A UNIFIED TREATMENT OF GRAVITATIONAL COLLAPSE IN GENERAL RELATIVITY

Paul Lasky & Anthony Lun

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Centre for Stellar and Planetary Astrophysics
MONASH UNIVERSITY

MONGRELS
MONash General RELativistS

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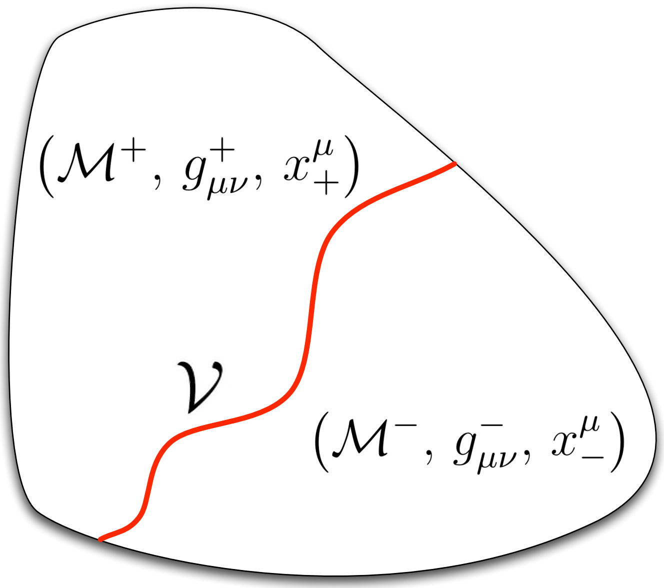
JUNCTION CONDITIONS



Consider 2 distinct spacetimes, $\mathcal{M}^+$ and $\mathcal{M}^-$, joined along some junction $\mathcal{V}$

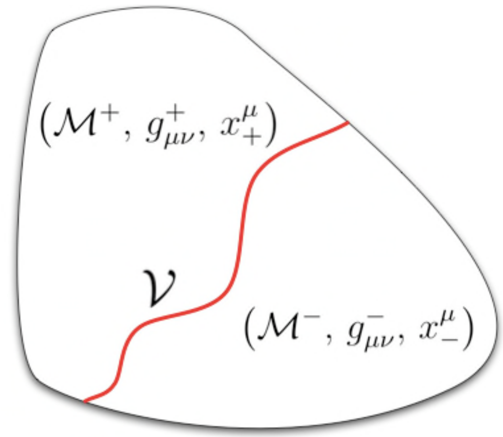
What conditions must be put on the metrics such that $g^+_{\mu\nu} \cup g^-_{\mu\nu}$ forms a valid solution of EFEs?

In general $x^\mu_+ \neq x^\mu_-$

Israel-Darmois junction conditions require the continuity of the first and second fundamental form across $\mathcal{V}$

ALTERNATIVE TO JUNCTIONS

What if $x^\mu_+ = x^\mu_-$ and even $g^+_{\mu\nu} = g^-_{\mu\nu}$



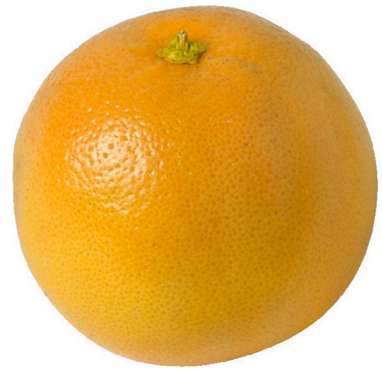
All difficulties with junction conditions are then swept under the carpet

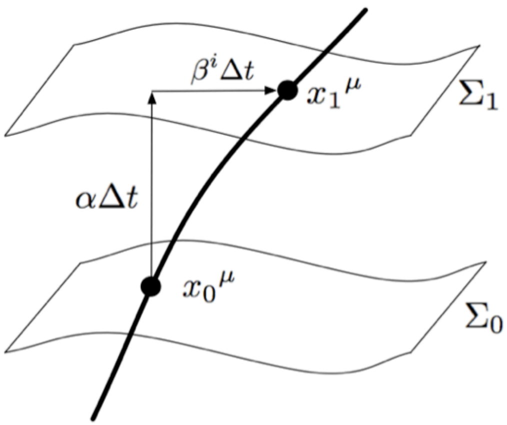
Only require usual continuity conditions acting on a single metric to be satisfied
(for example in Hawking & Ellis)

How does one establish such a spacetime?

INITIAL VALUE PROBLEM

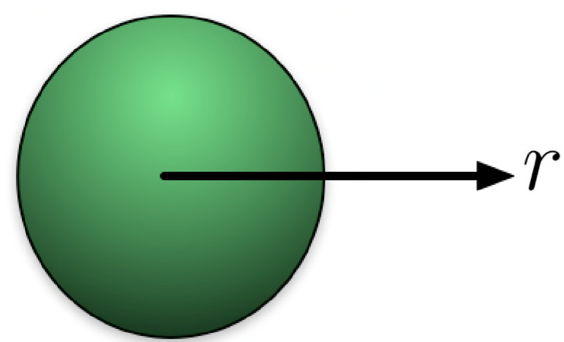
Consider spherical symmetry
(the paradigm is generalizable....stay tuned)





Use 3+1 formalism to establish initial value problem, with arbitrary matter distribution on an initial spacelike Cauchy hypersurface

On initial slice, let all matter terms vanish beyond some finite radius, $r_\star$ say, and evolve forward in time.



SPHERICAL SYMMETRY

Unique decomposition of energy momentum tensor is given by

$$T_{\mu\nu} = \rho n_\mu n_\nu + P \perp_{\mu\nu} + 2 j_{(\mu} n_{\nu)} + \Pi_{\mu\nu}$$

But **spherical symmetry** implies only 4 energy momentum variables

- ρ Mass-Energy density
- P Isotropic Pressure
- j "Heat" flux
- Π Anisotropic Stress

Thus 4 metric coefficients to keep line element as general as possible

- α Lapse function
- β Radial component of shift vector
- E Energy function in LTB dust
- R Radial function

$$dS^2 = -\alpha^2 dt^2 + \frac{(\beta dt + dr)^2}{1+E} + R^2 d\Omega$$

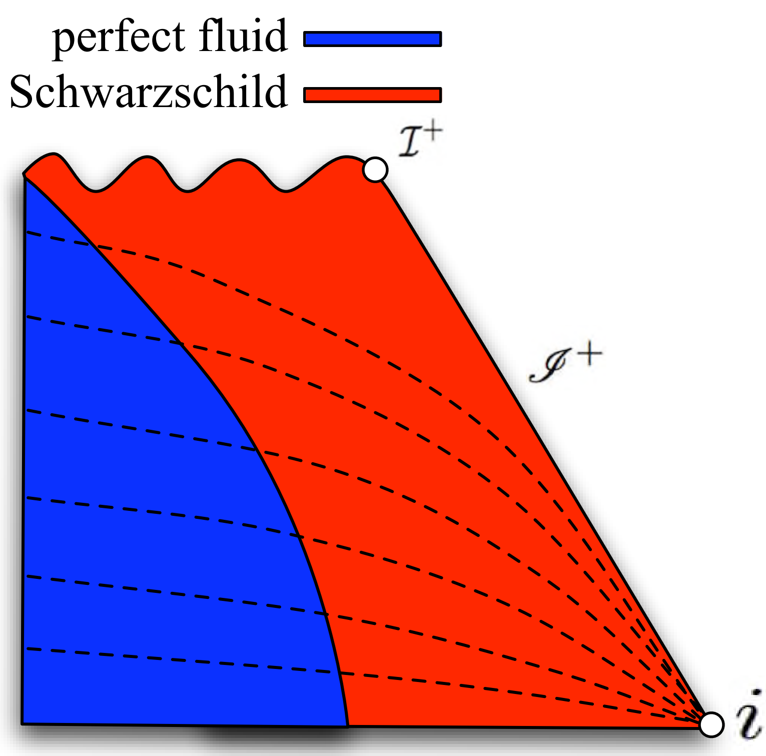
All functions depend only on t and r

PERFECT FLUID INTERIOR

Lasky & Lun (2006). Generalized Lemaitre-Tolman-Bondi solutions with pressure. *Phys. Rev. D* **74**, 084013

$$\text{Perfect fluid} \quad \Rightarrow \quad j = \Pi = 0$$

$$dS^2 = -\alpha^2 dt^2 + \frac{\left(\alpha\sqrt{E + 2M/r}\, dt + dr\right)^2}{1+E} + r^2 d\Omega^2$$



$$\frac{\partial M}{\partial r} := 4\pi\rho r^2$$

Lapse function determined by Euler equation

$$\frac{1}{\alpha}\frac{\partial \alpha}{\partial r} = \frac{-1}{\rho+P}\frac{\partial P}{\partial r}$$

Two evolution equations for E and M

$$\frac{\partial E}{\partial t} - \alpha\sqrt{E + \frac{2M}{r}}\left(\frac{\partial E}{\partial r} + 2\frac{1+E}{\rho+P}\frac{\partial P}{\partial r}\right) = 0$$

$$\frac{\partial M}{\partial t} - \alpha\sqrt{E + \frac{2M}{r}}\left(\frac{\partial M}{\partial r} + 4\pi P r^2\right) = 0$$

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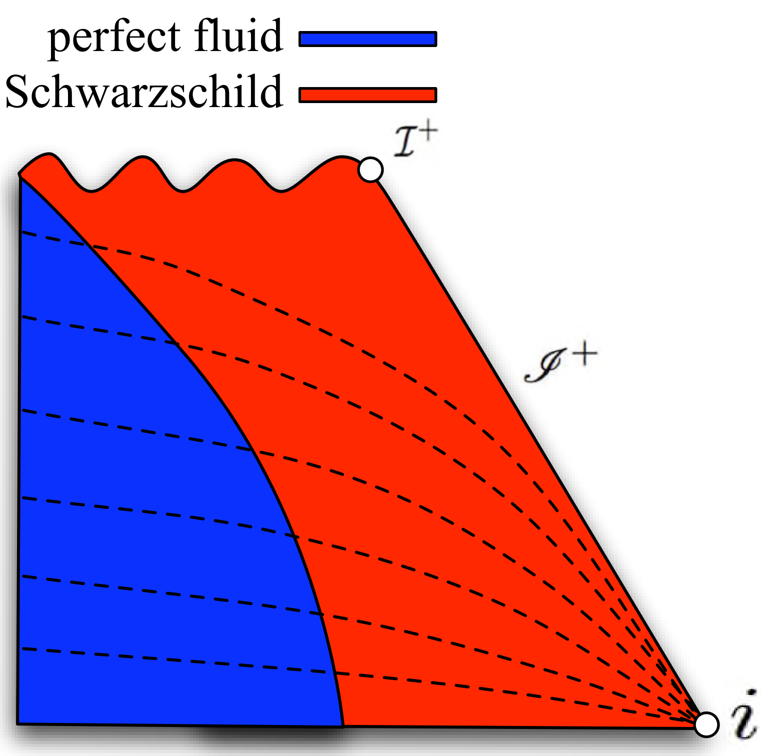


$$\text{let } P = \rho = 0 \text{ for all } r > r_\star$$

$$\Rightarrow \quad M(t, r > r_\star) = M_S$$

$$\text{and } \alpha = 1$$

We recover **Schwarzschild** spacetime for all $r > r_\star$

GENERAL FLUID INTERIOR

Lasky & Lun (2007). Spherically symmetric collapse of general fluids. *Phys. Rev. D* **75**, 024031

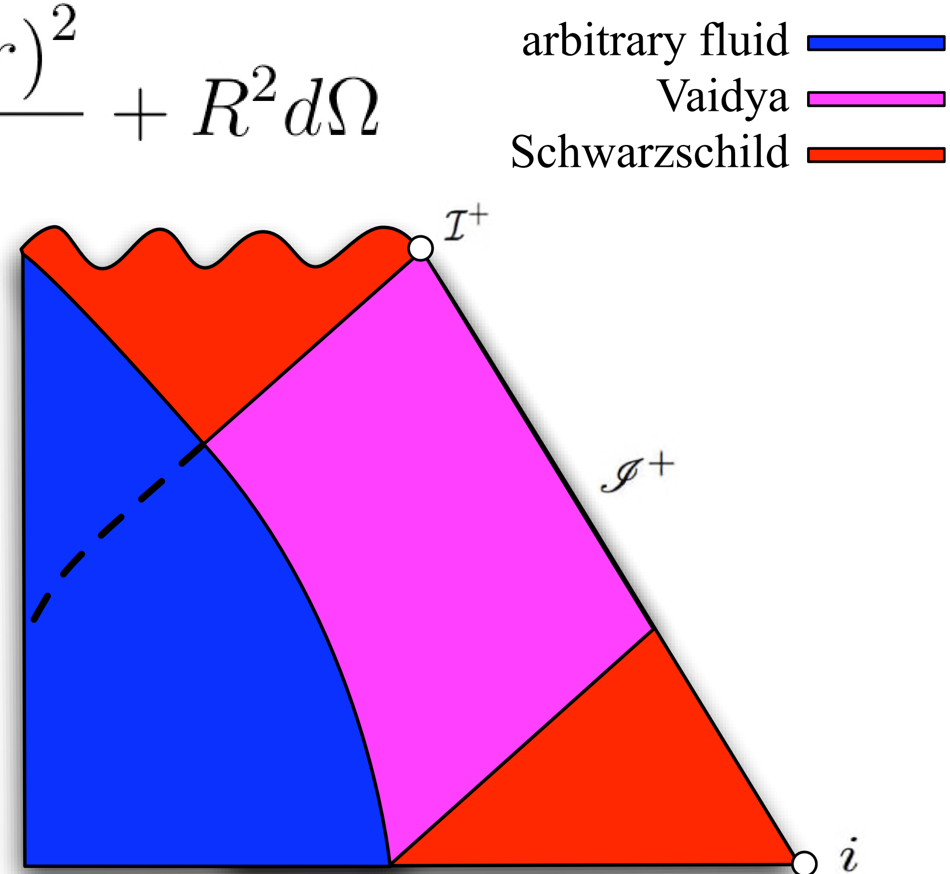
$$d\mathcal{S}^2 = -\alpha^2 dt^2 + \frac{(\beta dt + dr)^2}{1+E} + R^2 d\Omega$$

$$\rho,\ P,\ j,\ \Pi \neq 0$$

$$\frac{\partial M}{\partial r} := 4\pi \left(\rho \frac{\partial R}{\partial r} + j \mathcal{L}_n R\right) R^2$$

$$\left(\mathcal{L}_n := \frac{1}{\alpha}\frac{\partial}{\partial t} - \frac{\beta}{\alpha}\frac{\partial}{\partial r}\right)$$

Plus another five (hideous) equations



The Vaidya and Schwarzschild spacetimes are subsets of the general fluid equations

Thus, we can write **ALL** regions of this spacetime in a single coordinate system (i.e. as one metric)

PLASMA INTERIOR

Lasky & Lun (2007). Gravitational collapse of spherically symmetric plasmas in Einstein-Maxwell spacetimes. *Phys. Rev. D* **75**, 024031

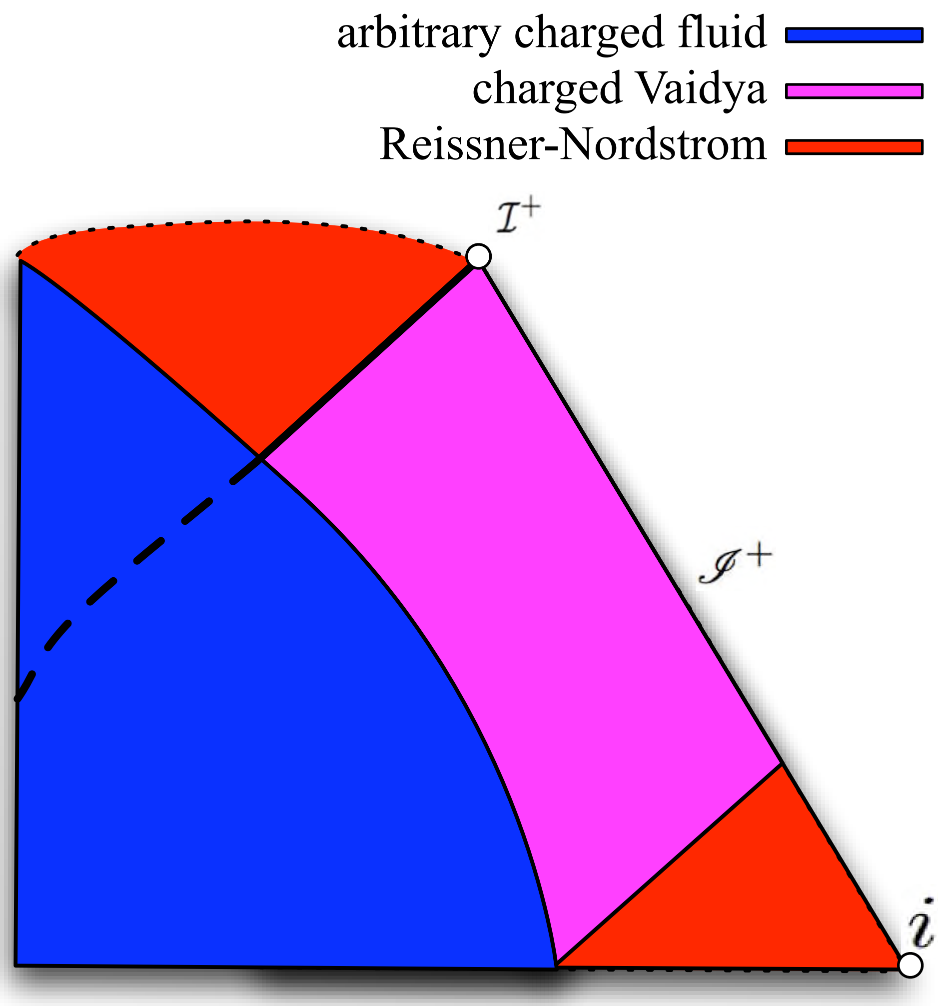
We can alter the uncharged model to the charged case simply by mapping

$$\rho \rightarrow \rho + \frac{Q^2}{8\pi R^4} \qquad P \rightarrow P + \frac{Q^2}{24\pi R^4}$$

$$j \rightarrow j \qquad \Pi \rightarrow \Pi + \frac{Q^2}{12\pi R^4}$$

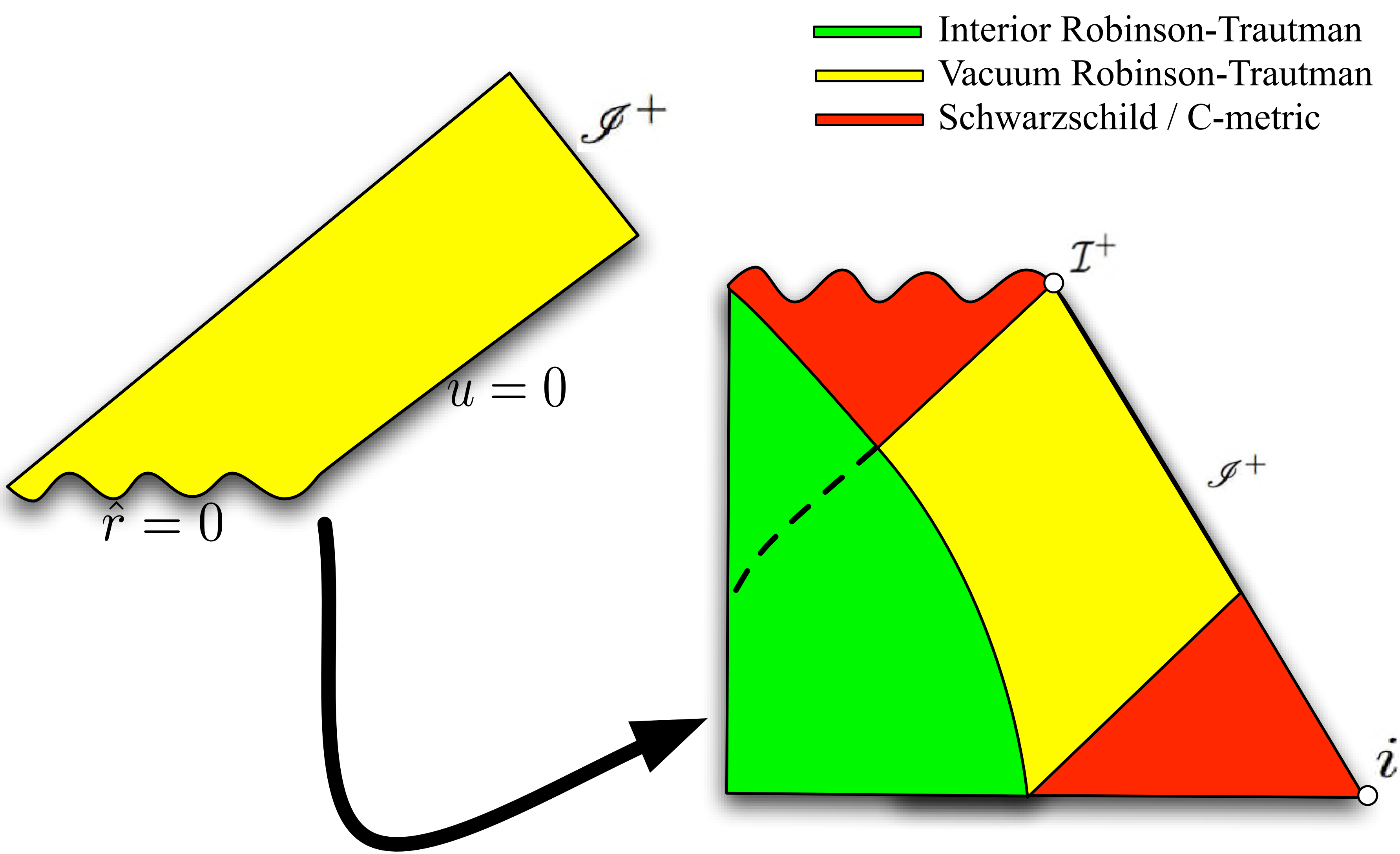
Where Q is charge per unit volume

Now, the charged Vaidya and Reissner-Nordstrom spacetimes are subsets of the general plasma equations



NON-SPHERICAL MODELS

(work in progress)



NON-SPHERICAL MODELS

(work in progress)

We begin with a similar metric ansatz

$$dS^2 = -\alpha^2 dt^2 + \frac{(\beta dt + dr)^2}{1+E} + 2R^2 d\zeta d\bar{\zeta}$$

where α, β, E & R are all functions of $(t,\, r,\, \zeta,\, \bar{\zeta})$

Dust energy momentum tensor

$$T_{\mu\nu} = \rho\, n_\mu n_\nu$$

implies

$$\alpha = 1$$

Geodesic, hypersurface orthogonal, shearfree null congruence

$$\kappa = \sigma = 0 \qquad \rho = \bar{\rho}$$

implies

$$\beta = \eta(t,r) - \sqrt{1+E}$$

Open Question:

Can we use this method to describe an interior **dust RT** spacetime, which reduces to the vacuum RT spacetime in non-null coordinates?