Baryogenesis from primordial black holes. String scale baryogenesis. Affleck-Dine (AD) baryogenesis. Hybridized AD baryogenesis. No-scale AD baryogenesis. Single field baryogenesis. Electroweak (EW) baryogenesis. Local EW baryogenesis. Non-local EW baryogenesis. EW baryogenesis at preheating. SUSY EW baryogenesis. String mediated EW baryogenesis. Baryogenesis via leptogenesis. Inflationary baryogenesis. Resonant baryogenesis. Spontaneous baryogenesis. Coherent baryogenesis. Gravitational baryogenesis. Defect mediated baryogenesis. Baryogenesis from long cosmic strings. Baryogenesis from short cosmic strings. Baryogenesis from collapsing loops. Baryogenesis through collapse of vortons. Baryogenesis through axion domain walls. Baryogenesis through QCD domain walls. Baryogenesis through unstable domain walls. Baryogenesis from classical force. Baryogenesis from electrogenesis. B-ball baryogenesis. Baryogenesis from CPT breaking. Baryogenesis through quantum gravity. Baryogenesis via neutrino oscillations. Monopole baryogenesis. Axino induced baryogenesis. Gravitino induced baryogenesis. Radion induced baryogenesis. Baryogenesis in large extra dimensions. Baryogenesis by brane collision. Baryogenesis via density fluctuations. Baryogenesis from hadronic jets. Baryogenesis from Q-

Rough classification

I Theoretical perversions.

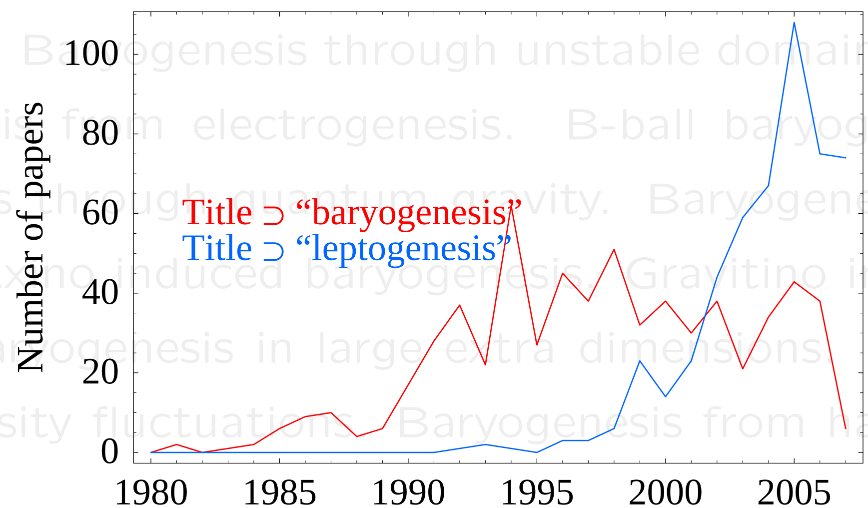
II Therapeutical accaniment. (Was plausible, now disfavoured).

III Plausible meta-physics. (It seems impossible to test).

This field is hard, because n_B/n_γ (1 number) is all experimental data, while theories contain more parameters. Lepton asymmetries and n_B anisotropies would be more data, but practically they cannot be measured.

Hope: link the theory of baryogenesis to other physics.

Leptogenesis is very plausible and links to neutrino masses.
Despite this, it seems type III.



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Neutrino masses

Neutrinos in the SM

Most generic renormalizable $\mathscr{L}$ built with SM fields: B, L_e, L_μ, L_τ are automatically conserved: p is stable, $\mu \not\to e\gamma$, ν are massless and fully described by

$$\bar{L} \not{D} L$$

$$(\bar{\nu}\not{\partial}\nu + \bar{\nu} Z \nu + \bar{\nu} W \ell_L)$$

Neutrino experiments discovered that **lepton flavour is violated**

What we surely know today?

Two direct evidences for violation of lepton flavour.

Anomaly	Solar	Atmospheric
first hint	1968	1986
confirmed	2002	1998
evidence	12σ	17σ
for	$\nu_e \to \nu_{\mu,\tau}$	$\nu_\mu \to \nu_\tau$
seen by	Cl,2Ga,SK,SNO,KL	SK,Macro,K2K,Minos
disappearance	seen	seen
appearance	seen	partly seen
oscillations	almost seen	almost seen
$\sin^2 2\theta$	0.85 ± 0.03	1.02 ± 0.04
Δm^2	$(8.0 \pm 0.3)10^{-5}\,\text{eV}^2$	$(2.5 \pm 0.3)10^{-3}\,\text{eV}^2$
sterile?	6σ disfavoured	7σ disfavoured

"a piece of 20th century physics that fell by chance into the 21th century"

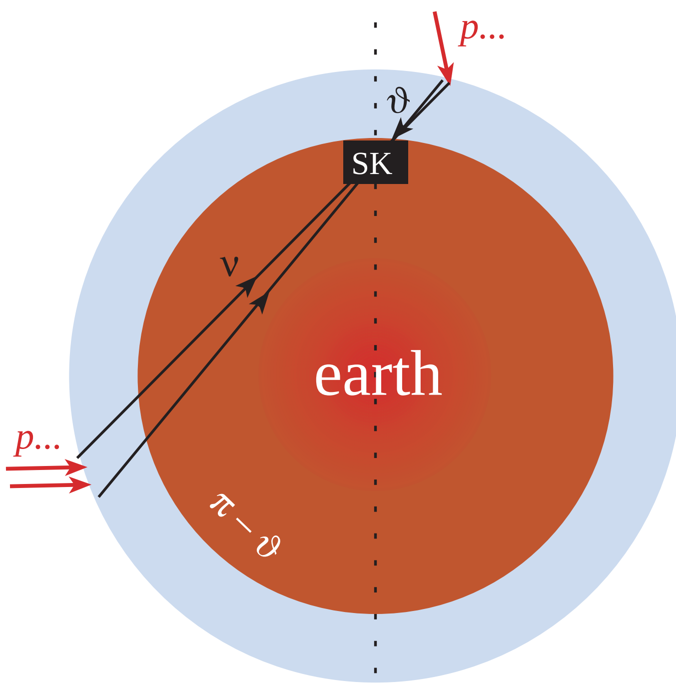
The atmospheric anomaly

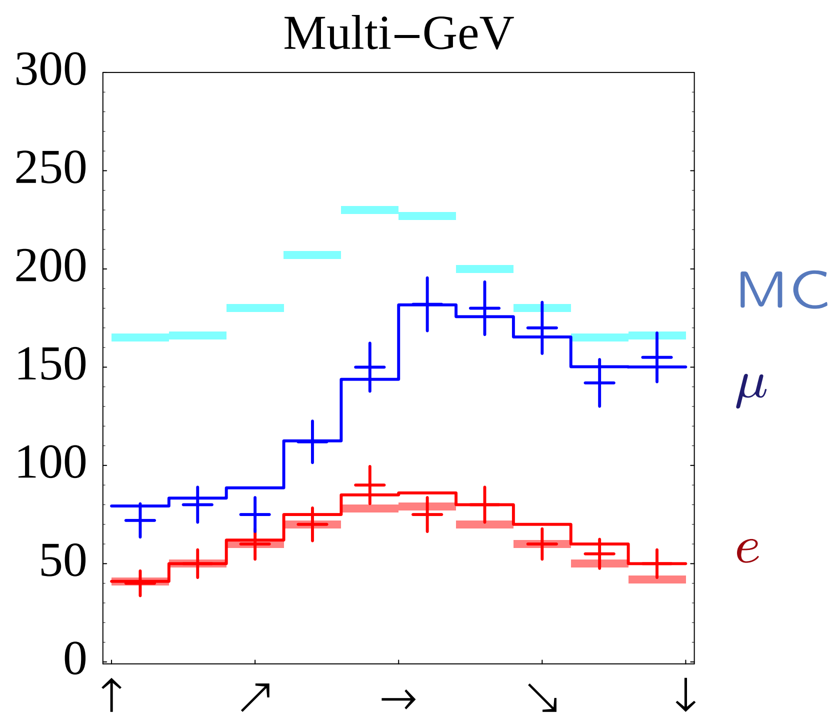
The atmospheric anomaly

SK detects $\nu_\ell N \to \ell N$ distinguishing μ from e. In the multi-GeV sample

$$E_\ell \lesssim E_\nu \sim 3\,\text{GeV}, \qquad \vartheta_\ell \sim \vartheta_\nu \pm 10^\circ$$

Without oscillations $N(\cos\vartheta_{\text{zenith}})$ is up/down symmetric





No doubt that there is an anomaly

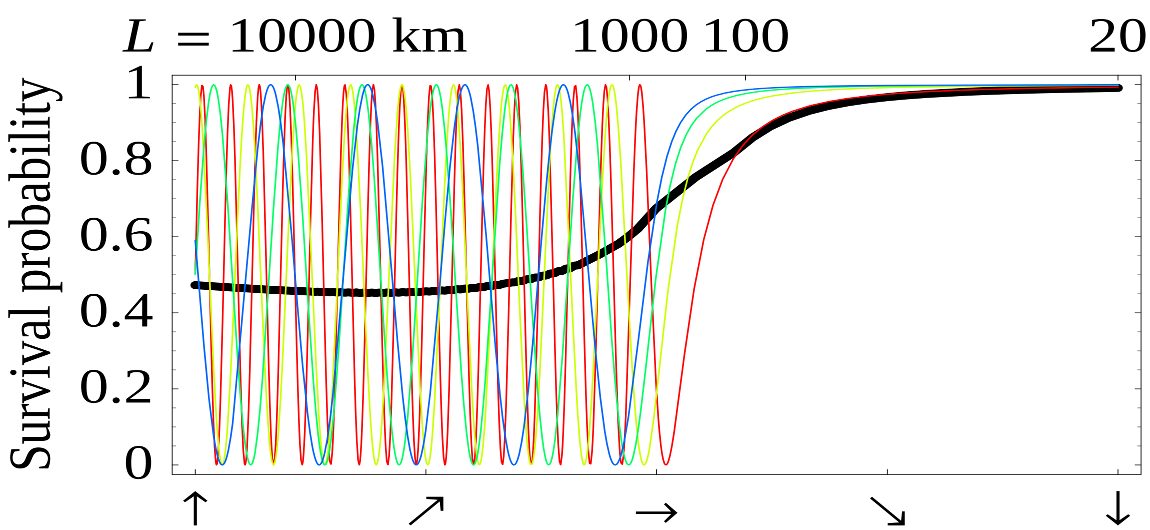
Atmospheric oscillations?

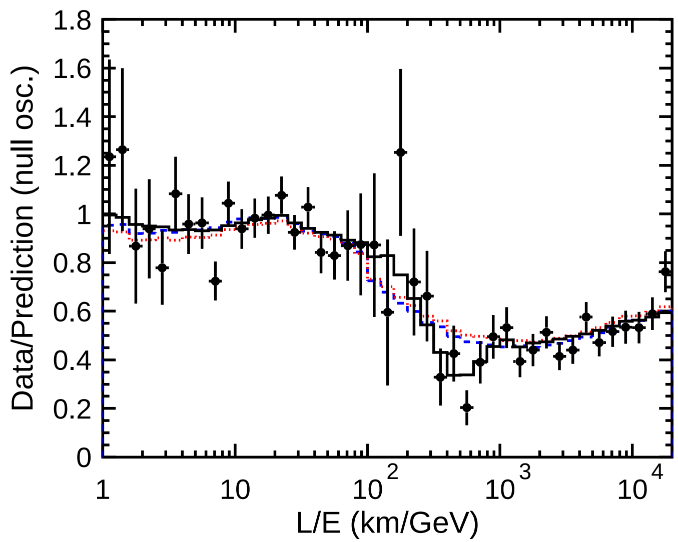
$$P_{ee} = 1 \qquad P_{e\mu} = 0 \qquad P_{\mu\mu} = 1 - \sin^2 2\theta_{\text{atm}} \ \sin^2 \frac{\Delta m^2_{\text{atm}} L}{4E_\nu}$$

- $\sin^2 2\theta_{\text{atm}} = 2 - 2\frac{N_\uparrow}{N_\downarrow} = 1 \pm 0.1$ i.e. $\theta_{\text{atm}} \sim 45$
- oscillatations start 'horizontal', $L \sim 1000\,\text{km}$: $\Delta m^2_{\text{atm}} \sim \frac{E_\nu}{L} \sim 3\ 10^{-3}\,\text{eV}^2$

$P_{\mu\mu}(E_\nu)$: the anomaly disappears at high energy, as predicted by oscillatons.

$P_{\mu\mu}(L)$: at SK $\sigma_{E_\nu} \sim E_\nu$: **oscillation dip** averaged out (ν_μ decay, decoeherence disfavoured at 4σ). Restricting to cleanest events, SK sees a hint





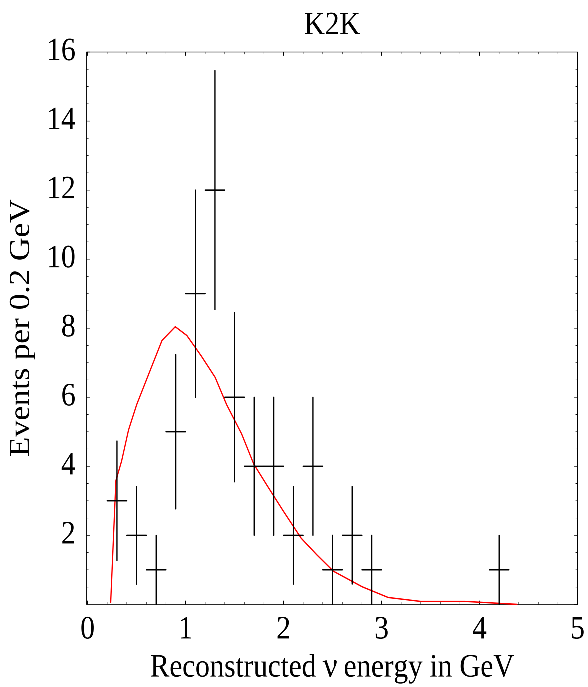
K2K and NuMi

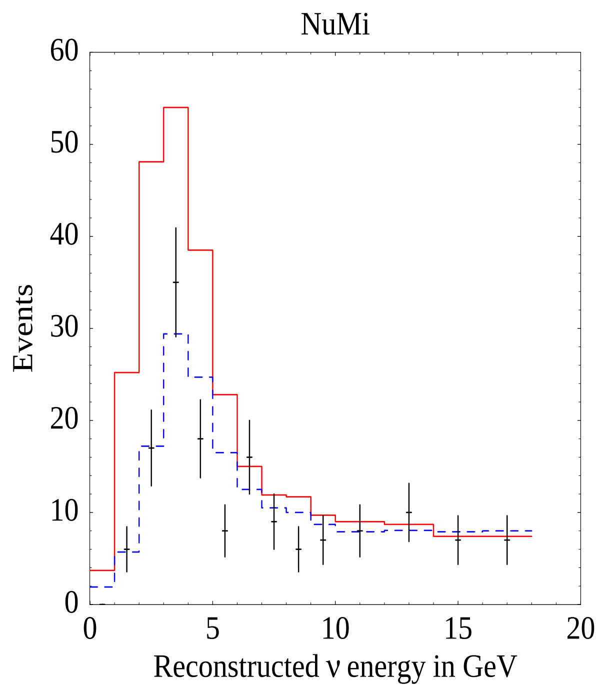
ν_μ beams:

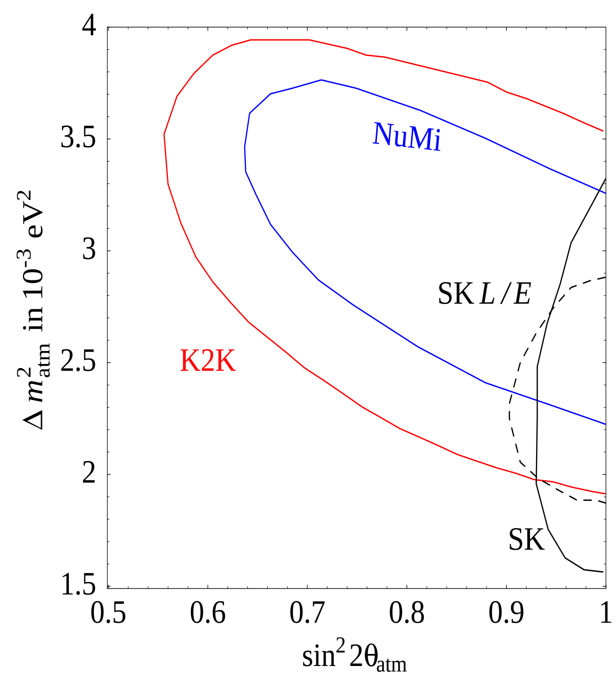
- Energy $E_\nu \sim m_p$ chosen such that $\vartheta_\mu \sim 1$.
- Distance $L \sim 500\,\text{km}$ chosen such that $\Delta m^2_{\text{atm}} L/E_\nu \sim 1$.

⋆ $\boldsymbol{E_\nu}$ **reconstructed** from E_μ, ϑ_μ since ν source known.

Result: deficit + hint of spectral distortion. Fit consistent with SK:







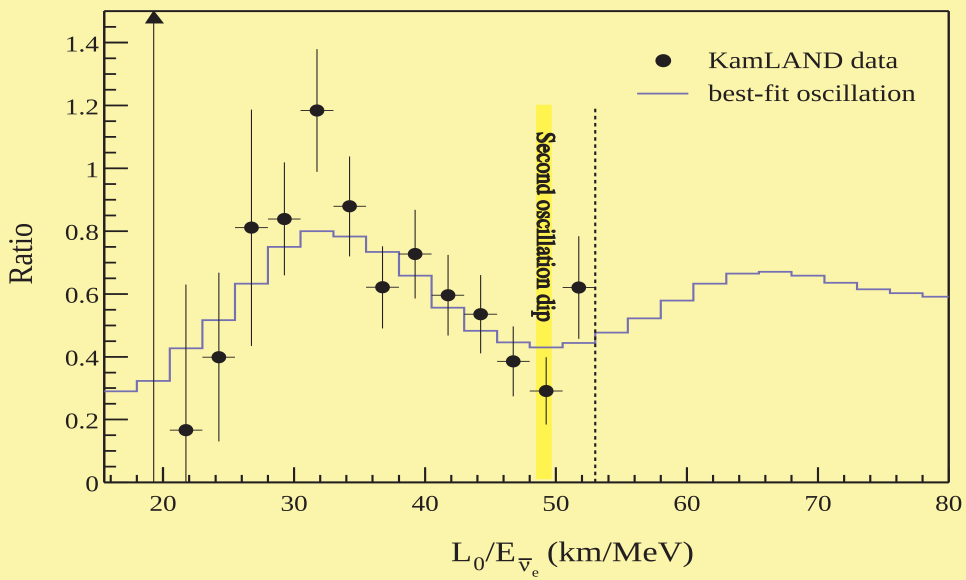
The solar anomaly

Fit without fit

Today we can focus on the best and simpler pieces of data

Solar mass splitting

Data dominated by KamLAND:



Theory: II dip of vacuum oscillations:

$$\Delta m^2 = 6\pi \left.\frac{E}{L}\right|_{\text{dip}} = (8.0 \pm 0.3) 10^{-5}\,\text{eV}^2$$

Solar mixing angle

Data dominated by SNO:

$$\langle P(\nu_e \to \nu_e)\rangle = 0.357 \pm 0.030.$$

Theory: at largest energies

$$P(\nu_e \to \nu_e) \simeq |\langle \nu_2|\nu_e\rangle|^2 = \sin^2\theta.$$

Small correction due to

$$\nu_e(\text{center of sun}) \neq \nu_2 :$$

$$\langle P(\nu_e \to \nu_e)\rangle \approx 1.15 \sin^2\theta$$

So:

$$\tan^2\theta = 0.45 \pm 0.05$$

Global fits needed to check if all the rest is consistent... and for movies

KamLAND

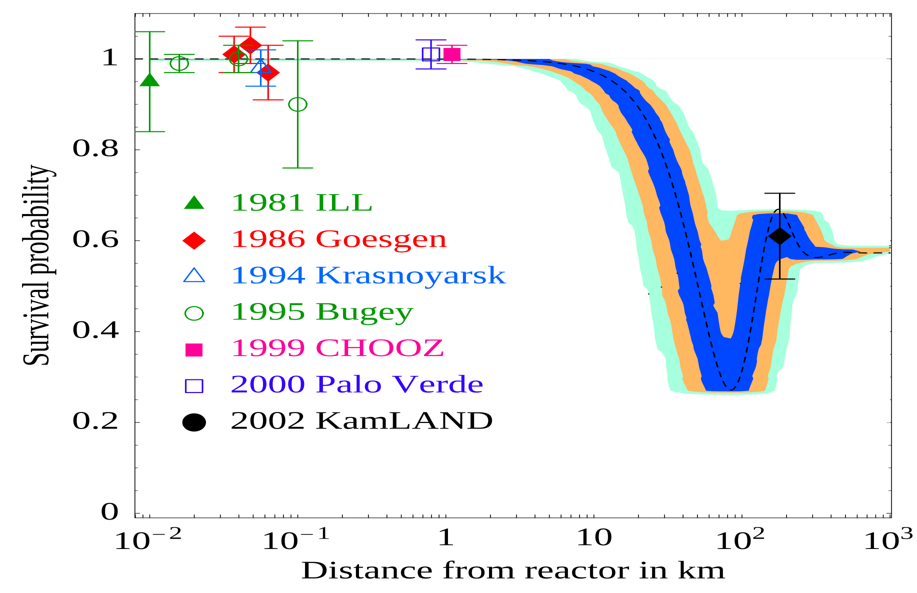
Čerenkov scintillator that detects $\bar{\nu}_e$ from terrestrial (japanese) reactors using $\bar{\nu}_e p \to \bar{e} n$

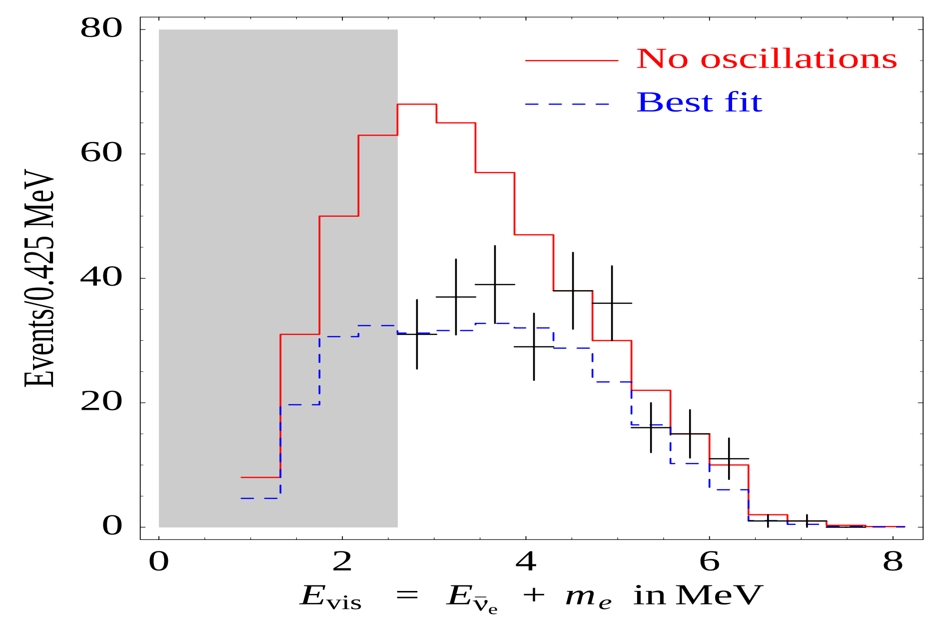
- Delayed $\bar{e}n$ coincidence: $\sim$ no bck
  (geo$\bar{\nu}_e$ background at $E_{\text{vis}} < 2.6\,\text{MeV}$)

- 258 events seen, 365 ± 24 expected
  Deficit seen at 4σ
  Errors will decrease to $(3 \div 4)\%$

- Most reactors at $L \sim$180 km.
  $E_{\bar{\nu}} \ll m_p$: $E_{\bar{\nu}} \approx E_e + m_n - m_p$:
  L/E distortion seen at 3σ

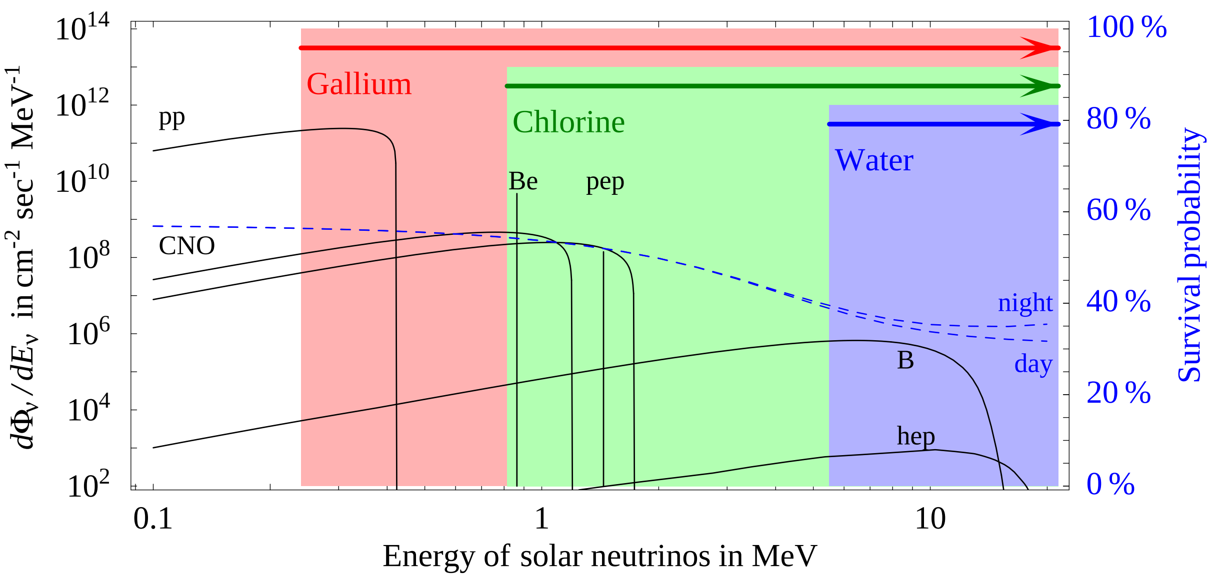
素晴らしい 結果

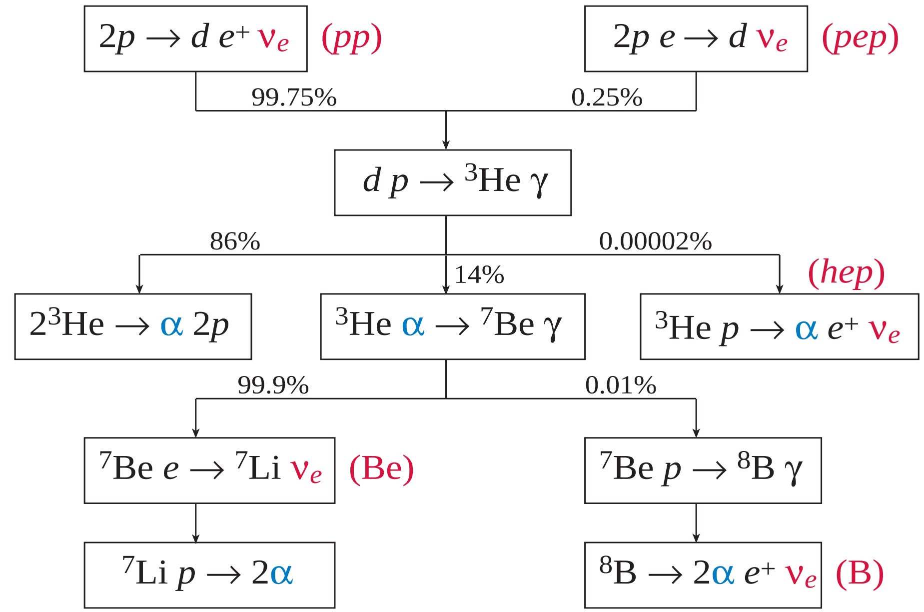




Solar ν fluxes

The sun shines as $4p + 2e \to {}^4\text{He} + 2\nu_e$ ($Q = 26.7\,\text{MeV}$).
Proceeds in steps giving a complex ν spectrum





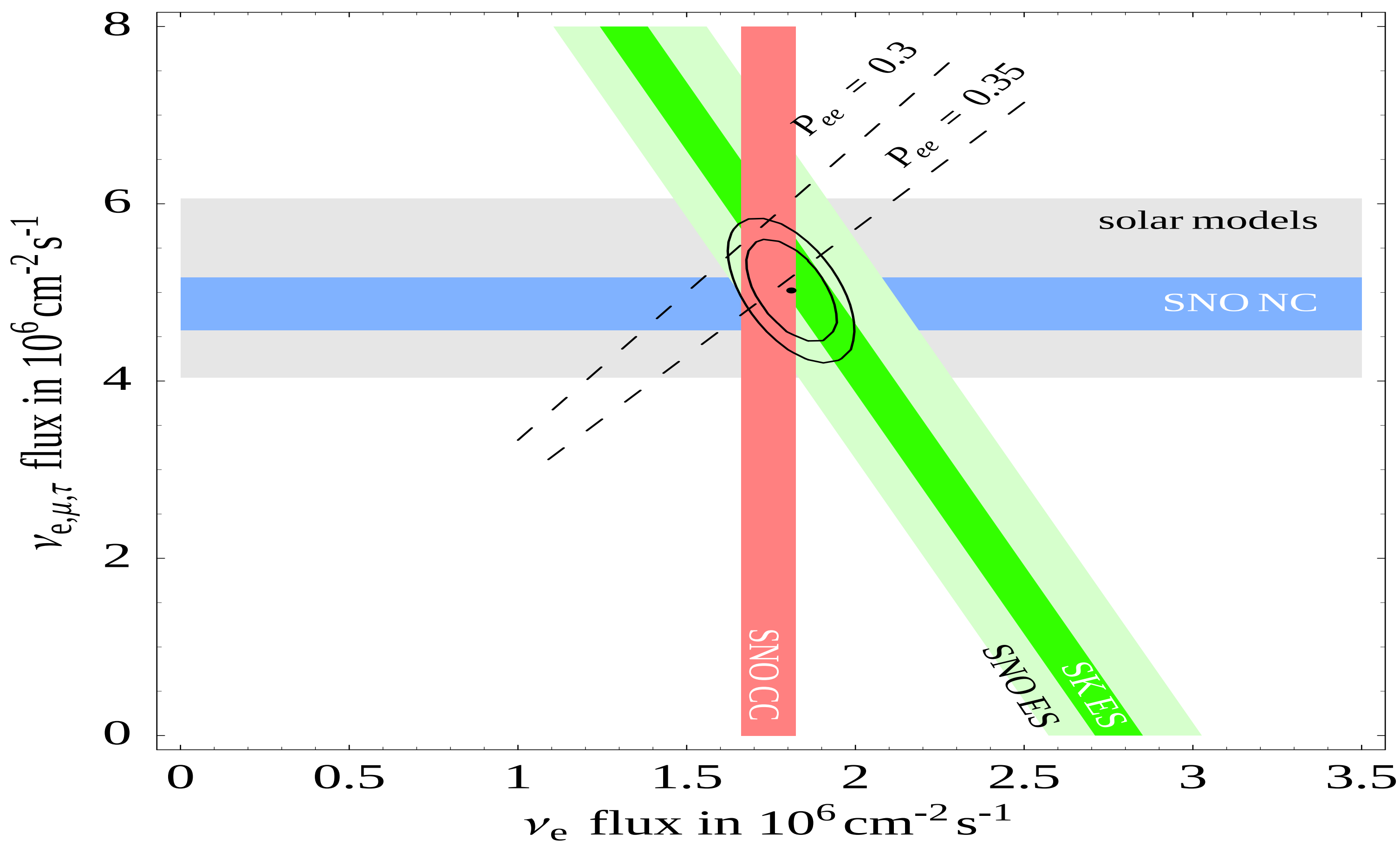
- pp: lowest energy $< 0.42\,\text{MeV} \sim 2m_p - m_d - m_e$ and precisly known flux $\Phi \sim 2K_\odot/Q \sim 6.5\cdot 10^{10}/\text{cm}^2\text{s}$. Seen only by radiochemical experiments. Vacuum oscillations: $P(\nu_e \to \nu_e) = 1 - \frac{1}{2}\sin^2 2\theta$.

- B: highest energy (detectable by SK, SNO), small flux predicted to $\pm 20\%$. Adiabatic MSW resonance: $P(\nu_e \to \nu_e) = \sin^2\theta$.

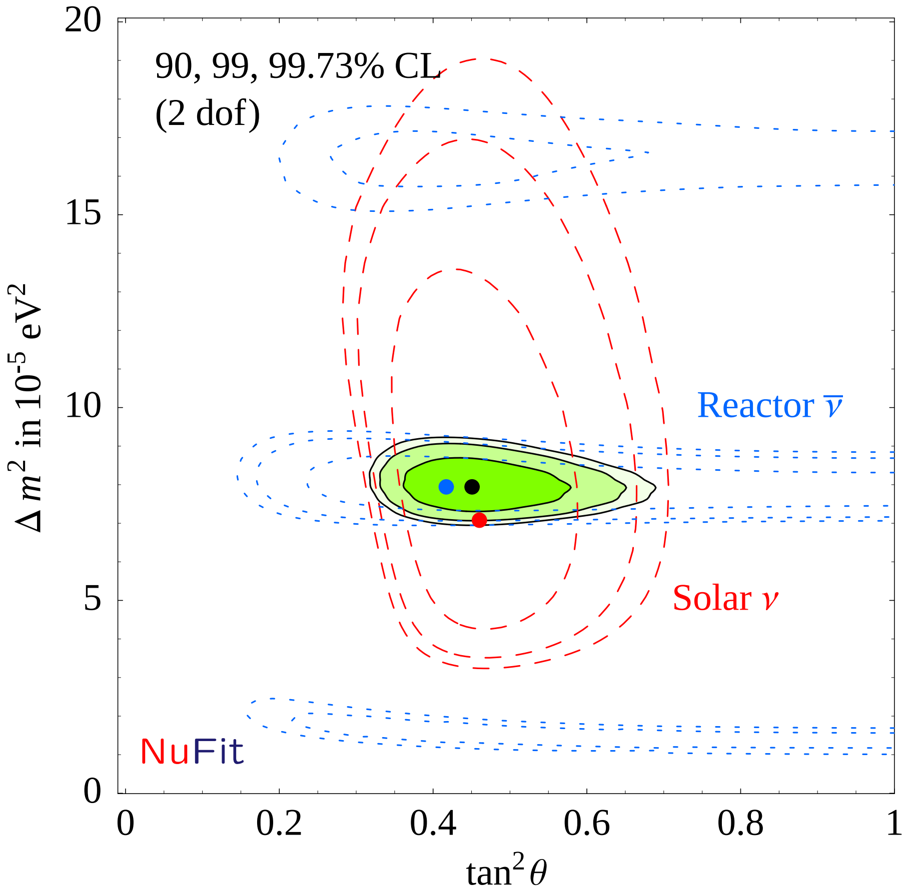
SNO

Čerenkov detector similar to SK (smaller, cleaner) with $H_2O \to D_2O$

$$\text{CC} + \frac{1}{6}\text{NC}: \ \nu e \to \nu e \qquad \text{CC}: \ \nu_e d \to ppe \qquad \text{NC}: \ \nu d \to \nu pn$$



Global fit



90, 99, 99.73% CL
(2 dof)
Reactor $\overline{\nu}$
Solar ν
NuFit
Δm^2 in 10^{-5} eV^2
$\tan^2\theta$
20
15
10
5
0
0
0.2
0.4
0.6
0.8
1

Understanding neutrino data

Plausible interpretation

Surely we saw violation of **lepton flavour** (absent in SM),
very likely due to **oscillations** induced by **neutrino masses** (absent in SM),
presumably of **Majorana** type ($\Delta L = 2$: $\mathscr{L} = \mathscr{L}_{\rm SM} + (LH)^2/\Lambda_L$),
maybe induced by new physics **around 10^{14} GeV** (see-saw?)...

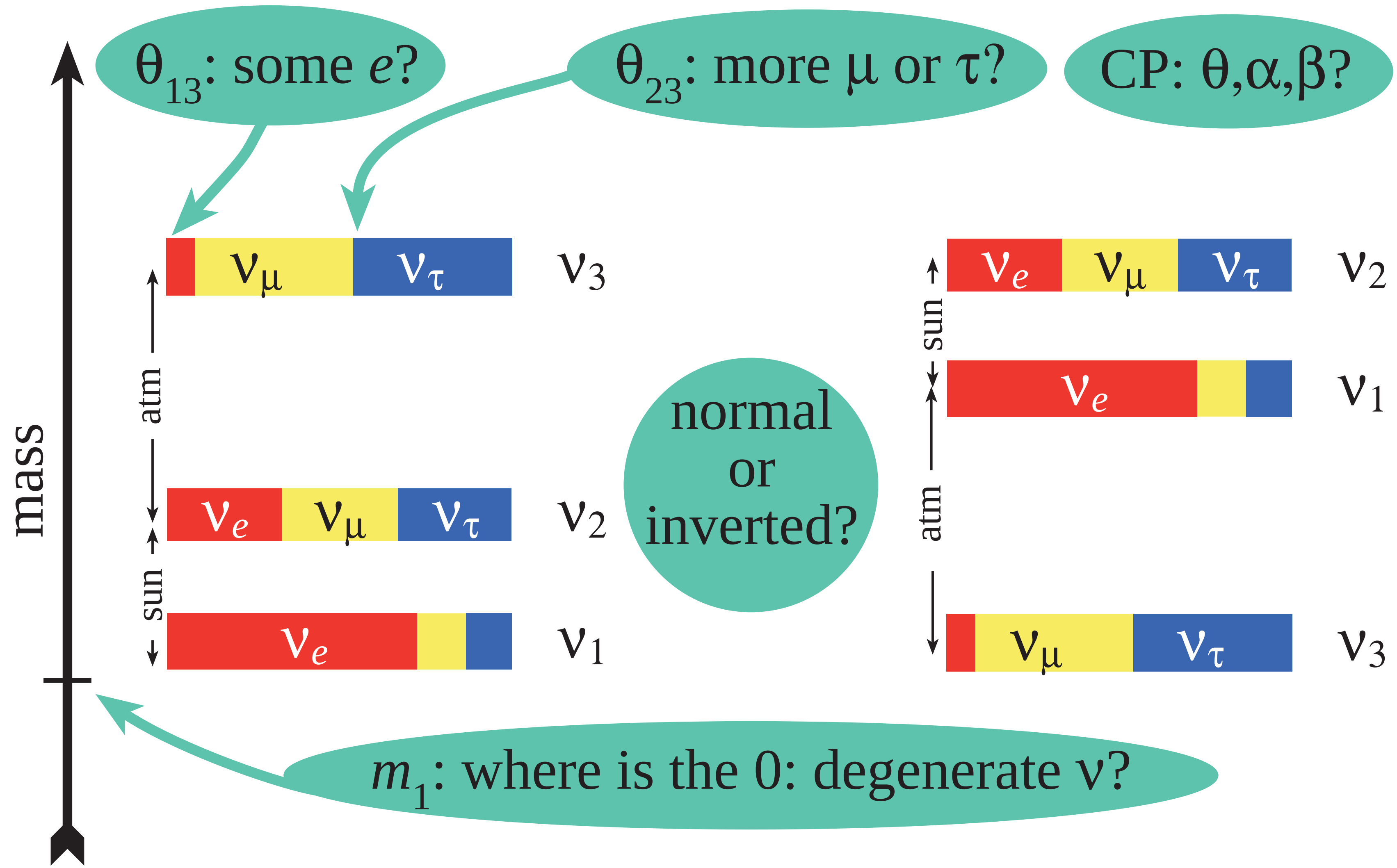
first manifestation of a new scale in nature, $\Lambda_L \sim 10^{14}$ GeV?

History: operators suppressed by the EW scale $\mathscr{L} = \mathscr{L}_{\rm QED} + (\bar{e}\nu)(\bar{p}n)/\Lambda^2_{\rm EW}$ first seen as β radioactivity by Rutherford in 1896. The SM, guessed in 1968, predicts operators in terms of 2 parameters, directly probed now at LEP, LHC.

Back to neutrinos: in next few $\times$ 10 yrs the 1st mostly experimental stage might be completed, seeing all 9 $(L_i H)(L_j H)$ operators accessible at low energy.

See-saw 'predicts' 9 Majorana ν parameters in terms of 18 parameters. bad
The physics behind m_ν seems either too heavy or too weakly coupled. worse
Leptogenesis or $\boldsymbol{\mu \to e\gamma}$ in SUSY-see-saw might give extra hints?

Issues to be solved by low energy experiments



Majorana or Dirac masses?

How to detect $m_\nu \gtrsim \sqrt{\Delta m^2_{\rm atm}} \approx$ 0.05 eV?

3 techniques are close to sensitivity; improvements are hard

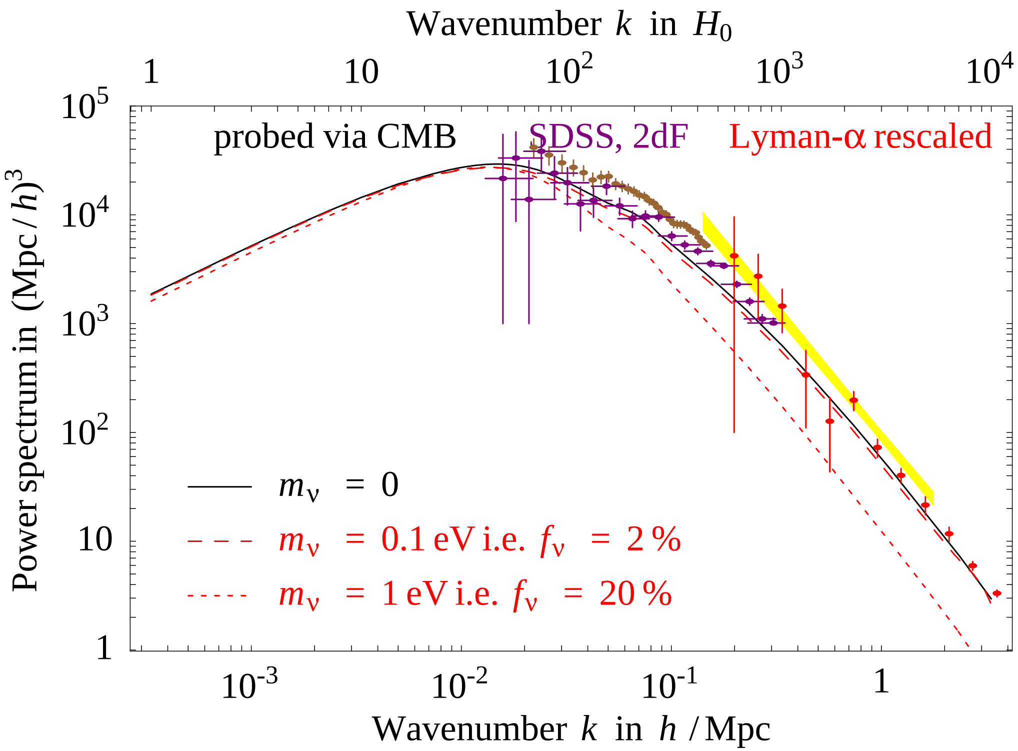
	Cosmology	β **decay**	$0\nu 2\beta$
Signal	LSS and CMB: reduced $P(k)$	End-point spectrum	Electrons with $E_{ee} = Q$-value
Needs	Simple cosmology	—	Majorana
Measures	$\sum m_\nu$	$(m^\dagger m)^{1/2}_{ee}$	m_{ee}
Today	$<$ 0.3 eV	$<$ 2 eV	$<$ 0.4h eV
From	WMAP,SDSS,Lyα	Mainz,Troitsk	HM,Igex,Cuoricino
Implies	$m_\nu \lesssim$ 0.1 eV	$m_\nu \lesssim$ 2 eV	$m_\nu/h \lesssim$ 1 eV
Sensitivity	0.03 eV	0.2 eV	0.05 eV
If normal	$(51 \div 66)$ meV	$(4.6 \div 10)$ meV	$(1.1 \div 4.5)$ meV
If inverted	$(83 \div 114)$ meV	$(42 \div 57)$ meV	$(12 \div 57)$ meV

Constraints and predictions at 99% C.L.

Cosmology

Neutrinos suppress clustering $P(k)$ in way which depends on m_ν because:
1) Heavier neutrinos contribute more: $\Omega_\nu \sim m_\nu/94\,\text{eV}$.
2) Lighter neutrinos travel more: ν non-relativistic at $z_{\text{NR}} \sim m_\nu/3\,\text{K} \sim 100$.

CMB starts seeing that $N_\nu > 0$ exist. Main probe is LSS: $m_\nu < (0.23 \div 1)\,\text{eV}$, improvable to $0.05\,\text{eV}$ with (10^7 galaxies, weak lensing)



Analytic approximation:

$$\frac{P(m_\nu,k)}{P(0,k)} \approx \begin{cases} 1 & k \lesssim k_{\text{NR}} \\ (k_{\text{NR}}/k)^p & k_{\text{NR}} \lesssim k \lesssim k_0 \\ (k_{\text{NR}}/k_0)^p & k \gtrsim k_0 \end{cases}$$

where $p \approx 5\Omega_\nu/2\Omega_{\text{DM}}$

$$k_{\text{NR}} = k_{\text{Jeans}}(a_{\text{NR}}) \approx 60\, H_0 \sqrt{m_\nu/\,\text{eV}}$$

$$k_0 = k_{\text{Jeans}}(a=1) \approx 5000\, H_0\, (m_\nu/\,\text{eV})$$

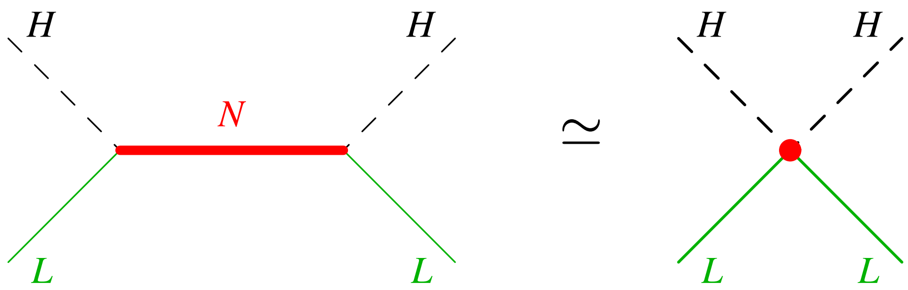
Theory of neutrino masses

See-saw

Add neutral 'right-handed neutrinos' N. The generic Lagrangian becomes

$$\mathscr{L} = \mathscr{L}_{\rm SM} + \bar{N}\partial\!\!\!/ N + M\frac{N^2}{2} + \lambda\, H L N$$

Exchange of heavy N gives the dimension-5 neutrino mass operator:



$$\simeq \quad = \quad \frac{\lambda^2}{M}\frac{(LH)^2}{2} \to \frac{(\lambda v)^2}{M}\frac{\nu^2}{2}$$

More explicit: the neutrino mass matrix is

$$\begin{array}{c} \\ \nu \\ N \end{array}\begin{array}{c} \begin{array}{cc} \nu & N \end{array} \\ \begin{pmatrix} 0 & \lambda v \\ \lambda v & M \end{pmatrix} \end{array}$$

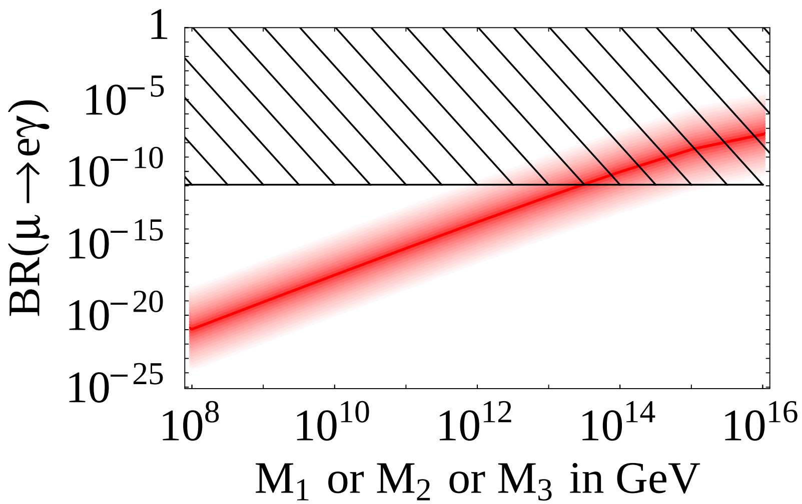
for $M \gg \lambda v$ the eigenvalues are $\simeq M$ and $m_\nu \simeq (\lambda v)^2/M$.

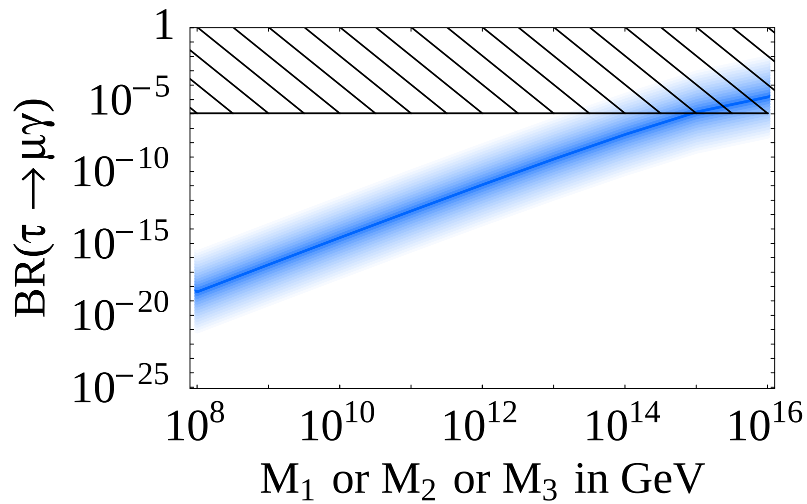
$\mu \to e\gamma$ from SUSY λ_ν

In the SM BR($\mu \to e\gamma$) $\sim (m_\mu/\Lambda_L)^2 \sim 10^{-40}$. **In SUSY see-saw quantum effects imprint LFV in slepton masses.** Starting from universal m_0^2 at $M_{\rm GUT}$

$$\boldsymbol{m}_{\tilde{L}}^2 = m_0^2 \mathbb{I} - \frac{3m_0^2}{(4\pi)^2} \boldsymbol{\lambda}_\nu^\dagger \ln(\frac{M_{\rm GUT}^2}{\boldsymbol{M}\boldsymbol{M}^\dagger}) \boldsymbol{\lambda}_\nu + \cdots$$

Even assuming large ν mixings also in λ_ν one gets loose predictions





because BR($\mu \to e\gamma$) $\sim 10^{-8}\lambda_\nu^4$ while $m_\nu = \lambda_\nu^2 v^2/M$ is measured.

Leptogenesis

Simple estimates

Neutrino masses are not enough

Baryogenesis via leptogenesis needs:

1. violation of L
2. violation of CP
3. out of equilibrium

Neutrino masses presumably directly provide 1. and 2. But 3. is still missing.

It is provided by the simplest mechanism for neutrino masses: see saw.

Right-handed ν can give both m_ν and n_B

(Not bad for the most trivial particle)

See-saw with three $N_{1,2,3}$ with Yukawa $\lambda_{1,2,3}$ and masses $M_1 < M_2 < M_3$.
$m_1 < m_2 < m_3$: ν masses $\tilde{m}_i \equiv \lambda_i^2 v^2 / M_i$ = 'N_i contribution to ν masses'

Maybe $\tilde{m}_1 = m_{\rm atm}$ or $\gtrsim m_{\rm sun}$ or $< m_{\rm sun}$ or anywhere between 0 and ∞.

The lepton asymmetry is generated at $T \lesssim M_1$ when $N_1 \to HL, H^* \bar{L}$

decays violate CP (ε) and proceed out of equilibrium (η):

$$6\ 10^{-10} = \frac{n_B}{n_\gamma} \approx \frac{\varepsilon\eta}{100}$$

Suppressed by 100 because only N_1 out of about 100 particles generates n_B.

The CP asymmetry ε

CP exchanges particles with antiparticles $p \leftrightarrow \bar{p}$. Broken by complex couplings

$$\mathscr{L} \ni \lambda_1 \, N_1 H L + \lambda_1^* \, N_1 H^* \bar{L} \ \stackrel{\text{CP}}{\to} \ \lambda_1 \, N_1 H^* \bar{L} + \lambda_1^* \, N_1 H L \neq \mathscr{L} \qquad \text{if } \lambda_1 \neq \lambda_1^*$$

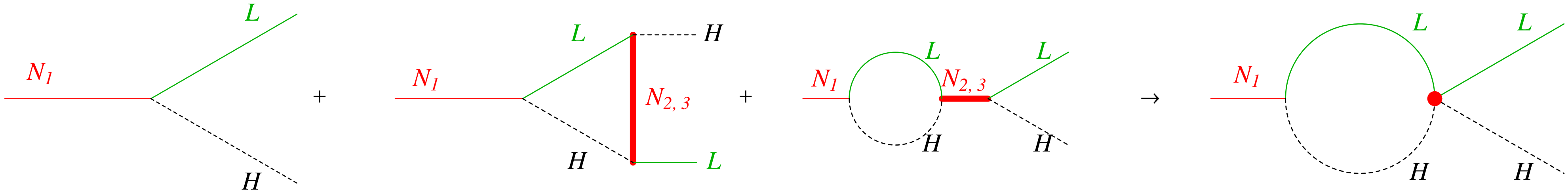
Physical effect

$$\varepsilon \equiv \frac{\Gamma(N_1 \to LH) - \Gamma(N_1 \to \bar{L}H^*)}{\Gamma(N_1 \to LH) + \Gamma(N_1 \to \bar{L}H^*)}$$

- from relative CP-violating phase between λ_1 and $\lambda_{2,3}$ in one-loop diagram

- with a CP-conserving complex loop factor $A \sim i(M_1/M_{2,3})/4\pi$

$$\Gamma(N_1 \to LH) \propto |\lambda_1 + A\lambda_1^* \lambda_{2,3}^2|^2 \qquad \neq \qquad \Gamma(N_1 \to \bar{L}H^*) \propto |\lambda_1^* + A\lambda_1 \lambda_{2,3}^{2*}|^2$$



For $M_{2,3} \gg M_1$ insertion of effective $\bullet = (LH)^2 \tilde{m}_{2,3}/v^2$ gives immediately

$$\varepsilon \simeq \frac{3}{16\pi} \frac{\tilde{m}_{2,3} M_1}{v^2} \sin\delta = 10^{-6} \frac{\tilde{m}_{2,3}}{0.05\,\text{eV}} \frac{M_1}{10^{10}\,\text{GeV}} \sin\delta$$

The efficiency η

Depends on expansion rate H vs decay rate Γ at $T \sim M_1$:

$$H \sim \frac{M_1^2}{M_{\rm Pl}} \qquad \Gamma \sim \lambda_1^2 M_1 \sim \frac{\tilde{m}_1 M_1^2}{v^2} \qquad \text{so} \qquad \frac{\Gamma}{H} \sim \frac{\tilde{m}_1}{m_*}$$

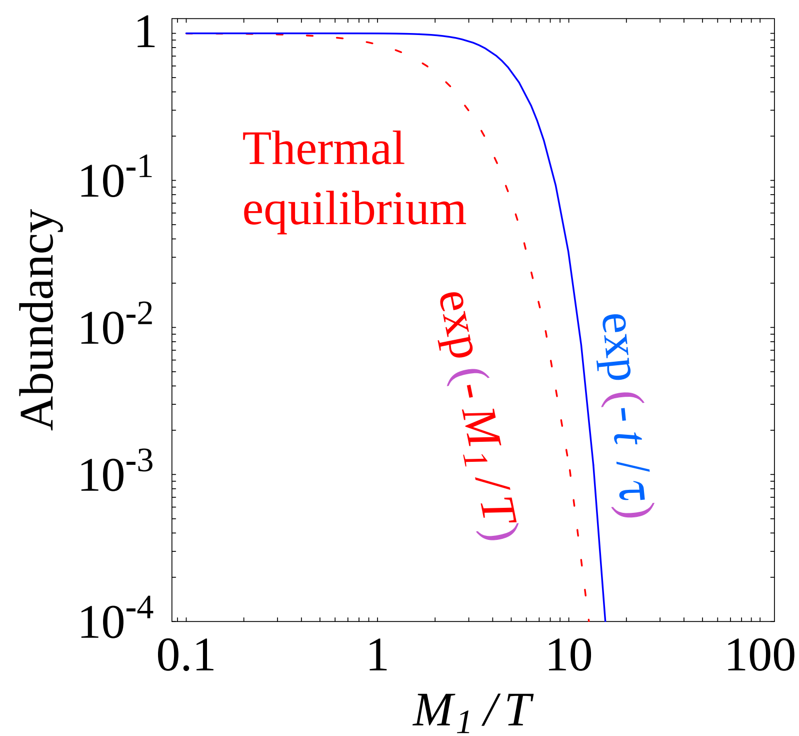
where $m_* \sim v^2/M_{\rm Pl} \sim 2\ 10^{-3}\,\text{eV}$ is comparable to neutrino masses!

$\eta = 1$ if $\Gamma \ll H$: decay is slower than expansion and N_1 decays out-of-equilibrium (starting from equilibrium abundance)

$\eta \ll 1$ if $\Gamma \gg H$: N_1 stay close to equilibrium until inverse-decay is enough Boltzmann suppressed: $e^{-M_1/T}\Gamma \sim H$, so

$$\eta \approx \exp(-M_1/T) \approx H/\Gamma \approx m_*/\tilde{m}_1.$$

More plausible since $m_{\text{sun,atm}} \gg m_*$.



Leptogenesis: precise computation

1) Boltzmann equations. 2) Boltzmann equations for leptogenesis

Boltzmann equations

A all-purpose tool in cosmology. Applications: DM, BBN, CMB, leptogenesis...

Each process changes the number of particles in a comoving volume V.
E.g. a $1 \leftrightarrow 2+3$ decay (in leptogenesis $N \leftrightarrow LH$) gives:

$$\frac{d}{dt}(n_1 V) = V \int d\vec{p}_1 \int d\vec{p}_2 \int d\vec{p}_3 \, (2\pi)^4 \delta^4(p_1 - p_2 - p_3) \times$$
$$\times |A|^2 [-f_1(1 \pm f_2)(1 \pm f_3) + (1 \pm f_1) f_2 f_3]$$

where $d\vec{p}_i = d^3 p_i / 2E_i (2\pi)^3$ and $|A|^2$ is summed over initial and final spins.

Simplify assuming **kinetic equilibrium**: $f(p) = f_{\rm eq}(p) \frac{n}{n_{\rm eq}}$ where $f_{\rm eq} = \frac{1}{e^{E/T} \pm 1}$.
Since $\langle E \rangle \sim 3T$ approximate FD, BE with Boltzmann: $f_{\rm eq} \simeq e^{-E/T}$ and $1 \pm f \simeq 1$.

$$n_{\rm eq} = g \int \frac{d^3 p}{(2\pi\hbar)^3} \, f_{\rm eq} = \frac{g M^2 T}{2\pi^2} {\rm K}_2(\frac{M}{T}) \stackrel{T \gtrsim M}{=} \frac{g T^3}{\pi^2}$$
$$\rho_{\rm eq} = g \int \frac{d^3 p}{(2\pi)^3} \, E f_{\rm eq} \stackrel{T \gtrsim M}{=} \frac{3 g T^4}{\pi^2}$$

g = degrees of freedom (spin, gauge...: $g_N = g_\gamma = 2$, $g_{G^a} = 16$, $g_{\rm SM} = 118$).

$$
\begin{aligned}
\frac{1}{V}\frac{d}{dt}(n_1 V) &= \int d\vec{p}_1 \int d\vec{p}_2 \int d\vec{p}_3\, (2\pi)^4 \delta^4(p_1 - p_2 - p_3) \times \\
&\quad \times |A|^2 [-\frac{n_1}{n_1^{\text{eq}}} e^{-E_1/T} + \frac{n_2}{n_2^{\text{eq}}}\frac{n_3}{n_3^{\text{eq}}} e^{-E_2/T} e^{-E_3/T}] \\
&= \langle \Gamma_1 \rangle n_1^{\text{eq}} \left[\frac{n_1}{n_1^{\text{eq}}} - \frac{n_2}{n_2^{\text{eq}}}\frac{n_3}{n_3^{\text{eq}}} \right]
\end{aligned}
$$

$\langle \Gamma_1 \rangle$ is the thermal average of the Lorentz-dilatated decay width

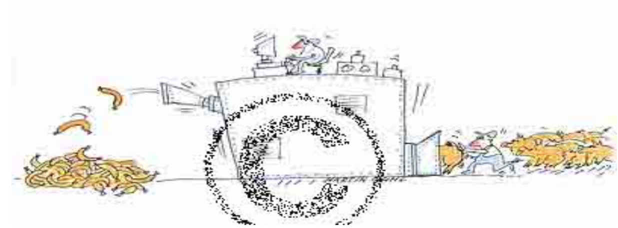
$$
\Gamma_1(E_1) = \frac{1}{2E_1} \int d\vec{p}_2\, d\vec{p}_3 (2\pi)^4 \delta^4(p_1 - p_2 - p_3) |A|^2
$$

If $\langle \Gamma_1 \rangle \gg H$ the term in square brackets vanish. **In general fast interactions force $\boldsymbol{n} = \boldsymbol{n}_{\text{eq}}$.** In the case of leptogenesis $2, 3 = L, H$ have fast gauge interactions. We do not have to evolve $n_{2,3}(t)$, because they are kept in equilibrium.

$$
\dot{n}_1 + 3Hn_1 = \langle \Gamma_1 \rangle (n_1 - n_1^{\text{eq}})
$$

having used $\dot{V}/V = -\dot{s}/s = 3H$.

Building a pig machine



To avoid big numbers, evolve $Y_i \equiv n_i/s$ as function of $z \equiv M_1/T$:

$$\frac{d}{dt}(sV) \overset{\text{eq}}{=} 0 \qquad a \propto T^{-1} \propto z \qquad \frac{d}{dt} = Ha\frac{d}{da} = Hz\frac{d}{dz}$$

$$sHz\frac{dY_1}{dz} = \sum_{\text{processes}} \Delta_1 \cdot \gamma_{\text{eq}}(12\cdots \leftrightarrow 34\cdots)\left[\frac{Y_1}{Y_1^{\text{eq}}}\frac{Y_2}{Y_2^{\text{eq}}}\cdots - \frac{Y_3}{Y_3^{\text{eq}}}\frac{Y_4}{Y_4^{\text{eq}}}\cdots\right]$$

- $\Delta_1 = -n$ for processes that destroy n units of 1 particles.
- γ_{eq} is the space-time density of scatterings in thermal equilibrium:

$$\gamma_{\text{eq}}(1 \to 23) = \int d\vec{p}_1\, f_1^{\text{eq}} \int d\vec{p}_2\, d\vec{p}_3\, (2\pi)^4\delta^4(p_1 - p_2 - p_3)|A|^2 = \gamma_{\text{eq}}(23 \to 1)$$

$$\gamma_{\text{eq}}(12 \leftrightarrow 34) = \int d\vec{p}_1\, d\vec{p}_2\, f_1^{\text{eq}} f_2^{\text{eq}} \int d\vec{p}_3\, d\vec{p}_4\, (2\pi)^4\delta^4(p_1 + p_2 - p_3 - p_4)|A|^2$$

Add symmetry factors $1/n!$ for any n identical particles in *final or initial* state:

$$\gamma_{\text{eq}}(11 \leftrightarrow 333) = \int \frac{d\vec{p}_1\, d\vec{p}'_1}{2!} f_1^{\text{eq}} f_{1'}^{\text{eq}} \int \frac{d\vec{p}_3\, d\vec{p}'_3 d\vec{p}''_3}{3!} (2\pi)^4\delta^4(p_1 + p'_1 - p_3 - p'_3 - p''_3)|A|^2$$

Main processes

Doing analytically as much $\int$ as possible one gets the formulæ used in practice:

For a **decay** (Γ is the decay width at rest, $\mathsf{K}_{1,2}$ are Bessel functions):

$$\gamma_{\rm eq}(1 \to 23\cdots) = \gamma_{\rm eq}(23\cdots \to 1) = n_1^{\rm eq}\frac{\mathsf{K}_1(M_1/T)}{\mathsf{K}_2(M_1/T)}\Gamma(1 \to 23\cdots)$$

For a **2 body scattering** (s is the squared center of mass energy)

$$\gamma_{\rm eq}(12 \to 34\cdots) = \frac{T}{32\pi^4}\int_{s_{\rm min}}^{\infty} ds\ s^{3/2}\,\lambda(1, M_1^2/s, M_2^2/s)\sigma(s)\ \mathsf{K}_1\left(\frac{\sqrt{s}}{T}\right)$$

= thermal average of relativistic $v \cdot \sigma$, summed over initial and final spins.

The N_1 decay width

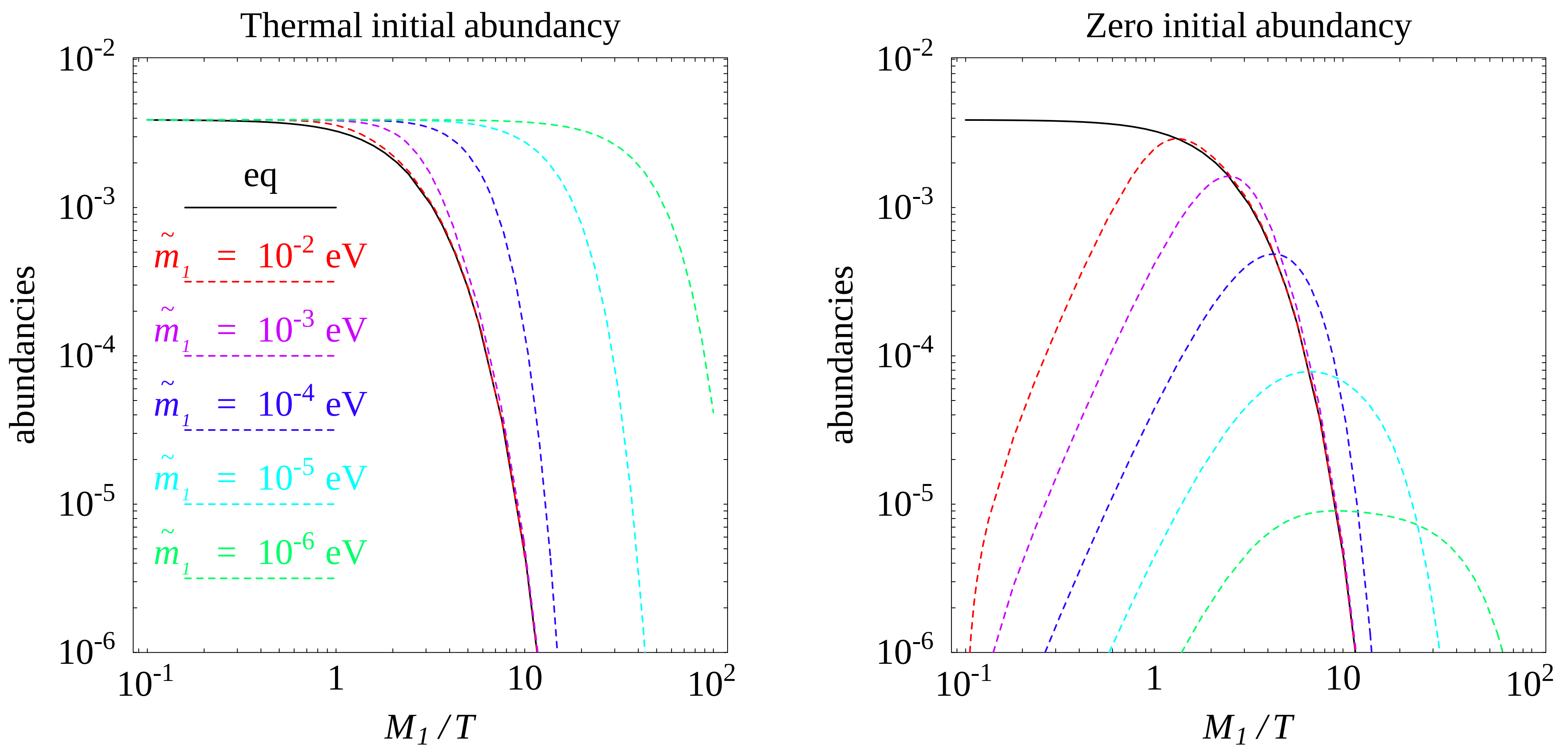
The Yukawa interaction $\lambda_1 \ N_1 L H$ gives

$$\mathscr{A}(N_1 \to L_i H_j) = \delta_{ij} \lambda_1 \bar{u}_N(P) P_L u_L(Q), \qquad \sum_{ij} |\mathscr{A}|^2 = 4\lambda_1^2 (P \cdot Q),$$

$$\Gamma(N_1 \to LH, \bar{L}\bar{H}) = \sum_{ij} \frac{1}{8\pi} \frac{|\mathscr{A}|^2 + |\bar{\mathscr{A}}|^2}{2M} = \frac{\lambda_1^2 M_1}{8\pi} = \tilde{m}_1 \frac{M_1^2}{8\pi v^2}$$

Evolution of the N_1 abundance

$$sHz\frac{dY_{N_1}}{dz} = -\gamma_D\left(\frac{Y_{N_1}}{Y_{N_1}^{\mathrm{eq}}} - 1\right)$$



Evolution of the lepton asymmetry

Start including only decays, that violate CP:

$$\gamma_{\text{eq}}(N \to LH) \overset{\text{CPT}}{=} \gamma_{\text{eq}}(\bar{L}\bar{H} \to N) \;=\; (1+\epsilon)\frac{\gamma_D}{2}$$
$$\gamma_{\text{eq}}(N \to \bar{L}\bar{H}) \overset{\text{CPT}}{=} \gamma_{\text{eq}}(LH \to N) \;=\; (1-\epsilon)\frac{\gamma_D}{2}$$

Boltzmann equations for leptons and anti-leptons

$$sHzY'_L \;=\; D \qquad D = \frac{\gamma_D}{2}\left[\frac{Y_N}{Y_N^{\text{eq}}}(1+\epsilon) - \frac{Y_L}{Y_L^{\text{eq}}}(1-\epsilon)\right]$$
$$sHzY'_{\bar{L}} \;=\; \bar{D} \qquad \bar{D} = \frac{\gamma_D}{2}\left[\frac{Y_N}{Y_N^{\text{eq}}}(1-\epsilon) - \frac{Y_{\bar{L}}}{Y_{\bar{L}}^{\text{eq}}}(1+\epsilon)\right]$$

and for the lepton number $Y_{\mathcal{L}} = Y_L - Y_{\bar{L}}$:

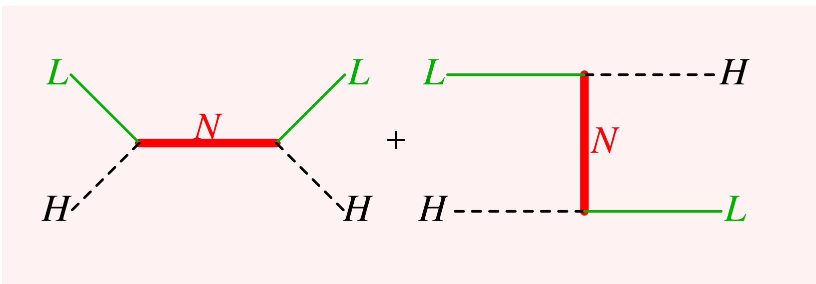
$$sHzY'_{\mathcal{L}} = D - \bar{D} = \epsilon\gamma_D(\frac{Y_N}{Y_N^{\text{eq}}} + 1) - \frac{Y_{\mathcal{L}}}{2Y_L^{\text{eq}}}\gamma_D$$

$\mathcal{L}$ asymmetry generated in thermal equilibrium!? Indeed CPT implies that if N decays preferentially produce L, than inverse decays preferentially destroy $\bar{L}$.

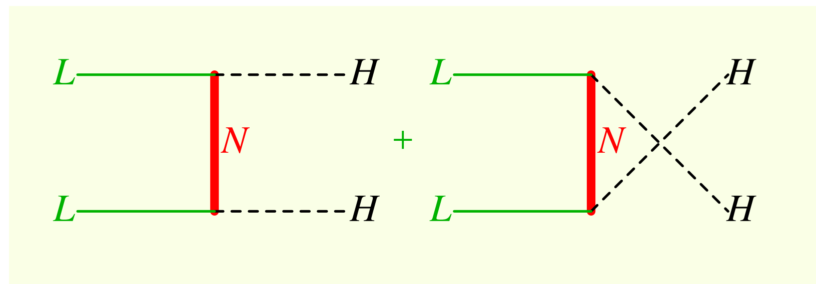
A subtelty

Consistent perturbative study: include all processes up to chosen order in λ:

$$\Delta L = \pm 1 : \begin{cases} D = [N \leftrightarrow LH] \\ \bar{D} = [N \leftrightarrow \bar{L}\bar{H}] \end{cases} \qquad \Delta L = \pm 2 : \begin{cases} N_s = [LH \leftrightarrow \bar{L}\bar{H}] \\ N_t = [LL \leftrightarrow \bar{H}\bar{H}] \\ \bar{N}_t = [\bar{L}\bar{L} \leftrightarrow HH] \end{cases}$$



$\gamma_{Ns} = \gamma_{eq}\,(LH \to \bar{L}\,\bar{H})$



$\gamma_{Nt} = \gamma_{eq}\,(LL \to \bar{H}\,\bar{H})$

$D \sim \lambda^2$, $D - \bar{D} \sim \lambda^4$: at this order $2 \leftrightarrow 2$ scatterings must be computed at tree level and are CP-conserving. Boltzmann equations:

$$sHzY_L' = D - N_s - 2N_t \qquad sHzY_{\bar{L}}' = \bar{D} + N_s - 2\overline{N}_t$$

$$sHzY_{\mathcal{L}}' = \epsilon\gamma_D\left(\frac{Y_N}{Y_N^{\rm eq}} + 1\right) - \frac{Y_{\mathcal{L}}}{2Y_L^{\rm eq}}[\gamma_D + 2\gamma_{N_s} + 4\gamma_{N_t}]$$

Still in trouble, but closer to the solution

Subtelty + anti-subtelty

$\gamma_{Ns} \sim \lambda^2$, not $\sim \lambda^4$ due to resonant enhancement.
Like at the Z-peak one has $\sigma_{\text{peak}} \sim \lambda^0/M_1^2$ in an energy range $\Delta E \sim \Gamma_{N_1} \sim \lambda^2$.
The exact result is

$$\gamma_{Ns}^{\text{on-shell}} = \gamma_D \cdot \text{BR}(LH \to N) \cdot \text{BR}(N \to \bar{L}\bar{H}) = \gamma_D/4$$

Is the non-sense cured by CP-violating corrections to $LH \to \bar{L}\bar{H}$?
No: scatterings are CP-conserving at one loop level.
Proof: unitarity demands $\sum_j |M(i \to j)|^2 = \sum_j |M(j \to i)|^2$, so

$$\sigma(LH \to LH) + \sigma(LH \to \bar{L}\bar{H}) = \sigma(LH \to LH) + \sigma(\bar{L}\bar{H} \to LH)$$

(at higher order states with more particles allow a negligible CP asymmetry).

Still in trouble, but closer to the solution

Solution: avoid over-counting

γ_{Ns} must be computed by subtracting the CP-violating contribution due to on-shell N_1 exchange, because in the Boltzmann equations this effect is already taken into account by successive decays, $LH \leftrightarrow N \leftrightarrow \bar{L}\bar{H}$. So

$$\gamma_{\rm eq}^{\rm sub}(LH \to \bar{L}\bar{H}) \;=\; \gamma_{Ns}^{\rm full} - \gamma_D \cdot \underbrace{{\rm BR}(LH \to N)}_{(1-\epsilon)/2} \cdot \underbrace{{\rm BR}(N \to \bar{L}\bar{H})}_{(1-\epsilon)/2} = \gamma_{Ns} + \epsilon\frac{\gamma_D}{2} + \cdots,$$

$$\gamma_{\rm eq}^{\rm sub}(\bar{L}\bar{H} \to LH) \;=\; \gamma_{Ns}^{\rm full} - \gamma_D \cdot \overbrace{{\rm BR}(\bar{L}\bar{H} \to N)}^{(1+\epsilon)/2} \cdot \overbrace{{\rm BR}(N \to LH)}^{(1+\epsilon)/2} = \gamma_{Ns} - \epsilon\frac{\gamma_D}{2} + \cdots$$

Final Boltzmann equation:

$$\begin{aligned} sHzY'_{\mathcal{L}} \;&=\; \gamma_D\epsilon(\frac{Y_N}{Y_N^{\rm eq}} - 1) - \frac{Y_{\mathcal{L}}}{Y_N^{\rm eq}}(\frac{\gamma_D}{2} + 2\gamma_{Ns}^{\rm sub} + 4\gamma_{Nt}) \\ &=\; \gamma_D\epsilon(\frac{Y_N}{Y_N^{\rm eq}} - 1) - \frac{Y_{\mathcal{L}}}{Y_N^{\rm eq}}(2\gamma_{Ns} + 4\gamma_{Nt}) \end{aligned}$$

Wash-out term contains the total γ_{Ns} and no γ_D: obvious a posteriori.
Correct result: $\gamma_{Ns}^{\rm sub} \sim \lambda^4$, so that $\gamma_D \sim \lambda^2$ is the only main process.

Sphalerons and Yukawas

- **Sphalerons** redistribute the L asymmetry to Q.
- SM **Yukawa couplings** redistribute to right-handed leptons and quarks.
- Furthermore so far we considered only one flavour (needs density matrix).

Redistributor process can be negligible at $T \sim M_1$ during leptogenesis
− Sphalerons and $\lambda_{t,b,c,\tau}$ start operating below $T \lesssim 10^{11\div 12}$ GeV.
− $\lambda_{\mu,s}$ below $T \lesssim 10^9$ GeV.

In theory one needs to enlarge Boltzmann equations adding these processes.

In practice: neglect slow processes, and include *very* fast processes by evolving $Y_{\mathcal{B}-\mathcal{L}}$ (conserved by all these extra processes), linked to $Y_{\mathcal{L}}$ by redistribution factors. E.g. $Y_{\mathcal{B}-\mathcal{L}} = -Y_{\mathcal{L}}$ if all redistributions are negligibly slow.

Redistribution factors

E.g. let us compute B at $T \sim$ TeV when all processes are fast.

Each particle $P = \{L, E, Q, U, D, H\}$ carries an asymmetry A_P.

Interactions equilibrate 'chemical potentials' $\mu_P \equiv A_P/g_P$ as

$$\begin{cases} ELH \text{ Yukawa}: & 0 = \mu_E + \mu_L + \mu_H \\ DQH \text{ Yukawa}: & 0 = \mu_D + \mu_Q + \mu_H \\ UQ\bar{H} \text{ Yukawa}: & 0 = \mu_U + \mu_Q - \mu_H \\ QQQL \text{ sphalerons}: & 0 = 3\mu_Q + \mu_L \\ \text{No electric charge}: & 0 = N_{\text{gen}}(\mu_Q - 2\mu_U + \mu_D - \mu_L + \mu_E) - 2N_{\text{Higgs}}\mu_H \end{cases}$$

(signs in my convention). 6 unknowns, 5 constraints: 1 independent asymm:

$$B = N_{\text{gen}}(2\mu_Q - \mu_U - \mu_D) = \frac{28}{79}(B - L)$$

$$\frac{n_{\mathcal{B}}}{s} = Y_{\mathcal{B}} = \frac{28}{79} Y_{\mathcal{B}-\mathcal{L}} \equiv -\frac{28}{79}\epsilon\eta Y^{\text{eq}}_{N_1}(T \gg M_1) = -\frac{28}{79}\epsilon\eta\frac{2/4}{118}$$

Since today $s = 7.04 n_\gamma$,

$$\left.\frac{n_{\mathcal{B}}}{n_\gamma}\right|_{\text{today}} = -\frac{\epsilon\eta}{103.}$$

Details, details, details...

Flavour?

Leptogenesis depends on how the total $\mathcal{B}-\mathcal{L}$ is shared among the 3 flavours
A single Boltzmann equation for $Y_{\mathcal{B}-\mathcal{L}}$ is an approximation.

To be correct one needs to evolve the 3×3 density matrix ρ (not its diagonal elements $Y_{\mathcal{B}/3-\mathcal{L}_i}$) as the following example shows. Suppose N_1 decays into $|\nu_3\rangle = |\nu_\mu\rangle + |\nu_\tau\rangle$. Washout scatterings act on $|\nu_2\rangle = |\nu_\mu\rangle - |\nu_\tau\rangle$. What happens?

(A) All is erased, because $\langle \nu_{\mu,\tau}|\nu_{2,3}\rangle \neq 0$.

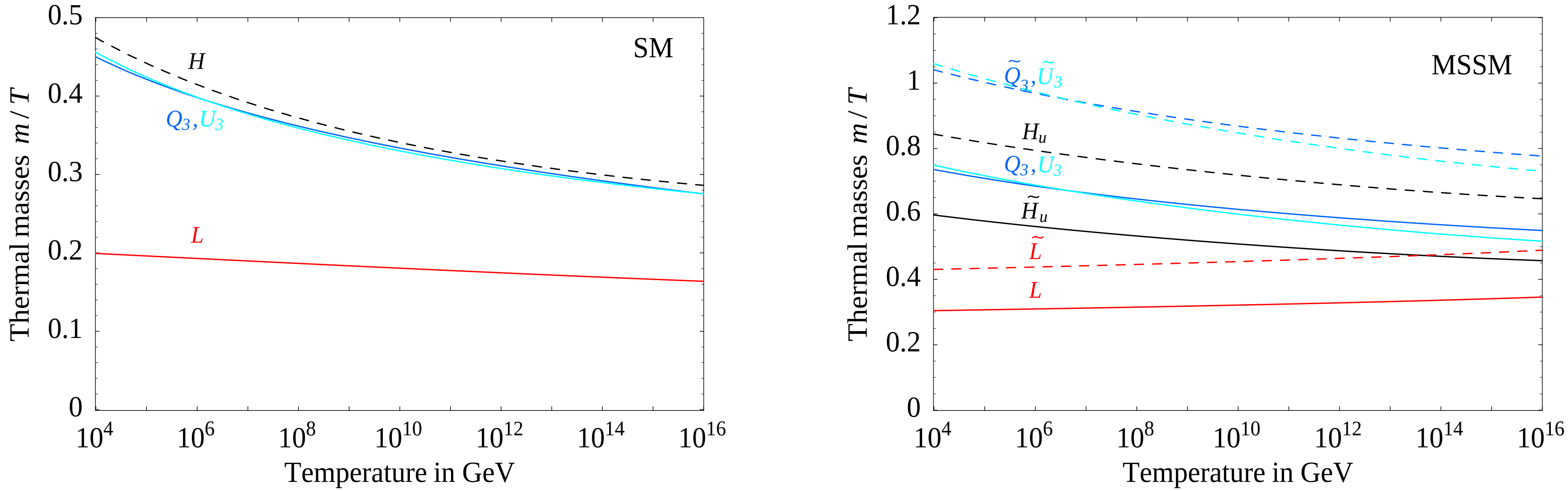
(N) Nothing is erased, because $\langle \nu_2|\nu_3\rangle = 0$.

(A) is correct if λ_τ interactions are in thermal equilibrium (i.e. $T \lesssim 10^{11}\,\text{GeV}$), because they kill coeherences in ρ. Otherwise (N) is correct.

Knowing the full equation for ρ one finds which single equation for $Y_{\mathcal{B}-\mathcal{L}}$ is a good approx (e.g. $\Delta L = 2$ scatterings are controlled by $\langle m_i^2\rangle$, not by $\sum m_i^2$).

Thermal corrections

Many effects. Big ones affect propagators. Resummation gives thermal masses: a particle that collides with others at temperature T gets a minimal energy gT.



Important at $T >$ few M_1: e.g. $N \to HL$ replaced by $H \to NL$.

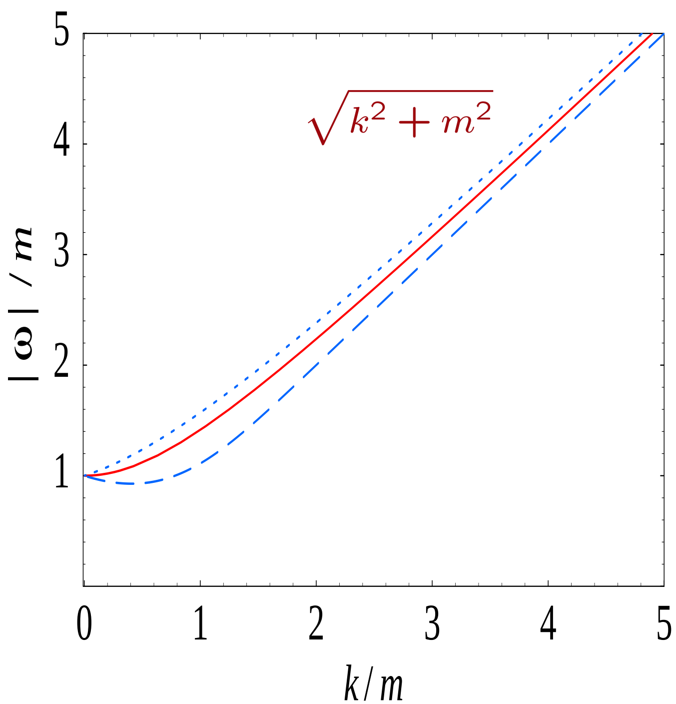
Important also at low T (exchanging light particles gives long range forces)

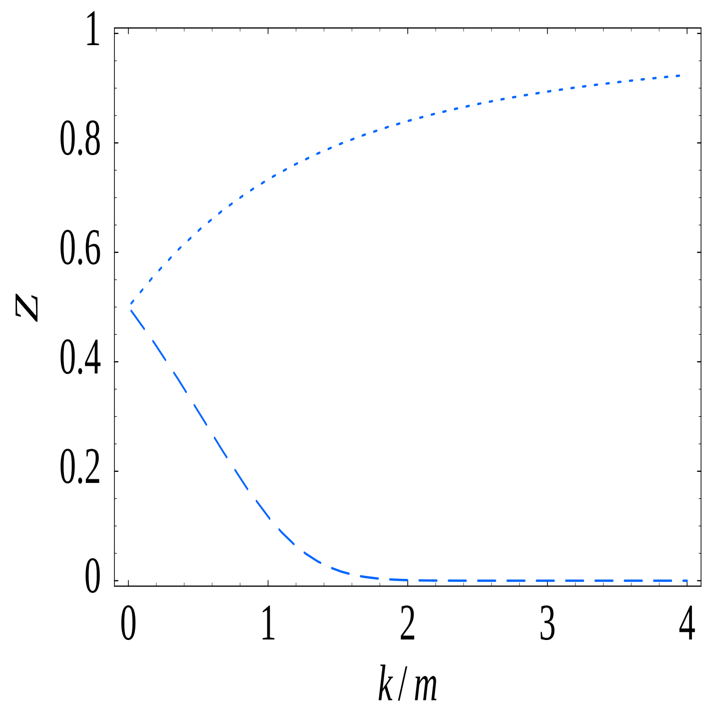
$$\ln \frac{M_1}{M_Z} \sim 20 \to \ln \frac{M_1}{T} \sim 1$$

Thermal masses...

...of spin 1/2 and 1 are not the usual relativistic masses (e.g. $m\,\bar\Psi\gamma_0\Psi$ in the plasma rest frame). But particle (dotted)+hole (dashed) $\approx$ normal particle:

dispersion relation coupling

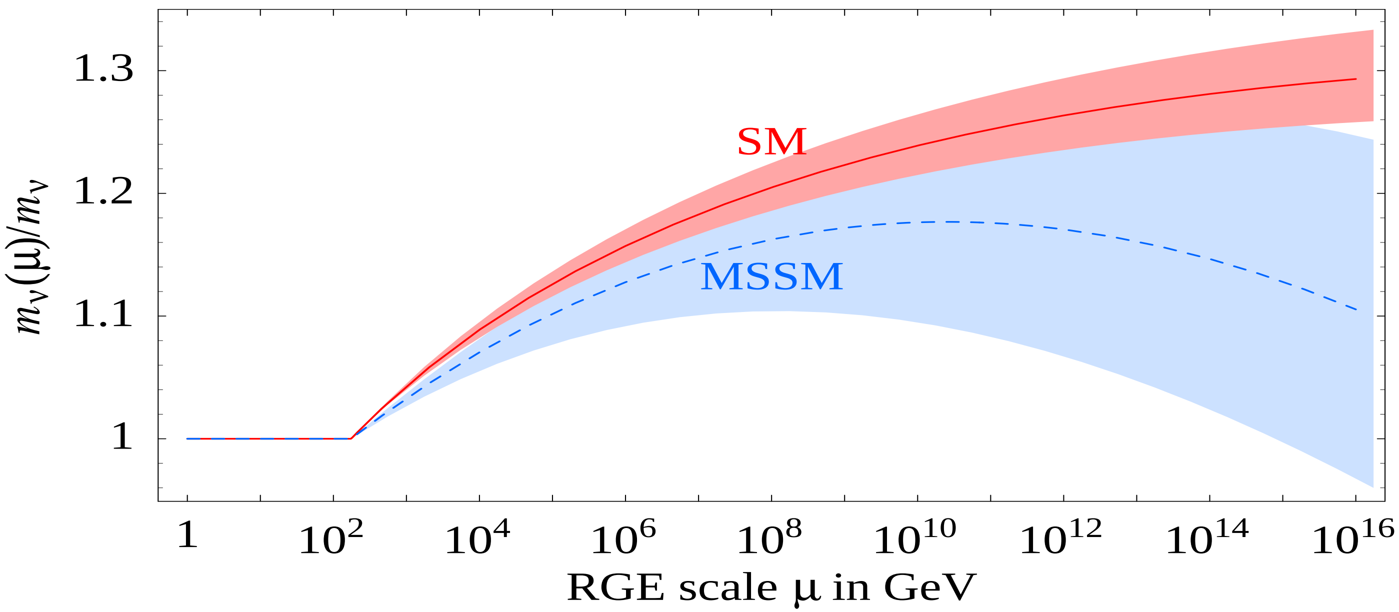




Processes with external fermions (e.g. $N \to HL$) are simple: their wave-functions remain equal as for massless fermions. Processes with virtual fermions or gauge bosons depend on motion with respect to the plasma. Thermal averages become too cumbersome. Simplify assuming $g^2 \ll 1$ or $T \ll M_1$.

Radiative corrections

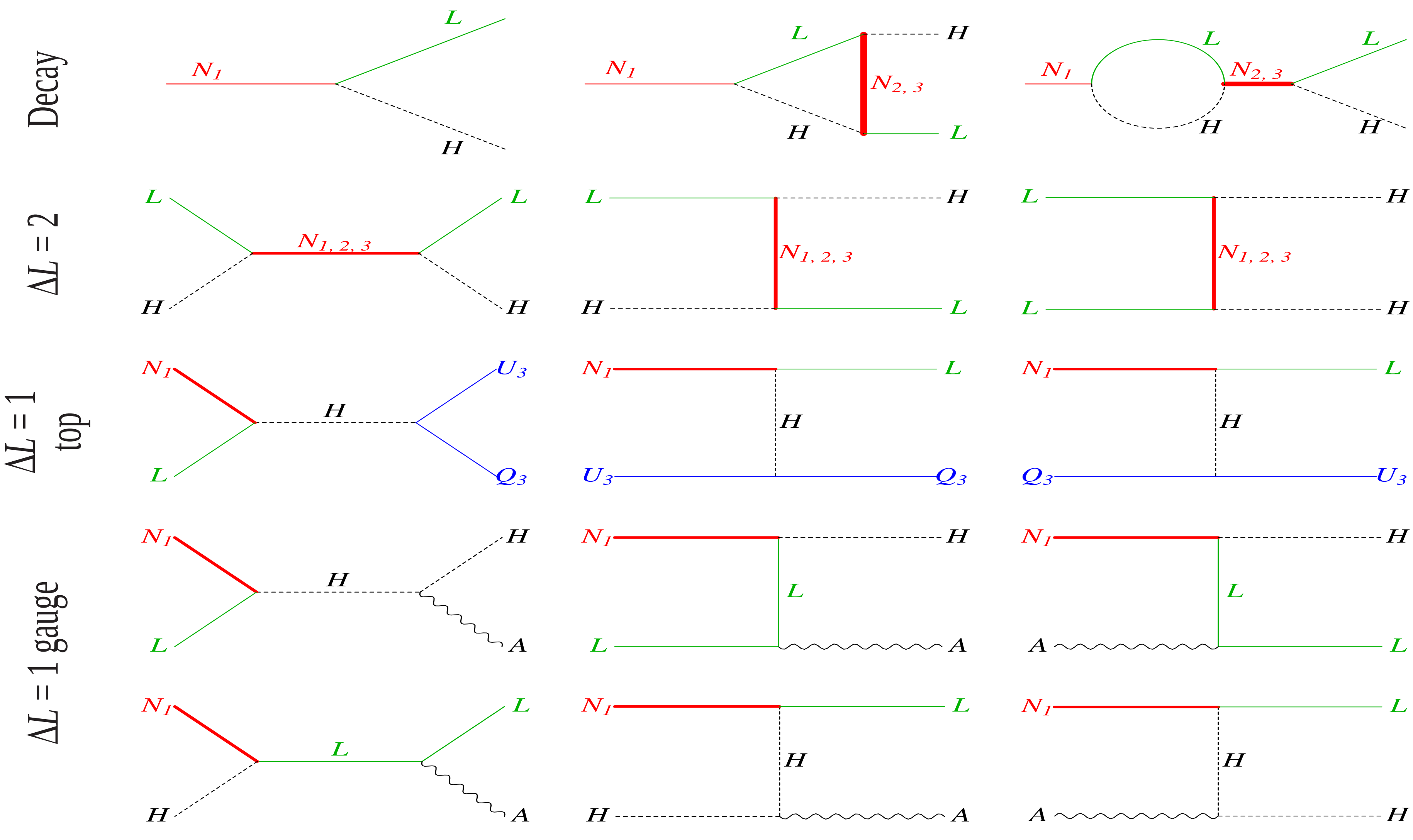
Use couplings renormalized at $\sim 2\pi T$, not at M_Z.



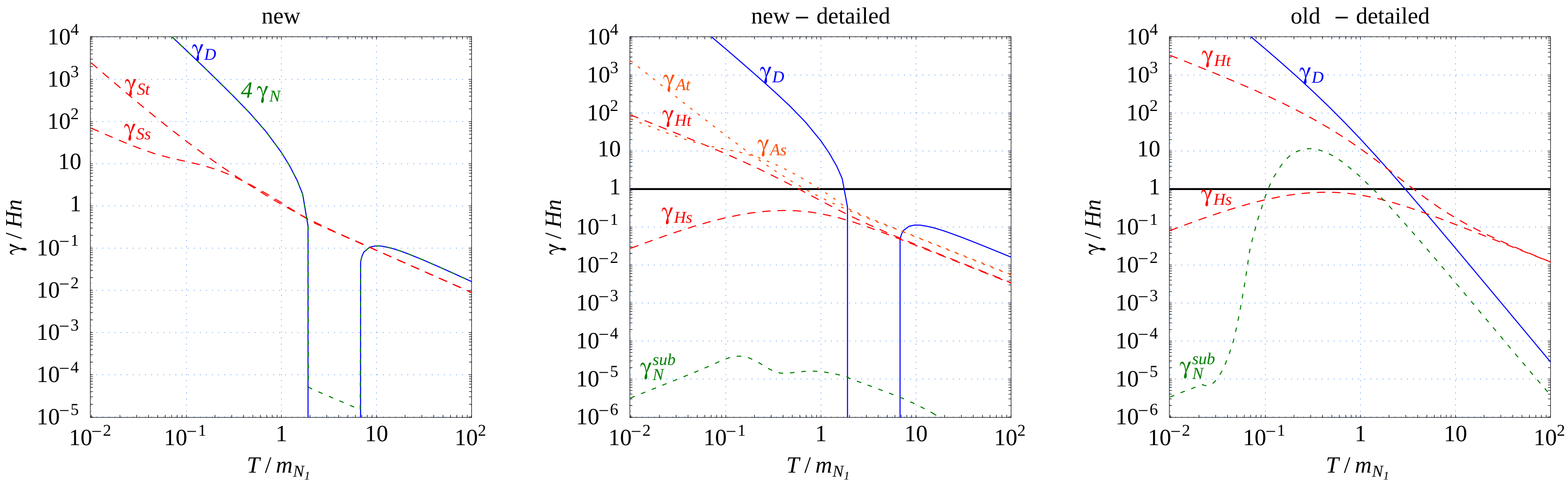
makes $\nu \sim 25\%$ heavier and $\lambda_t \lesssim g_2 \sim 0.5$ smaller

Gauge and top Yukawa couplings

Add $\Delta L = 1$ gauge scatterings, more important than top scatterings



Rates for $\tilde{m}_1 = 0.06\,\text{eV}$ and $M_1 = 10^{10}\,\text{GeV}$



γ_D is dominant.

- $\Delta L = 2$ is not resonantly enhanced off-shell.

- Without thermal masses at $T \gg M$ the scattering rates $\gamma_S \sim g^2 T^4/(4\pi)^2$ were more important that $\gamma_D \propto T^2 M^2$ (one power of T lost because $\Gamma_{\text{at rest}} \propto M$, another power due to Lorentz contraction M/T). It has never been precisely computed at $T \sim M$ (quasi-particles, continuum).

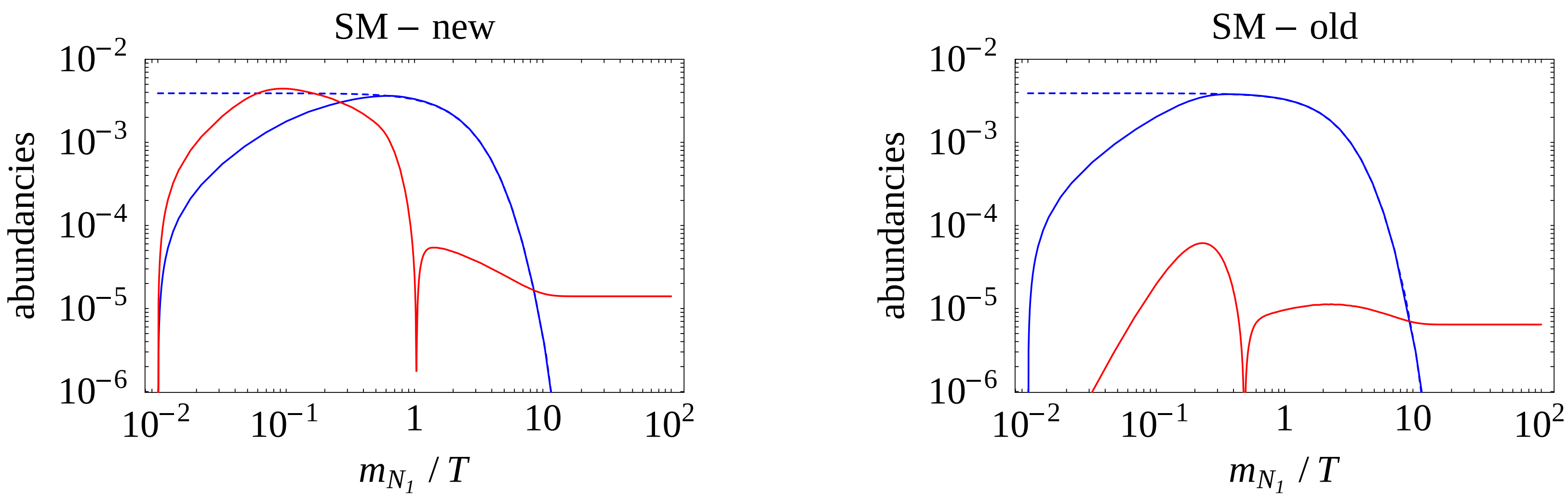
Results

Finally, n_B

Put all non trivial physics in η:

$$Y_B = \frac{n_B}{s} = -1.38 \times 10^{-3} \cdot \varepsilon(T=0) \cdot \eta$$

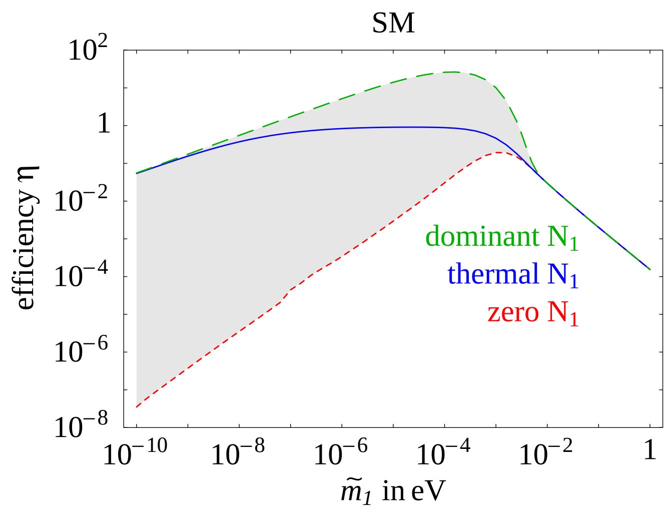
Evolution of Y_{N_1} and of Y_B/ε at the 'atmospheric' sample point $\tilde{m}_1 = 0.06\,\text{eV}$:

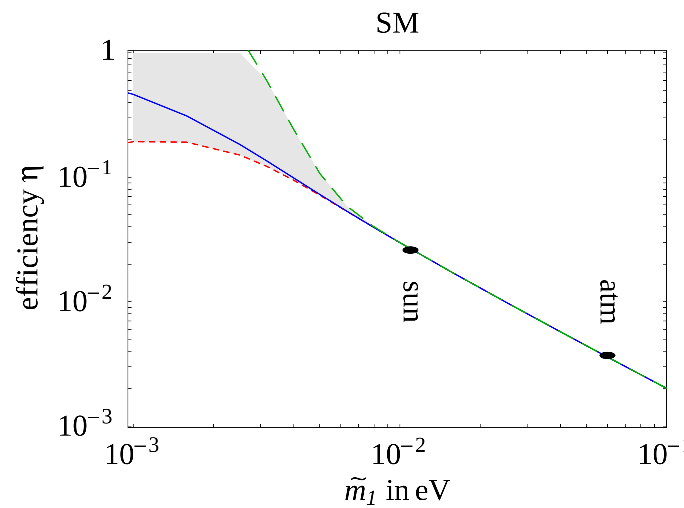


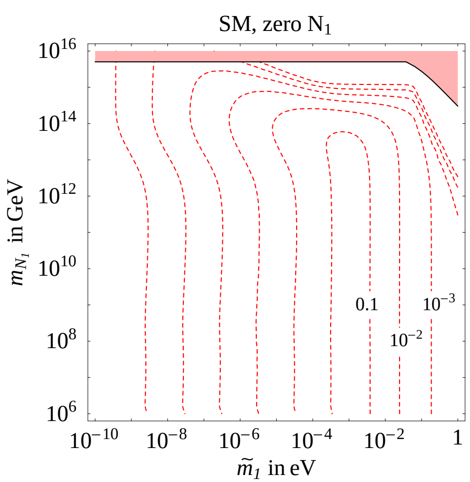
Result: $\eta = 0.0036$

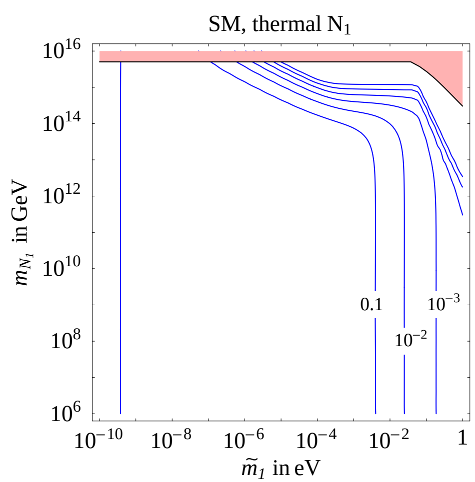
Efficiency of SM leptogenesis

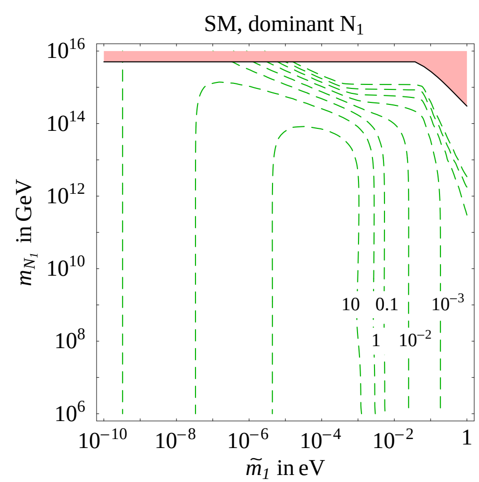
η does not depend on the initial conditions if N_1 gets close to thermal equilib.





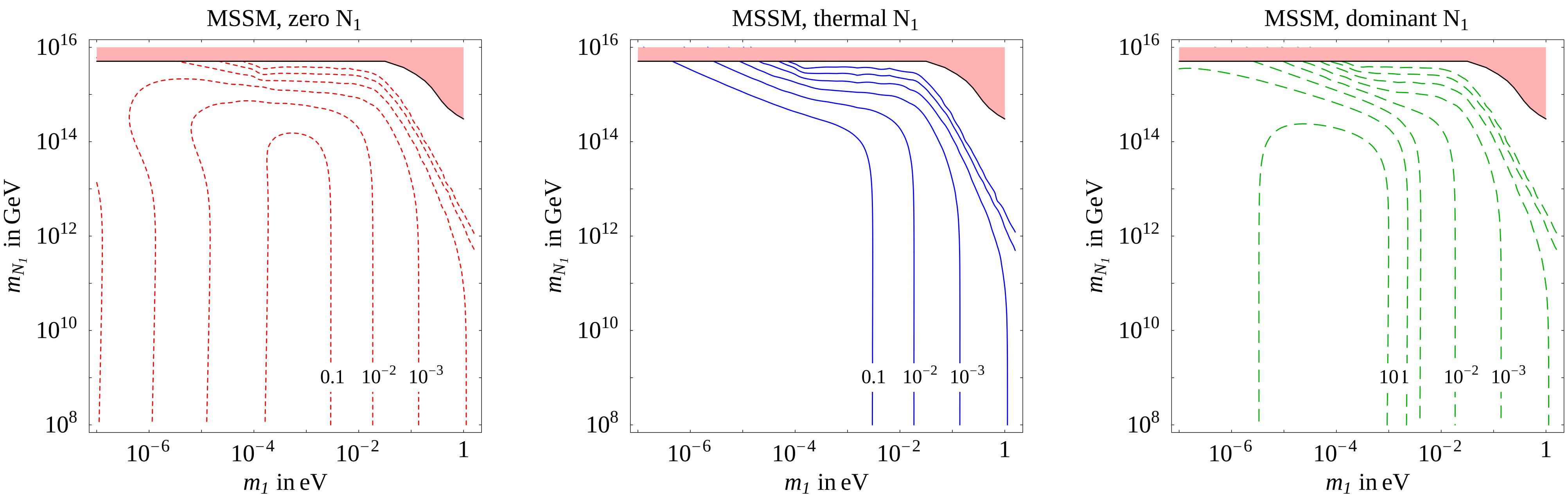






SM → MSSM

SUSY breaking likely to be irrelevant. Even using $M_{N_1} = M_{\tilde{N}_1}$ and $\Gamma_{N_1} = \Gamma_{\tilde{N}_1}$ MSSM remains a mess. Since sparticle masses unknown we just subtract resonances fully, renormalize couplings, $\varepsilon(T)$, add IR-enhanced thermal effects



$\tilde{N}_1$ could have large vev and dominate

Higher order corrections

The gauge and top Yukawa couplings give sizable effects at $T \sim M$: e.g. $M \sim gT$. Hopefully $\tilde{m}_1 \gg m^*$ so that only $T \ll M$ is relevant and leptogenesis does not depend on initial conditions. Then NLO corrections are of order $\mathcal{O}(g^2, \lambda_t^2)/\pi^2 \lesssim 10\%$. Many corrections to be included:

- To the expansion rate: $\rho_{\rm SM} = \left[\frac{427}{4}\frac{\pi^2}{30} - \frac{7}{4}g_3^2 - \ldots\right] T^4$.

- To the N interaction rate: scatterings (e.g. $AN \to LH$); 3 body decays (e.g. $N \to LHA$); one loop corrections to $N \to LH$. It is precisely defined as the imaginary part of the N propagator at finite temperature, and the KLN theorem tells that the result must not depend on infrared details (masses of L, H, A), so the result is simple:

$$\gamma_N = \gamma_D^{\rm tree\ level}\left[1 + \frac{15}{16\pi}(3\alpha_2 + \alpha_Y) + \mathcal{O}(g^2)\frac{T^2}{M^2} + \text{top effects}\right]$$

- to the CP asymmetry and washouts (is CPT violated at $T \neq 0$?)

TESTING LEPTOGENESIS?

Is a precise computation useless?

peut-être..

GUT, see-saw, leptogenesis, (SUSY) form the Invincible Armada.
Main hard problem is discriminating right/wrong/'not even wrong'.

Leptogenesis allows to compute n_B in terms of particle physics.
But see-saw 'predicts' 9 Majorana ν parameters in terms of 18 parameters.
(Im)possible ways of testing leptogenesis:

- theorists could understand flavour with symmetries/numerology/zerology
- hope that $M \sim$ TeV with large enough couplings
- exps could discover δ, m_{ee}, SUSY, $\mu \to e\gamma$, $\tau \to \mu\gamma$, allowing archeology

In the meantime, leptogenesis (+ extra assumptions) gives concrete bounds

A concrete attempt: most minimal see-saw

Ignore • elegant postdictions • predictions up to $\mathcal{O}(1)$ factors • predictions involving $\theta_{23}-\pi/4$ and $\not{C}\not{P}$ because hard to test precisely • fine-tunings

One possibility is see-saw texture with $N_{1,2}$ (I could explain 0 in a decent way)

$$\lambda_N = \begin{matrix} & L_e & L_\mu & L_\tau \\ N_1 & * & * e^{i\star} & 0 \\ N_2 & 0 & * & * \end{matrix} \qquad M_N = \begin{pmatrix} * & 0 \\ 0 & * \end{pmatrix} \qquad \lambda_E = \begin{pmatrix} * & 0 & 0 \\ 0 & * & 0 \\ 0 & 0 & * \end{pmatrix}$$

It predicts

$$\theta_{13} \simeq \frac{1}{2}\sqrt{\frac{\Delta m^2_{\text{sun}}}{\Delta m^2_{\text{atm}}}} \sin 2\theta_{12} \tan\theta_{23} = 0.085 \pm 0.013 \qquad |m_{ee}| = (2.4 \pm 0.2)\,\text{meV}$$

Depending on the relative sign between CP-violation in ν osc and leptogenesis (**not** predicted, even if the model has a single phase), it predicts either

$$\text{BR}(\mu \to e\gamma) = 10^{-14} (\frac{\tan\beta}{10})^2 (\frac{150\,\text{GeV}}{m_{\text{SUSY}}})^4$$

or $\tau \to \mu\gamma$ (if/once m_{SUSY} is measured we can be precise).

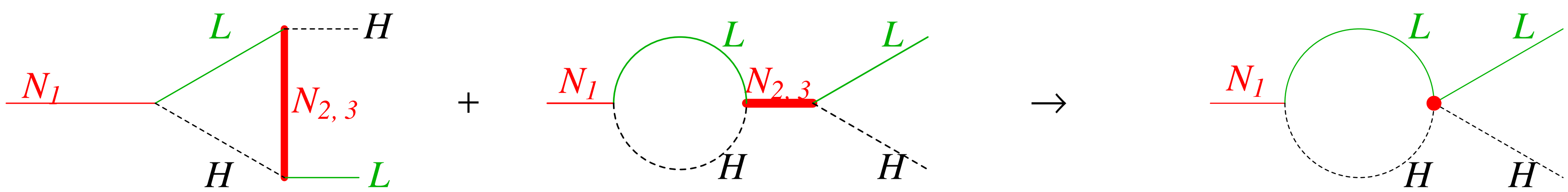
Constraints from leptogenesis

For $M_{2,3}/M_1 = \infty$

Maximal ε for $M_{2,3}/M_1 = \infty$

Assuming infinitely hierarchical ν_R, ε is directly related to $\tilde{\boldsymbol{m}}_2 + \tilde{\boldsymbol{m}}_3$.

($\tilde{\boldsymbol{m}}_i \equiv \nu$ mass matrix generated by N_i ; $\boldsymbol{m}_\nu = \tilde{\boldsymbol{m}}_1 + \tilde{\boldsymbol{m}}_2 + \tilde{\boldsymbol{m}}_3$)



Rigorous bound [Davidson-Ibarra]:

$$|\varepsilon| = \frac{3}{16\pi}\frac{M_1}{v^2}\frac{|\mathrm{Im\,Tr}\,\tilde{\boldsymbol{m}}_1^\dagger(\tilde{\boldsymbol{m}}_2 + \tilde{\boldsymbol{m}}_3)|}{\tilde{m}_1} \le \frac{3}{16\pi}\frac{M_1}{v^2}(m_3 - m_1)$$

m_3 because $\tilde{m}_2 + \tilde{m}_3 = m_\nu - \tilde{m}_1$, in any model.

$m_3 - m_1$ because 3 degenerate ν imply flavor-orthogonality.

Implications

ν_R cannot be too light because $\varepsilon \propto M_1$

ν_L cannot be too heavy: $m_3 - m_1 \simeq \Delta m^2_{\rm atm}/2m_\nu$

Maximal n_B for $M_{2,3}/M_1 = \infty$

A refinement.

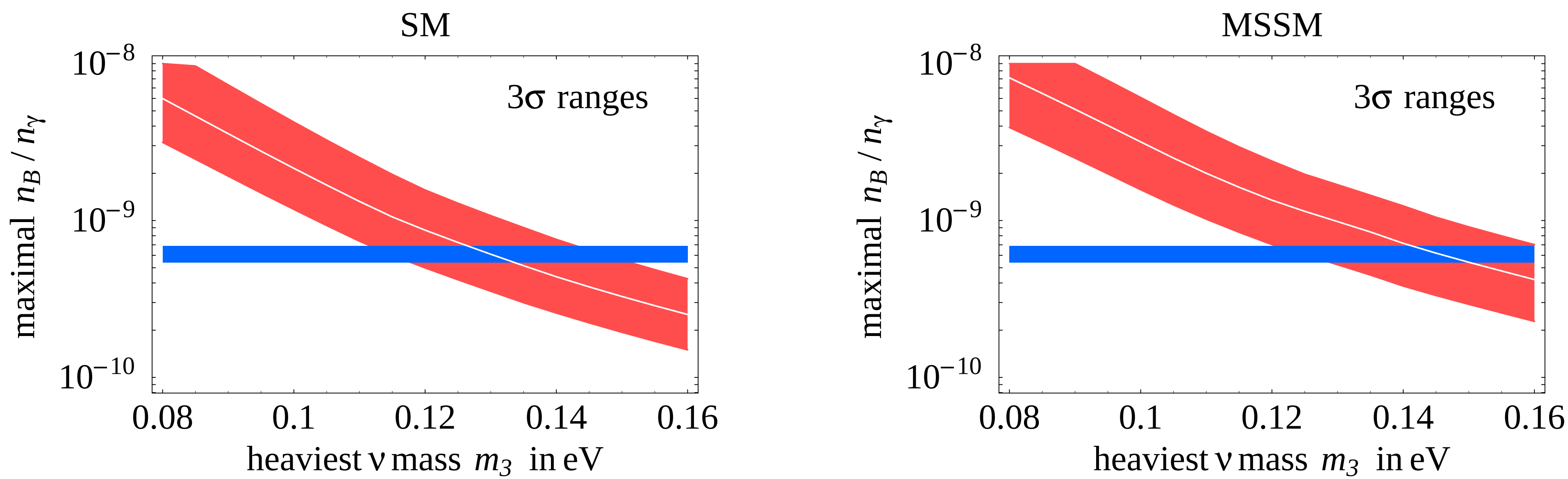
We measure $n_B/n_\gamma \approx \varepsilon\eta/100$, not ε.
η depends on $\tilde{m}_1$: we need the maximal ε at fixed $\tilde{m}_1$.
Since $\tilde{m}_1 > m_1$ this is relevant for large m_1. We get

$$|\varepsilon| \le \varepsilon_{\rm DI}^{\rm max} \times \begin{cases} \sqrt{1 - m_1^2/\tilde{m}_1^2} & \text{for quasi-degenerate } \nu\text{: large } m_1 \simeq m_3 \\ 1 - m_1/\tilde{m}_1 & \text{for hierarchical } \nu\text{: } m_1 \ll m_3 \\ \text{longer expression in the generic case} & \end{cases}$$

Constraint on ν_L masses for $M_{2,3}/M_1 = \infty$

Thermal leptogenesis fails if neutrinos are too heavy and degenerate due to:

- Flavor orthogonality: small $\varepsilon \propto m_3 - m_1 \simeq \Delta m^2_{\text{atm}}/2m_1$.
- Wash-out: small $\eta \simeq m_*/\tilde{m}_1$ and $\tilde{m}_1 > m_1$.

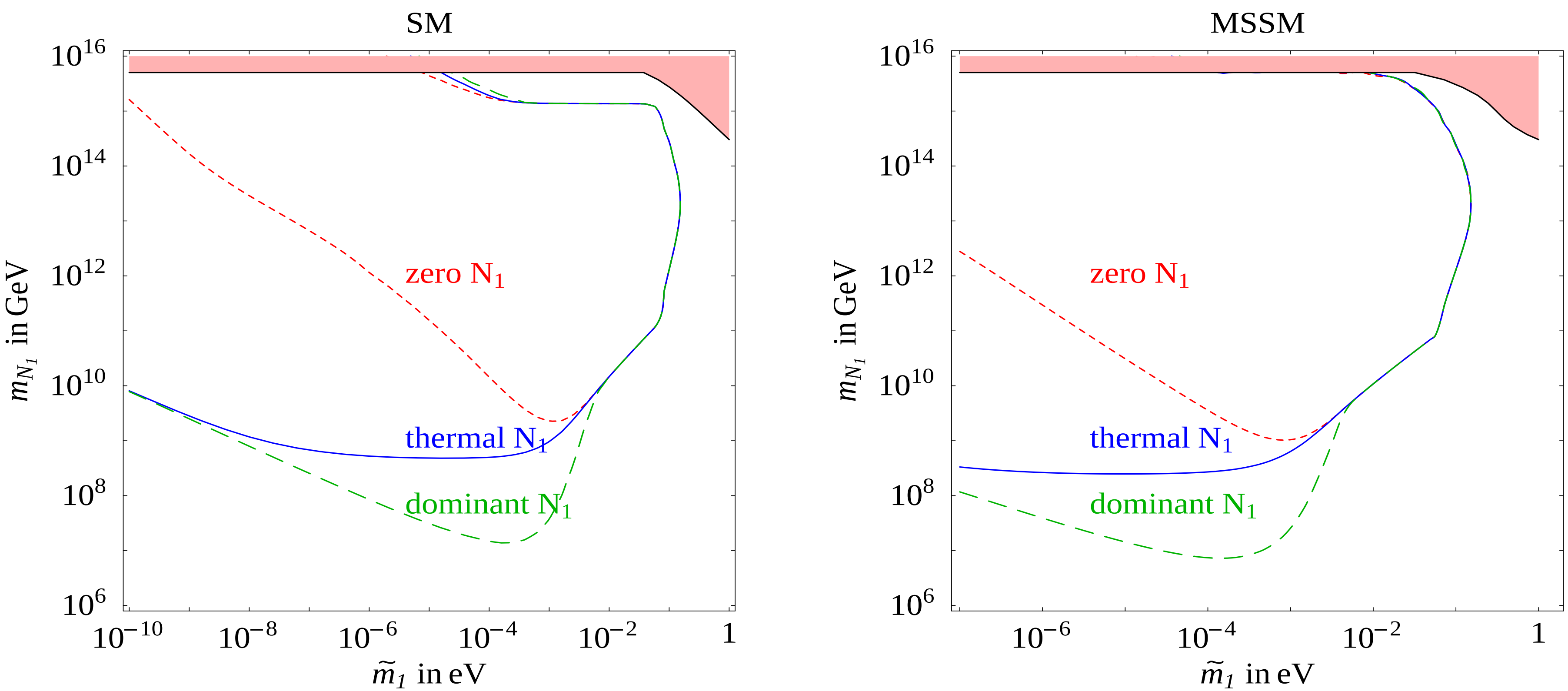


$m_\nu < 0.15\,\text{eV}$ at 3σ in the SM

Other 95% CL: • $m_\nu < 2.2\,\text{eV}$ from β • $m_\nu < 1.0h$ eV from $0\nu 2\beta$ + Majorana
• $m_\nu < 0.2\,\text{eV}$ from cosmology (LSS + WMAP + ΛCDM+ minimal inflation)

Constraint on ν_R masses for $M_{2,3}/M_1 = \infty$

Assume $m_3 = \max(\tilde{m}_1, m_{\text{atm}})$ and $\xi = m_3/\tilde{m}_1$ (detail related to $\Delta L = 2$...)



In the SM $M_1 > \dfrac{4.5 \times 10^8 \text{ GeV}}{\eta} > \begin{cases} 24 \times 10^8 \text{ GeV} & \text{in case (0)} \\ 4.9 \times 10^8 \text{ GeV} & \text{in case (1)} \\ 0.17 \times 10^8 \text{ GeV} & \text{in case } (\infty) \end{cases}$

In MSSM a similar constraint, possibly in **conflict** with gravitinos $T \lesssim 10^8$ GeV.

The gravitino constraint

- Gravitinos $\tilde{G}$ are (expected to be) spin-3/2 partners of gravitons.
- Gravitational couplings to matter: $(q\tilde{q} + g\tilde{g})\tilde{G}/M_{\rm Pl}$.
- Expected mass: $\text{eV} \lesssim m_{\tilde{G}} \lesssim 100\,\text{TeV}$.
- The gravitino might be the stable LSP or slowly decay after BBN

$$\tau_{\tilde{G}} \sim \frac{M_{\rm Pl}^2}{m_{\tilde{G}}^3} \sim \text{sec}\left(\frac{100\,\text{TeV}}{m_{\tilde{G}}}\right)^3$$

- Rate of thermal gravitino production

$$\gamma_{\tilde{G}}(T) \sim \frac{T^6}{M_{\rm Pl}^2} \qquad \frac{n_{\tilde{G}}}{n_\gamma} \sim \frac{\gamma_{\tilde{G}}}{H n_\gamma} \sim \frac{T_{\rm max}}{M_{\rm Pl}} \qquad \Omega_{\tilde{G}} \sim \frac{m_{\tilde{G}}}{\text{TeV}} \frac{T_{\rm max}}{10^{10}\,\text{GeV}}$$

- So $T_{\rm max} \lesssim 10^9\,\text{GeV}$ from $\Omega_{\tilde{G}} < \Omega_{\rm DM}$.
  Possibly stronger bound: $\tilde{G}$ or NLSP decays can damage BBN.

Constraints on ν_R masses

For $M_{2,3} \gg M_1$ but not ∞

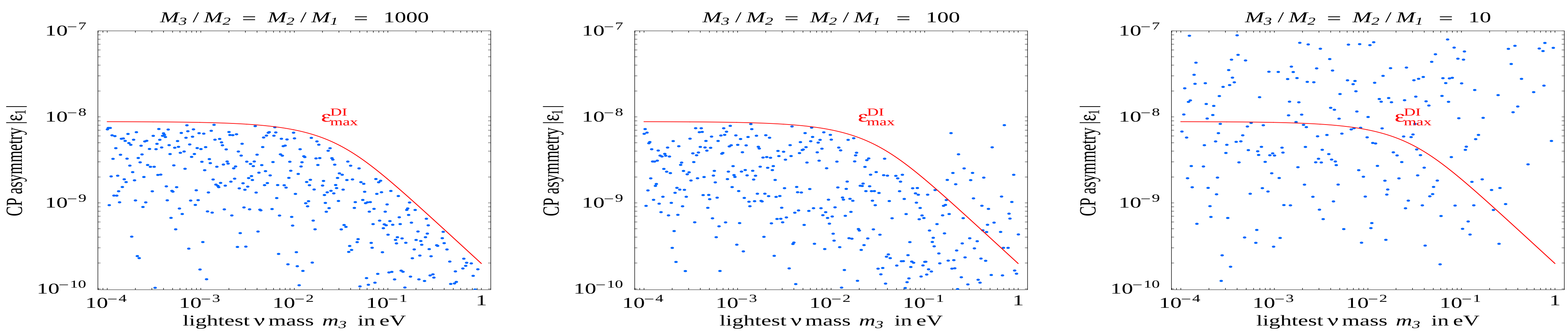
Maximal ε for $M_{2,3}/M_1$ big but $< \infty$

Higher order terms in $M_1/M_{2,3}$ are not directly related to ν masses:

$$\varepsilon \sim \frac{3}{16\pi}\frac{M_1}{v^2}\left[\tilde{m}_2\left(1+\frac{M_1^2}{M_2^2}\right)+\tilde{m}_3\left(1+\frac{M_1^2}{M_3^2}\right)\right] \lesssim \max\left(\varepsilon^{\rm DI}_{\rm max}, \frac{M_1^3}{M_3 M_2^2}\right)$$

$\tilde{m}_2+\tilde{m}_3$ cannot be big and complex, while $\tilde{m}_2$ and $\tilde{m}_3$ can.
Enhancement limited only by $\lambda_{2,3} \lesssim 4\pi$.

Confirmed by random sampling: for $M_1 = 10^8$ GeV ($m_3/m_2 \lesssim 6$ in ν_L):



Minimal leptogenesis in minimal see-saw is compatible with minimal SUSY.

Gravitino problem avoided if $N_{2,3}$ give large contributions $\tilde{m}_{2,3} \gg m_{2,3}$ to neutrino masses, which cancel out among themselves. An unnatural pattern?

Constraints on ν_L masses

For $M_{2,3} \sim M_1$

$M_1 \simeq M_2$: resonant leptogenesis

If the 2 lightest right-handed ν are quasi-degenerate, there is a new effect: CP-violation in N_1/N_2 mixing, analogous to CP-violation in $K^0/\bar{K}^0$ mixing:

$$\varepsilon_1 \approx \frac{\Delta M^2_{12} \cdot M_1 \Gamma_2}{(\Delta M^2_{12})^2 + (M_1 \Gamma_2)^2}$$

It gives maximal $\varepsilon_1 \sim 1$ for $M_2 - M_1 \sim \Gamma$, allowing $M_1 \sim$ TeV.

With supersymmetry one more analogous possibility: 'soft leptogenesis'.

Constraint on ν_L masses

Good taste suggests that quasi-degenerate ν_L come from quasi-degenerate ν_R.

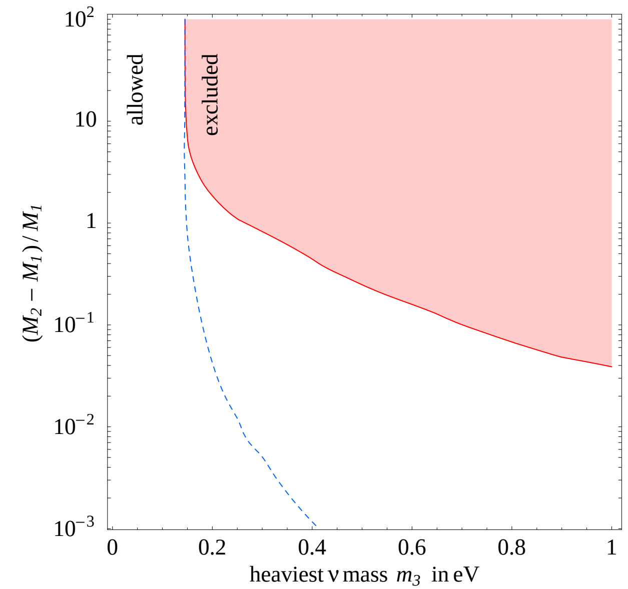
Maximal m_3 depends on why ν_L should be quasi-degenerate:

A) no flavour symmetry acts on $\nu_{L,R}$ so that $\theta \sim 1$

$$m_1 \approx m_2 \approx m_3 \approx \tilde{m}_1 \approx \tilde{m}_2 \approx \tilde{m}_3 \qquad M_1 \approx M_2 \approx M_3$$

ϵ can be resonantly enhanced and no longer suppressed by $\sim 1 - m_1/m_3$: this is the conservative case

$m_3 <$ eV for 10% degeneracy



B) SO(3)-like flavour symmetry keeps **all** quasi-degenerate

$$\Delta\tilde{m} \approx \Delta m, \qquad \frac{\Delta M}{M} \approx \frac{\Delta m}{m} \approx 10^{-3}$$

ϵ can be resonantly enhanced but suppressed by $\sim (1 - m_1/m_3)^{3/2}$:

$m_3 < 0.6\,\text{eV}$ or larger if loose $\approx$

Summary of constraints from thermal leptogenesis

Constraint on ν_L mass:

A) $M_{2,3} \gg M_1$: $m_\nu < 0.15\,\text{eV}$ at 3σ

B) $M_{2,3} \sim M_1$ and $M_2 \simeq M_1$: no relevant constraint.
$m_\nu \lesssim 0.6$ eV making reasonable aggressive assumptions.

Constraint on ν_R mass:

A) $M_{2,3} \gg M_1$:
$M_1 > 4.9\ 10^8\,\text{GeV}$ if N_1 initially has (sub-)thermal abundance.
(In SUSY models, this likely conlicts with gravitino over-abundance).
$M_1 > 0.17\ 10^8\,\text{GeV}$ if N_1 dominates energy density.

B) $M_{2,3} \lesssim 10 M_1$: no constraint. Natural in models with detectable SUSY-LFV.

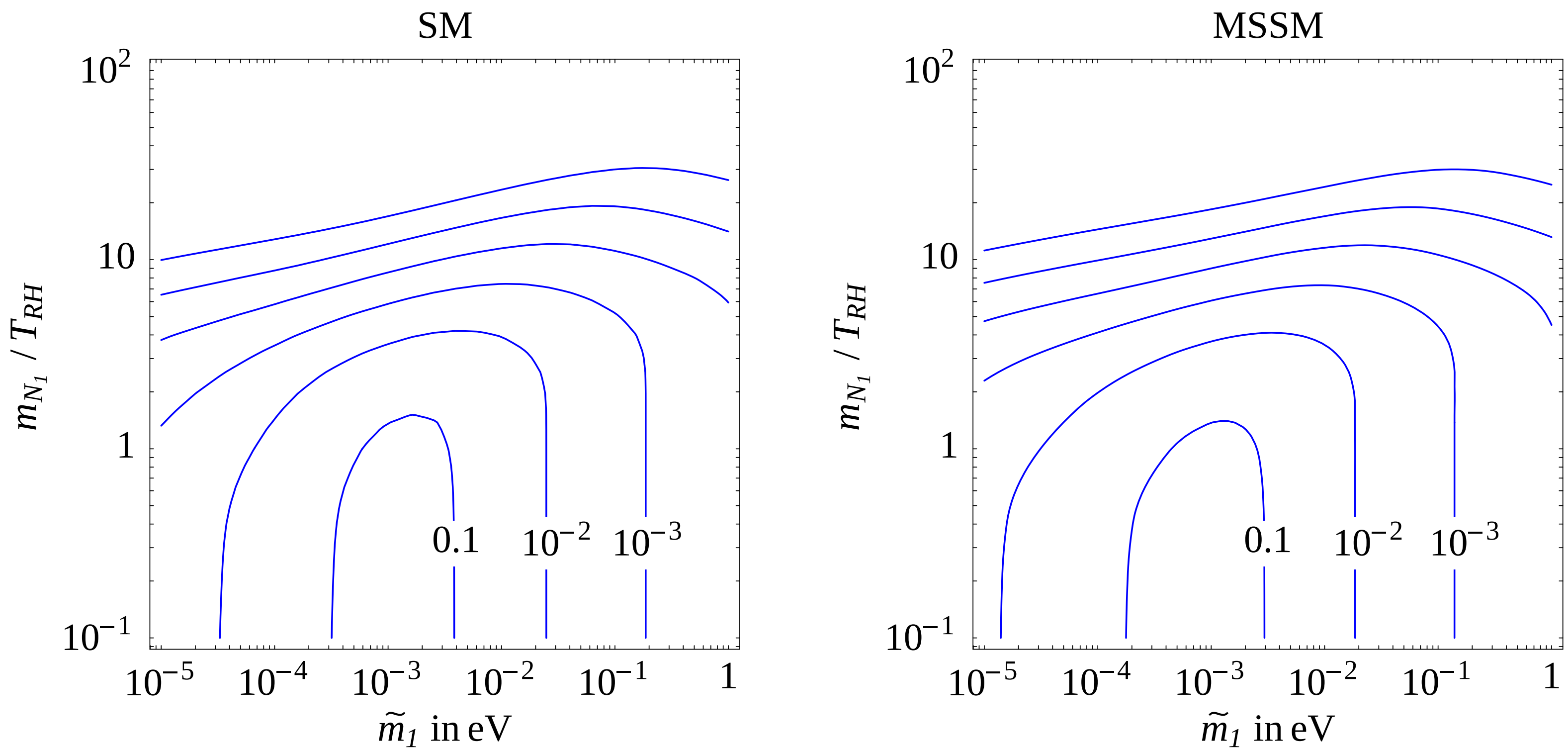
C) $M_2 \simeq M_1$: no constraint.

Leptogenesis during reheating

Inflaton reheating

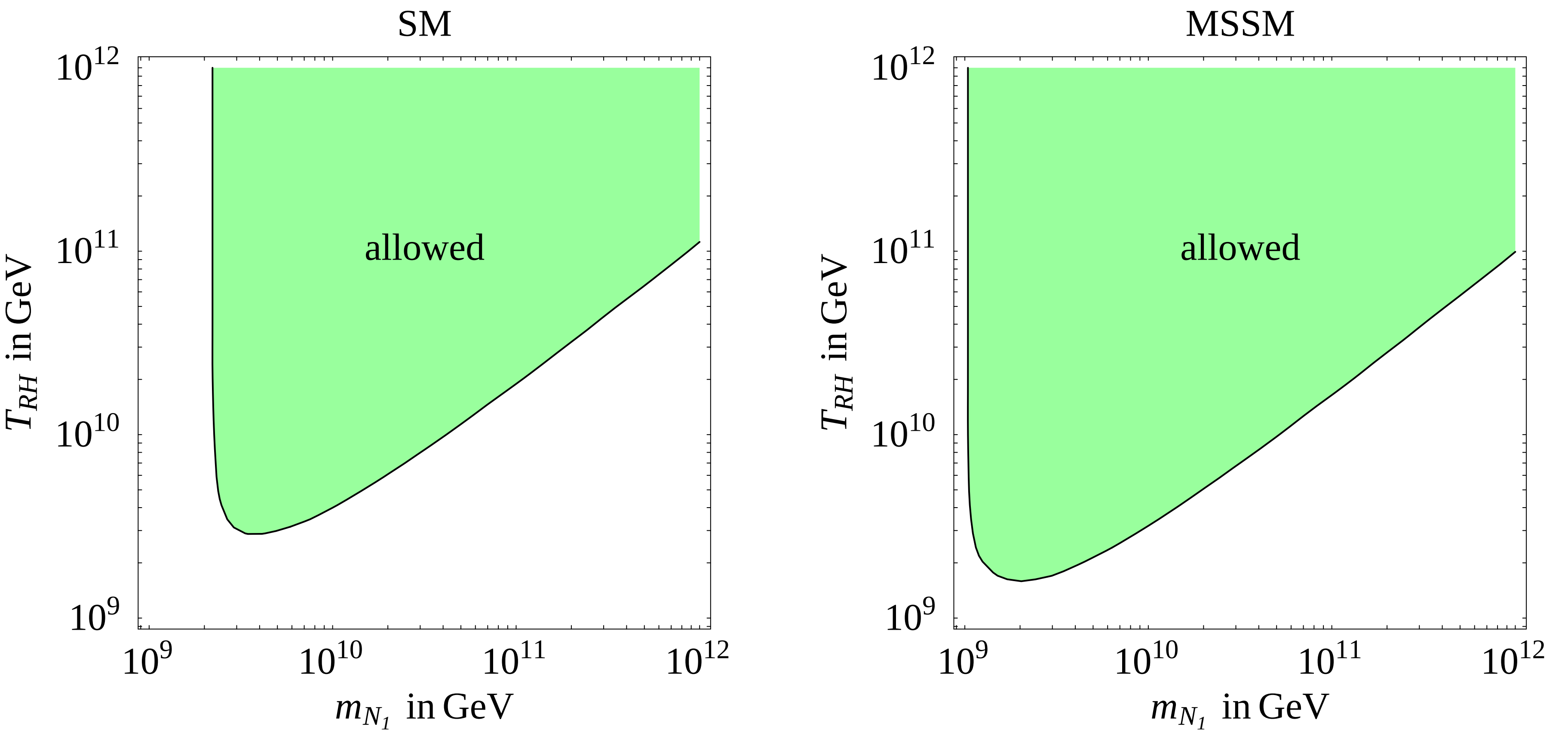
Leptogenesis might proceed while inflaton is reheating universe. Described by a unique extra parameter, the temperature T_{RH} at which inflaton 'decays'. Assume it reheats SM particles but not directly N_1:

$\eta =$



2 main effects. • inflaton decays generating extra SM particles giving a $\sim (M_1/T_{\text{RH}})^5$ dilution of n_B. • inflaton ρ makes expansion faster: $H/H_{\text{standard}} \approx (T/T_{\text{RH}})^2$ increasing the value of $\tilde{m}_1$ at which leptogenesis is maximally efficient.

Bound on $T_{\rm RH}$ for $M_{2,3}/M_1 = \infty$



$T_{\rm RH} \gtrsim 2\ 10^9$ GeV if inflaton decays to SM particles

Triplet leptogenesis

The minimal alternatives to the standard scenario

Alternative ν masses: $2 \times 2 = 3 + 1$

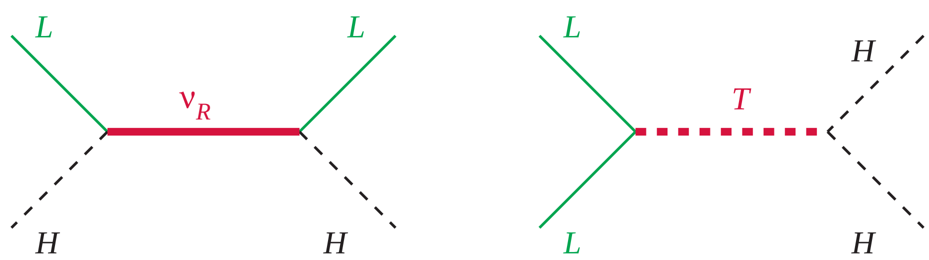
Generic Majorana ν masses can be mediated by tree-level exchange of:

N) At least three fermion singlets ('right-handed neutrinos').

$$\mathscr{L} = \mathscr{L}_{\mathrm{SM}} + \lambda_N^{ij} N_i L_j H + \frac{M_N^{ij}}{2} N_i N_j. \qquad \text{18 param.s}$$

N^a) At least three fermion SU(2)$_L$ triplets:

$$\mathscr{L} = \mathscr{L}_{\mathrm{SM}} + \lambda_N^{ij}\ \tau^a_{\alpha\beta} N_i^a L_j^\alpha H^\beta + \frac{M_N^{ij}}{2} N_i^a N_j^a \qquad \text{18 param.s}$$



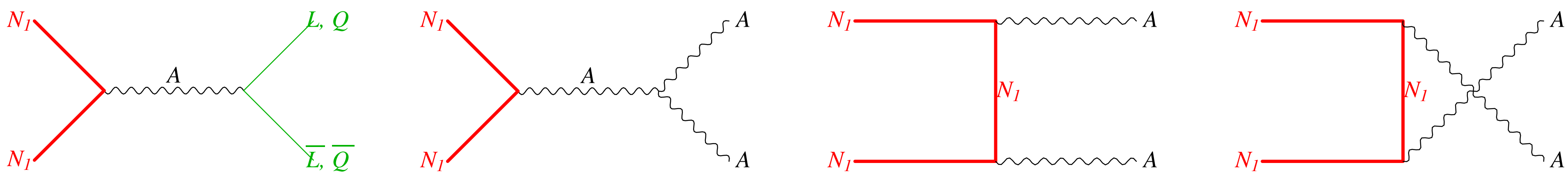
T^a) At least one scalar ('Higgs') triplet T with $Y = 1$:

$$\mathscr{L} = \mathscr{L}_{\mathrm{SM}} + \lambda_T^{ij} L^i L^j T - M_T^2 |T|^2 + M\ HHT^*. \qquad \text{11 param.s}$$

Alternative leptogenesis?

Naïve expectation: leptogenesis works only for neutral ν_R, because charged N^a or T^a are kept in thermal equilibrium by gauge scatterings $\gamma_A \sim g^2 \gg \gamma_D \sim \lambda^2$:

$$sHz\frac{dY}{dz} = -(\frac{Y}{Y_{\rm eq}} - 1)\gamma_D - 2(\frac{Y^2}{Y_{\rm eq}^2} - 1)\gamma_A$$



True result: γ_A involves 2 massive N^a or T^a and is doubly Boltzmann suppressed at $T \ll M$. Leptogenesis efficient enough even for $M \sim$ TeV.

$$\eta(\text{fermion singlet}) \approx \min\left[\frac{H}{\Gamma}, X\right] \qquad X = \{1, \Gamma/H, g_{\rm SM}\}$$

$$\eta(\text{fermion triplet}) \approx \min\left[\frac{H}{\Gamma}, \frac{M}{10^{12}\,\text{GeV}}\max(1, \frac{\Gamma}{H})\right]$$

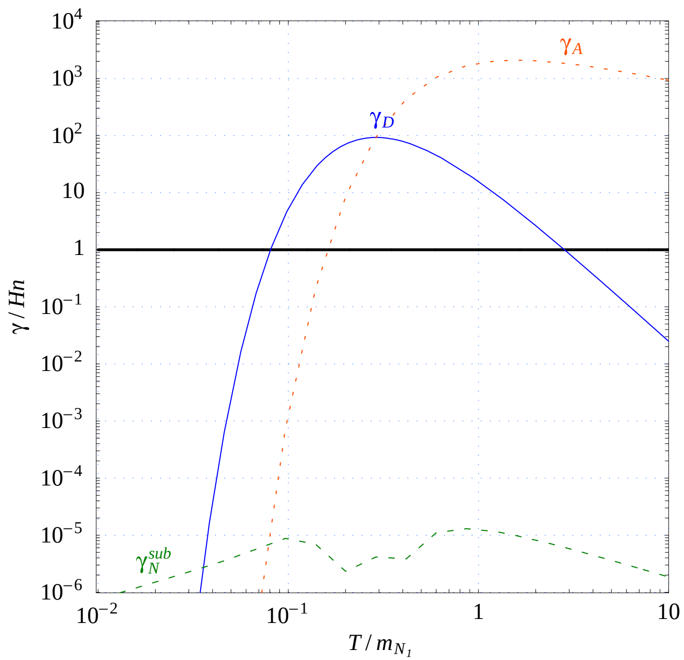
max because when $\Gamma \gg H$, gauge scatterings have to compete with Γ

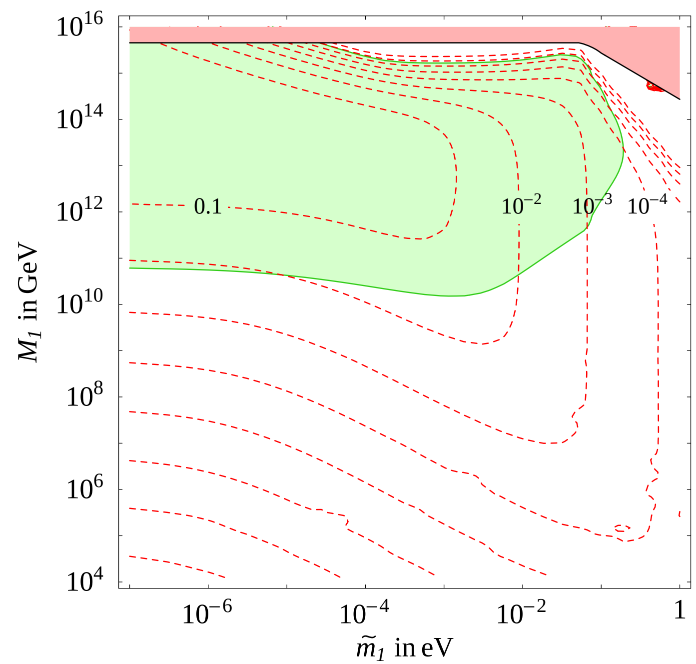
Fermion triplet

Effects of gauge scatterings:

At $T \gg M_1$ thermalize N_1 abundance, making leptogenesis more predictive.

At $T \sim M_1$ annihilate most N_1, but a fraction $M_1/10^{11}$ GeV survives. These N^a decay later (if small $\tilde{m}_1$) or during (large $\tilde{m}_1$) annihilations producing n_B.





$M_1 \gtrsim 1.5\ 10^{10}$ GeV $\qquad m_3 < 0.12$ eV $\qquad$ if $M_{2,3}/M_1 = \infty$

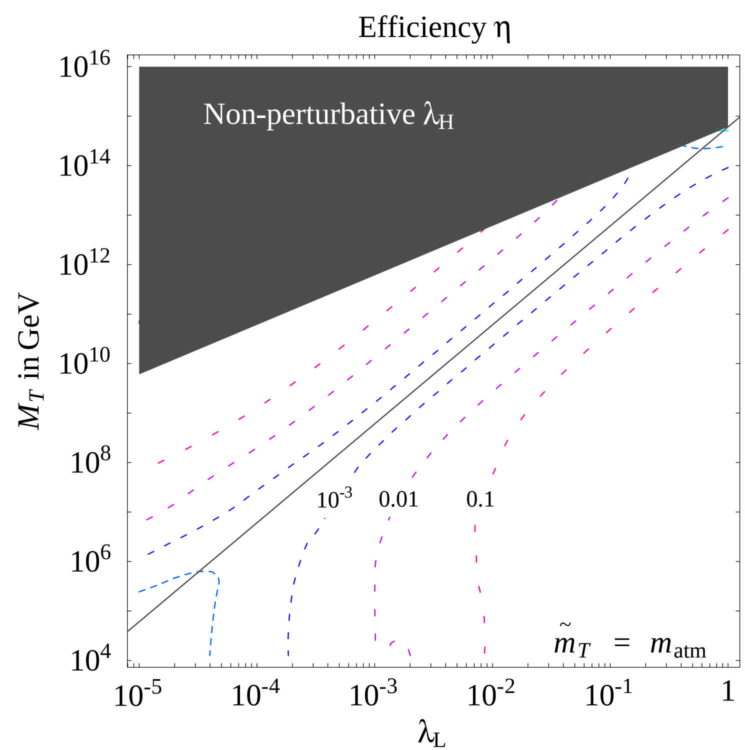
Scalar triplet

Decay channes: $T \to LL$ and $T \to H^*H^*$ with BR B_L and B_H. CP asymmetry

$$\varepsilon_L \equiv 2\frac{\Gamma(\bar{T} \to LL) - \Gamma(T \to \bar{L}\bar{L})}{\Gamma_T + \Gamma_{\bar{T}}} = \frac{1}{4\pi}\frac{M_T}{v^2}\sqrt{B_L B_H}\frac{\mathrm{Im}\,\mathrm{Tr}\,\boldsymbol{m}_T^\dagger \boldsymbol{m}_{\text{heavier}}}{\tilde{m}_T}$$

where $\boldsymbol{m}_\nu = \boldsymbol{m}_T + \boldsymbol{m}_{\text{heavier}}$. New main features:

- Gauge scatterings keep Y_T close to thermal equilibrium. Irrelevant if $\gamma_D \gg \gamma_A$.
- Big efficiency η even for large γ_D. Lepton number is violated by the contemporaneous presence of λ_L and λ_H, so that the lepton asymmetry is washed-out only when both partial decay rates to leptons and Higgses are faster than the expansion rate.



Summary

We want to understand what generates ν masses.
We want to understand what generates baryons.

See-saw is a plausible common answer.

Precise computations of thermal leptogenesis completed. In the SM it works.

The key issue is finding a way of testing leptogenesis.

Unfortunately this is much more difficult than proposing or computing it.

Supplement with: assumptions, flavour models, GUT, archeology. . .

In SUSY possible incompatibility: enough baryons need too much gravitinos.
One natural and predictive minimal way of avoiding the conflict.