Braneworlds: gravity & cosmology

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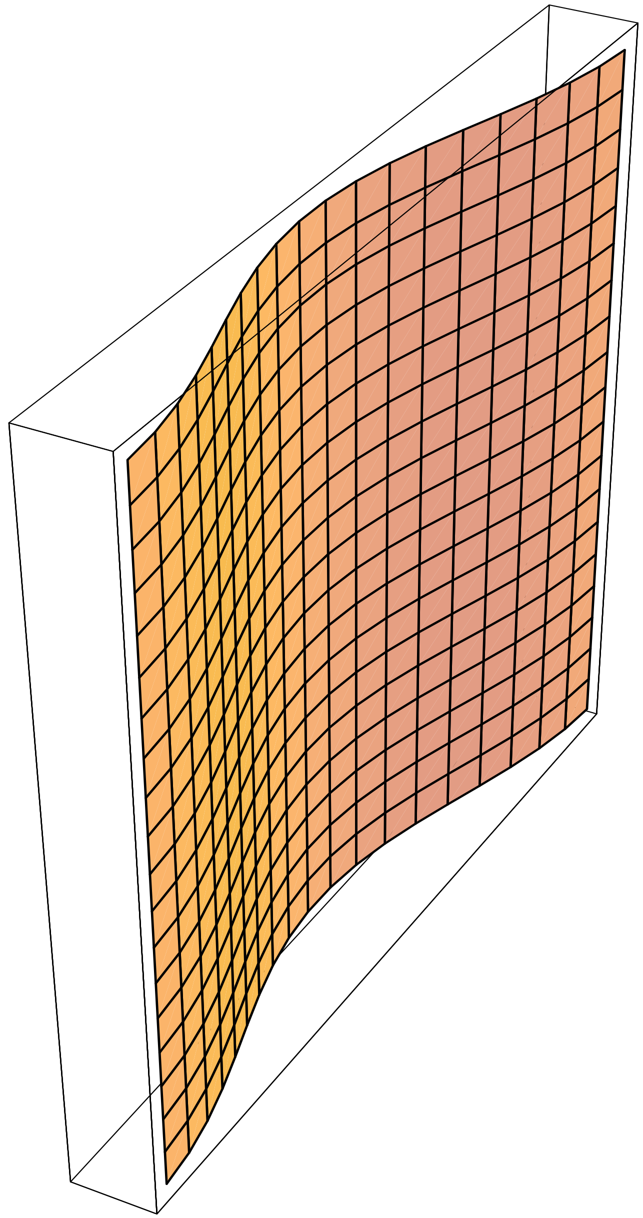
Astroparticules et Cosmologie

Outline

- Introduction
- Extra dimensions and gravity
  - Large (flat) extra dimensions
  - Warped extra dimensions
- Homogeneous brane cosmology
- Brane cosmology and cosmological perturbations
- Other models



Brane-world: introduction

- Based on two ideas:
  - Extra dimensions ⇨ higher dim spacetime: the "bulk"
  - Confinement of matter to a subspace: the "brane" (in contrast with Kaluza-Klein approach)

- Motivations
  - Strings: D-branes, Horava-Witten supergravity; AdS/CFT
  - Particle physics: hierarchy problem (why $M_{EW} \ll M_{Pl}$?), etc…
  - **Gravity**: new compactification scheme

- Many models; essentially two categories:
  - flat compact extra-dimensions
  - **warped extra-dimensions**

Kaluza-Klein approach

- Single extra-dimension

Compactification $y \to y + 2\pi R$

- Example of matter field: scalar field

$$\phi(x^\mu, y) = \sum_n e^{i\frac{ny}{R}} \phi_n(x^\mu)$$

$$\partial^2_{(5)}\phi - m^2\phi = 0 \quad \Longrightarrow \quad \partial^2_{(4)}\phi_n - \underbrace{\left(m^2 + \frac{n^2}{R^2}\right)}_{\text{effective 4d mass}} \phi_n = 0$$

5d Klein-Gordon eq

- Experimental constraints

$$R_{KK} < (10^2 \text{ GeV})^{-1} \sim 10^{-20} \text{ m}$$

Large (flat) extra dimensions

Arkani-Hamed, Dimopoulos, Dvali '98

- Matter fields are confined to a 3-brane
- Flat compact extra dimensions: $D = 4 + n$

$$y^I \rightarrow y^I + 2\pi R \quad \Rightarrow \quad V_{(n)} = (2\pi R)^n$$

- **Gravity:**

  Newton's law with extra dimensions: $F(r) = G_{(n)} \frac{m}{r^{2+n}}$

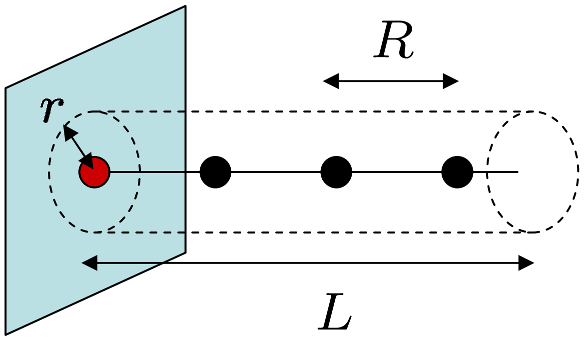
  but this applies only on small scales, much below R.

On larger scales, compactness must be taken into account.

Large (flat) extra dimensions

- Poisson equation

$$\Delta_{(3+n)}\phi = \Omega_{(2+n)} \; G_{(n)} \; \rho_m$$



$$\Rightarrow \quad \int_{\mathcal{C}} F \; dS = \Omega_{(2+n)} \; G_{(n)} \; \mathsf{Mass}(\mathcal{C})$$

- On large distances $r >> R$

$$F(r) \; L^n \; (4\pi r^2) = \Omega_{(2+n)} G_{(n)} \; m \; \left(\frac{L}{R}\right)^n$$

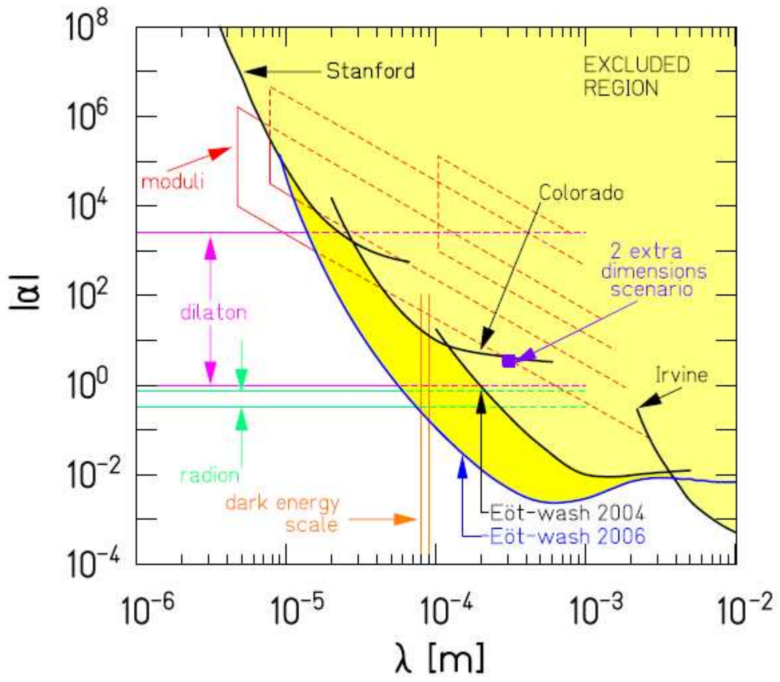
$$\Rightarrow \quad F(r) = G_{(4)} \frac{m}{r^2} \qquad G_{(4)} = \frac{\Omega_{(2+n)}}{4\pi} \frac{G_{(n)}}{R^n}$$

$$M_{\mathsf{Pl}}^2 \sim M_{(4+n)}^{2+n} R^n$$

Deviations from the 4D Newton's law below scales of order R.

Deviations from Newton's law

$$V(r) = G\,\frac{m_1 m_2}{r}\,\left(1 + \alpha\, e^{-r/\lambda}\right)$$



[From Kapner et al., 06]

Present constraints: $\lambda < 56\ \mu\text{m} \qquad (|\alpha| = 1)$

Warped geometries

- Instead of a flat bulk spacetime with metric

$$ds^2 = \eta_{AB} dx^A dx^B = \eta_{\mu\nu} dx^\mu dx^\nu + \delta_{IJ} dy^I dy^J$$

one can envisage more complicated metrics, **warped metrics**, of the form

$$ds^2 = a^2(y) \tilde{g}_{\mu\nu} dx^\mu dx^\nu + h_{IJ} dy^I dy^J$$

- Simplest example: one extra-dimension

$$ds^2 = a^2(y) \eta_{\mu\nu} dx^\mu dx^\mu + dy^2$$

Randall-Sundrum model

- Empty 5D bulk with cosmological constant
- Self-gravitating brane with tension
- "Mirror" symmetry with respect to the brane
- The static metric $ds^2 = a^2(y)\eta_{\mu\nu}\, dx^\mu dx^\nu + dy^2$
  is a solution of the 5D Einstein eqs

$$G_{ab} + \Lambda g_{ab} = \kappa^2 T_{ab}^{\text{brane}} \qquad \left[\kappa^2 = M_5^{-3}\right]$$

provided one has $\Lambda + \frac{\kappa^4}{6}\sigma^2 = 0$

Randall-Sundrum model

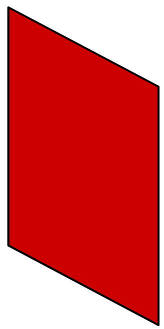
- Lengthscale ℓ such that $\Lambda = -\frac{6}{\ell^2}$

  Einstein's equations in the bulk $\Rightarrow$ $a(y) = e^{\pm y/\ell}$

- Energy-momentum tensor of the brane (located at $y = 0$)

$$T_{AB} = \delta^{\mu}_{A}\delta^{\nu}_{B}T_{\mu\nu}\delta(y) \qquad T_{\mu\nu} = -\sigma g_{\mu\nu}$$

- Einstein's equations imply



$$G_{\mu\nu} = 3\left(aa'' + a'^2\right)\eta_{\mu\nu} = \kappa^2 T_{\mu\nu}\delta(y) + \frac{6}{\ell^2}a^2\eta_{\mu\nu}$$

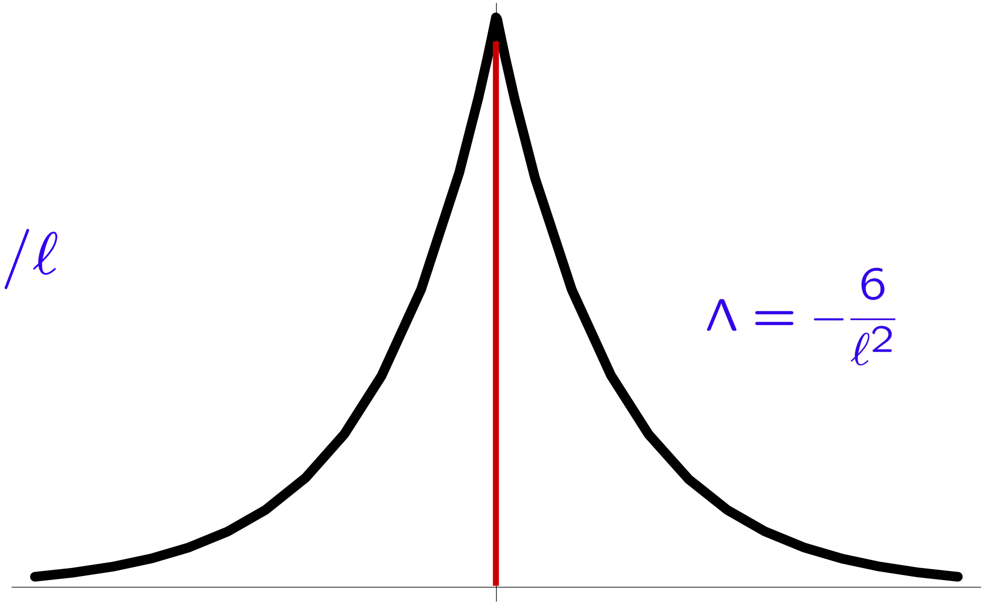
$$\Rightarrow \quad 3\left[aa'\right]^{+\epsilon}_{-\epsilon} = -\kappa^2\sigma$$

$$[a'] \equiv a'(0^+) - a'(0^-) = -\frac{\kappa^2}{3}\sigma \quad \Rightarrow \quad \sigma = \pm\frac{6}{\kappa^2\ell}$$

Randall-Sundrum (2) model

- Scale factor

$$a(y) = e^{-|y|/\ell}$$



$$\Lambda = -\frac{6}{\ell^2}$$

$$\sigma = \frac{6}{\kappa^2 \ell}$$

$$ds^2 = a^2(y)\eta_{\mu\nu}dx^\mu dx^\mu + dy^2$$

Portion of anti-de Sitter spacetime

Randall-Sundrum: effective gravity

- Metric of the form

$$ds^2 = a^2(y)\,\tilde{g}_{\mu\nu} dx^\mu dx^\mu + dy^2$$

- 5D action

$$S_{\rm grav} = \frac{M_5^3}{2}\int d^5x\sqrt{-g}\,R = \frac{M_5^3}{2}\int_{-\infty}^{+\infty} dy\ a^2(y)\int d^4x\sqrt{-\tilde{g}}\tilde{R}$$

$$M_{Pl}^2 = M_5^3\int_{-\infty}^{+\infty} dy\ a^2(y) = 2M_5^3\int_0^{+\infty} dy\ e^{-2y/\ell} = M_5^3\ell.$$

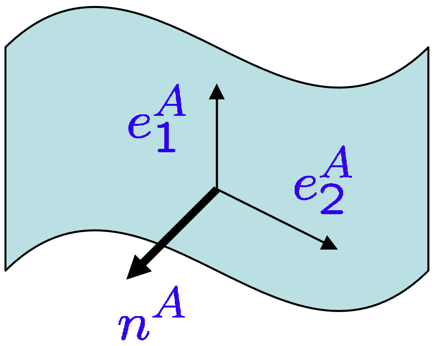
Compactification without compactification !

$$\sigma = \frac{6}{\kappa^2\ell} \quad \Rightarrow \quad 8\pi G \equiv \frac{\kappa^2}{\ell} = \frac{\kappa^4}{6}\sigma$$

Geometry of hypersurfaces

- Embedding of a hypersurface in a spacetime

$$X^A = X^A(x^\mu)$$



- Tangent vectors

$$e_\mu^A = \frac{\partial X^A}{\partial x^\mu}$$

- Unit normal vector

$$g_{AB}\, e_\mu^A\, n^B = 0, \qquad g_{AB}\, n^A\, n^B = 1$$

- Induced metric on the brane

$$h_{AB} = g_{AB} - n_A n_B, \quad h_{\mu\nu} = h_{AB}\, e_\mu^A\, e_\nu^B = g_{AB}\, e_\mu^A\, e_\nu^B$$

Junction conditions

- Extrinsic curvature tensor $$K_{AB} = h_A^C \nabla_C n_B,$$

  or $$K_{AB} = \frac{1}{2}\mathcal{L}_{\mathbf{n}} h_{AB} \equiv \frac{1}{2}\left(n^C \nabla_C h_{AB} + h_{CB} \nabla_A n^C + h_{AC} \nabla_B n^C\right)$$

- Intrinsic curvature ⬌ bulk curvature & extrinsic curvature

  Gauss-Codacci equations

- Gaussian Normal coordinates

  $$ds^2 = h_{\mu\nu} dx^\mu dx^\nu + dy^2 \quad \Rightarrow \quad K_{\mu\nu} = \frac{1}{2}\partial_y h_{\mu\nu}$$

- Integrate Einstein's eqs around the brane with $T_{AB} = S_{AB}\delta(y)$

  $$[K_{AB} - K h_{AB}] = -\kappa^2 S_{AB}$$

  Darmois-Israel junction conditions

- One also gets $D_A S_B^A = -\left[T_{AC}\; n^C\; h_B^A\right]$

Randall-Sundrum: linearized gravity

- Metric perturbations: $g_{ab} = \bar{g}_{ab} + h_{ab}$

Linearized Einstein's equations yield the equation of motion

$$\left(a^{-2}\partial^2_{(4)} + \partial^2_y - \frac{4}{\ell^2} + \frac{4}{\ell}\delta(y) \right) h_{\mu\nu} = 0,$$

which is separable.

- Eingenmodes of the form

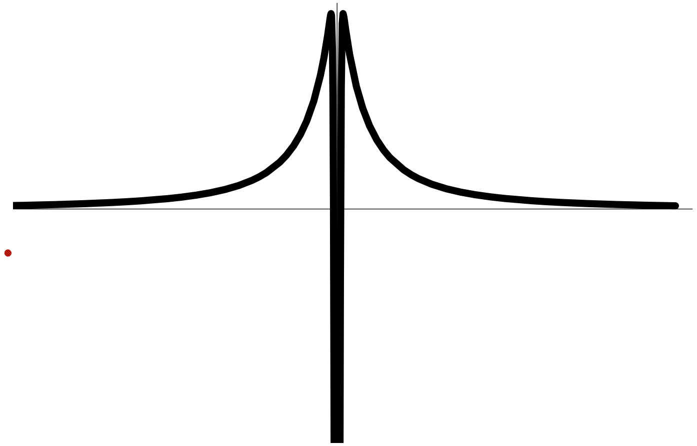
$$h(x^\mu, y) = u_m(y) e^{ip_\mu x^\mu}, \quad p_\mu p^\mu = -m^2$$

Effective gravity in the brane

- The mode dependence on the fifth dimension is obtained by solving a Schroedinger-like equation for $\psi_m = a^{-1/2} u_m$ (with $z = \int dy/a(y)$):

$$\frac{d^2\psi_m}{dz^2} - V(z)\psi_m = -m^2\psi_m$$

$$V(z) = \frac{15}{4(|z|+\ell)^2} - \frac{3}{\ell}\delta(z).$$



- One finds
  - **a zero mode (m=0)** concentrated near the brane ⇨ 4d **GR !**
  - a **continuum of massive modes** weakly coupled to the brane.

- Outside a spherical source of mass M,

$$V(r) \simeq -\frac{GM}{r}\left(1+\frac{2\ell^2}{3r^2}\right), \quad r \gg \ell$$

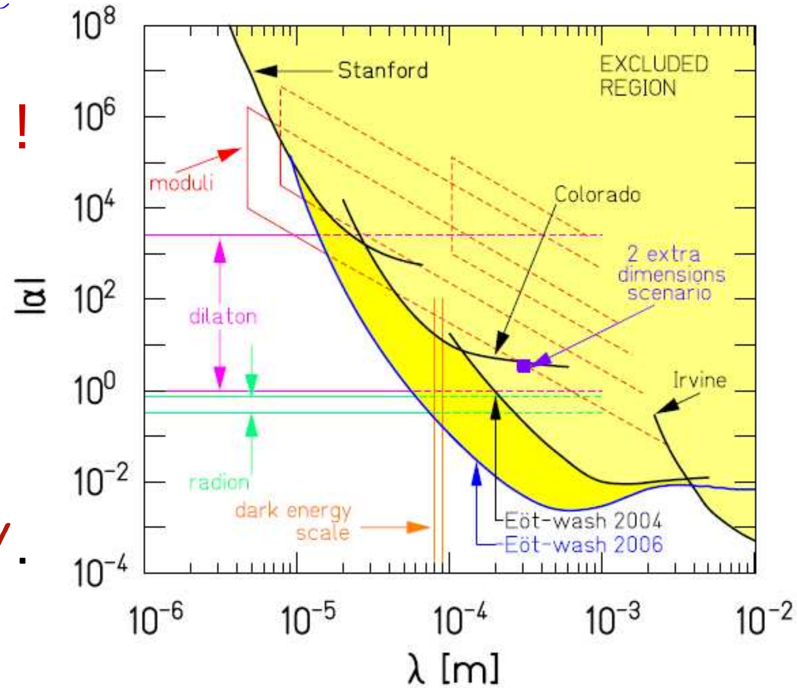
Standard gravity on scales $r \gg \ell$!

- Tests of Newton's law

$$\left[\Delta V/V = \alpha e^{-r/\lambda}\right]$$

$$\ell < 0.1\text{mm} \implies M_5 > 10^8 \text{ GeV}.$$

$$M_{Pl}^2 = M_5^3 \ell.$$



- This holds beyond linearized gravity (2nd order calculations, numerical gravity).
- However, things are more complicated for black holes.

  Conjecture, based on the AdS/CFT correspondence, that RS black holes should evaporate classically.

  Life time of a black hole: $\tau \simeq 10^2\,(M/M_\odot)^3\,(\ell/1\text{mm})^{-2}$ years

Brane cosmology

- **Cosmological symmetry**: 3d slices with maximal symmetry, i.e. homogeneous and isotropic.

- Generalized cosmological metric

$$ds^2 = -n^2(t,y)dt^2 + a^2(t,y)d\Sigma_k^2 + dy^2$$

[Gaussian Normal coordinates, with the brane at y=0.]

- Brane energy-momentum tensor

$$T_a^b = Diag(-\rho_b(t), P_b(t)\delta_i^j, 0)\delta(y)$$

- 5D Einstein eqs

$$G_{ab} + \Lambda g_{ab} = \kappa^2 T_{ab}^{\text{brane}}$$

Brane cosmology

- One can solve explicitly Einstein's equations

- In the brane,

$$ds_b^2 = -n_b(t)^2 dt^2 + a_b(t)^2 d\Sigma_k^2, \quad [n_b(t) \equiv n(t,0), a_b(t) \equiv a(t,0)]$$

and the scale factor satisfies the modified Friedmann equation

$$H_b^2 \equiv \frac{\dot{a}_b^2}{a_b^2} = \frac{\Lambda}{6} + \frac{\kappa^4}{36}\rho_b^2 + \frac{\mathcal{C}}{a_b^4} - \frac{k}{a_b^2}$$

- Conservation equation unchanged (empty bulk)

$$\nabla_\mu T^\mu_\nu = 0 \quad \Rightarrow \quad \dot{\rho}_b + 3H(\rho_b + P_b) = 0$$

- Example

$$\Lambda = 0,\ \mathcal{C} = 0,\ k = 0,\ p_b = w\,\rho_b \quad \Rightarrow \quad a_b(t) \propto t^{\frac{1}{3(1+w)}}$$

In contrast with $a(t) \propto t^{\frac{2}{3(1+w)}}$

Viable brane cosmology

- Generalize the Randall-Sundrum setup:
  - Minkowski brane: $\rho_b = \sigma \equiv \frac{6M_5^3}{\ell}$
  - Cosmological brane: $\rho_b(t) = \sigma + \rho(t)$,

$$\frac{\Lambda}{6} + \frac{\kappa^4}{36}(\sigma+\rho)^2 = \frac{\kappa^4}{18}\sigma\rho + \frac{\kappa^4}{36}\rho^2$$

- Friedmann equation:

$$H_b^2 = \frac{8\pi G}{3}\rho - \frac{k}{a_b^2} + \frac{\kappa^4}{36}\rho^2 + \frac{\mathcal{C}}{a_b^4}$$

- Two new features:
  - a ρ^2 **term**, which becomes significant at high energy;
  - a radiation-like term, C/a_b^4, usually called **dark radiation**.

Viable brane cosmology

- Friedmann

$$H_b^2 = \frac{8\pi G}{3}\rho - \frac{k}{a_b^2} + \frac{\kappa^4}{36}\rho^2 + \frac{\mathcal{C}}{a_b^4}$$

- Conservation equation

$$\dot{\rho} + 3H(\rho + P) = 0$$

- Example: $\mathcal{C} = 0,\ k = 0,\ p = w\,\rho$

$$a(t) \propto t^{1/q}\left(1 + \frac{q\,t}{2\ell}\right)^{1/q}, \qquad q = 3(1+w)$$

- Constraints: nucleosynthesis in the low energy regime

$$\sigma^{1/4} > 1\,\text{MeV} \;\Rightarrow M_5 > 10^4\text{GeV}$$

but $M_5 > 10^8\text{GeV}$ already required from gravity constraints

Effective 4D Einstein equations

- Decompose matter on the brane

$$S_{\mu\nu} = -\sigma h_{\mu\nu} + \tau_{\mu\nu}$$

- One can write $\quad {}^{(4)}G_{\mu\nu} + \Lambda_4\, g_{\mu\nu} = 8\pi G \tau_{\mu\nu} + \kappa^2 \Pi_{\mu\nu} - E_{\mu\nu}$

with $\quad \Lambda_4 \equiv \frac{1}{2}\left(\Lambda + \frac{\kappa^4}{6}\sigma^2\right), \quad 8\pi G \equiv \frac{\kappa^4}{6}\sigma$

$$\Pi_{\mu\nu} = -\frac{1}{4}\tau_{\mu\sigma}\tau^{\sigma}{}_{\nu} + \frac{1}{12}\tau\,\tau_{\mu\nu} + \frac{1}{8}\left(\tau_{\rho\sigma}\tau^{\rho\sigma} - \frac{1}{3}\tau^2\right)h_{\mu\nu}$$

$E_{\mu\nu} = C_{y\mu y\nu}$ where C_{abcd} is the bulk Weyl tensor

$$\left[\; R_{ABCD} = C_{ABCD} + \frac{2}{3}\left(g_{A[C}R_{D]B} - g_{B[C}R_{D]A}\right) - \frac{1}{6} g_{A[C}g_{D]B}\, R \;\right]$$

Another point of view

- The five-dimensional metric can be rewritten as

$$ds^2 = -n^2(t,r)dt^2 + b^2(t,r)dr^2 + R^2(t,r)d\Sigma_k^2, \quad (k = 0, \pm 1)$$

This is analogous to a spherically symmetric ansatz in 4D.

- In vacuum, one gets the analog of Birkhoff's theorem and the metric is of the form

$$ds^2 = -f(R)dT^2 + \frac{dR^2}{f(R)} + R^2 d\Sigma_k^2, \quad f(R) = k + \frac{R^2}{\ell^2} - \frac{\mathcal{C}}{R^2}.$$

(integration constant C analogous to the Schwarzschild mass)

And the brane is moving !

Dark radiation

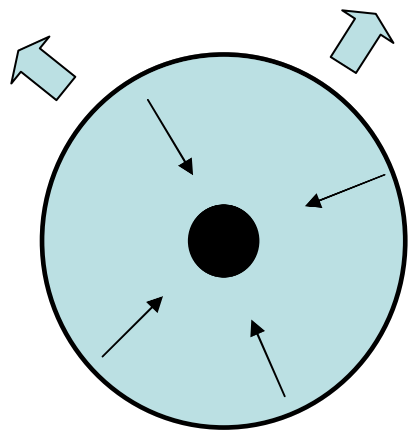
- For a strictly empty bulk, $$H^2 = \frac{8\pi G}{3}\left(\rho + \frac{\rho^2}{2\sigma} + \rho_D\right),$$
  with $\frac{8\pi G}{3}\rho_D = \frac{\mathcal{C}}{a^4}.$
- Constraint on extra radiation from nucleosynthesis

$$\Delta N_\nu \lesssim 0.2 \Longrightarrow \epsilon_D \equiv \rho_D/\rho_r \lesssim 0.03\,(g_*/g_{*N})^{1/3}$$

[$\epsilon_D \lesssim 0.09$ with d.o.f. of the standard model]

- However, interactions of brane particles will generate **bulk gravitons**.
  And the brane loses energy: $\dot{\rho} + 4H\rho \propto -\hat{g}(T)\kappa^2 T^8$
- Exactly solvable model: 5D Vaidya

Dark radiation



- 5d AdS-Vaidya metric

$$ds^2 = -\left(k + \frac{r^2}{\ell^2} - \frac{\mathcal{C}(v)}{r^2}\right) dv^2 + 2drdv + r^2 d\mathbf{x}^2$$

solution of 5d Einstein's eqs with

$$T_{ab} = \psi\, k_a k_b \quad (k_c k^c = 0)$$

- Cosmological evolution governed by the coupled eqs:

$$\left.\begin{aligned} &\frac{d\hat{\rho}}{d\hat{t}} + 4\hat{H}\hat{\rho} = -\alpha\hat{\rho}^2, \\ &\hat{H}^2 = 2\hat{\rho} + \hat{\rho}^2 + \frac{\hat{\mathcal{C}}}{a^4}, \\ &\frac{d\hat{\mathcal{C}}}{d\hat{t}} = 2\alpha a^4 \hat{\rho}^2 \left(1 + \hat{\rho} - \hat{H}\right). \end{aligned}\right\}$$

in the low energy limit

$$(\hat{\rho} \equiv \rho/\sigma \ll 1)$$

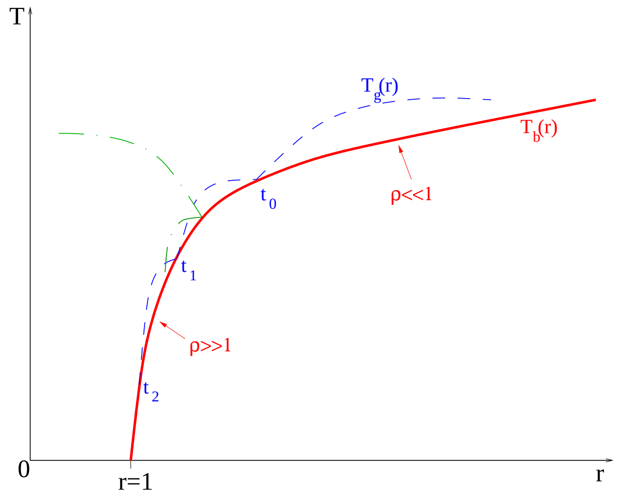
$$\mathcal{C} \longrightarrow \mathrm{const}$$

- But the bulk gravitons are only radial…

- The bulk is filled with the gravitons produced by the brane.

  ⇨ Effective bulk energy-momentum tensor

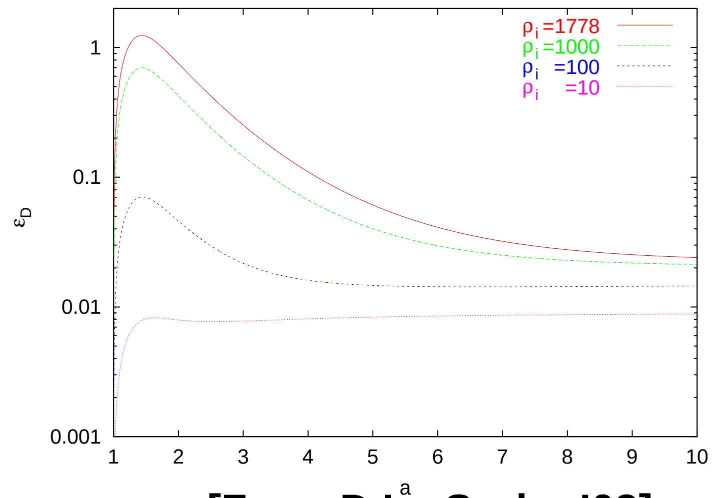
$$\mathcal{T}_{ab} = \int d^5p\ \delta\left(p_e p^e\right)\sqrt{-g}\, f\, p_a p_b,$$



- The evolution of the ``dark component'' ρ_D is given by

$$\dot{\rho}_D + 4H\rho_D = \underbrace{-2\left(1+\rho/\sigma\right)\mathcal{T}_{ab}u^a n^b}_{\textbf{Energy flux (>0)}} \underbrace{-\,2H\ell\,\mathcal{T}_{ab}n^a n^b}_{\textbf{Transverse pressure (<0)}}\,.$$

- Late in the low energy regime, ρ_D behaves like radiation, i.e. the production of bulk gravitons becomes negligible.



[From D.L., Sorbo ’03]

De Sitter brane

- Obtained by "detuning" the brane tension: $\rho_b = \sigma + \rho$
- The metric in GN coordinates is separable:

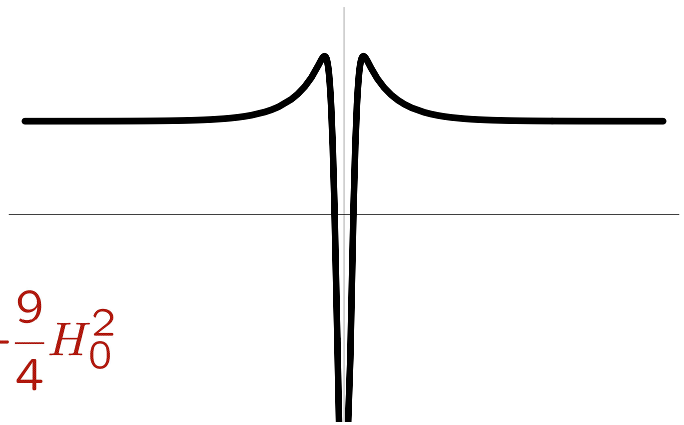
$$ds^2 = \mathcal{A}(y)^2\left(-dt^2 + e^{2Ht}d\mathbf{x}^2\right) + dy^2$$

with

$$\mathcal{A}(y) = \cosh(y/\ell) - \left(1 + \frac{\rho}{\sigma}\right)\sinh(|y|/\ell)$$

- The **tensor modes** satisfy a **wave** equation, which is **separable**. The y-dependent part can be rewritten as

$$\frac{d^2\Psi_m}{dz^2} - V(z)\Psi_m = -m^2\Psi_m$$



$$V(z) = \frac{15H_0^2}{4\sinh^2(H_0 z)} - \frac{3}{\ell}(1 + \rho/\sigma)\,\delta(z - z_{\mathrm{b}}) + \frac{9}{4}H_0^2$$

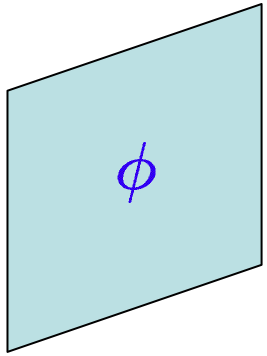
⇨ **Gap** between the **zero mode** and the **massive continuum**

Brane inflation

- 4D scalar field localized on the brane

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi), \qquad p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

Slow-roll: $H^2 \simeq \frac{8\pi G}{3} V \left(1 + \frac{V}{2\sigma}\right), \quad \dot{\phi} \simeq -\frac{V'}{3H}$



- Scalar spectrum

Extension of the 4D formula $\quad \delta\phi \sim \frac{H}{2\pi} \qquad \mathcal{P}_{\mathsf{S}}^{1/2} \propto \frac{H}{\dot{\phi}}\delta\phi \propto \frac{H^2}{\dot{\phi}}$

$$\mathcal{P}_{\mathsf{S}} \propto \frac{H^6}{V'^2} \qquad \mathcal{P}_{\mathsf{S}} = \mathcal{P}_{\mathsf{S}}^{(4\mathsf{D})} \left(1 + \frac{V}{2\sigma}\right)^3$$

- Tensor spectrum

$$\mathcal{P}_{\mathsf{T}} = \mathcal{P}_{\mathsf{T}}^{(4\mathsf{D})} F^2(H\ell) \qquad F(x) = \left\{\sqrt{1+x^2} - x^2 \ln\left[\frac{1}{x} + \sqrt{1+\frac{1}{x^2}}\right]\right\}^{-1/2}$$

Cosmological perturbations

- Direct link with cosmological observations, in particular
  - large scale structures
  - **CMB anisotropies**

- To determine completely the predictions of brane cosmology, a 5D analysis is required.

  The evolution of (metric) perturbations is governed by *partial differential* equations, which are not separable in general.

Numerical approach

Evolution of tensor modes

- The 4D tensor modes satisfy the wave equation

$$\ddot{h}_{4d} + 3H\dot{h}_{4d} + (k^2/a^2)h_{4d} = 0$$

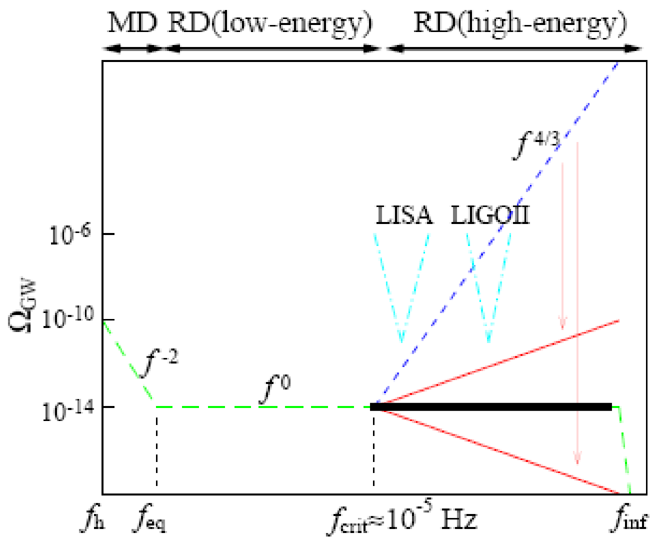
- For k≪ aH, h_{4d} is constant, whereas, inside the Hubble radius, h_{4d} behaves like a^{-1}.

- Taking into account only the change of homogeneous cosmology, one finds $\Omega_{gw} \propto k^{4/3}$ for modes entering the Hubble radius in the high energy (radiation) era.
- Five-dimensional effects:

$k \ll aH$: h_{5d} constant

$\frac{k}{a} > H > \ell^{-1}$ massive modes produced

⇨ damping of the zero mode



[From Hiramatsu '06]

- What is the relative strength of the two effects ? They cancel each other !

$$\Omega_{gw} \equiv \frac{1}{\rho_c}\frac{d \ln \rho_{gw}}{d \ln k} \propto k^{\frac{3w-1}{3w+2}}$$

Cosmological perturbations: the brane point of view

- Effective Einstein's equations on the brane

$$G_{\mu\nu} + \Lambda_4 g_{\mu\nu} = \kappa_4^2 \tau_{\mu\nu} + \kappa^2 \Pi_{\mu\nu} - E_{\mu\nu},$$

- Projected Weyl tensor $E_{\mu\nu}$ ⇨ Effective energy-momentum tensor for a "Weyl" fluid

- The equations governing the evolution of the perturbations are modified in two ways:
  - background corrections (negligible in the low energy limit)
  - additional contributions due to the Weyl fluid

Brane induced gravity

DGP model (Dvali-Gabadadze-Porrati)

$$S = \int d^5x \sqrt{-g} \frac{{}^{(5)}R}{2\kappa_5^2} + \int d^4x \sqrt{-\gamma} \left[\frac{{}^{(4)}R}{2\kappa_4^2} + \mathcal{L}_m \right]$$

- Critical lengthscale $r_c = \dfrac{\kappa_5^2}{2\kappa_4^2}$
- Pertubative treatment breaks down below $r_V = \left(r_g r_c^2\right)^{1/3}$, $r_g = 2GM$

Brane induced gravity

- Junction conditions $$K^{\mu}_{\nu} - K\delta^{\mu}_{\nu} = -\frac{\kappa^2}{2}\left(T^{\mu}_{\nu} - \kappa_4^{-2} G^{\mu}_{\nu}\right)$$

- Cosmological solutions

  Friedmann $$H^2 - \epsilon \frac{H}{r_c} = \frac{8\pi G}{3}\rho$$

  - $\epsilon = +1$: self-accelerating solution
  - $\epsilon = -1$: “normal” solution

- Ghost problem !

  Linearized perturbations about self-accelerating solution

  ⇨ Ghost

Codimension 2 branes

- Einstein-Maxwell theory in the 6D bulk

$$S_{\text{bulk}} = \int d^6x \sqrt{-g_{(6)}} \left[\frac{R}{2\kappa_6^2} - \Lambda_6 - \frac{1}{4} F_{AB} F^{AB} \right]$$

- Flux compactification

$$\begin{aligned} ds_6^2 &= \eta_{\mu\nu}\, dx^\mu\, dx^\nu + \gamma_{ij}(y) dy^i dy^j \\ &= \eta_{\mu\nu}\, dx^\mu\, dx^\nu + a_0^2 \left(d\theta^2 + \sin^2\theta d\varphi^2 \right) \end{aligned}$$

$$F_{ij} = \sqrt{\gamma}\, B_0\, \epsilon_{ij}$$

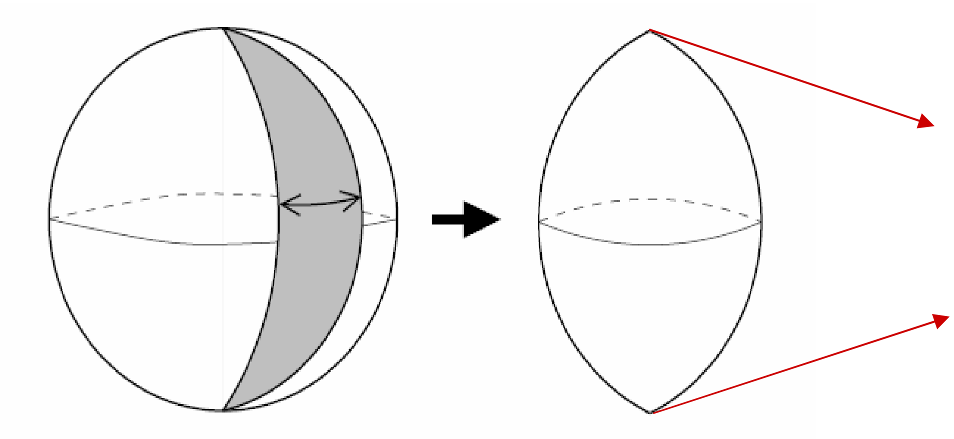
with $B_0^2 = 2\Lambda_6, \quad a_0^2 = \dfrac{M_6^4}{2\Lambda_6}$

Codimension 2 branes

- Introducing branes of tension σ …

$$\gamma_{ij}(y)dy^i dy^j = a_0^2\left(d\theta^2 + \alpha^2 \sin^2\theta d\varphi^2\right)$$

Deficit angle : $\alpha = 1 - \frac{\sigma}{2\pi M_6^4}$



[From Carroll & Guica '03]

Conical defects:
codimension 2 branes

Rugby-ball geometry

Codimension 2 branes

- Einstein-Maxwell theory in the 6D bulk

$$S_{\text{bulk}} = \int d^6x \sqrt{-g_{(6)}} \left[\frac{R}{2\kappa_6^2} - \Lambda_6 - \frac{1}{4} F_{AB} F^{AB} \right]$$

- Warped solution

$$ds_6^2 \;=\; \rho^2 \eta_{\mu\nu} dx^\mu dx^\nu + \frac{d\rho^2}{f(\rho)} + c_0^2 f(\rho) d\varphi^2 \,, \qquad F_{\rho\varphi} = -\frac{b_0}{\rho^4}$$

$$f(\rho) = -\frac{\Lambda_0}{10}\rho^2 - \frac{b_0^2}{12 c_0^2 \rho^6} + \frac{\mu_0}{\rho^3} \,.$$

Assume $f(\rho)$ has two roots $\rho_+ > \rho_- > 0$

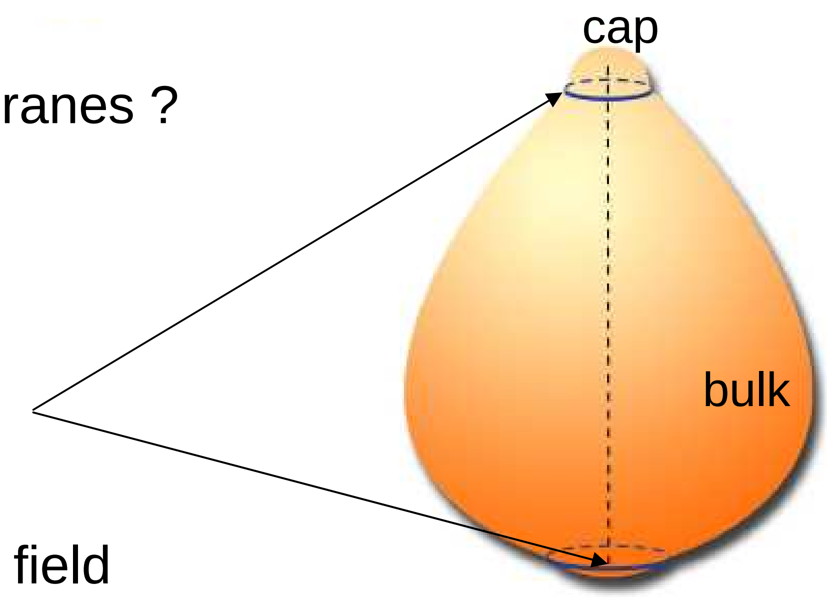
Two codim 2 branes at $\rho = \rho_-$ and $\rho = \rho_+$

Regularized codimension 2 branes

- Codim 2 branes support only a tension-like energy momentum tensor in Einstein gravity

- Effective gravity on codim 2 branes ?

- Regularization
  Replace codim 2 branes with
  ring-like codim 1 branes



[From Kobayashi, Minamitsuji '07]

- Brane matter: complex scalar field

$$S_{b,\pm} = -\int d^5x\sqrt{-q}\left(V(|\phi_\pm|) + \frac{1}{2}D_a\phi_\pm(D^a\phi_\pm)^*\right)$$

Usual 4d gravity recovered on large scales

Conclusions

- Is our Universe brany ?
  - modifications of gravity at small (or large) scales
  - collider physics
  - cosmology…

- Cosmology of the simplest warped model:
  - modification of Friedmann equation
  - change in amplitude of primordial spectra
  - dark radiation (nucleosynthesis constraints)
  - cosmological perturbations: no full predictions yet

  Randall-Sundrum model satisfies so far all observational constraints

- Other models: brane induced gravity, two extra dims

- Look at old or new problems with a different point of view…