String Theory, Black holes and the AdS/CFT duality

Samir D. Mathur

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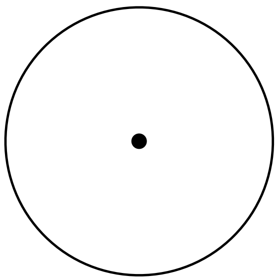
The entropy problem

Black holes behave as if they have an entropy given by their surface area



$$S_{bek} = \frac{A}{4G}$$

(Bekenstein, 72)

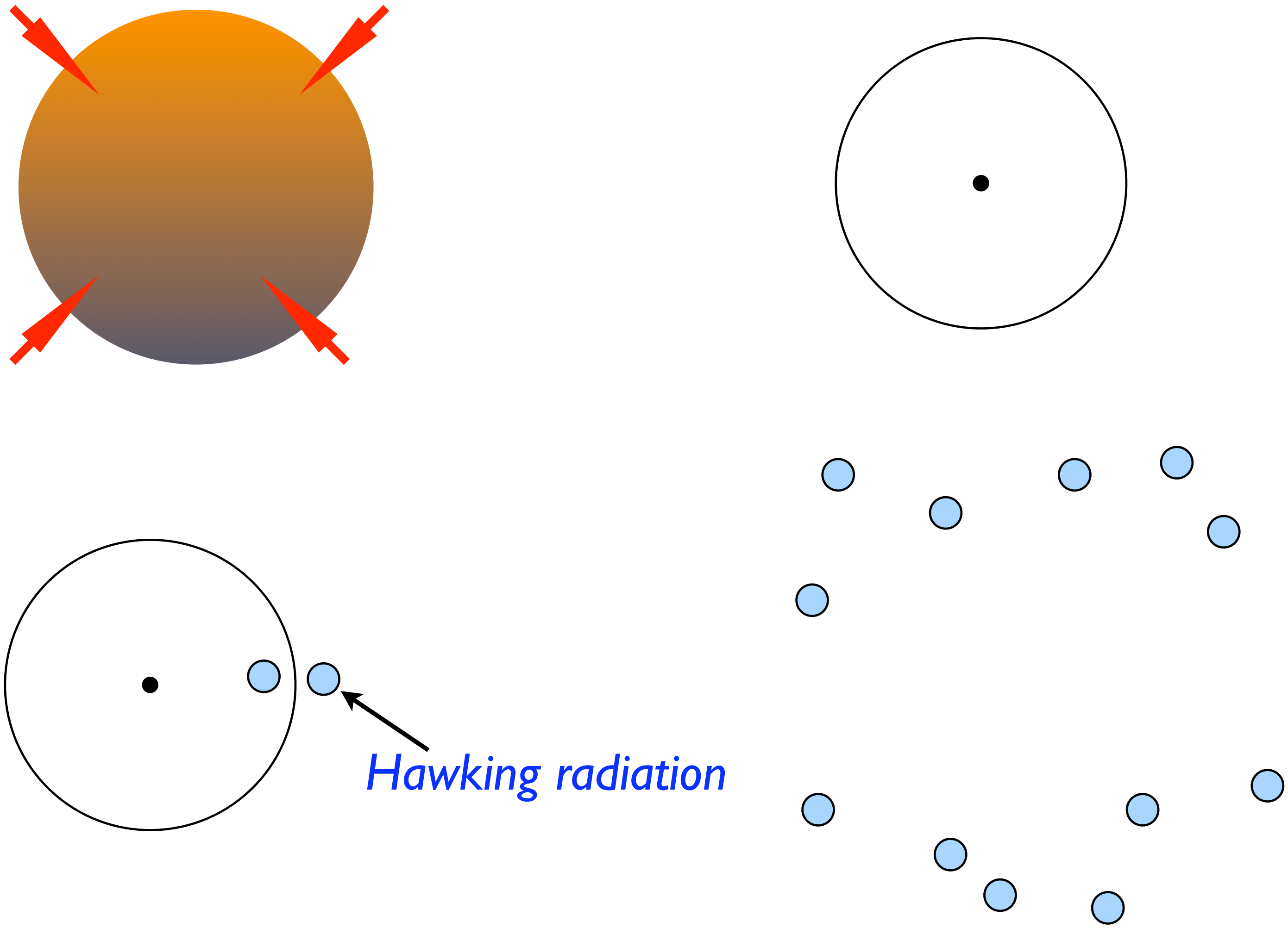
But statistical mechanics then says that there should be

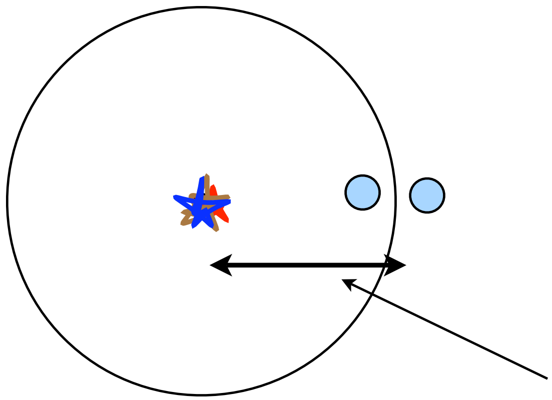
$e^{S_{bek}}$ states of the hole for the same mass and charge

Can we show that there are $e^{S_{bek}}$ states of the hole ?

(Classical relativity finds that black holes have no hair, so there is only *one* state)

The information problem





Large distance
(much bigger than planck length)

How can the Hawking radiation carry
the information of the initial matter ?

If the radiation does not carry the information,
then the final state cannot be determined from the initial state, and there is no Schrodinger type evolution equation for the whole system.

So we lose quantum theory ...

Plan of the lectures

(A) How do you make black holes in string theory ?
(solving the entropy problem)

(B) Understanding the origins of AdS/CFT duality

(C) How do we solve the information puzzle in string theory? (The fuzzball picture of the black hole)

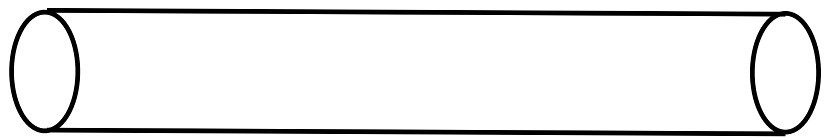
(D) Understanding black hole states using the AdS/CFT duality

(A) How do you make black holes in string theory ?
(solving the entropy problem)

Some facts from string theory :

(a) Strings live in 9+1 dimensions

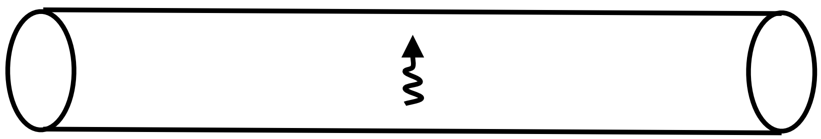
We see only 3+1 dimensions, so the others
must be small compact directions



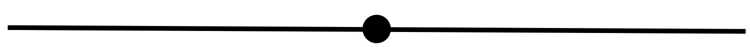
(b) There are many kinds of elementary excitations in the theory, for example gravitons, strings, branes ...

We must make our black holes using these objects ...

Consider a graviton running along the compact direction



To a person who cannot resolve the compact circle,
this looks like a point mass in the noncompact directions



This point mass also carries a ‘winding charge’,
from the usual idea of Kaluza-Klein reduction

$$g_{y\mu} = A_\mu$$

compact   non-compact

(c) Such objects are ‘BPS objects’, i.e., they have
‘mass = charge’

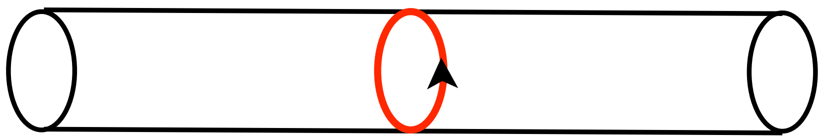
Newtonian mechanics

Gravitational attraction: $-\frac{GM^2}{r^2}$

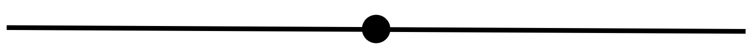
Electromagnetic repulsion: $\frac{Q^2}{r^2}$

These should exactly cancel

We can also wrap a string around the compact directions



Again, to a person who cannot resolve the compact circle, this looks like a point mass in the noncompact directions



$$B_{y\mu} = \tilde{A}_\mu$$

The string radiates a 2-form gauge field, which looks like a usual gauge field in the non-compact directions

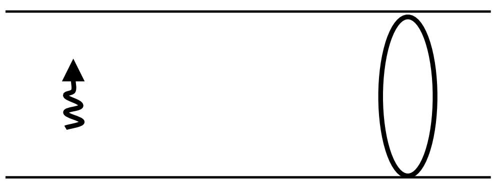
So this string is also a charged object ... it has ‘mass=charge’ (BPS)

A simple example: 2-charge holes

(Susskind, Sen, Vafa '94-'95)

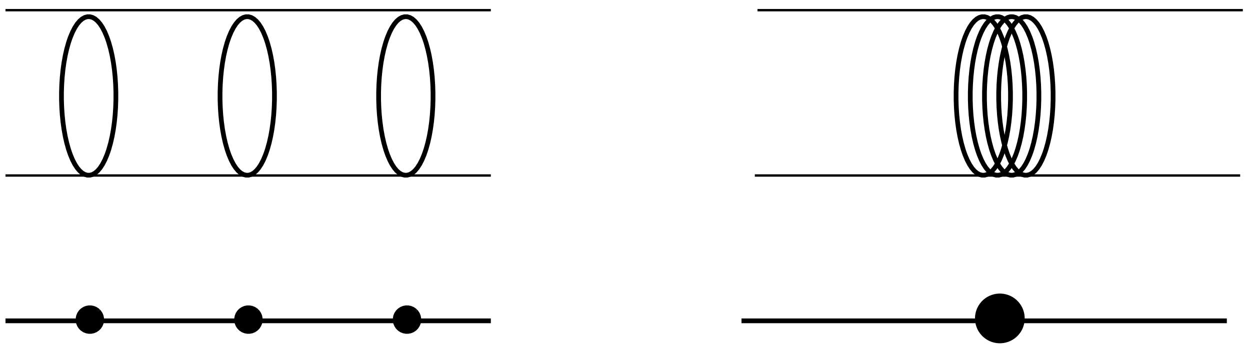
In string theory, we must make black holes from the objects present in string theory.

Let us compactify spacetime as $M_{9,1} \to M_{4,1} \times T^4 \times S^1$



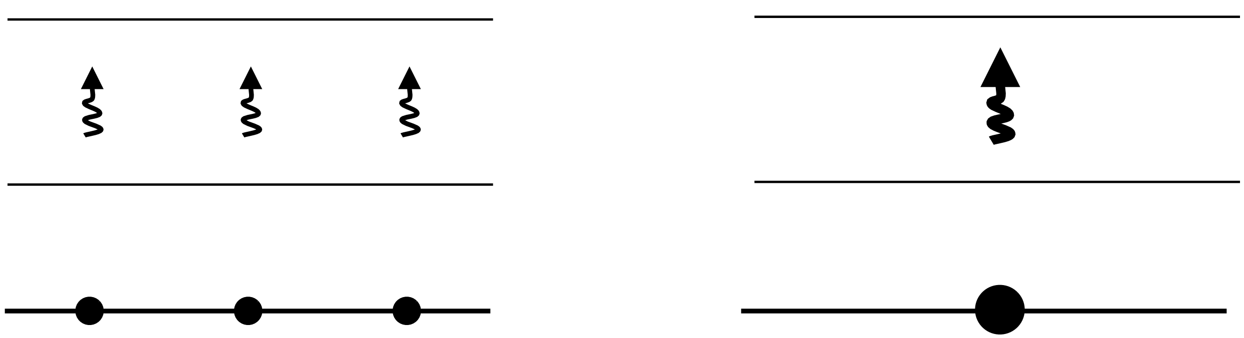
momentum mode P

winding mode NS1



Winding charge n_1

Mass = Charge



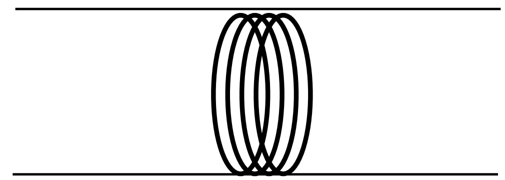
Momentum charge n_p

Mass = Charge

A black hole with winding charge only

$$S_{micro} = \ln[256] \sim 0$$

(Does not grow with n_1)

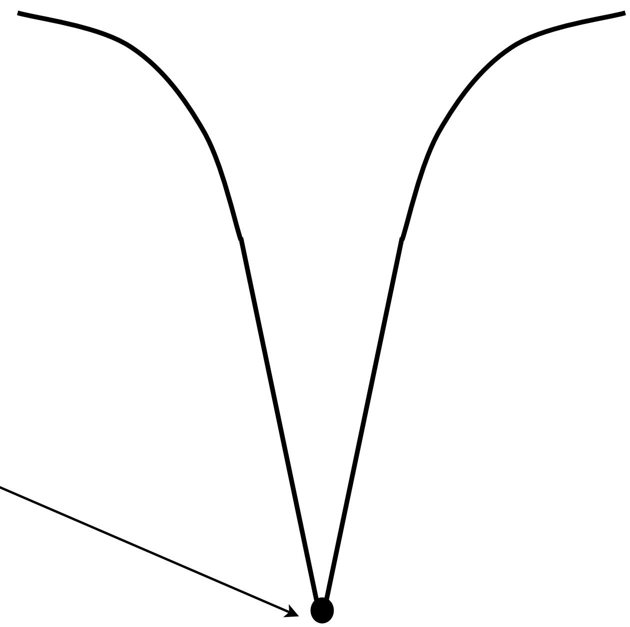


n_1

Horizon is singular

$$A \ = \ 0$$

Bekenstein entropy vanishes

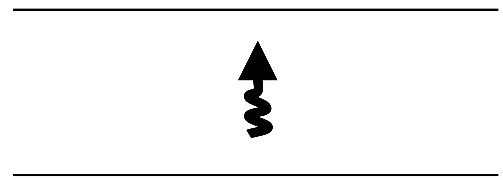


$$S_{micro} = S_{bek} = 0$$

A black hole with momentum charge only

$$S_{micro} = \ln[256] \sim 0$$

(Does not grow with n_p)



n_p

Horizon is singular

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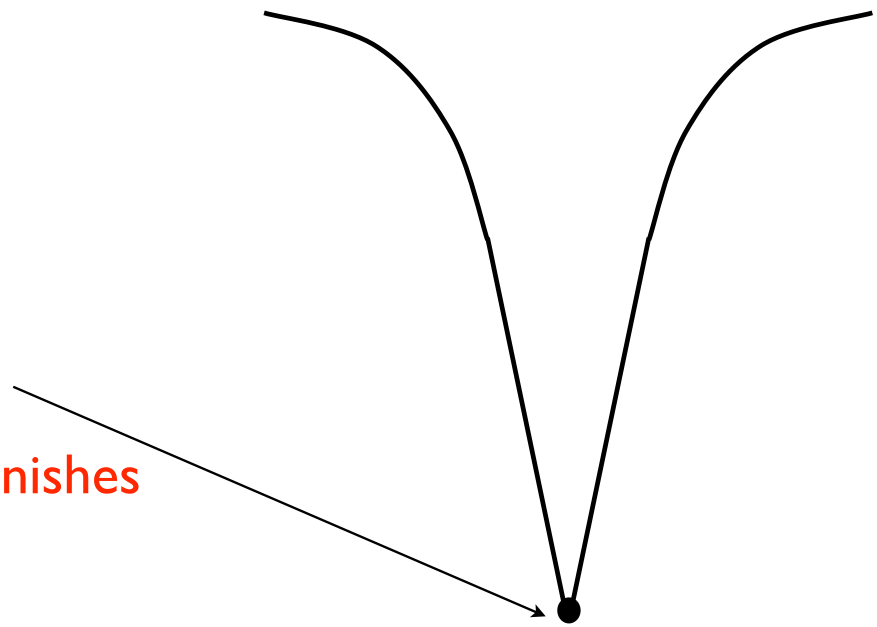
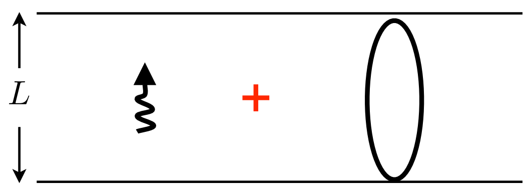
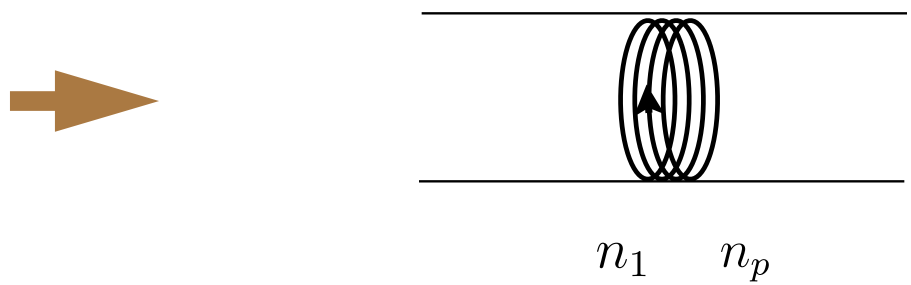
Bekenstein entropy vanishes



$$S_{micro} = S_{bek} = 0$$

A black hole with winding AND momentum charge

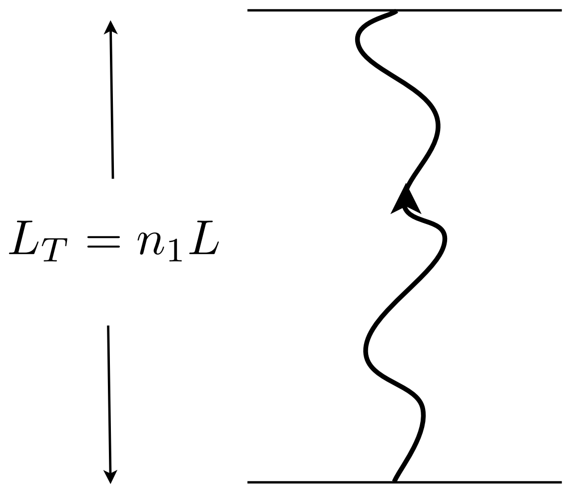




The momentum charge is carried as traveling waves on the 'multiwound string'

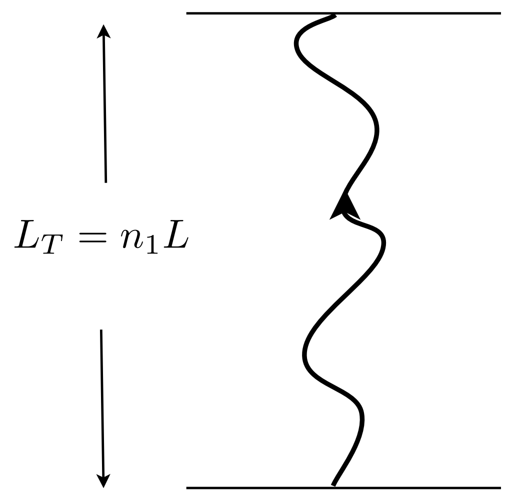
But there are many ways to do this ...

For example, we can put all the energy in the lowest harmonic, or some in the first and some in the second harmonic etc ....



Computing the entropy

Each quantum of harmonic k carries momentum $\frac{2\pi k}{L_T}$



Total momentum

$$P = \frac{2\pi n_p}{L} = \frac{2\pi(n_1 n_p)}{L_T}$$

So we have to count 'partitions' of $n_1 n_p$

$$\sum k \; n_k = n_1 n_p$$

8 bosonic + 8 fermionic degrees of freedom

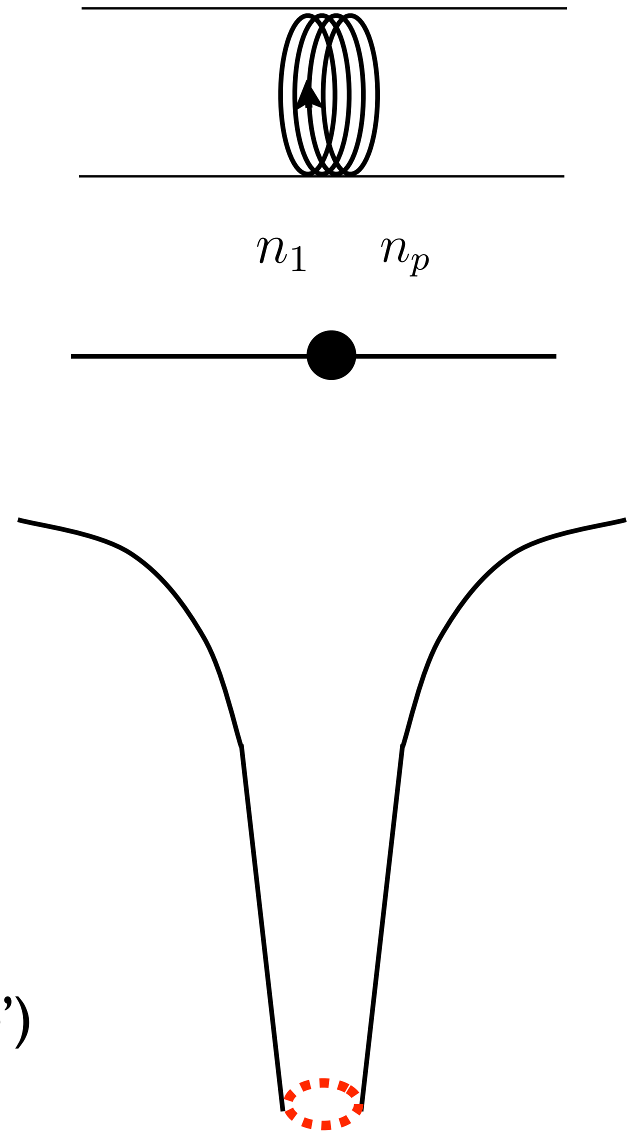
$e^{2\pi\sqrt{2}\sqrt{n_1 n_p}}$ *states* →

$$S_{micro} = 2\pi\sqrt{2}\sqrt{n_1 n_p} \qquad T^4 \times S^1$$

$$S_{micro} = 4\pi\sqrt{n_1 n_p} \qquad K3 \times S^1$$

(Susskind '93, Sen '94)

Now let us make a black hole with these charges ...



For $K3 \times S^1$ compactification, geometry gives a Bekenstein - Wald entropy

$$S_{bek} = \frac{A}{2G} = 4\pi\sqrt{n_1 n_p} = S_{micro}$$

(Dabholkar '04)

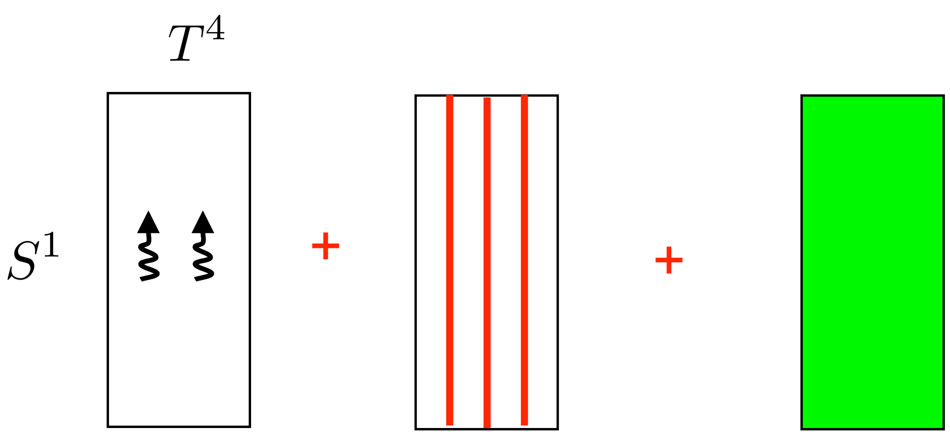
So the 2-charge hole ('Sen-Vafa hole')
gives a complete story for
black hole entropy in string theory ......

The 2-charge black hole is called the 'small black hole' since R corrections to the action affect its horizon area

To get a black hole whose area is given by just the usual Einstein action R, we need 3 charges ...

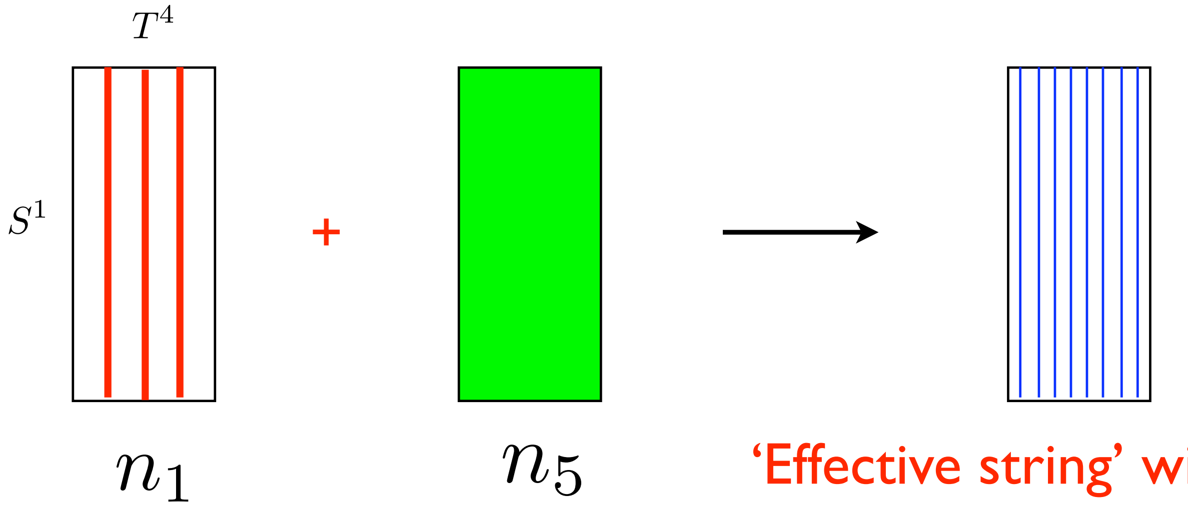
Recall that we had compactified spacetime as

$$M_{9,1} \to M_{4,1} \times T^4 \times S^1$$

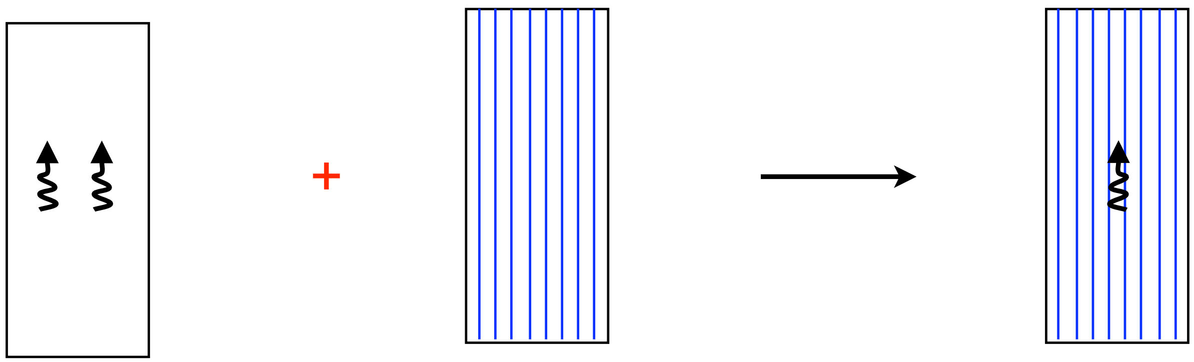


Make a bound state of 3 kinds of charges

5-branes



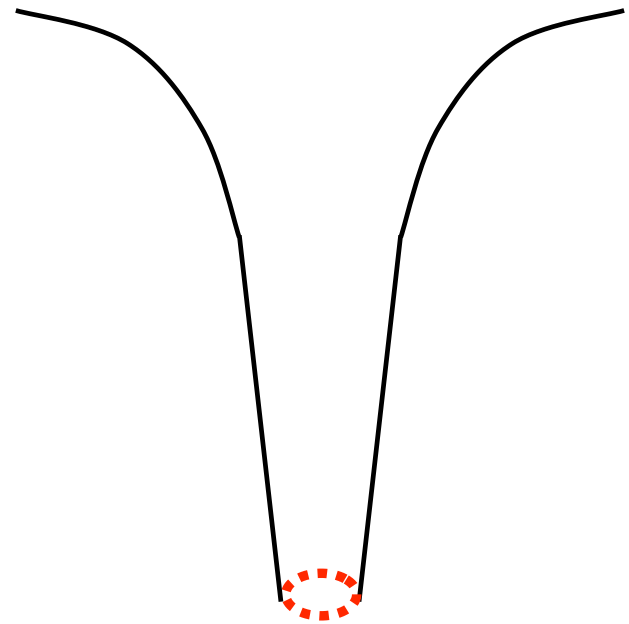
'Effective string' with total winding number $n_1 n_5$



Vibrations of effective string are partitioned into harmonics in usual way

$$S_{micro} = 2\pi\sqrt{n_1 n_5 n_p}$$

Make a black hole with momentum, winding, 5-brane charges ....



$$S_{bek} = \frac{A}{4G} = 2\pi\sqrt{n_1 n_5 n_p} = S_{micro}$$

(Strominger and Vafa, 1996)

Thus at least for extremal black holes (mass=charge) we understand something about the entropy from a microscopic viewpoint ...

Near-extremal holes have also been understood ... (Callan and Maldacena 1996)

Some general lessons about entropy

(a) To understand the entropy of black holes in 4+1 noncompact dimensions, we have to see how branes wrap around the OTHER 5 (compact) dimensions

(b) People used to think that string theory is very wasteful ... apart from the graviton, we get all these other excited vibration modes of the string ... but now we see that exactly these vibrations account for the entropy of black holes ... this is a big validation of string theory

(B) Understanding the origins of AdS/CFT duality

The AdS/CFT correspondence

This is a very remarkable relation.

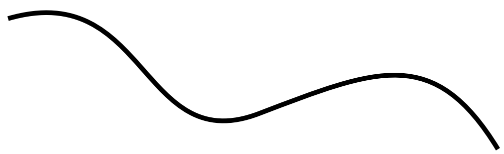
Gravity is a very funny theory,
nobody was able to quantize it for a long time

This is because the graviton is a spin 2 particle

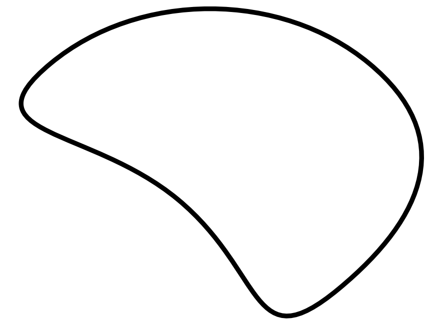
We can quantize gauge theories,
where the basic particle, the photon, is spin 1

But this correspondence says that we can
regard gravity as a gauge theory ... !!
(The relation between variables in the two descriptions
must be somewhat complicated ....)

Open and closed strings



open string

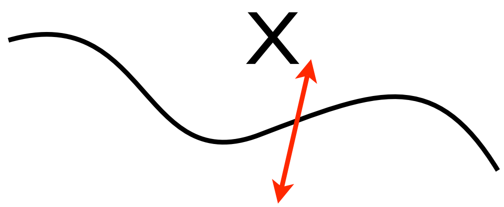


closed string

Open string:

The energy of vibration must not flow off the end of the string.

For this either (a) the endpoint must be fixed or (b) the endpoint must be moving with the speed of light



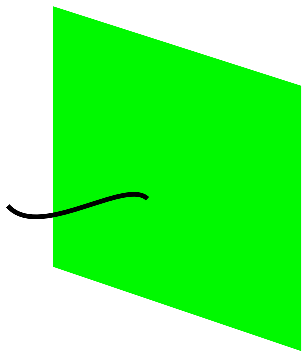
If endpoint is fixed, we call it Dirichlet boundary condition (D),

If spatial derivative is fixed, then we call it Neumann boundary condition (N)

So we can have DD, ND, NN open strings

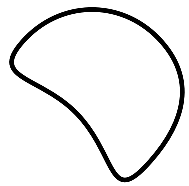
The NN open strings can move wherever they want, so initially these were the only ones studied

But then Polchinski realized that a D boundary condition meant that the open string ended on some source ...



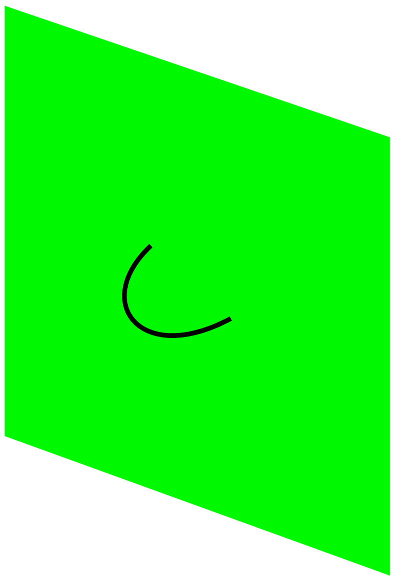
DD open strings

What is a closed string ?



We have a background spacetime, and a closed string moving on this background is an excitation, for example a graviton ...

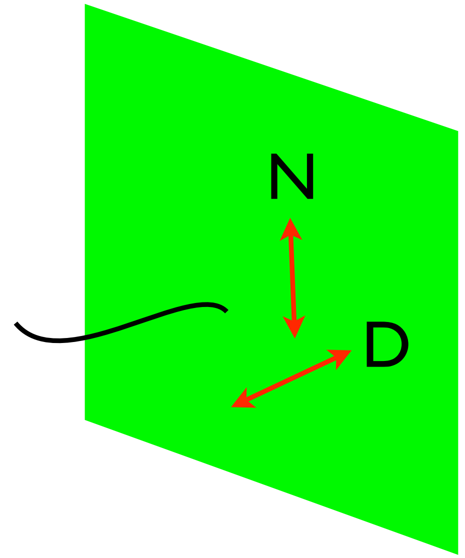
Now suppose there is a soliton on this background ...



The soliton has low energy excitation modes ...

These are described by an open string attached to the soliton ...

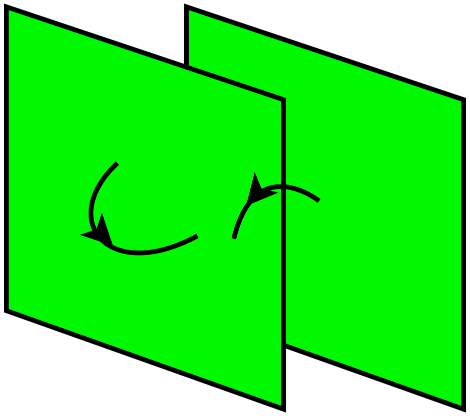
The string lives in 10 spacetime dimensions, and each direction can be N or D, and we have this choice at each endpoint



The number of N directions tells us how many directions are ‘along’ the brane. So if 3 space and one time directions are N, then the directions along the brane (the brane ‘world-volume’) is 3+1 dimensional ...

This gives a D3 brane

Let there be N solitions (D3 branes)



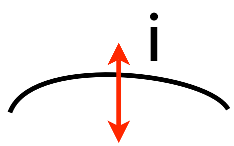
The open string can start on any brane and end on any other brane

The open string has an orientation, indicated by the arrow ...

We will now be interested in the low energy excitations of the solitons

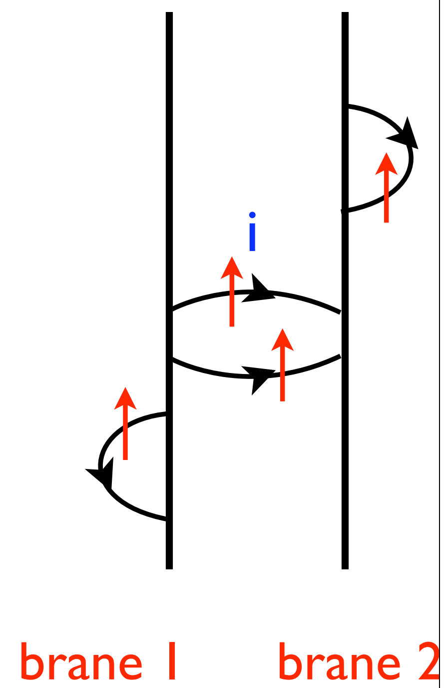
Thus we look at the lowest energy mode of the open string

When we quantize the superstring carefully, we find that this lowest mode has one unit of vibration, which is described by one index i, with i=0,1, ... 9



Let i be a direction along the brane, i=0, 1, 2, 3

We wish to write down the 'strength' of open strings of each type



$$\hat{A}_\mu = \begin{pmatrix} A_\mu^{11} & A_\mu^{12} \\ A_\mu^{21} & A_\mu^{22} \end{pmatrix}$$

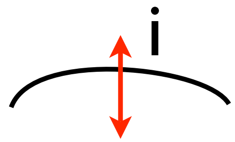
This is the field content of SU(2) Yang-Mills gauge theory

But we also have i=4,5,6,7,8,9. This gives 6 scalars

$$\hat{X}_a = \begin{pmatrix} X_a^{11} & X_a^{12} \\ X_a^{21} & X_a^{22} \end{pmatrix}$$

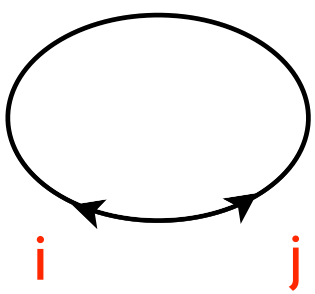
We also have fermions.
All together, we get n=4 supersymmetric Yang-Mills with gauge group SU(N)

We have seen that the ground state of the open string has one excitation, which is described by a direction of vibration i



The closed string can have propagating clockwise as well as anticlockwise, (Left and Right moving vibrations)

For each kind of mode, the ground state has one unit of excitation, so the ground state of the closed string is described by two polarizations i, j = 0, 1, ... 9



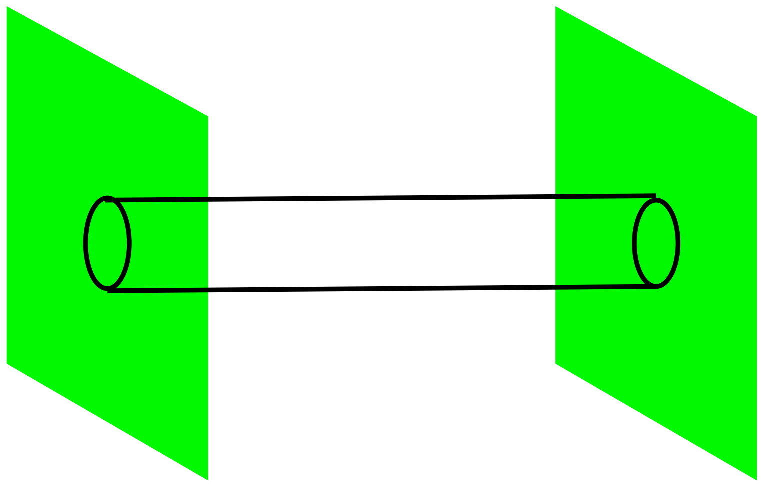
Number of closed strings with polarization i,j $\longrightarrow$ S_{ij}

Symmetric traceless part: graviton h_{ij}

Antisymmetric part: 2-form gauge field B_{ij}

Trace: dilaton ϕ

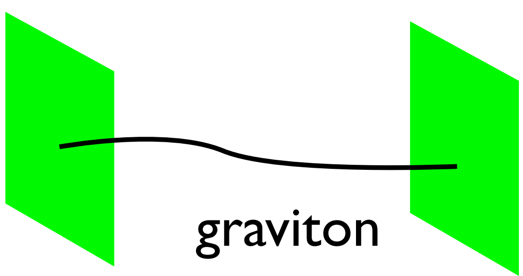
Open closed duality



We can think of this in two ways:

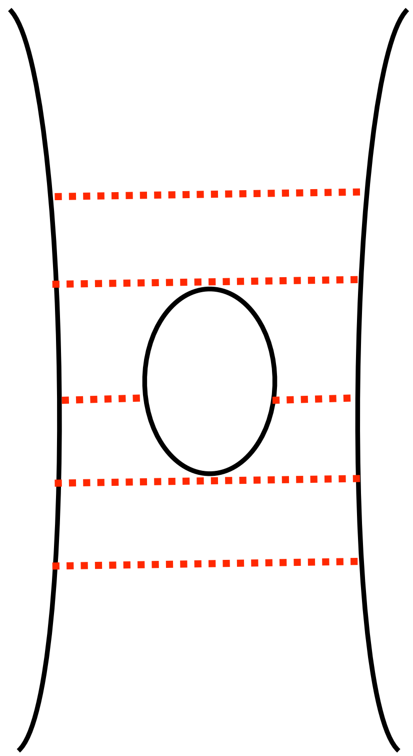
(a) a 1-loop diagram of open strings: gauge theory

(b) a tree level diagram of closed strings:
the first brane emits a closed string, (e.g. gravtion),
the other one absorbs it

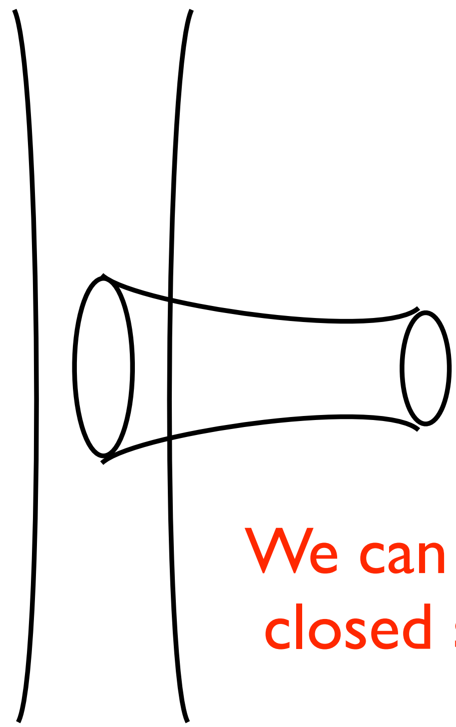


So the same process can be thought of as a gauge theory process or a gravity process

We cannot separate open and closed string theories ...

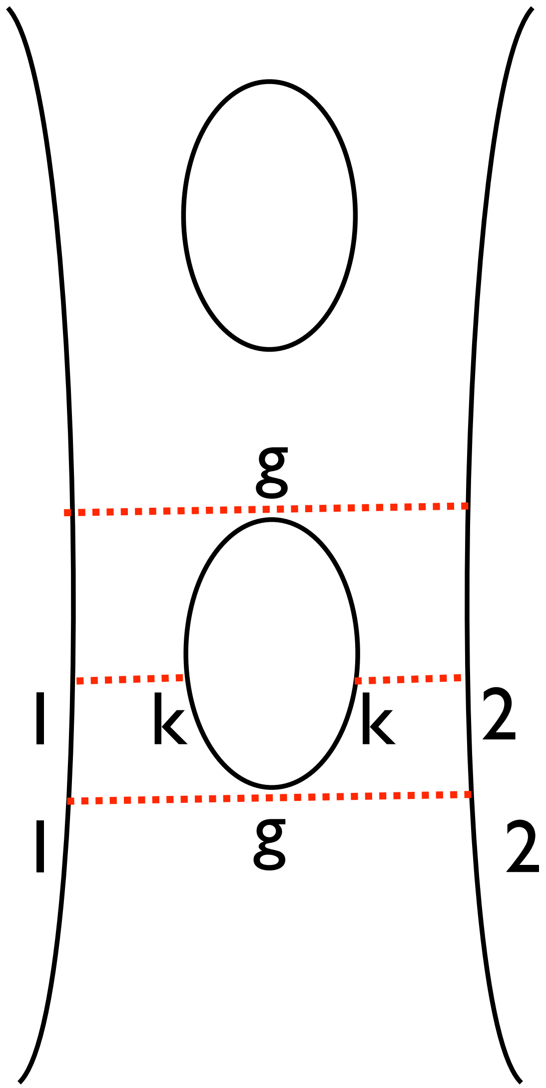


An open string splits into two open strings, and joins back to one open string



We can regard this as a closed string being emitted

But we still want to take a limit where the open strings separate, in some way, from the closed strings ... this is called ‘decoupling’



Let there be a large number N of D-branes

Then k = 1, 2, ... N

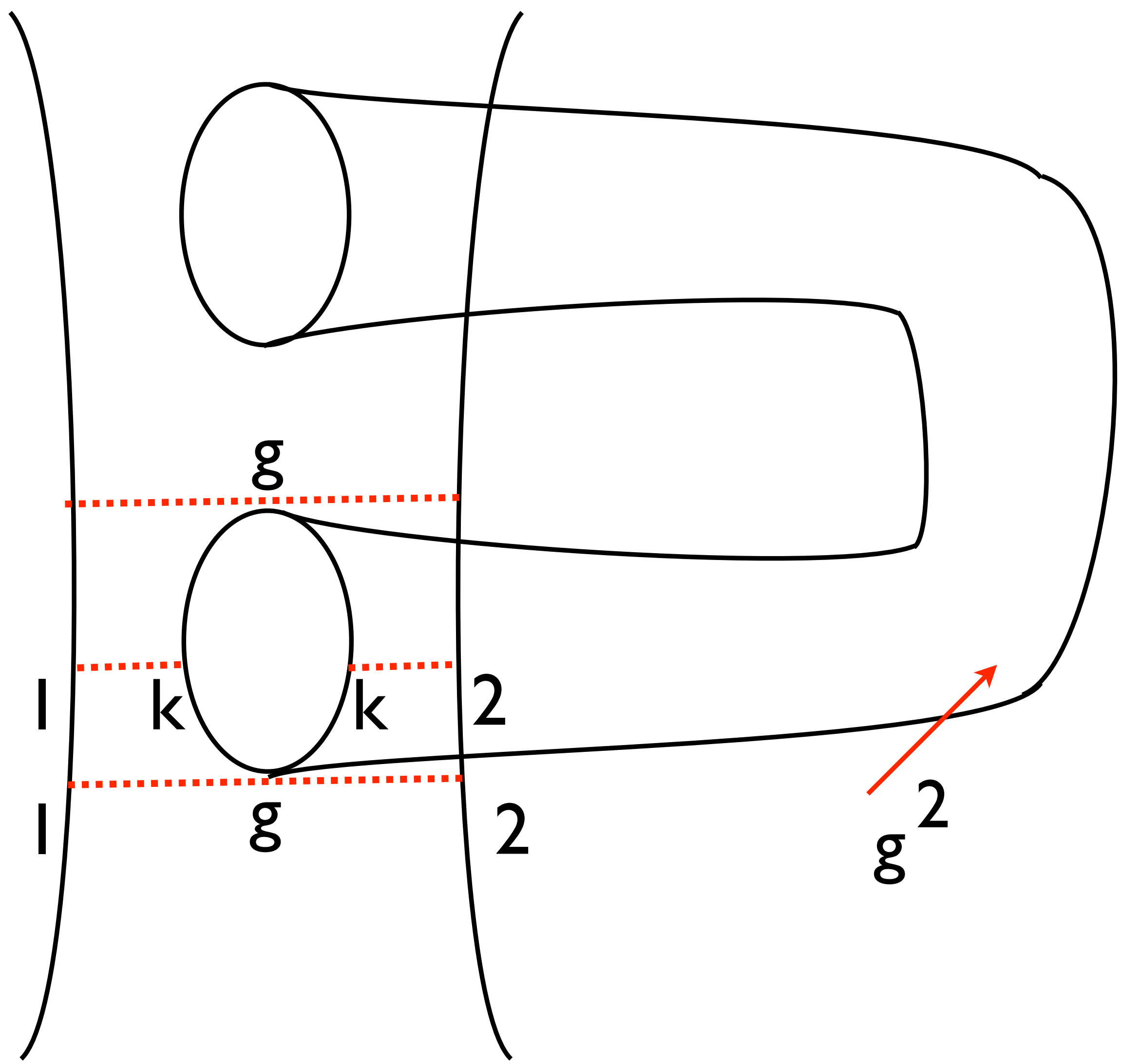
Take the limits:

$g \longrightarrow 0$

$N \longrightarrow$ infinity

$g^2 N = \lambda$ finite

λ is called the ‘t Hooft coupling in SU(N) gauge theory



Closed string interaction costs

g^2 extra,

so it vanishes in our limit.

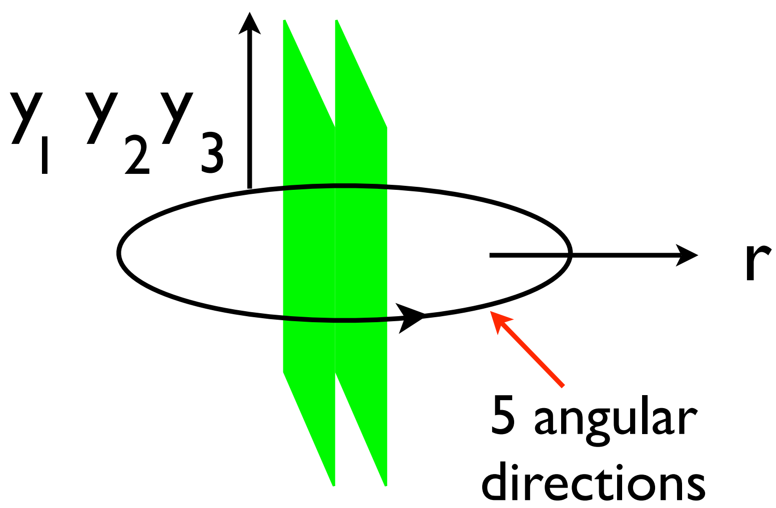
So we have only open string interactions ...

This is called the decoupling limit, because the closed string interactions decouple from the open string physics ..

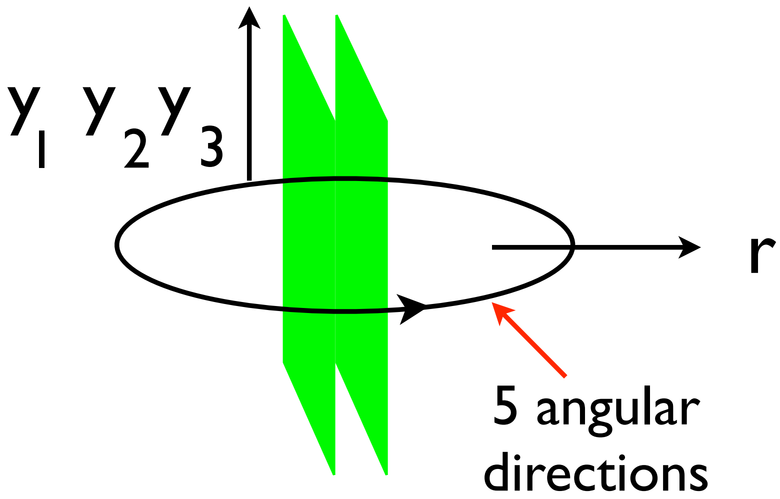
This was the gauge theory side of the story ...

Now let us look at the gravity side

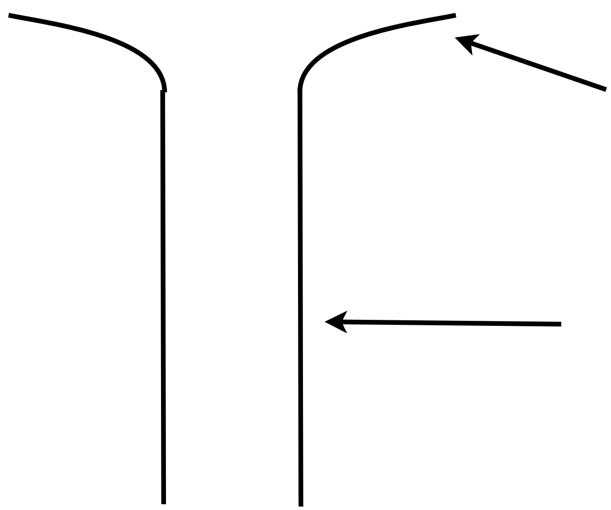
We have taken N D3 branes and kept them at a point



In string theory the coupling g is a tunable parameter. If g is small, we just have D3 branes sitting in flat space. If g is large, what metric do we get ?



$$ds^2 = \frac{(-dt^2 + dy_a dy_a)}{(1+\frac{Q}{r^4})^{\frac{1}{2}}} + (1+\frac{Q}{r^4})^{\frac{1}{2}}[dr^2 + r^2 d\Omega_5^2]$$



Flat space for large r

Small r

$$1 + \frac{Q}{r^4} \rightarrow \frac{Q}{r^4}$$

$$AdS_5 \times S^5$$

$$ds^2 = \frac{(-dt^2 + dy_a dy_a)}{(1 + \frac{Q}{r^4})^{\frac{1}{2}}} + (1 + \frac{Q}{r^4})^{\frac{1}{2}}[dr^2 + r^2 d\Omega_5^2]$$

$$1 + \frac{Q}{r^4} \to \frac{Q}{r^4}$$

$$ds^2 \approx \frac{r^2}{\sqrt{Q}}(-dt^2 + dy_a dy_a) + \frac{\sqrt{Q}}{r^2} dr^2 \quad + \quad \sqrt{Q}\, d\Omega_5^2$$

AdS_5

S^5

S^5

$$x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2 + x_6^2 = \sqrt{Q}$$

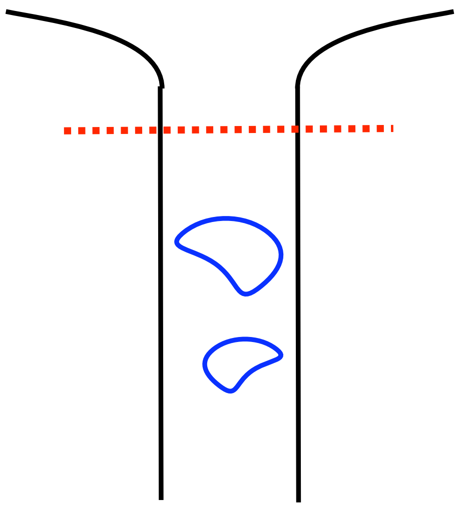
symmetry group $SO(6)$

AdS_5

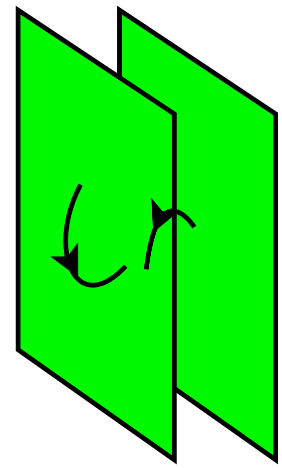
$$-x_1^2 - x_2^2 + x_3^2 + x_4^2 + x_5^2 + x_6^2 = -\sqrt{Q}$$

symmetry group $SO(4,2)$

Statement of AdS/CFT duality (Maldacena 97)



Two descriptions of
the SAME thing



String theory on

AdS_5 X S^5

n=4 supersymmetric
SU(N) Yang-Mills
gauge theory,
obtained from N D3 branes
in the decoupling limit

This is all very mysterious ...

How does it happen ?

What does it mean ?

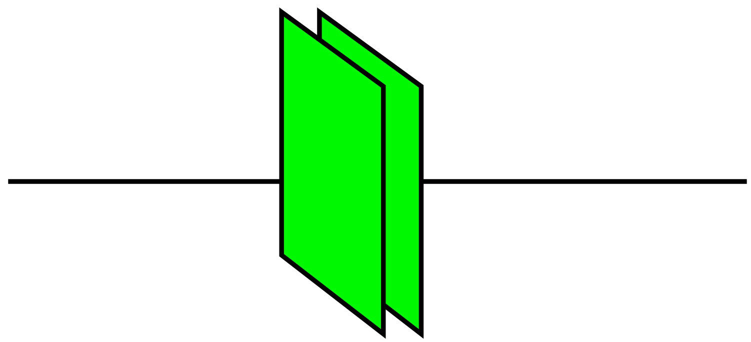
How do we use this duality ?

To investigate all this, we will return to black holes,
and ask the very basic question:

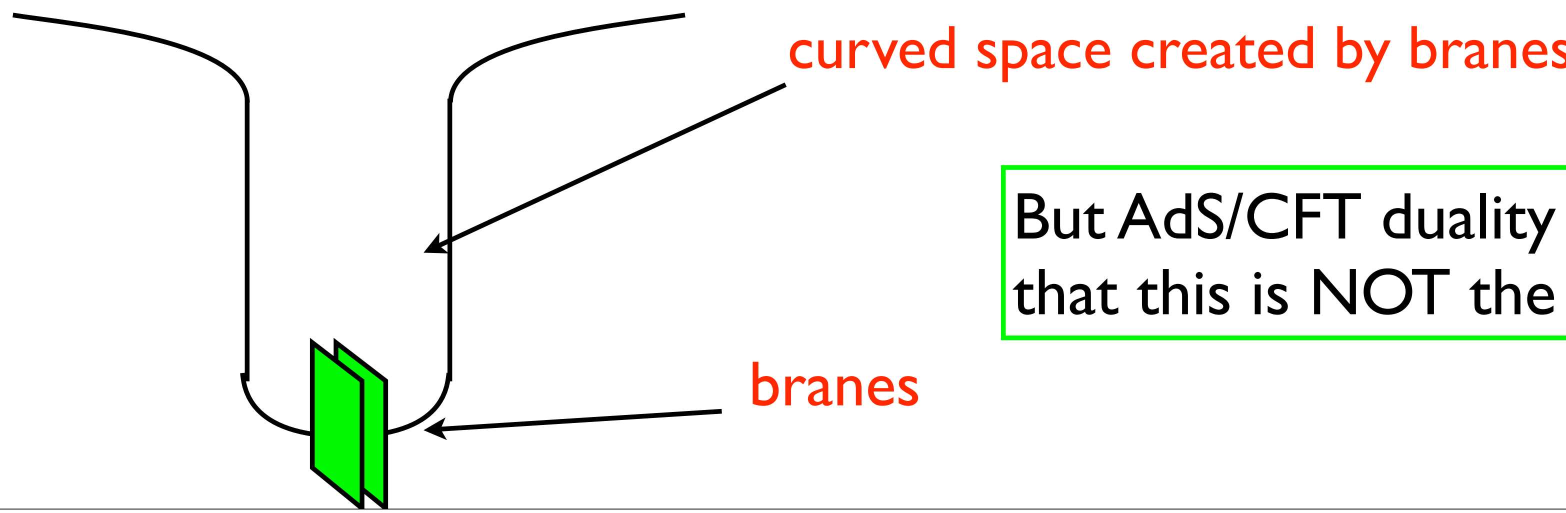
What happens to something that falls into a black hole ?

We have started with N D3 branes kept at a point

At small coupling g we just had branes on flat space

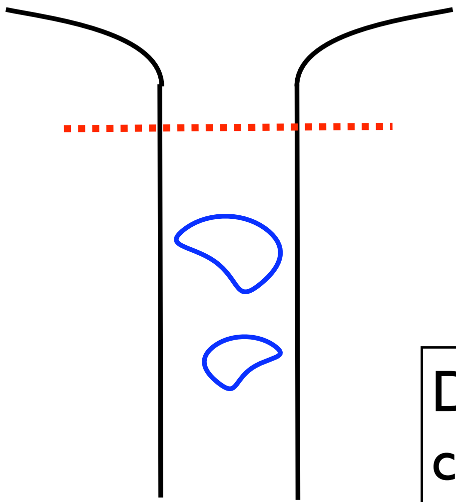


At larger coupling we expect to get a deformation of the metric around the branes

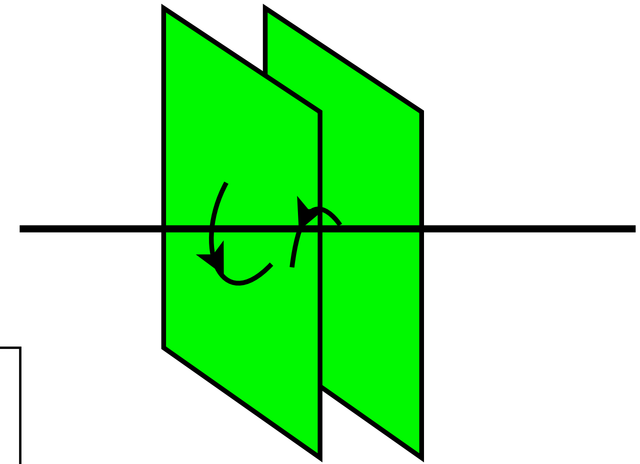


But AdS/CFT duality says that this is NOT the case ...

EITHER



OR



Duality means that we can take either picture, but not both

Gravity (string theory) on curved space

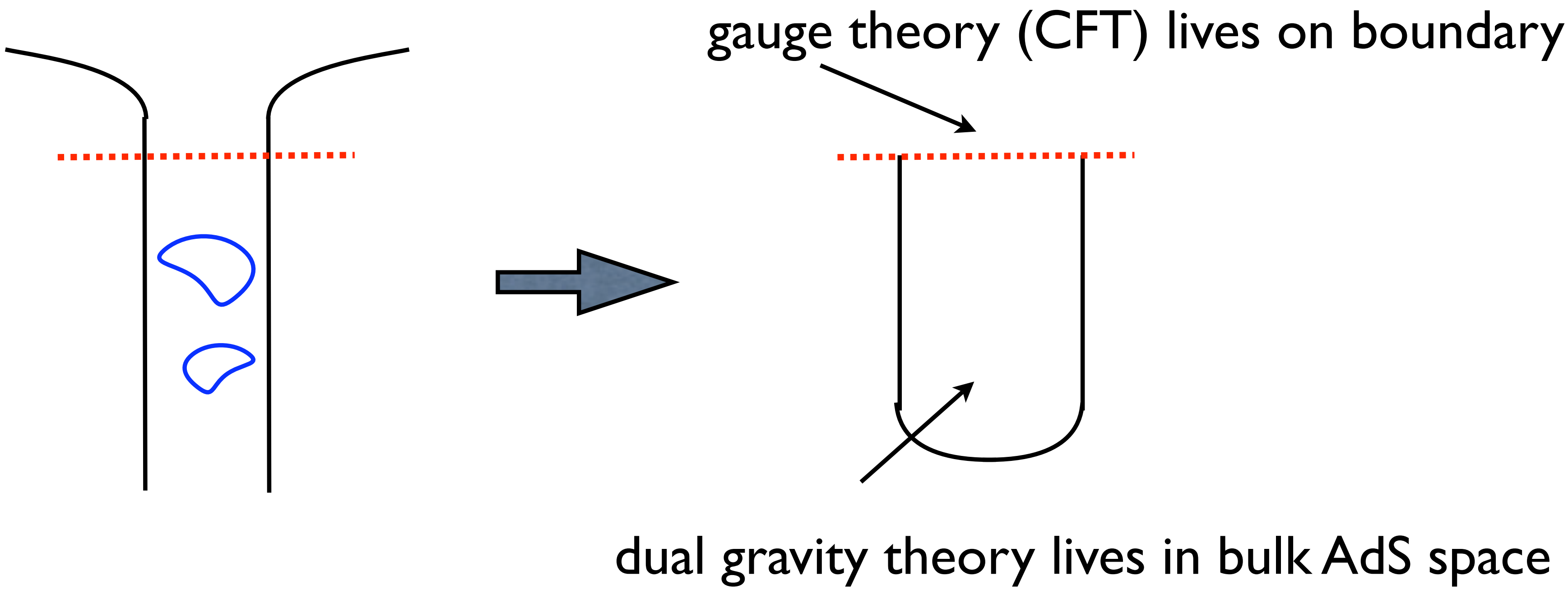
AdS_5 X S^5

D brane dynamics on flat space, open strings, SU(N) gauge theory (Conformal field theory CFT)

We need to know what happens at the end of the AdS space ...

How do we use this correspondence ?

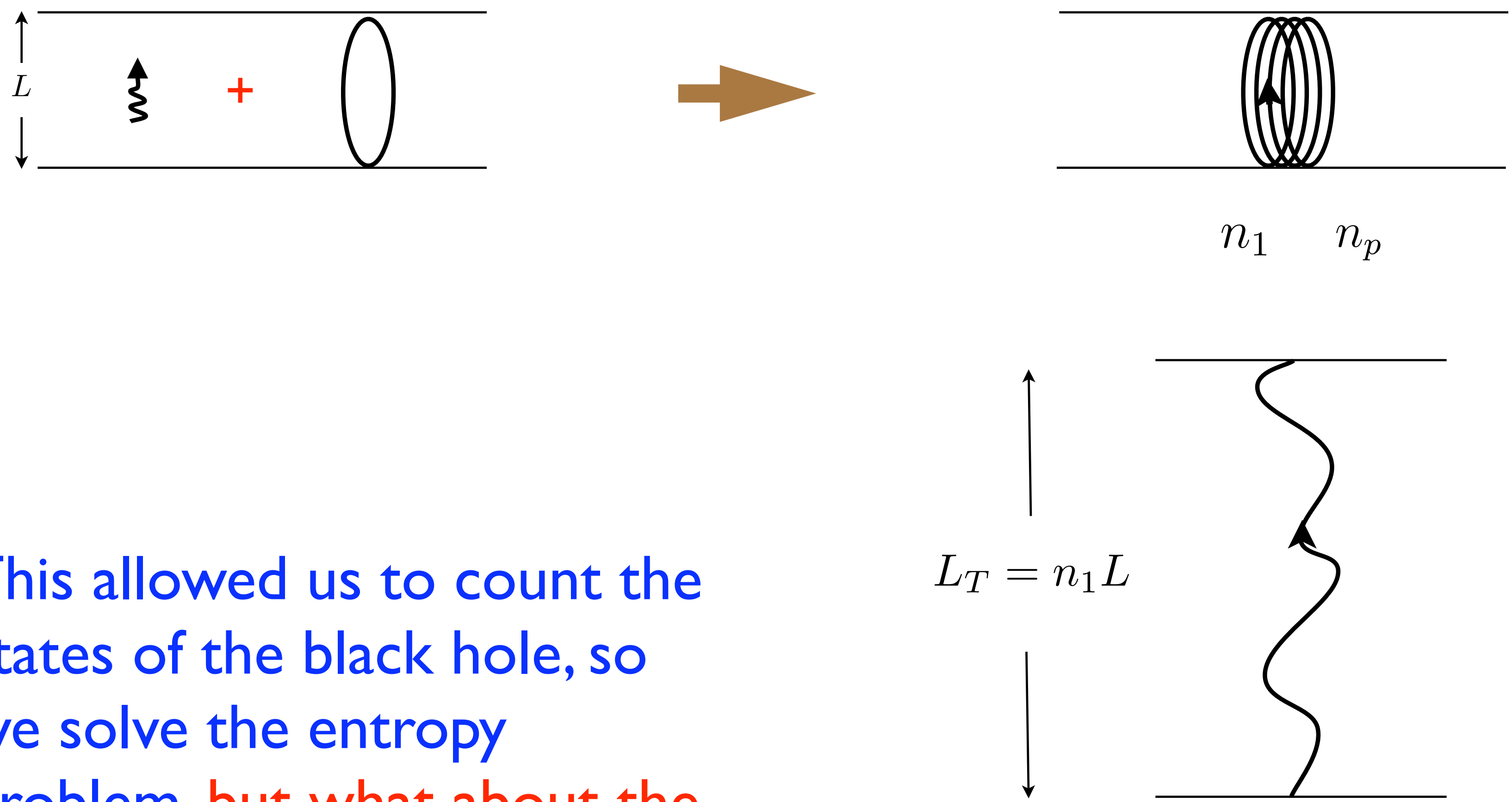
Witten 98: Go Euclidean ... then there is no horizon, just a smooth ‘ball’ shaped region of Euclidean AdS space



WHERE are the branes ?

(C) How do we solve the information puzzle in string theory? (The fuzzball picture of the black hole)

Recall the way we made the 2-charge black hole ...

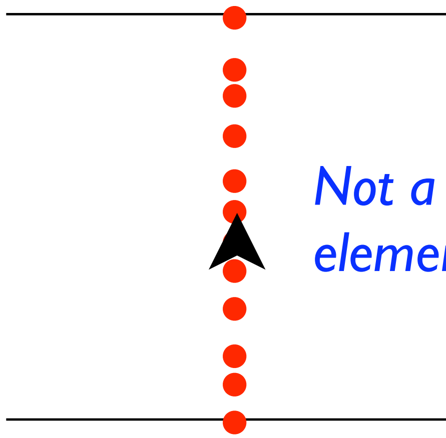


This allowed us to count the states of the black hole, so we solve the entropy problem, but what about the information puzzle?

A key point

The elementary string (NS1) does not have any LONGITUDINAL vibration modes

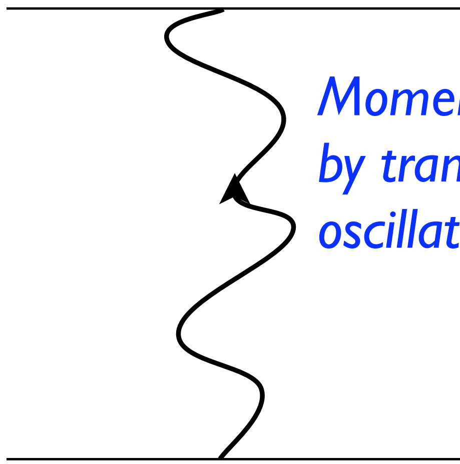
This is because it is not made up of 'more elementary particles'



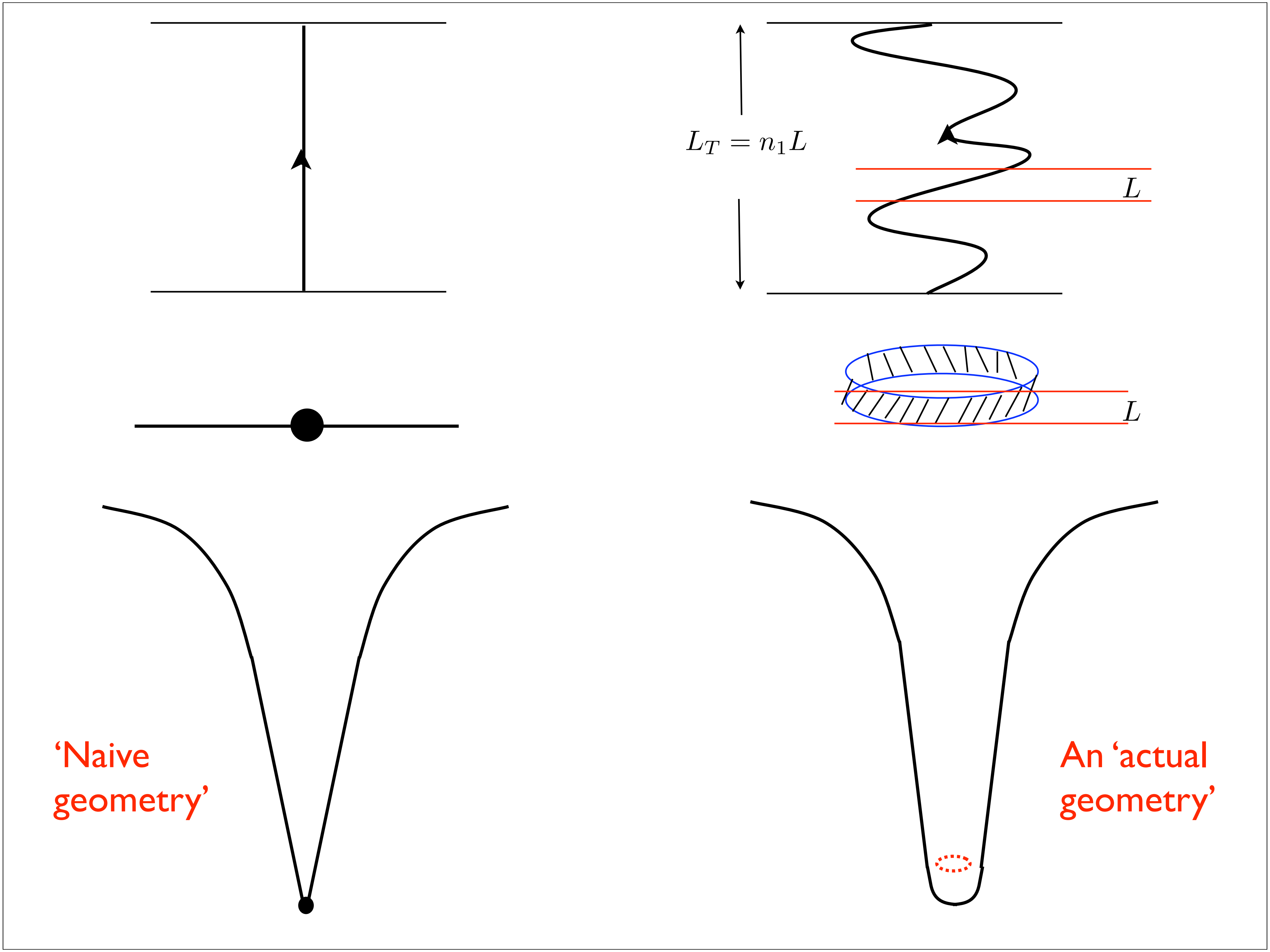
Not a mode for the elementary string

Thus only transverse oscillations are permitted

This causes the string to spread over a nonzero transverse area



Momentum is carried by transverse oscillations



$L_T = n_1 L$
L
L
'Naive geometry'
An 'actual geometry'

Making the geometry

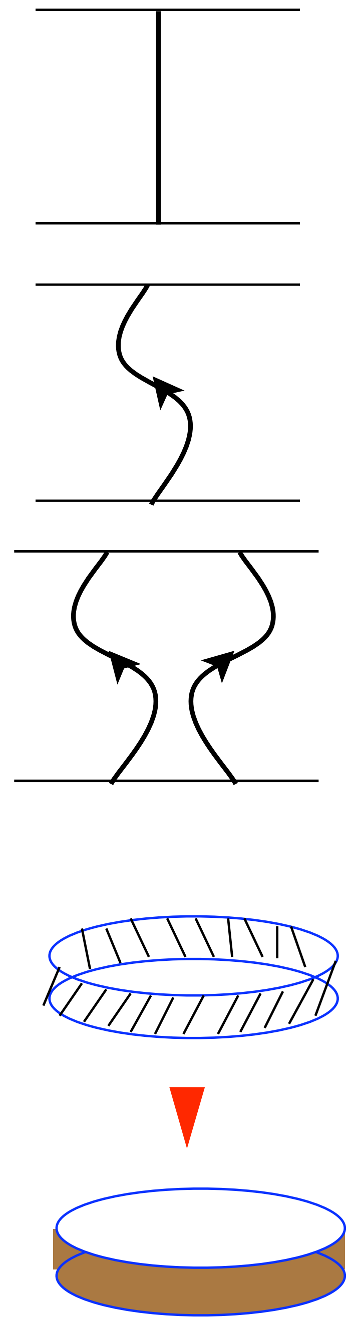
We know the metric of one straight strand of string

We know the metric of a string carrying a wave -- 'Vachaspati transform'

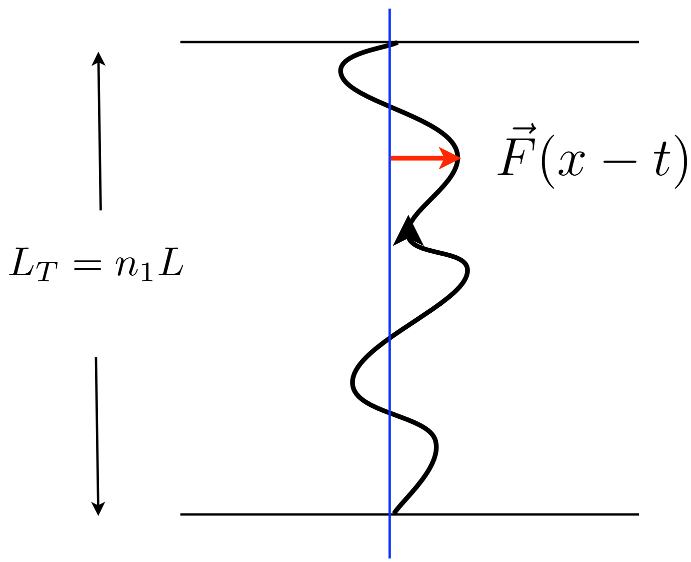
We get the metric for many strands by superposing harmonic functions from each strand

(Dabholkar, Gauntlett,Harvey,Waldram '95, Callan,Maldacena,Peet '95)

In our present case, we have a large number of strands, so we 'smear over them to make a continuous 'strip' (Lunin+SDM '01)

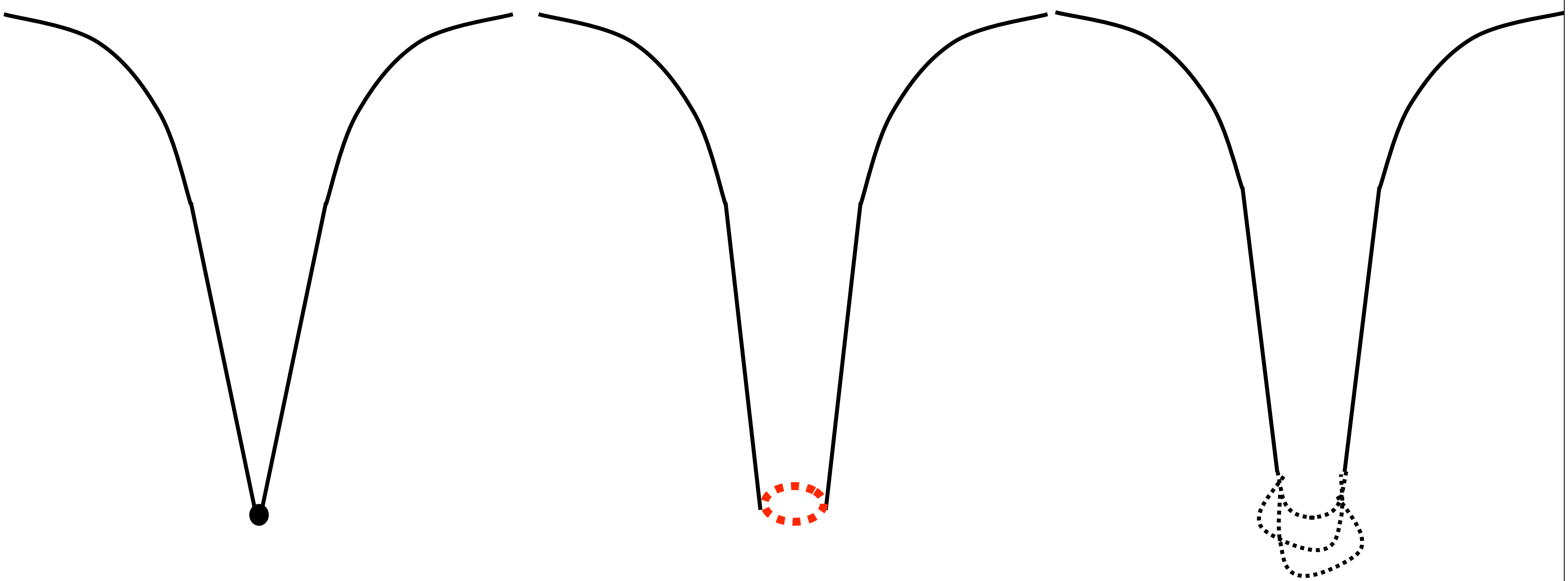


$$
\begin{aligned}
ds^2_{string} &= H[-dudv + Kdv^2 + 2A_i dx_i dv] + \sum_{i=1}^{4} dx_i dx_i + \sum_{a=1}^{4} dz_a dz_a \\
B_{uv} &= \frac{1}{2}[H-1], \qquad B_{vi} = HA_i \\
e^{2\phi} &= H
\end{aligned}
$$



$$
\begin{aligned}
H^{-1} &= 1 + \frac{Q_1}{L_T}\int_0^{L_T} \frac{dv}{|\vec{x}-\vec{F}(v)|^2} \\
K &= \frac{Q_1}{L_T}\int_0^{L_T} \frac{dv(\dot{F}(v))^2}{|\vec{x}-\vec{F}(v)|^2} \\
A_i &= -\frac{Q_1}{L_T}\int_0^{L_T} \frac{dv\dot{F}_i(v)}{|\vec{x}-\vec{F}(v)|^2}
\end{aligned}
$$

Thus we see that for the 2-charge extremal hole (‘Sen-Vafa hole’)
we get not a singularlty or horizon but ‘fuzzball caps’

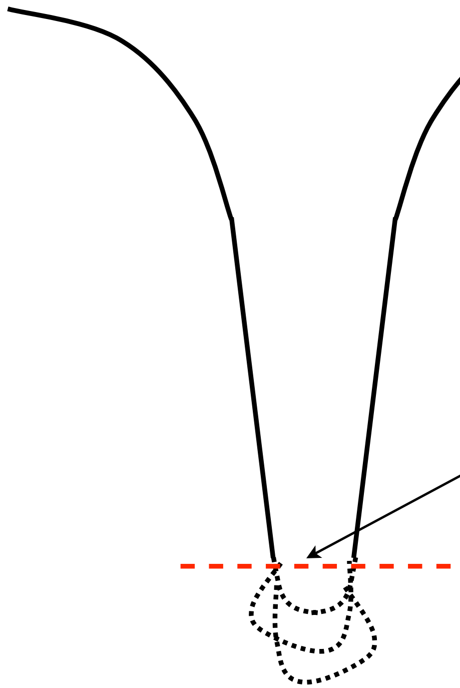


Naive geometry,
classical action

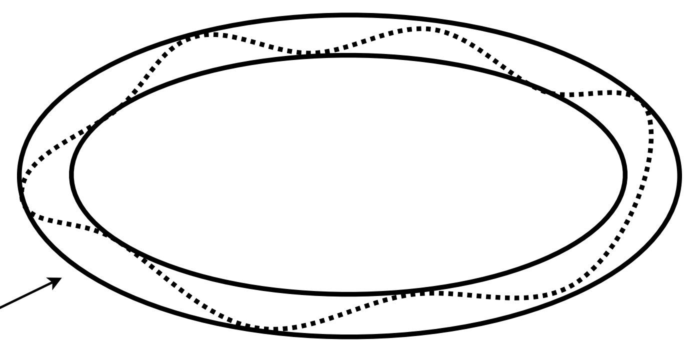
Naive geometry,
higher derivative
terms included

Actual states,
throat ends in
fuzzball cap

Scale of the 'fuzzball'



$$\frac{A}{G} \sim \sqrt{n_1 n_p} \sim S_{micro} = 2\pi\sqrt{2}\sqrt{n_1 n_p}$$



(Lunin+SDM '02)

$$\frac{A}{G} \sim \sqrt{n_1 n_p - J} \sim S_{micro} = 2\pi\sqrt{2}\sqrt{n_1 n_p - J}$$

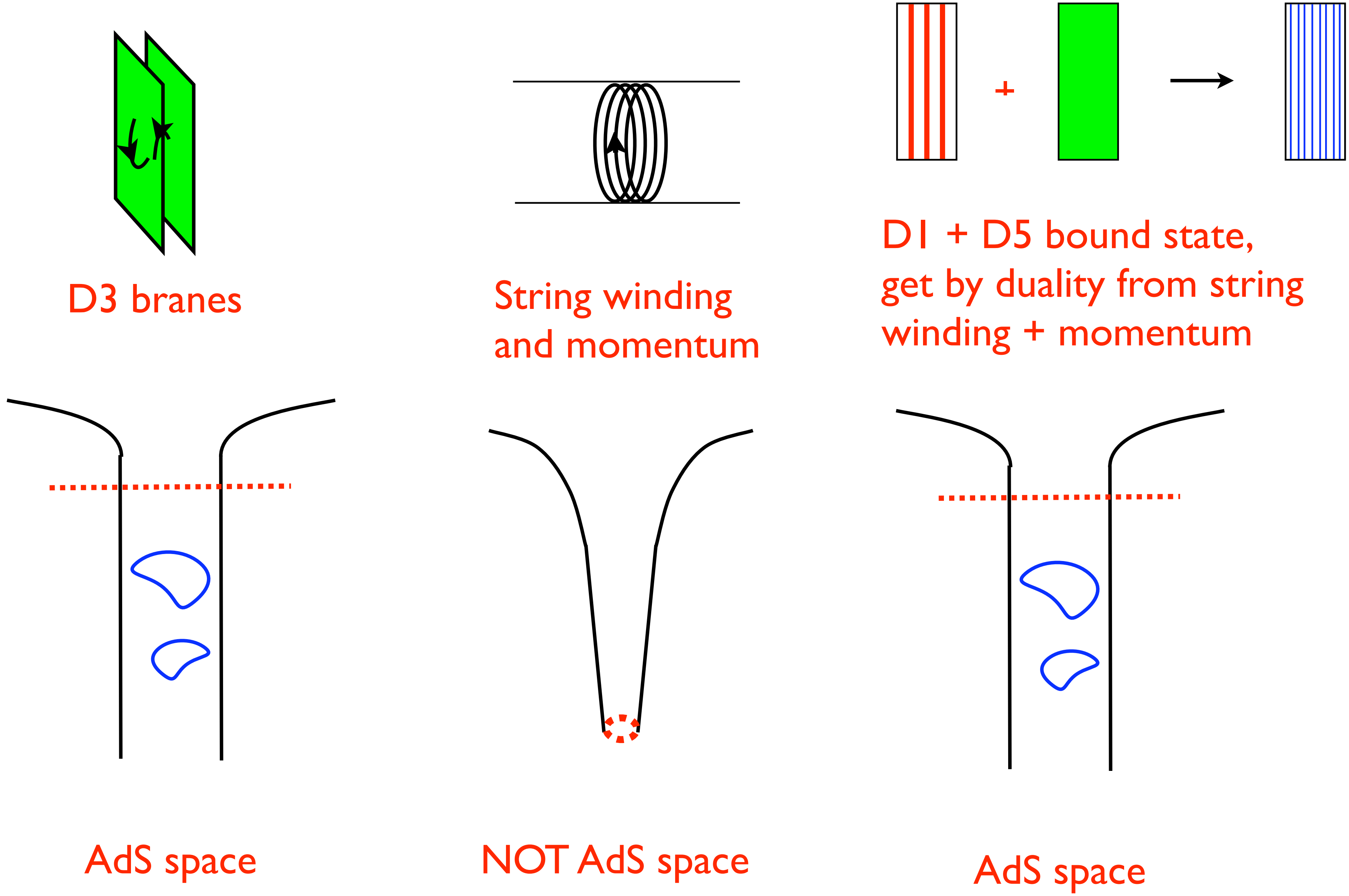
(D) Understanding black hole states using the AdS/CFT duality

S,T Dualities

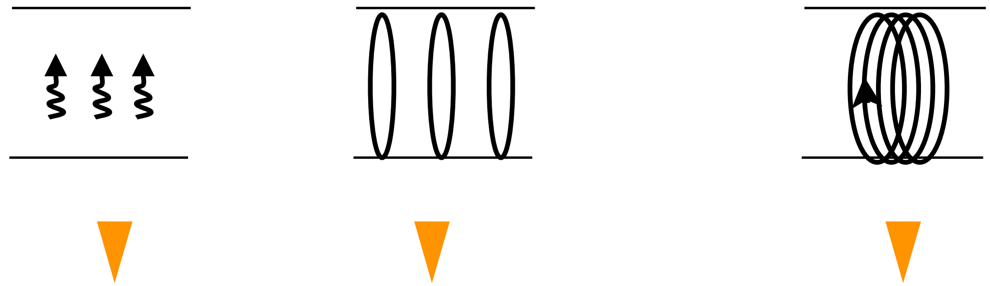
These are some simple dualities of classical supergravity, which extend to the full string theory

These dualities permute the various objects in string theory among each other ....

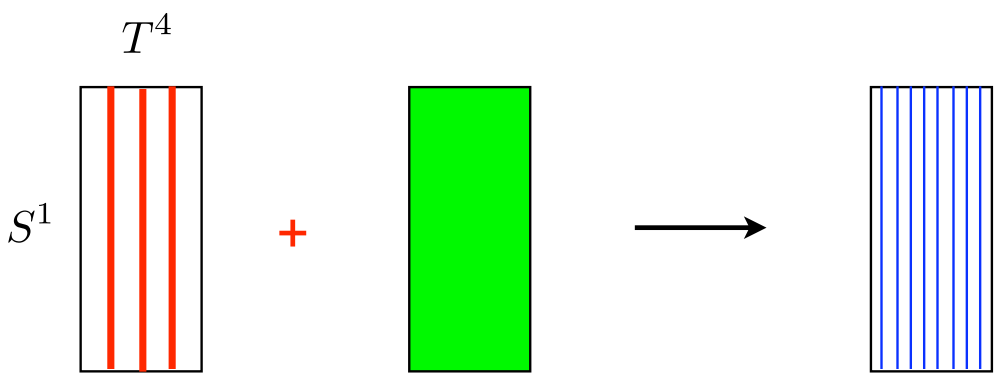
Getting AdS space



A final step: Use dualities to map to D1-D5



$$\sum k \; n_k = n_1 n_p$$

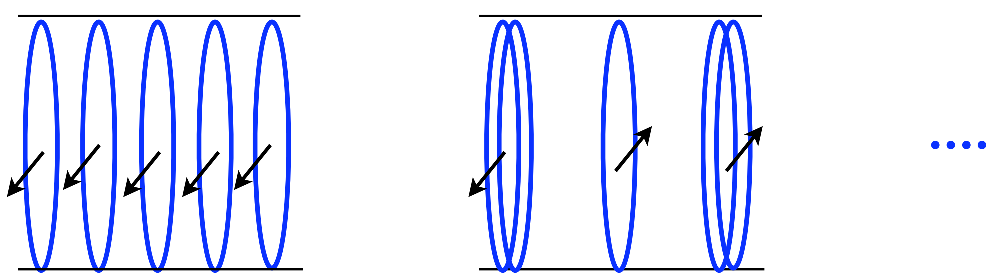


$$\sum k \; n_k = n_1' n_5'$$

$n_1' = n_p$
D1 branes

$n_5' = n_1$
D5 branes

'Effective string' with total winding number
$n_1' n_5' = n_1 n_p$



Geometry for D1-D5

$$ds^2 = \sqrt{\frac{H}{1+K}}[-(dt - A_i dx^i)^2 + (dy + B_i dx^i)^2] + \sqrt{\frac{1+K}{H}} dx_i dx_i + \sqrt{H(1+K)} dz_a dz_a$$

$$H^{-1} = 1 + \frac{Q}{L_T}\int_0^{L_T} \frac{dv}{|\vec{x} - \vec{F}(v)|^2}$$

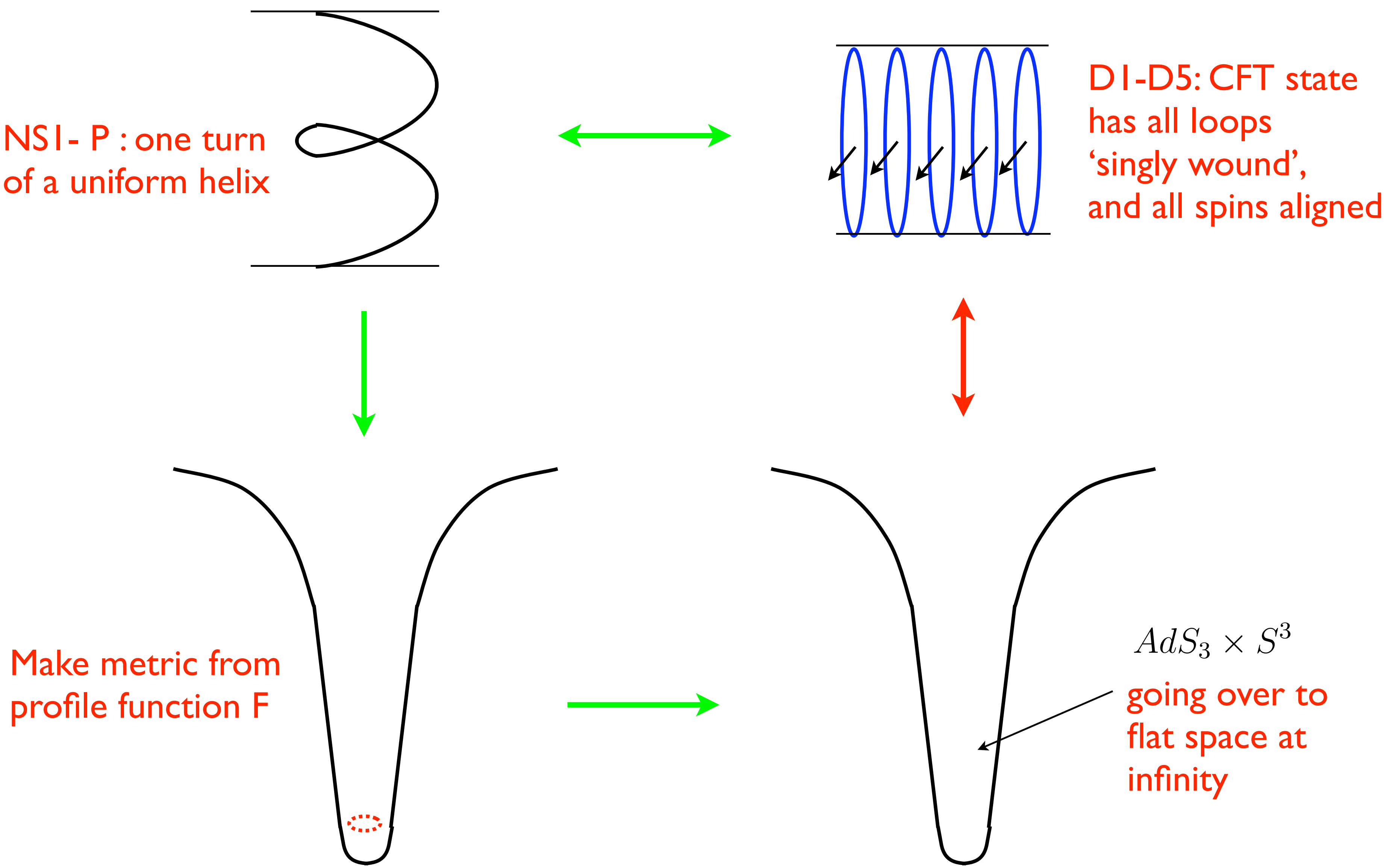
$$K = \frac{Q}{L_T}\int_0^{L_T} \frac{dv(\dot{F}(v))^2}{|\vec{x} - \vec{F}(v)|^2}$$

$$A_i = -\frac{Q}{L_T}\int_0^{L_T} \frac{dv\ \dot{F}_i(v)}{|\vec{x} - \vec{F}(v)|^2}$$

$$dB = - *_4 dA$$

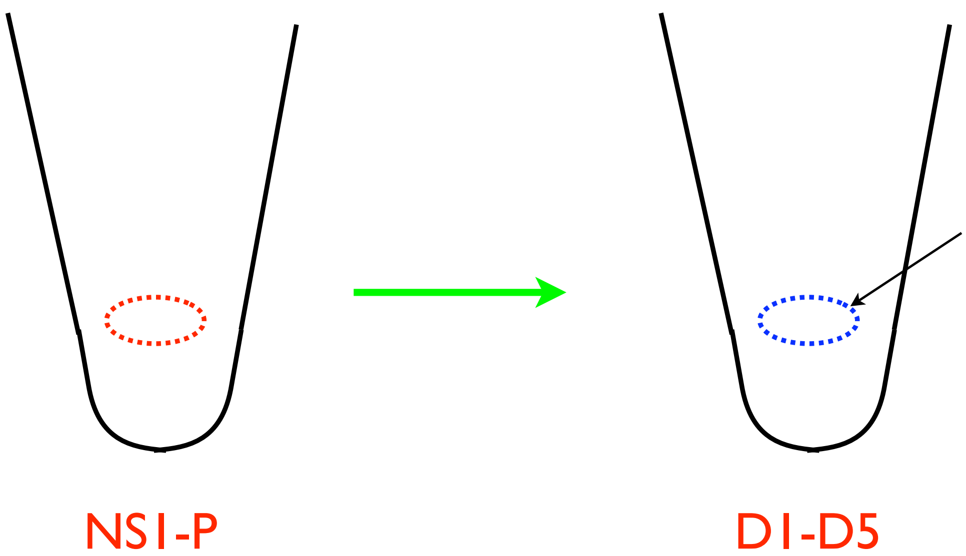
(Lunin+SDM '01,
also
'Supergravity supertubes'
(Emparan+Mateos+Townsend '01)

A simple example



$$
\begin{aligned}
ds^2 &= -H_1^{-1}(dt^2-dy^2)+H_5 f\left(d\theta^2+\frac{dr^2}{r^2+a^2}\right)-\frac{2a\sqrt{Q_1'Q_5'}}{H_1 f}\left(\cos^2\theta dy d\psi+\sin^2\theta dt d\phi\right)\\
&+ H_5\left[\left(r^2+\frac{a^2Q_1'Q_5'\cos^2\theta}{H_1H_5f^2}\right)\cos^2\theta d\psi^2+\left(r^2+a^2-\frac{a^2Q_1'Q_5'\sin^2\theta}{H_1H_5f^2}\right)\sin^2\theta d\phi^2\right]\\
&+ dz_a dz_a
\end{aligned}
$$

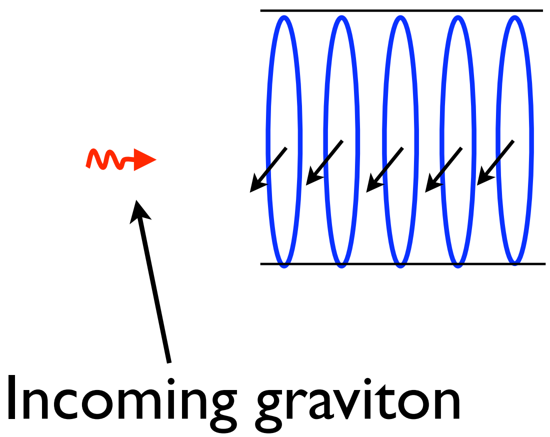
$$
f=r^2+a^2\cos^2\theta, \qquad H_1=1+\frac{Q_1'}{f}, \quad H_5=1+\frac{Q_5'}{f}
$$



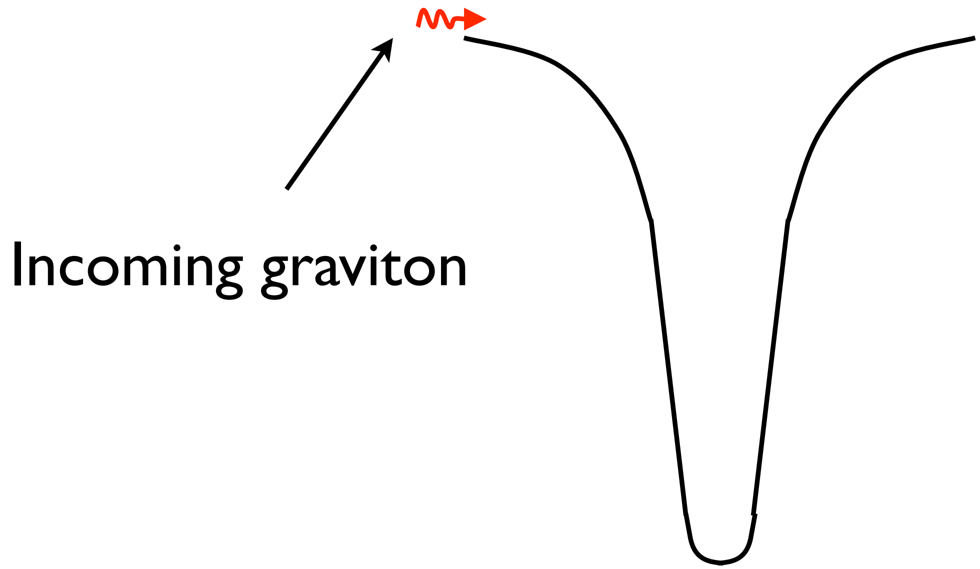
Each point on the ring is the center of a Kaluza-Klein monopole

No net KK monopole charge : KK is a *dipole charge*

An illustration of AdS/CFT duality



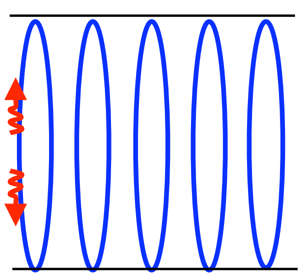
CFT picture



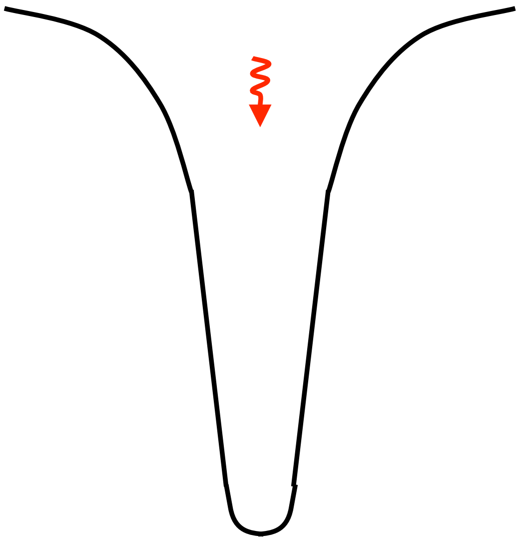
Gravity picture

We want to study what happens in each picture ...

Absorption

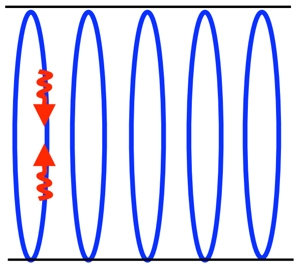


With an absorption probability P, the energy of the graviton gets converted to a pair of vibration modes on one of the pieces of the effective string



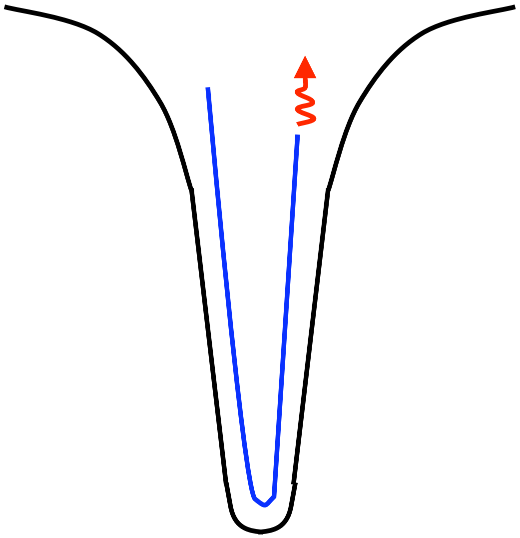
With probability P, the graviton enters the throat of the geometry

Time delay



The excitations travel around the loop in a time T and re-collide

The colliding excitations can lead to re-emission of the graviton with probability P



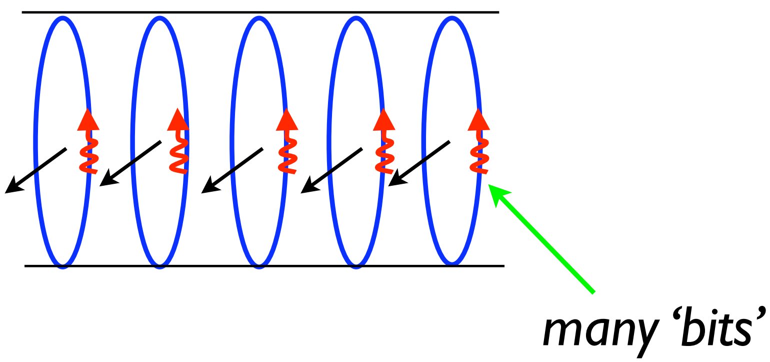
The graviton travels down the throat, bounces off the 'cap' and comes back up in a time T

It can re-emerge from the throat with probability P

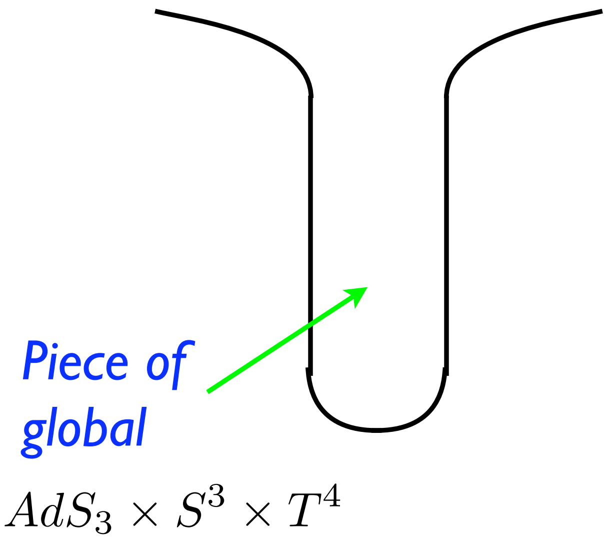
The horizon of 2-charge holes needed higher derivative corrections

Consider D1-D5-P, which does not need such corrections at leading order

$$|k\rangle^{total} = (J^{-,total}_{-(2k-2)})^{n_1 n_5} (J^{-,total}_{-(2k-4)})^{n_1 n_5} \dots (J^{-,total}_{-2})^{n_1 n_5} \; |1\rangle^{total}$$



Field theory representation of brane state



Geometry created by this state

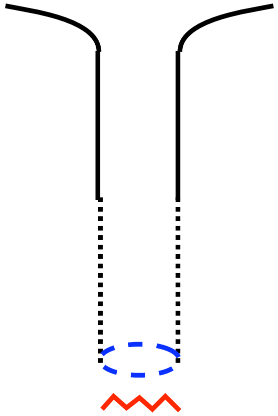
$$
\begin{aligned}
ds^2 &= -\frac{1}{h}(dt^2-dy^2)+\frac{Q_p}{hf}\left(dt-dy\right)^2+hf\left(\frac{dr_N^2}{r_N^2+a^2\eta}+d\theta^2\right)\\
&+ h\left(r_N^2-na^2\eta+\frac{(2n+1)a^2\eta Q_1Q_5\cos^2\theta}{h^2f^2}\right)\cos^2\theta d\psi^2\\
&+ h\left(r_N^2+(n+1)a^2\eta-\frac{(2n+1)a^2\eta Q_1Q_5\sin^2\theta}{h^2f^2}\right)\sin^2\theta d\phi^2\\
&+ \frac{a^2\eta^2Q_p}{hf}\left(\cos^2\theta d\psi+\sin^2\theta d\phi\right)^2\\
&+ \frac{2a\sqrt{Q_1Q_5}}{hf}\left[n\cos^2\theta d\psi-(n+1)\sin^2\theta d\phi\right](dt-dy)\\
&- \frac{2a\eta\sqrt{Q_1Q_5}}{hf}\left[\cos^2\theta d\psi+\sin^2\theta d\phi\right]dy+\sqrt{\frac{H_1}{H_5}}\sum_{i=1}^{4}dz_i^2
\end{aligned}
$$

$$
\begin{aligned}
f &= r_N^2-a^2\eta\, n\sin^2\theta+a^2\eta\,(n+1)\cos^2\theta\\
h &= \sqrt{H_1H_5},\;\; H_1=1+\frac{Q_1}{f},\;\; H_5=1+\frac{Q_5}{f}
\end{aligned}
$$

$$
\eta\equiv\frac{Q_1Q_5}{Q_1Q_5+Q_1Q_p+Q_5Q_p}
$$

What happens to the no hair 'theorem' ?

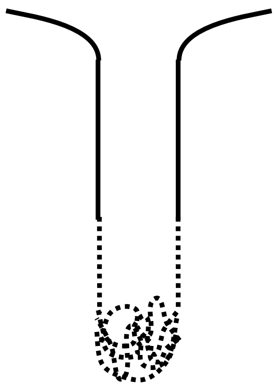
If we take a spherically symmetric ansatz for the metric, we get the black hole with horizon



But none of the actual microstates are spherically symmetric

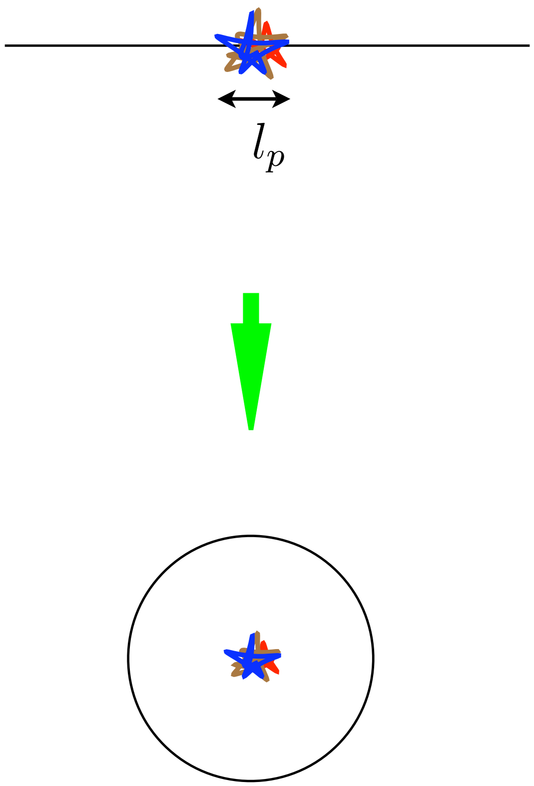
The compact directions 'twist over the angular sphere', and the geometry 'caps off'

Different ways of twisting give different states



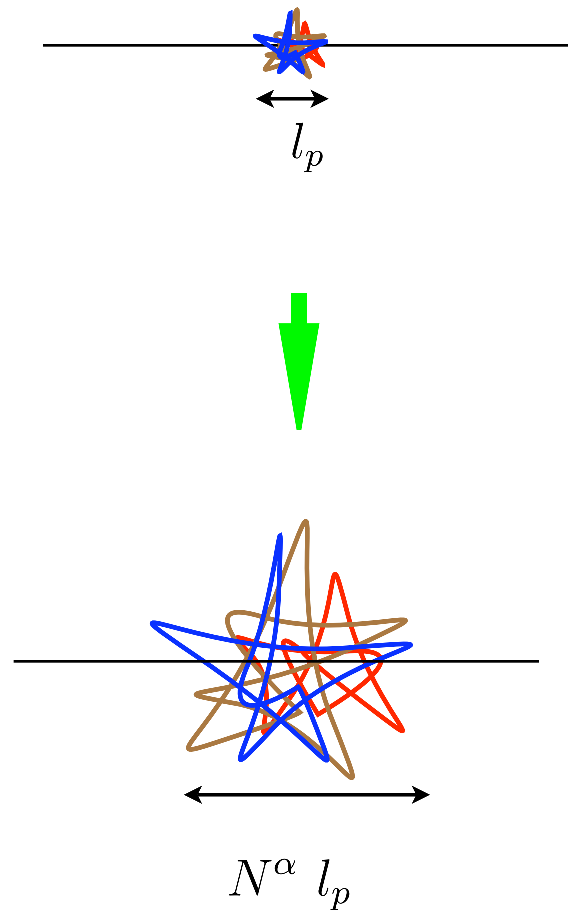
If a horizon formed, we would have 'no hair', but the throat 'caps-off' before the horizon forms ...

Summary of the fuzzball picture



small g

large g



Conventional picture

Fractionation → 'Fuzzball'

The structure of extremal holes