COSMIC MICROWAVE BACKGROUND ANISOTROPIES

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OUTLINE

Fundamentals of CMB physics (today)

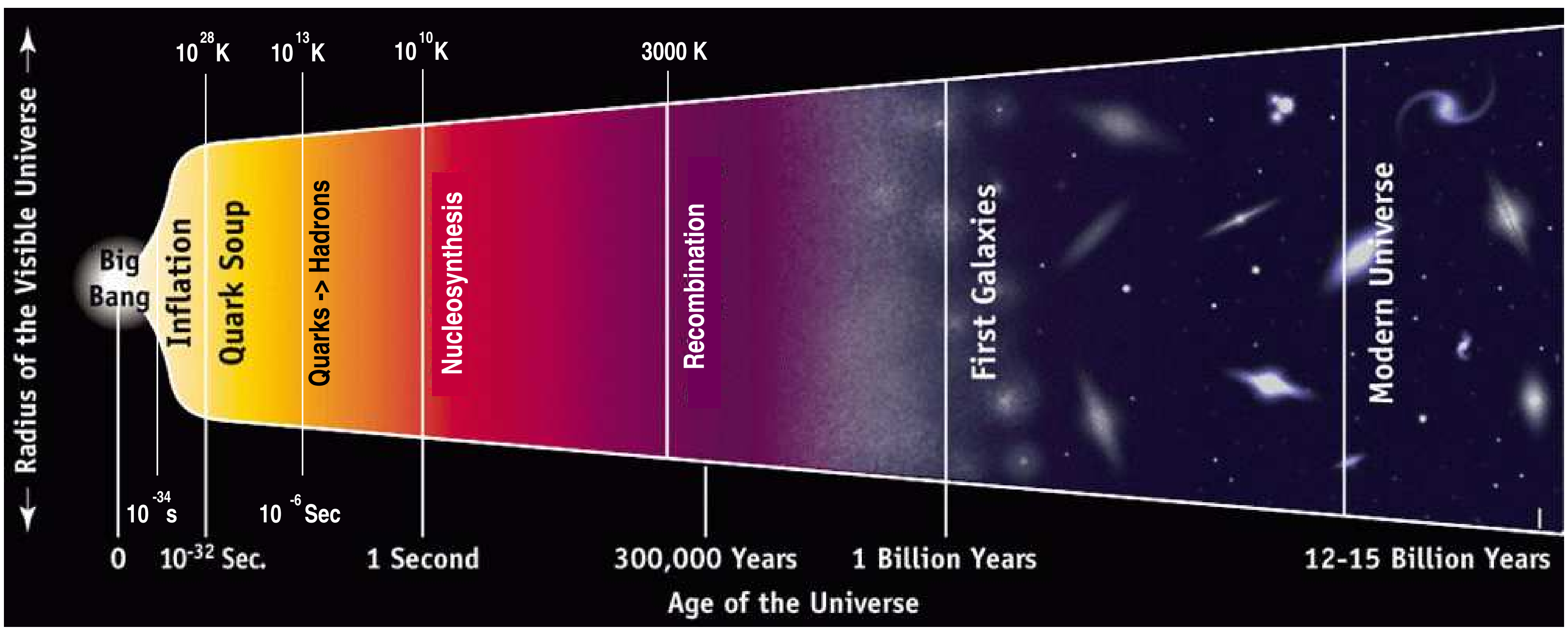
- Brief history
- Spectrum and statistics of anisotropies
- Anisotropy generation
- Acoustic physics
- Complications to simple picture:
  - Photon diffusion
  - ISW
  - Reionization
  - Gravitational waves
  - Isocurvature modes
- Polarization

What have we learned from the CMB? (tomorrow)

- Parameters from the CMB
- Current measurements
- Major milestones passed
- CMB constraints on inflation
- The future:
  - Planck
  - Secondary anisotropies
  - Gravitational waves

THERMAL HISTORY

- CMB and matter plausibly produced during reheating at end of inflation
- CMB decouples around recombination, 300 kyr later
- Universe starts to reionize once first stars (?) form (somewhere in range $z = 10\text{–}20$) and 10% of CMB re-scatters

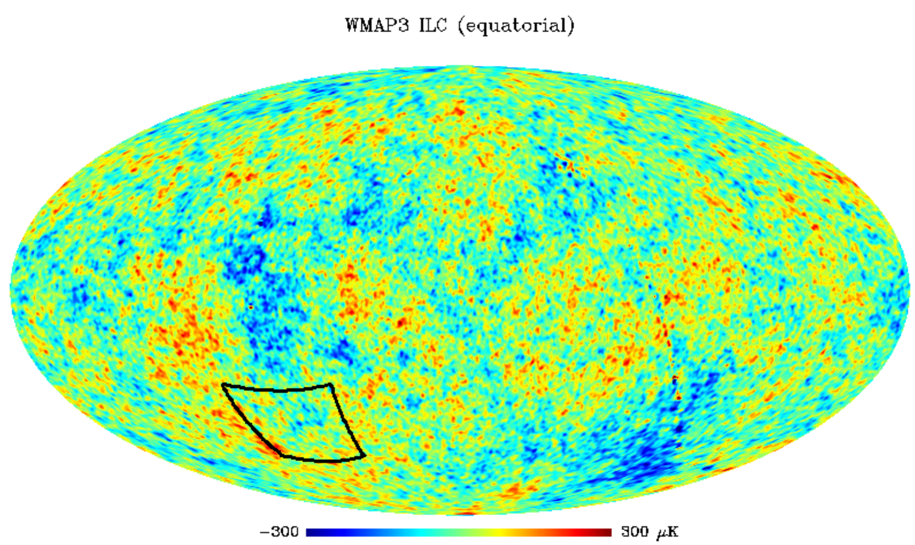


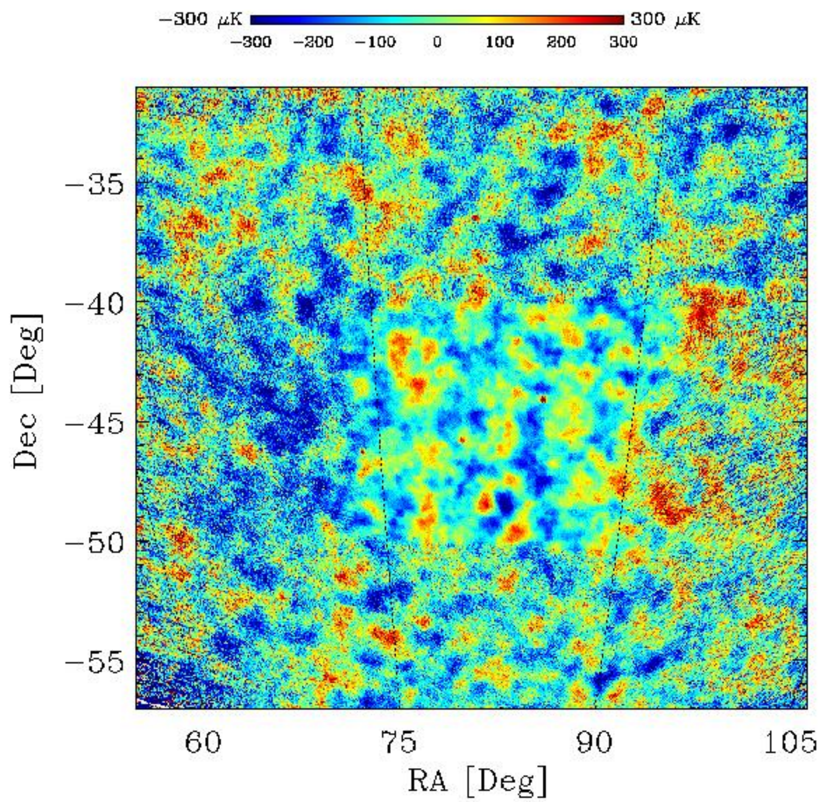
History of CMB physics

- 1939: CMB detected indirectly through $\sim 2.3\,\mathrm{K}$ temperature of interstellar Cyanogen (Adams & McKellar)
- 1948: CMB predicted following work on synthesis of light elements (Alpher & Herman)
- 1965: CMB discovered serendipitously discovered (Penzias & Wilson) and interpreted as relic radiation (Dicke et al)
- 1967: Sachs & Wolfe predict CMB anisotropies as clumpiness $\Rightarrow$ redshift variations
- 1977: CMB dipole detected (Smoot et al)
- Early 1980s: Anisotropy predictions for CDM universe (Peebles; Bond & Efstathiou)
- 1990: Definitive measurement of CMB spectrum by COBE-FIRAS (Mather et al)
- 1992: COBE-DMR detects anisotropy at 10^{-5} level (Smoot et al)
- Late 1990s: First acoustic peak detected from ground (MAT/TOCO experiment)
- 2002: Polarization detected by DASI (Kovac et al)
- 2003, 2006: WMAP1 release and WMAP3 release

Mapping the CMB

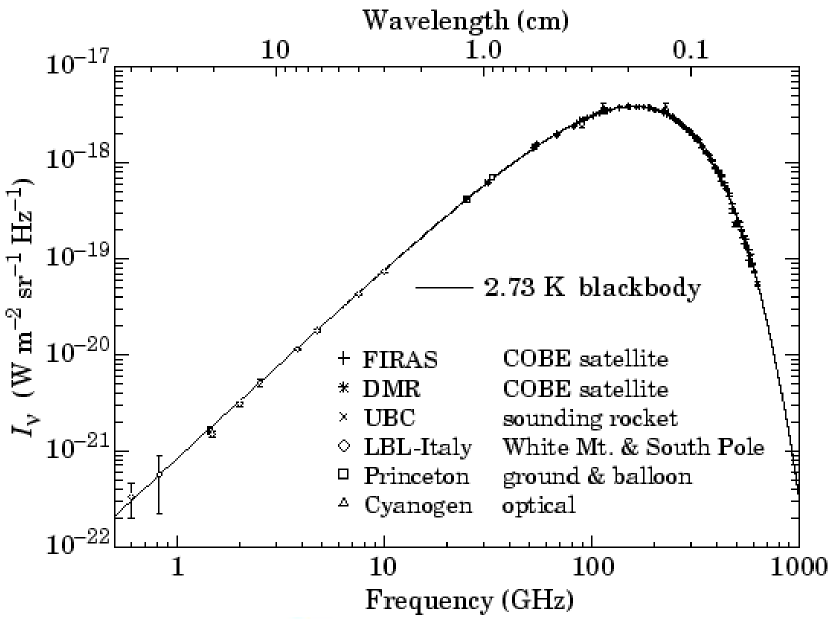
- WMAP3 internal-linear combination map (left) and BOOMERanG03 (right)
- Aim to answer in these lectures:
  - What is the physics behind these images?
  - What have we learned about cosmology from them?

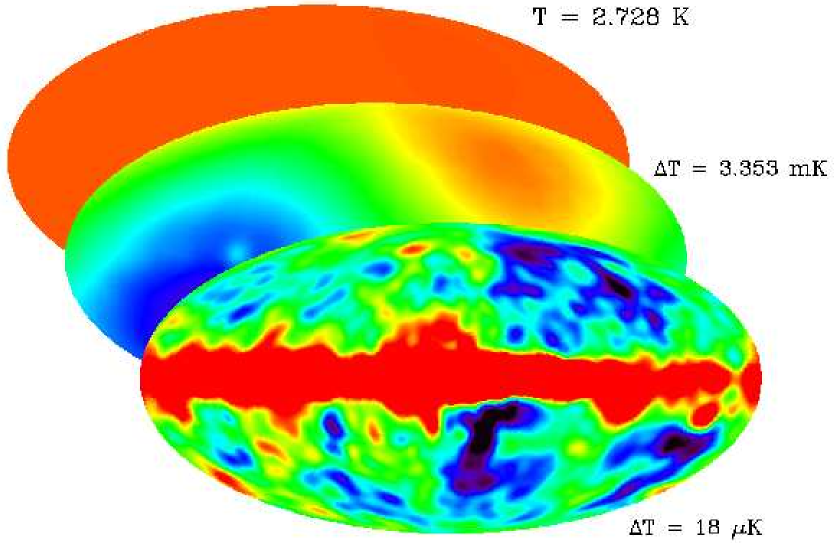




CMB SPECTRUM AND DIPOLE ANISOTROPY

- Microwave background almost perfect blackbody radiation
  - Temp. (COBE-FIRAS) 2.725 K
- Dipole anisotropy $\Delta T/T = \beta \cos\theta$ implies solar-system barycenter has velocity $v/c \equiv \beta = 0.00123$ relative to 'rest-frame' of CMB
- Variance of intrinsic fluctuations first detected by COBE-DMR: $(\Delta T/T)_{\rm rms} = 16\mu$K smoothed on 7° scale





Anisotropies and the Power Spectrum

- Decompose temperature anisotropies in spherical harmonics

$$\Theta \equiv \Delta T(\hat{\boldsymbol{n}})/T = \sum_{lm} a_{lm} Y_{lm}(\hat{\boldsymbol{n}})$$

  - Under a rotation (R) of sky $a_{lm} \to D^l_{mm'}(R) a_{lm'}$
  - Demanding *statistical isotropy* requires, for 2-point function

$$\langle a_{lm} a^*_{l'm'} \rangle = D^l_{mM} D^{l'*}_{m'M'} \langle a_{lM} a^*_{l'M'} \rangle \quad \forall R$$

  - Only possible (from unitarity $D^{l*}_{Mm} D^l_{Mm'} = \delta_{mm'}$) if

$$\langle a_{lm} a^*_{l'm'} \rangle = C_l \delta_{ll'} \delta_{mm'}$$

  - Symmetry restricts higher-order correlations also, but for *Gaussian* fluctuations all information in *power spectrum* C_l

- Estimator for power spectrum $\hat{C}_l = \sum_m |a_{lm}|^2/(2l+1)$ has mean C_l and *cosmic variance*

$$\text{var}(\hat{C}_l) = \frac{2}{2l+1} C_l^2$$

Thermal History — Details

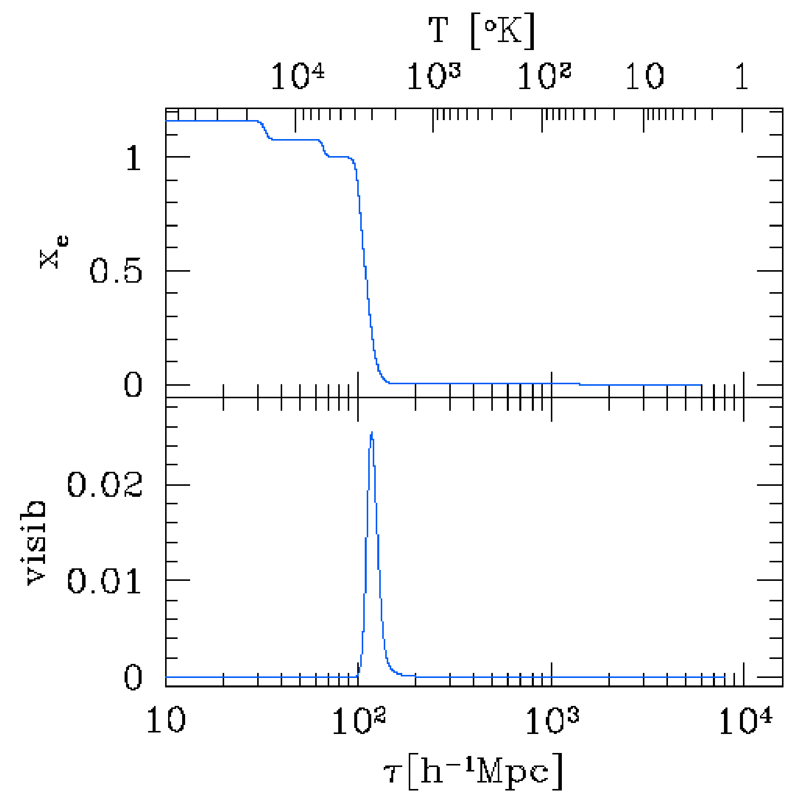
- Dominant element hydrogen recombines rapidly around $z \approx 1000$
  - Prior to recombination, Thomson scattering efficient and mean free path short cf. expansion time
  - Little chance of scattering after recombination $\rightarrow$ photons free stream keeping imprint of conditions on last scattering surface

- Optical depth back to (conformal) time η_0 for Thomson scattering:

$$\tau(\eta) = \int_{\eta}^{\eta_0} a n_e \sigma_T \, d\eta'$$

- Visibility is probability of last scattering at η per $d\eta$:

$$\text{visibility}(\eta) = -\dot{\tau} e^{-\tau}$$



Anisotropy generation: gravity

- For (dominant) scalar perturbations, work in CNG (and assume $K = 0$)

$$ds^2 = a^2(\eta)[-(1+2\psi)d\eta^2 + (1-2\phi)d\boldsymbol{x}^2]$$

  - $\phi = \psi$ in GR if no anisotropic stress (e.g. more exotic dark energy) but not so in modified gravity ($f(R)$, DPG etc.)

- *Comoving energy* $\epsilon = aE$ constant in background but evolves due to gravitational perturbations:

$$d\epsilon/d\eta = -\epsilon d\psi/d\eta + \epsilon(\dot{\phi} + \dot{\psi})$$

  - First term on left gives additional redshift, hence $\Delta T/T$, due to differences in potential ψ between last scattering point and reception
    - * Negative contribution to $\Delta T/T$ from potential wells (matter over-densities) at last scattering
  - Second term gives *integrated Sachs-Wolfe* contribution

$$(\Delta T/T)_{\mathrm{ISW}} = \int (\dot{\phi} + \dot{\psi})\, d\eta$$

- Holds irrespective of metric theory of gravity assumed

Anisotropy generation: scattering

- Thomson scattering ($k_B T \ll m_e c^2$) around recombination and reionization dominant scattering mechanism to affect CMB:

$$\frac{d\Theta}{d\eta} = \underbrace{-an_e\sigma_T\Theta}_{\text{out-scattering}} \underbrace{+\frac{3an_e\sigma_T}{16\pi}\int d\hat{\boldsymbol{m}}\,\Theta(\epsilon,\hat{\boldsymbol{m}})[1+(\boldsymbol{e}\cdot\hat{\boldsymbol{m}})^2]}_{\text{in-scattering}} \underbrace{+an_e\sigma_T\boldsymbol{e}\cdot\boldsymbol{v}_b}_{\text{Doppler}}$$

- Neglecting anisotropic nature of Thomson scattering,

$$\frac{d\Theta}{d\eta} \approx -an_e\sigma_T(\Theta - \Theta_0 - \boldsymbol{e}\cdot\boldsymbol{v}_b)$$

  so scattering tends to isotropise in rest-frame of electrons: $\Theta \to \Theta_0 + \boldsymbol{e}\cdot\boldsymbol{v}_b$

- Doppler effect arises from electron bulk velocity $\boldsymbol{v}_b$
  - Enhances $\Delta T/T$ for $\boldsymbol{v}_b$ towards observer
  - Linear effect only important from recombination; non-linear effects from reionization avoid peak-trough cancellation

Temperature Anisotropies

- On degree scales, scattering time short c.f. wavelength of fluctuations and (local!) temperature is uniform plus dipole: $\Theta_0 + \mathbf{e} \cdot \mathbf{v}_b$
- Observed temperature anisotropy is snapshot of this at last scattering but modified by gravity:

$$[\Theta(\hat{\mathbf{n}}) + \psi]_R = \underbrace{\Theta_0|_E}_{\text{temp.}} + \underbrace{\psi|_E}_{\text{gravity}} + \underbrace{\mathbf{e} \cdot \mathbf{v}_b|_E}_{\text{Doppler}} + \underbrace{\int_E^R (\dot{\psi} + \dot{\phi})\, d\eta}_{\text{ISW}}$$

  with line of sight $\hat{\mathbf{n}} = -\mathbf{e}$, and Θ_0 isotropic part of Θ

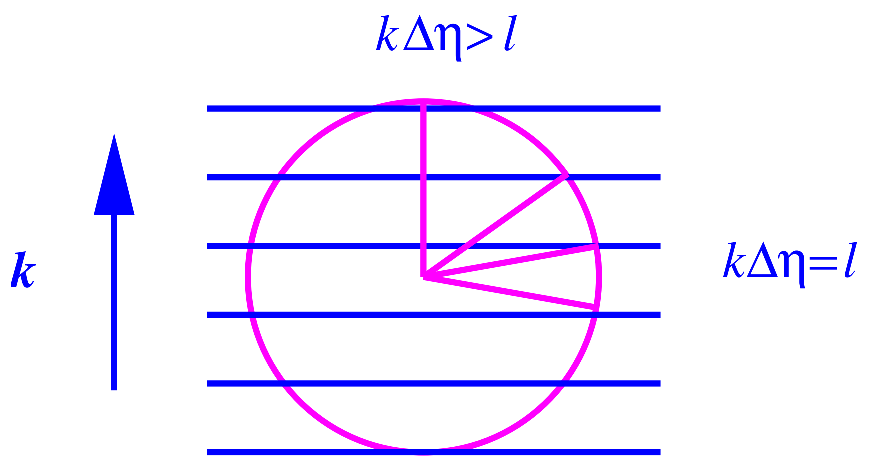
  - Ignores anisotropic scattering, finite width of visibility function (i.e. last-scattering surface) and reionization
    - $*$ Will fix these omissions shortly

SPATIAL-TO-ANGULAR PROJECTION

- Consider angular projection at origin of potential $\psi(\boldsymbol{x}, \eta_*)$ over last-scattering surface; for a single Fourier component

$$\psi(\hat{\boldsymbol{n}}) = \psi(\hat{\boldsymbol{n}}\Delta\eta, \eta_*) \qquad \Delta\eta \equiv \eta_0 - \eta_*$$
$$= \psi(\boldsymbol{k}, \eta_*) \sum_{lm} 4\pi i^l j_l(k\Delta\eta) Y_{lm}(\hat{\boldsymbol{n}}) Y^*_{lm}(\hat{\boldsymbol{k}})$$
$$\psi_{lm} \sim 4\pi\psi(\boldsymbol{k}, \eta_*) i^l j_l(k\Delta\eta) Y^*_{lm}(\hat{\boldsymbol{k}})$$

- $j_l(k\Delta\eta)$ peaks when $k\Delta\eta \approx l$ but for given l considerable power from $k > l/\Delta\eta$ also (wavefronts perpendicular to line of sight)



  - CMB anisotropies at multipole l mostly sourced from fluctuations with linear wavenumber $k \sim l/\Delta\eta$ where conformal distance to last scattering $\approx$ 14 Gpc

ACOUSTIC PHYSICS

- Photon isotropic temperature Θ_0 and electron velocity v_b at last scattering depend on acoustic physics of pre-recombination plasma

- Large-scale approximation: ignore diffusion and slip between CMB and baryon bulk velocities (requires scattering rate $\gg k$)

  – Photon-baryon plasma behaves like perfect fluid responding to gravity (drives infall to wells), Hubble drag of baryons, gravitational redshifting and baryon pressure (resists infall):

$$\ddot{\Theta}_0 + \underbrace{\frac{\mathcal{H}R}{1+R}\dot{\Theta}_0}_{\text{Hubble drag}} + \underbrace{\frac{1}{3(1+R)}k^2\Theta_0}_{\text{pressure}} = \underbrace{\ddot{\phi}}_{\text{redshift}} + \frac{\mathcal{H}R}{1+R}\dot{\phi} - \underbrace{\frac{1}{3}k^2\psi}_{\text{infall}}$$

  where $R \equiv 3\rho_b/(4\rho_\gamma) \propto a$

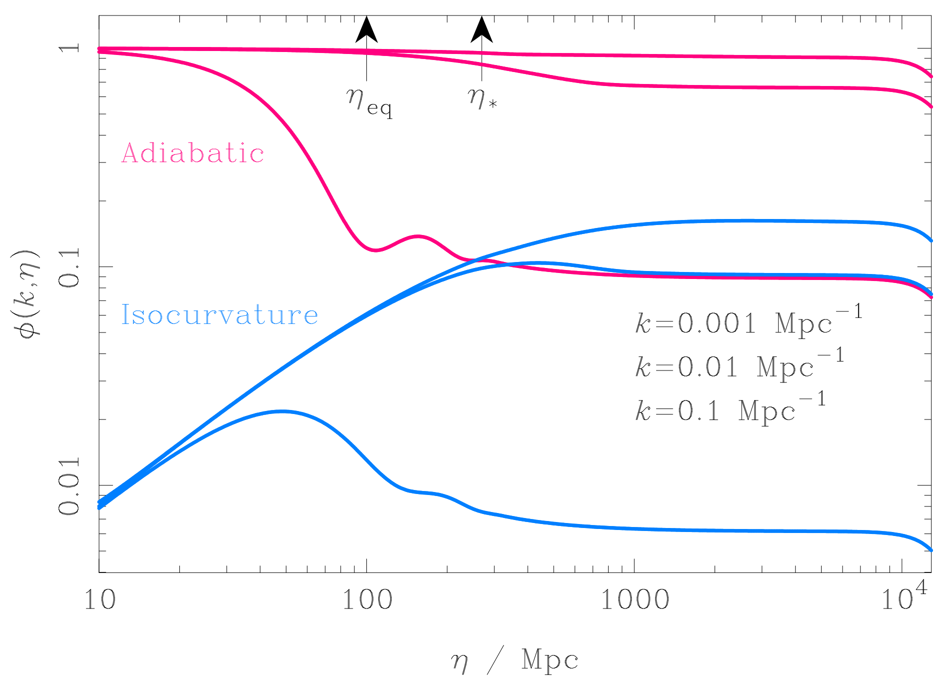
  – WKB solutions of homogeneous equation:

$$(1+R)^{-1/4}\cos kr_s \quad , \quad (1+R)^{-1/4}\sin kr_s$$

  with *sound horizon* $r_s \equiv \int_0^\eta \frac{d\eta'}{\sqrt{3(1+R)}}$

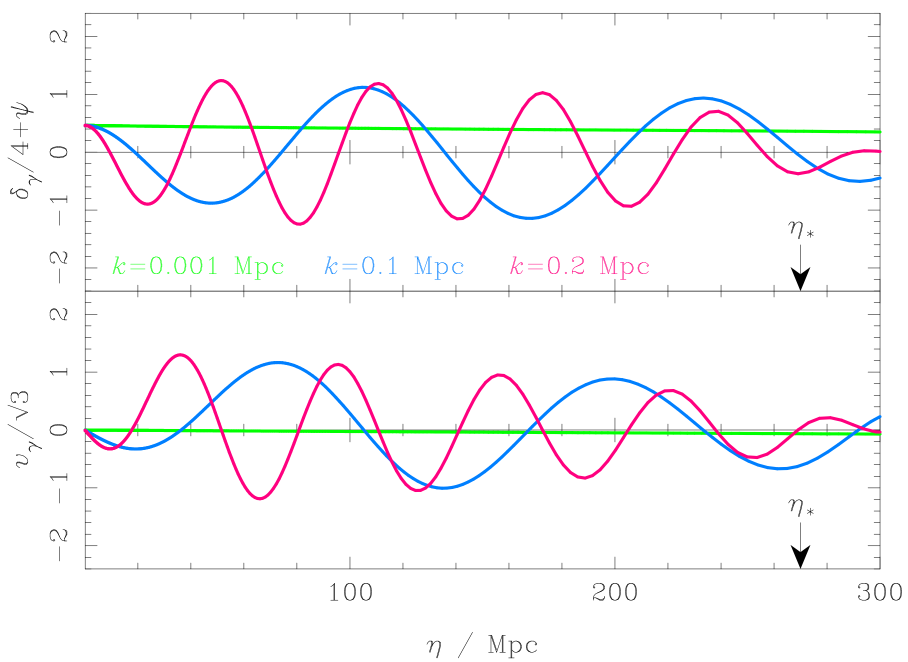
GRAVITATIONAL POTENTIALS AND ACOUSTIC DRIVING

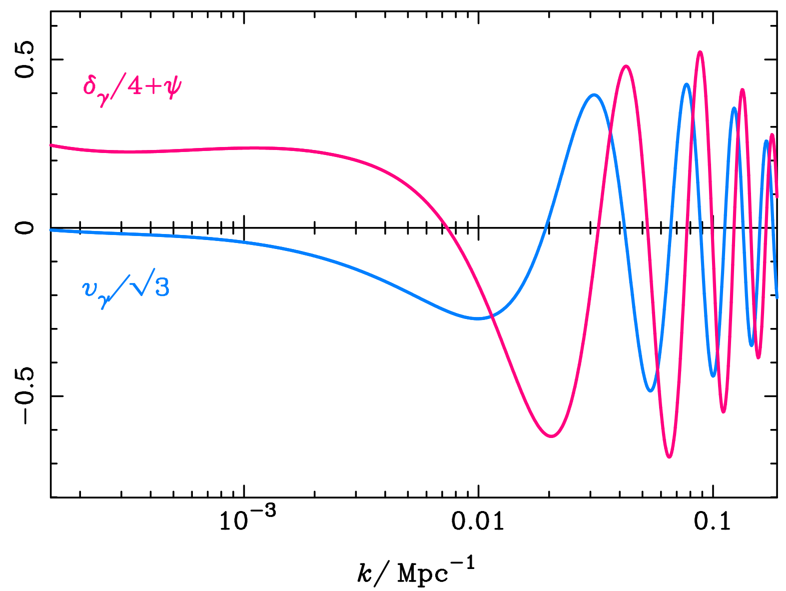
- For adiabatic initial conditions (e.g. simple inflation models), no *relative* perturbations between number densities of species
  - Density perturbations of all species vanish on same hypersurface — its curvature equals comoving curvature $\mathcal{R}$ on super-Hubble scales
- Adiabatic driving term mimics $\cos k\eta$
  - Oscillator is resonantly driven inside sound horizon whilst CDM sub-dominant
  - Potentials constant in matter domination then decay as DE dominates dominates



ACOUSTIC OSCILLATIONS: ADIABATIC MODELS

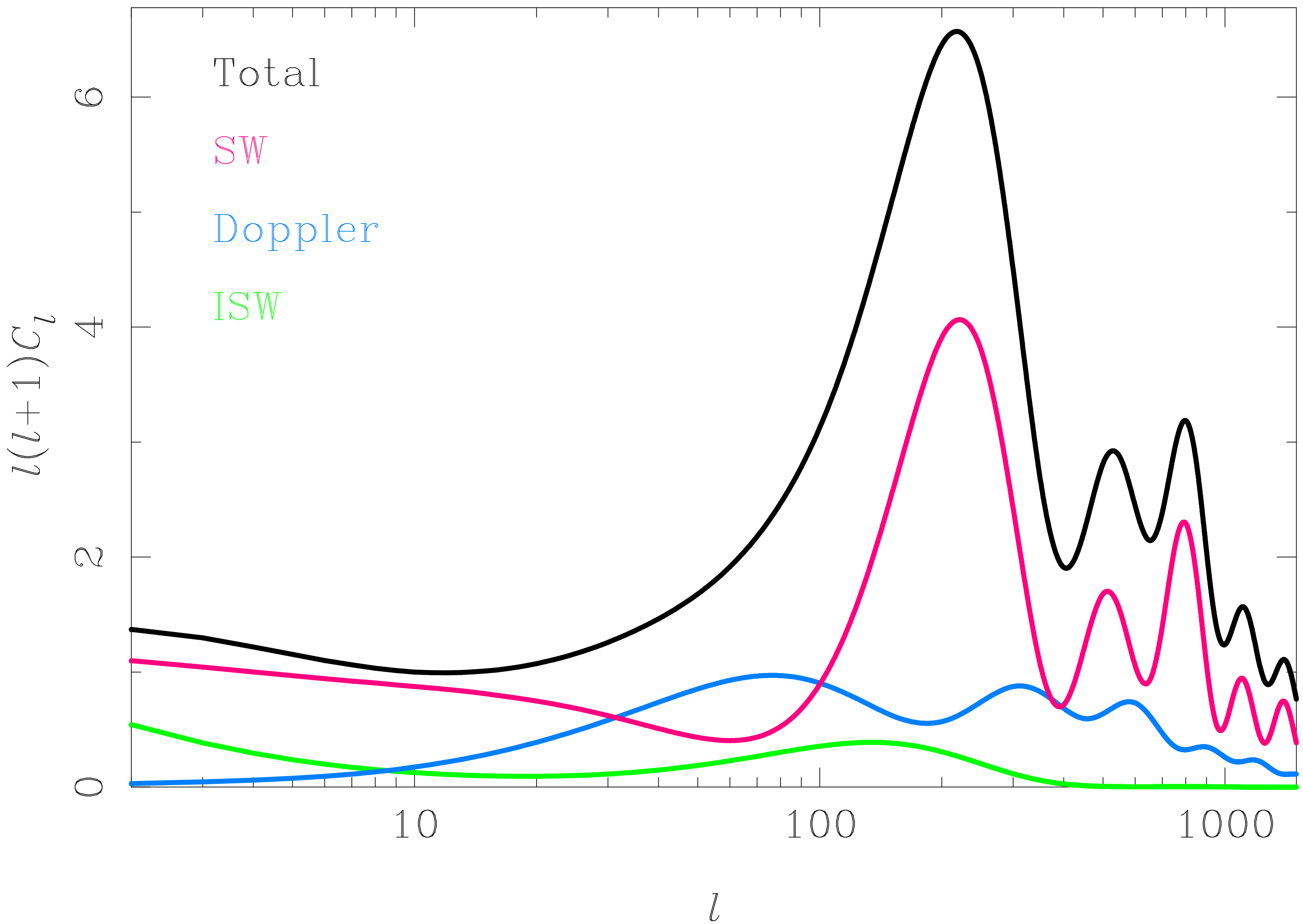
- $\delta_\gamma/4 \equiv \Theta_0$ starts out constant at $-\psi(0)/2 \Rightarrow$ cosine oscillation $\cos kr_s$ about equilibrium point $-(1+R)\psi$
  - Modes with $k\int_0^{\eta_*} c_s\, d\eta = n\pi$ are at extrema at last scattering $\Rightarrow$ acoustic peaks in power spectrum
  - $v_b \approx v_\gamma$ follows from continuity equation ($\pi/2$ out of phase with Θ_0 so Doppler effect 'fills in' zeroes of $\Theta_0 + \psi$)





Adiabatic Anisotropy Power Spectrum

- Temperature power spectrum for scale-invariant curvature fluctuations



Complications: Photon Diffusion

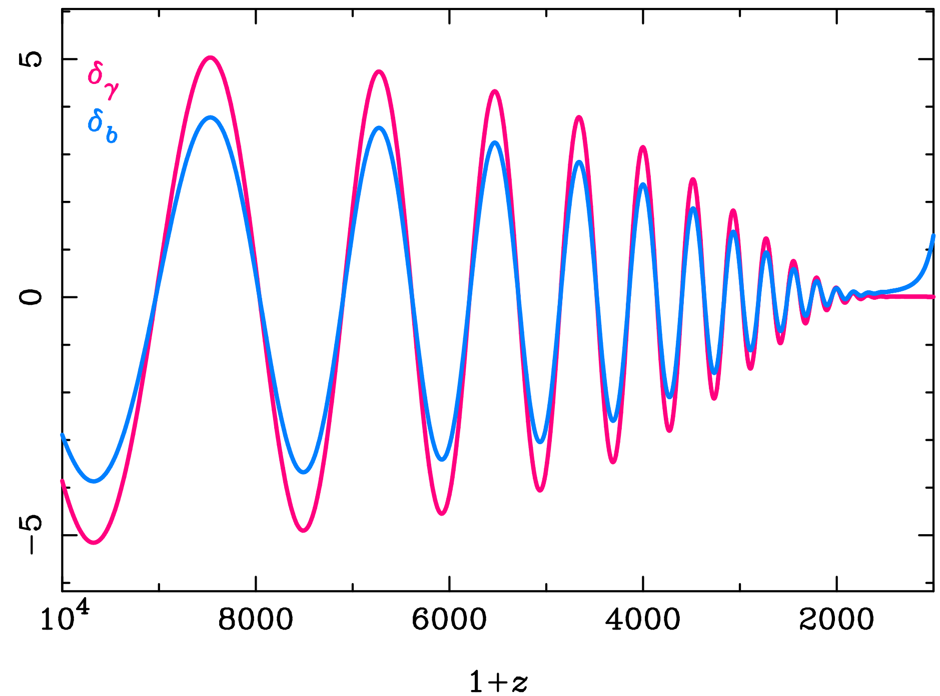
- Photons diffuse out of dense regions damping inhomogeneities in Θ_0 (and creating higher moments of Θ)
  - In time $d\eta$, when mean-free path $\ell = (an_e\sigma_T)^{-1} = 1/|\dot\tau|$, photon random walks mean square distance $\ell d\eta$
  - Defines a diffusion length by last scattering:

$$k_D^{-2} \sim \int_0^{\eta_*} |\dot\tau|^{-1} d\eta \approx 0.2(\Omega_m h^2)^{-1/2}(\Omega_b h^2)^{-1}(a/a_*)^{5/2}\,\mathrm{Mpc}^2$$

- Get exponential suppression of photons (and baryons)

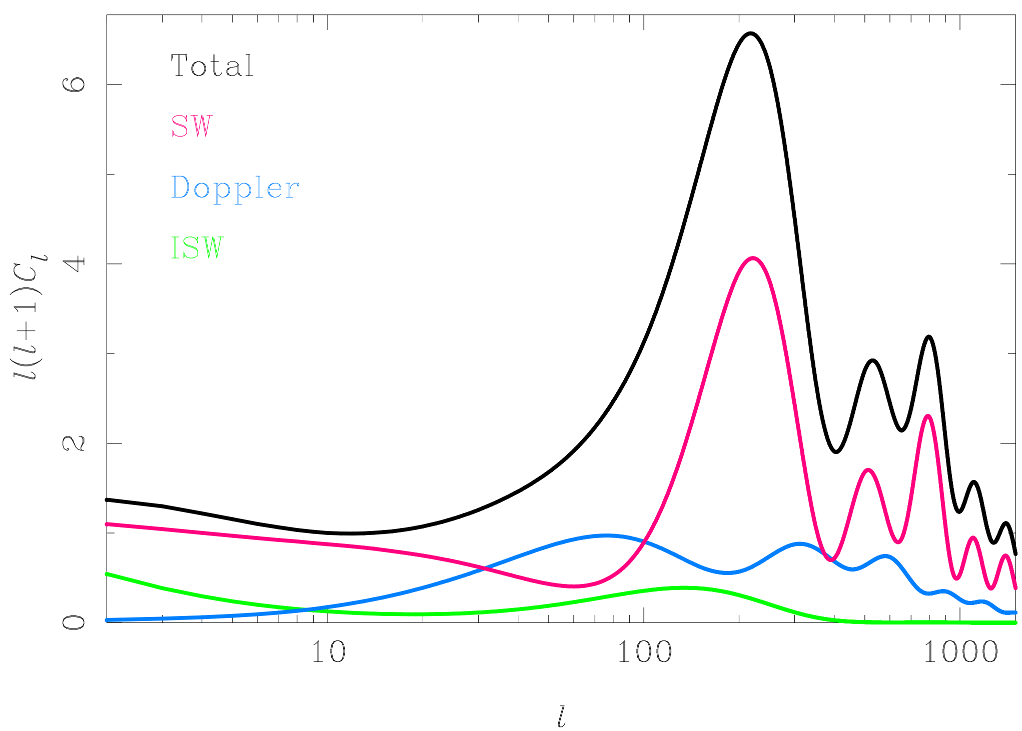
$$\Theta_0 \propto e^{-k^2/k_D^2} \cos kr_s$$

  on scales below $\sim 30\,\mathrm{Mpc}$ at last scattering
  - Implies e^{-2l^2/l_D^2} *damping tail* in power spectrum



INTEGRATED SACHS-WOLFE EFFECT

- Linear $\Theta_{\rm ISW} \equiv \int (\dot\phi + \dot\psi)\, d\eta$ from late-time dark-energy domination and residual radiation at η_*; non-linear small-scale effect from collapsing structures
  - In adiabatic models early ISW adds coherently with SW at first peak since $\Theta_0 + \psi \sim -\psi/2$ same sign as $\dot\psi$
  - Late-time effect is large scale (integrated effect ⇒ peak–trough cancellation suppresses small scales)
  - Late-time effect in dark-energy models produces positive correlation between large-scale CMB and LSS tracers for $z < 2$



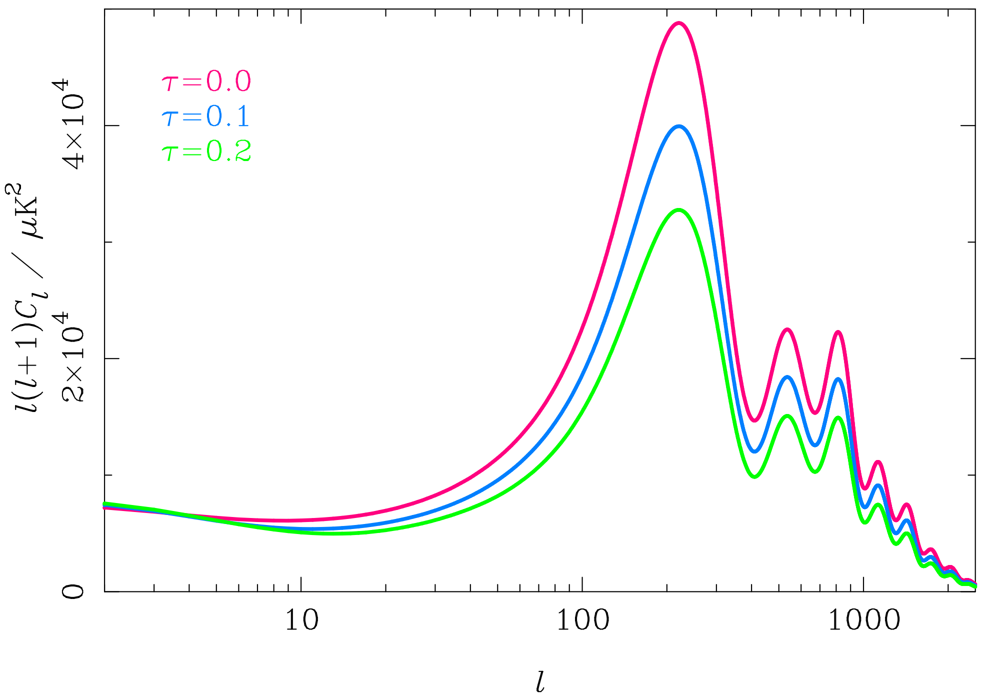
Reionization

- CMB re-scatters off re-ionized gas; ignoring anisotropic (Doppler and quadrupole) scattering terms, locally at reionization have

$$\Theta(\boldsymbol{e}) + \psi \to e^{-\tau}[\Theta(\boldsymbol{e}) + \psi] + (1 - e^{-\tau})(\Theta_0 + \psi)$$

  - Outside horizon at reionization, $\Theta(\boldsymbol{e}) \approx \Theta_0$ and scattering has no effect
  - Well inside horizon, $\Theta_0 + \psi \approx 0$ and observed anisotropies

$$\Theta(\hat{\boldsymbol{n}}) \to e^{-\tau}\Theta(\hat{\boldsymbol{n}}) \quad \Rightarrow \quad C_l \to e^{-2\tau} C_l$$

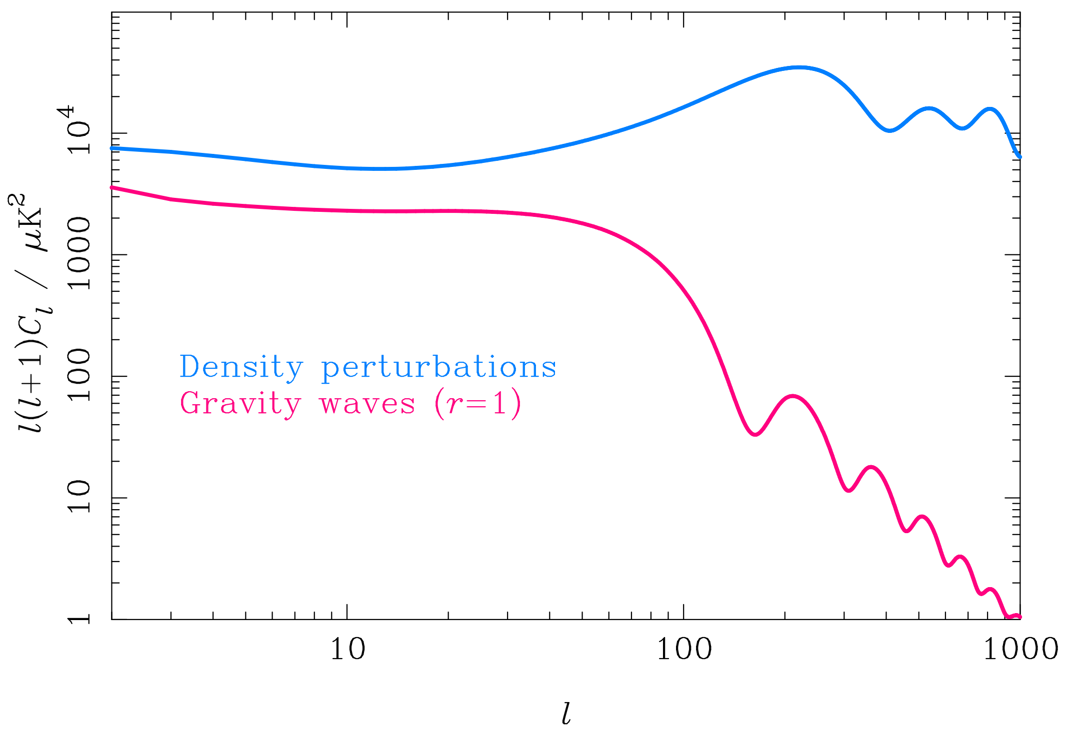


Gravitational waves

- Tensor metric perturbations $ds^2 = a^2[d\eta^2 - (\delta_{ij} + h_{ij})dx^i dx^j]$ with $\delta^{ij}h_{ij} = 0$
  - Shear $\propto \dot{h}_{ij}$ gives anisotropic redshifting $\Rightarrow$

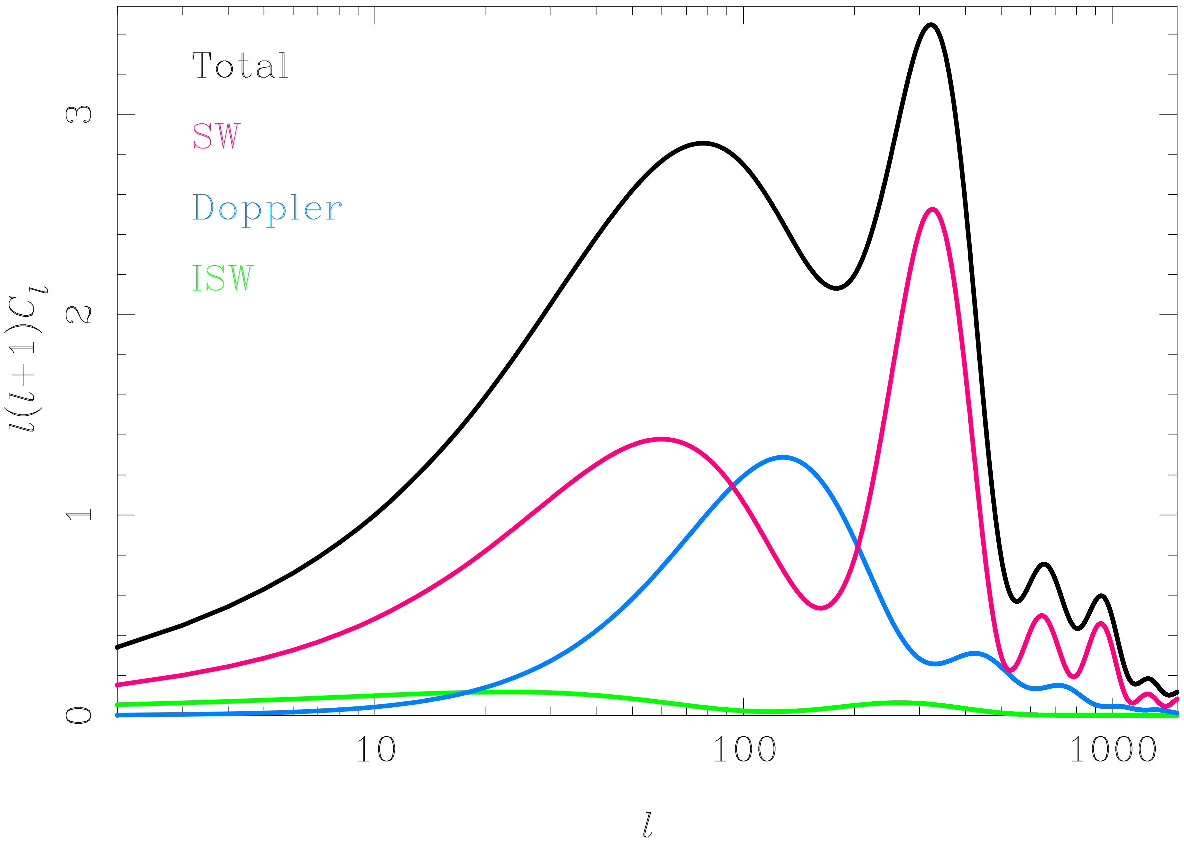
$$\Theta(\hat{\boldsymbol{n}}) \approx -\frac{1}{2}\int d\eta\, \dot{h}_{ij}\hat{n}^i\hat{n}^j$$

  - Only contributes on large scales since h_{ij} decays like a^{-1} after entering horizon



Isocurvature Modes

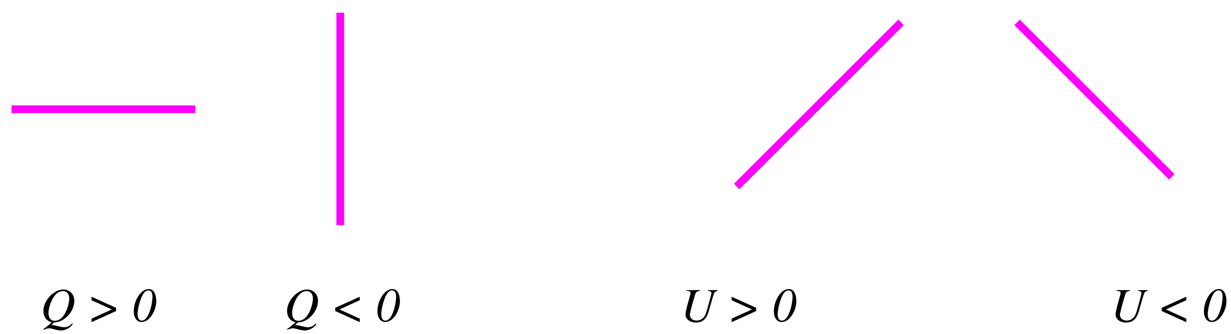
- CDM isocurvature most physically motivated (perturb CDM relative to everything else)
- Starts off with $\delta_\gamma(0) = \phi(0) = 0$ so matches onto $\sin kr_s$ modes
- Temperature power spectrum for $n_{\rm iso} = 1$ entropy fluctuations (CDM isocurvature mode)



CMB POLARIZATION: STOKES PARAMETERS

- For plane wave along z, symmetric trace-free (STF) correlation tensor of electric field $\boldsymbol{E}$ defines (transverse) linear polarization tensor:

$$\mathcal{P}_{ab} \equiv \begin{pmatrix} \frac{1}{2}\langle E_x^2 - E_y^2 \rangle & \langle E_x E_y \rangle \\ \langle E_x E_y \rangle & -\frac{1}{2}\langle E_x^2 - E_y^2 \rangle \end{pmatrix} \equiv \frac{1}{2} \begin{pmatrix} Q & U \\ U & -Q \end{pmatrix}$$



- Under right-handed rotation of x and y through ψ about propagation direction (z)

$$Q \pm iU \to (Q \pm iU)e^{\mp 2i\psi} \quad \Rightarrow \quad Q + iU \text{ is spin -2}$$

E AND B MODES

- Decomposition into E and B modes (use θ, $-\phi$ basis to define Q and U)

$$\mathcal{P}_{ab}(\hat{\boldsymbol{n}}) = \nabla_{\langle a}\nabla_{b\rangle}P_E + \epsilon^c{}_{(a}\nabla_{b)}\nabla_c P_B$$
$$\Rightarrow \quad Q + iU = \bar{\eth}\bar{\eth}(P_E - iP_B)$$

  – Spin-lowering operator: $\bar{\eth}\,{}_s\eta = -\sin^{-s}\theta(\partial_\theta - i\mathrm{cosec}\partial_\phi)(\sin^s\theta\,{}_s\eta)$

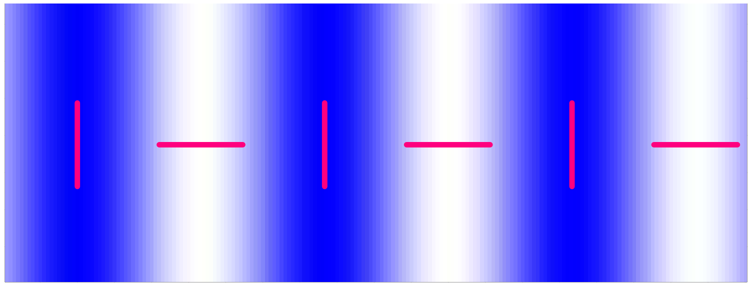
- Expand P_E and P_B in spherical harmonics, e.g.

$$P_E(\hat{\boldsymbol{n}}) = \sum_{lm}\sqrt{\frac{(l-2)!}{(l+2)!}}E_{lm}Y_{lm}(\hat{\boldsymbol{n}}) \quad \Rightarrow \quad (Q \pm iU)(\hat{\boldsymbol{n}}) = \sum_{lm}(E_{lm} \mp iB_{lm})_{\mp 2}Y_{lm}(\hat{\boldsymbol{n}})$$

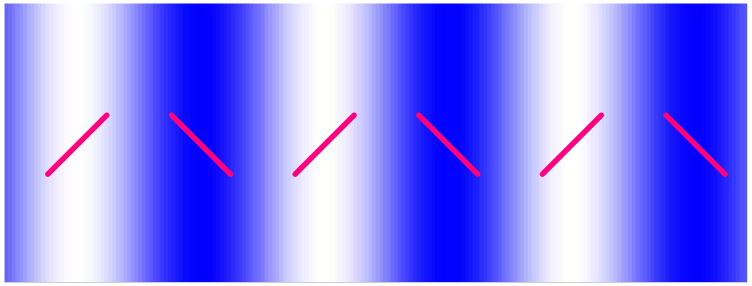
  – Spin-weight harmonics ${}_sY_{lm}$ provide orthonormal basis for spin-s functions

- Only three power spectra if parity respected in mean: C_l^E, C_l^B and C_l^{TE}

Pure E mode

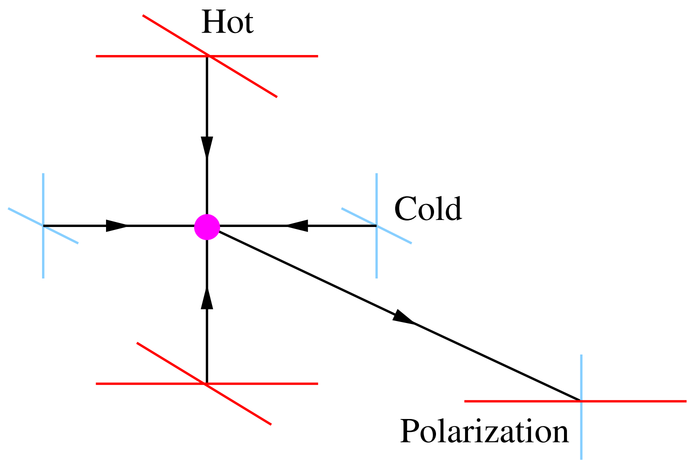


Pure B mode



CMB POLARIZATION: THOMSON SCATTERING

- Photon diffusion around recombination $\rightarrow$ local temperature quadrupole
  - Subsequent Thomson scattering generates (partial) linear polarization with r.m.s. $\sim 5\,\mu\mathrm{K}$ from density perturbations



- Thomson scattering of radiation quadrupole produces linear polarization (dimensionless temperature units!)

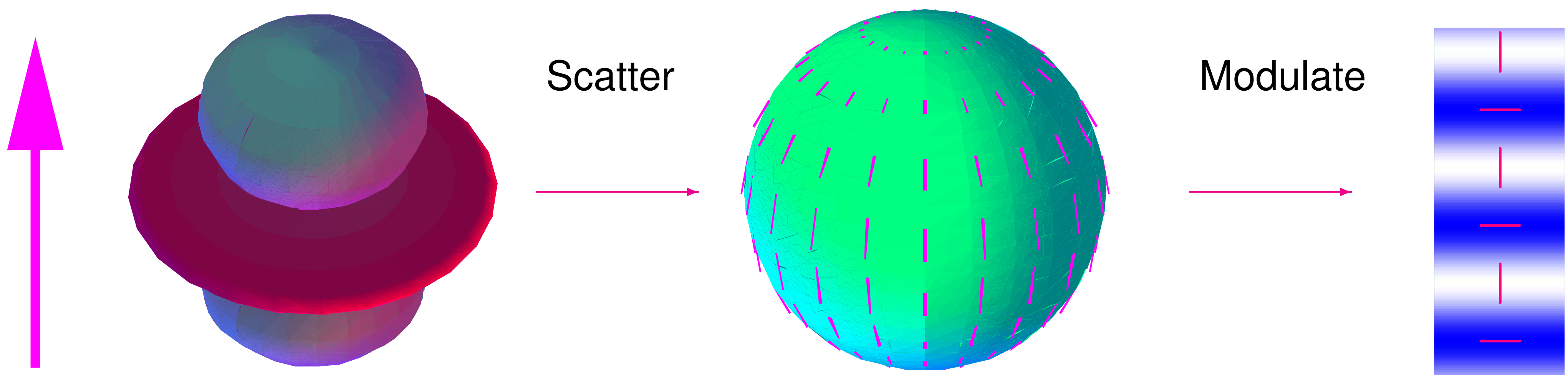
$$d(Q \pm iU)(\boldsymbol{e}) = \frac{3}{5} a n_e \sigma_T d\eta \sum_m {}_{\pm 2}Y_{2m}(\boldsymbol{e}) \left(E_{2m} - \sqrt{\frac{1}{6}} \Theta_{2m} \right)$$

  - Purely electric quadrupole ($l = 2$)

- In linear theory, generated $Q + iU$ then conserved for free-streaming radiation
  - Suppressed by $e^{-\tau}$ if further scattering at reionization

PHYSICS OF CMB POLARIZATION: SCALAR PERTURBATIONS

- Single plane wave of scalar perturbation has $\Theta_{2m} \propto Y^*_{2m}(\hat{\boldsymbol{k}}) \Rightarrow$ with $\hat{\boldsymbol{k}}$ along z, $dQ \propto \sin^2\theta$ and $dU = 0$

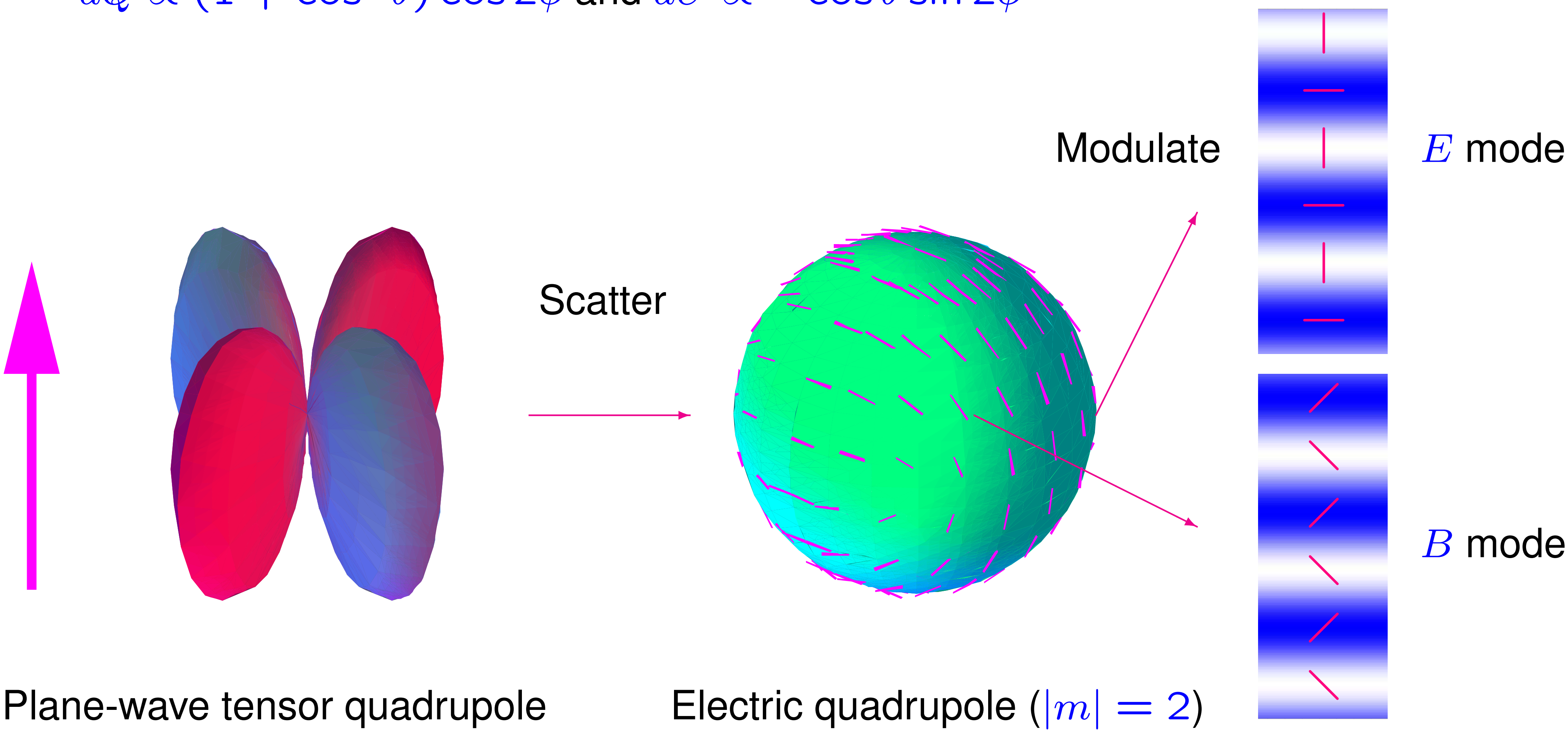


Plane-wave scalar quadrupole — Electric quadrupole ($m = 0$) — Pure E mode

- Linear scalar perturbations produce only E-mode polarization
- Mainly traces baryon velocity at recombination $\Rightarrow$ peaks at troughs of ΔT

Physics of CMB Polarization: Gravitational Waves

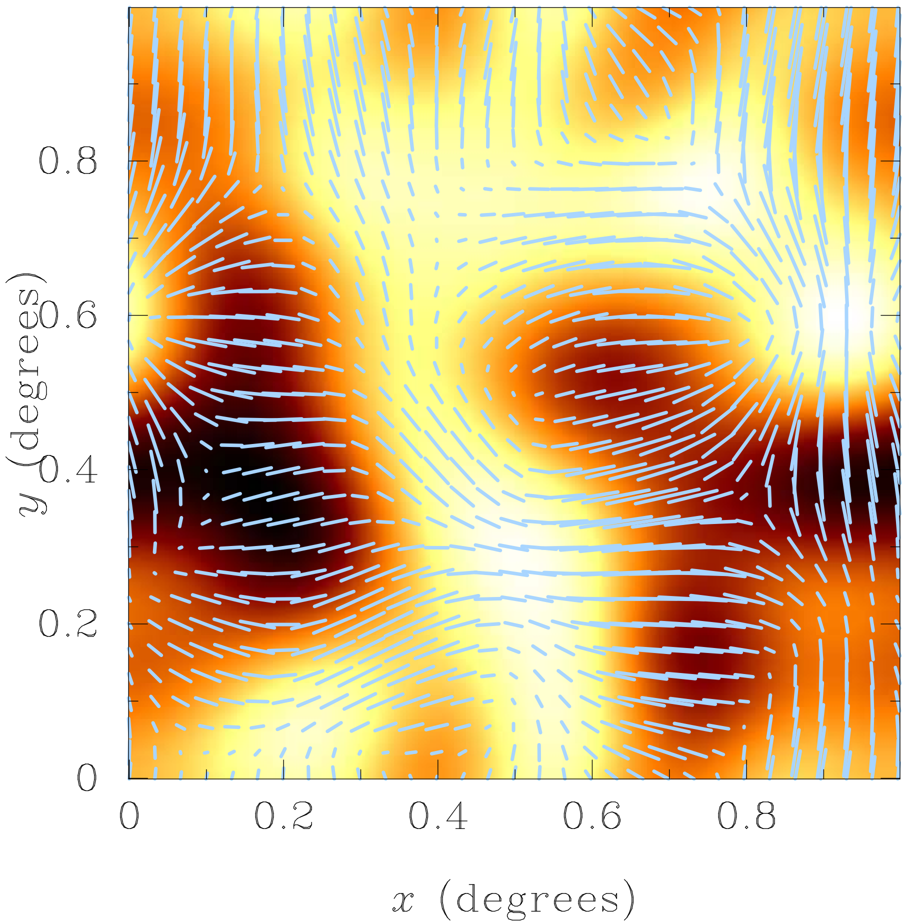
- For single $+$-polarized gravity wave with $\hat{\boldsymbol{k}}$ along z, $\Theta_{2m} \propto \delta_{m2} + \delta_{m\,-2}$ so $dQ \propto (1 + \cos^2\theta)\cos 2\phi$ and $dU \propto -\cos\theta \sin 2\phi$



- Gravity waves produce both E- and B-mode polarization (with roughly equal power)

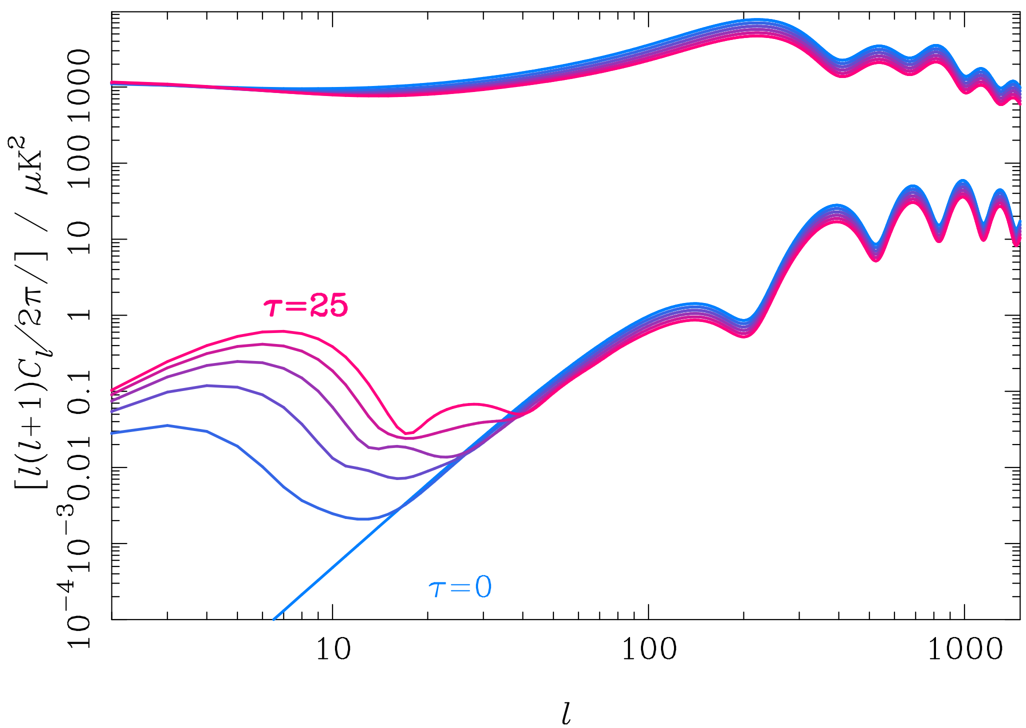
Correlated Polarization in Real Space

- On largest scales, infall into potential wells at last scattering generates e.g. tangential polarization around large-scale hot spots
- Sign of correlation scale-dependent inside horizon



Large-angle polarization from reionization

- Temperature quadrupole at reionization peaks around $k(\eta_{\rm re} - \eta_*) \sim 2$
  - Re-scattering generates polarization on this linear scale → projects to $l \sim 2(\eta_0 - \eta_{\rm re})/(\eta_{\rm re} - \eta_*)$
  - Amplitude of polarization $\propto$ optical depth through reionization → best way to measure τ with CMB



SCALAR AND TENSOR POWER SPECTRA ($r = 0.28$)

- For scalar perturbations (left), δ_γ oscillates $\pi/2$ out of phase with $v_\gamma \Rightarrow C_l^E$ peaks at minima of C_l^T