Hawking Radiation from Extra Dimensional Rotating Black Holes

- **Phys.Rev.D75:084043,2007**
- **arXiv:0707.1768** , accepted in **P.R. D**
- **arXiv:0709.0241** , submitted in **P.R.L.**

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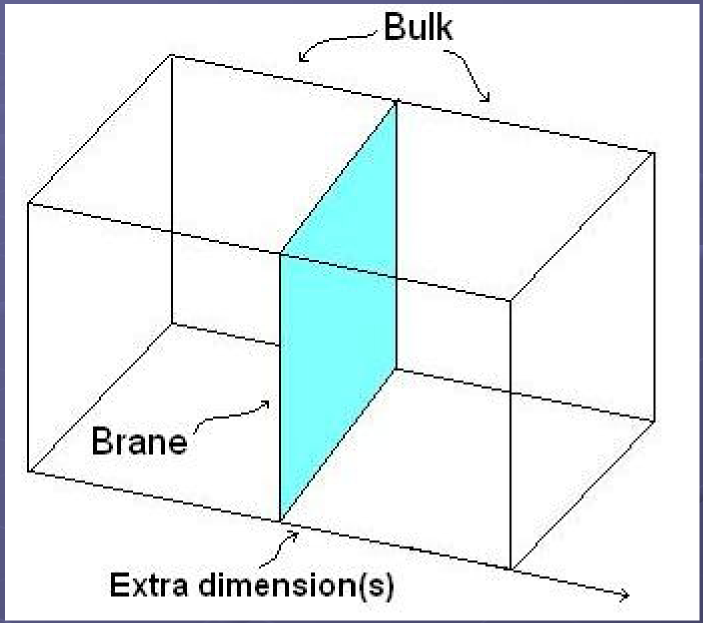
Plan of the talk

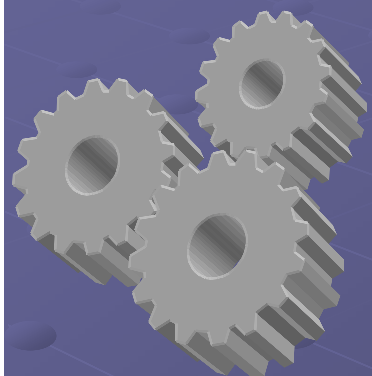
- Describe the Brane World scenario
- Write down equations describing propagation of **brane** fields around a rotating black hole
- Solve the equations analytically, compute the absorption probability, which characterizes Hawking radiation emission from the black hole
- Do the same for a **bulk** scalar field
- Results-Conclusions

The Brane world Scenario



- SM particles live on the brane
- Gravity - scalar fields propagate on (4+n) dimensions (Bulk)
- Extra dimensions: spacelike, compact

Black holes can be produced in ground based colliders (even LHC) or cosmic ray interactions



Can be detected through Hawking radiation on the Brane

Which will give information about the nature of spacetime

If B.H. are created in the lab, they will go through a number of phases :

- **The <u>balding phase</u>** : BH emits mainly gravitational radiation , loses all "hair" from the original particles and all asymmetries coming from the production process
- **The <u>spin-down phase</u>** *:* The BH loses energy and angular momentum through the emission of Hawking radiation
- **The *<u>Schwarzschild phase</u>*** : A spherically symmetric BH loses energy through Hawking radiation
- **The *<u>Planck phase</u>*** *:* The BH's mass approaches M_* , the characteristic scale of gravity. Quantum gravity needed to study this phase.

The Schwarzschild phase has been studied extensively in the past both analytically and numerically

In these works we focused on the **spin down** phase

FRAMEWORK:

- The geometry of a higher dimensional **uncharged, rotating** BH was found by Myers and Perry:

$$ds^2 = (1-\frac{\mu}{\Sigma r^{n-1}})dt^2 + \frac{2a\mu\sin^2\theta}{\Sigma r^{n-1}}dtd\phi - \frac{\Sigma}{\Delta}dr^2 - \Sigma d\theta^2$$

$$-(r^2 + a^2 + \frac{a^2\mu\sin^2\theta}{\Sigma r^{n-1}})\sin^2\theta d^2\phi - r^2\cos^2\theta d\Omega^2{}_n$$

- with $\Delta = r^2 + a^2 - \frac{\mu}{r^{n-1}}, \Sigma = r^2 + a^2\cos^2\theta$
- With **n** the number of extra dimensions.
- event horizon $\mathbf{r_H}$ found by Δ=0.
- BH's mass and angular momentum are given by

$$\mathrm{M_{BH}} = \frac{(n+2)(2\pi)^{(n+3)/2}}{16\pi G\Gamma[(\nu+3)/2]}\mu \qquad J = \frac{2}{n+2}\mathrm{M_{BH}}\alpha$$

Hawking Radiation

- The number of particles emitted can be written as:

$$\frac{d^2N}{dtd\omega} = \frac{1}{2\pi}\sum_{l,m}\frac{1}{\exp(k/T_H)-1}\left|A_{l,m}\right|^2$$

with

$$k = \omega - \frac{ma}{{r_h}^2 + a^2} \qquad T_H = \frac{(n+1)+(n-1)(a/r_h)^2}{4\pi(1+a^2/{r_h}^2)r_h}$$

$\left|A_{l,m}\right|^2$

- the absorption probability
- Is what differentiates Hawking radiation from perfect blackbody radiation
- Depends on n, particle's energy ω , and quantum numbers l,m
- Can be computed from the equations of motion for a given type of particle as the probability of it being absorbed by the black hole, when coming from infinity

So in order to study a BH, we should compute the absorption probability

How to do that?

- Write down the equations of motion for a type of field in the gravitational background of a rotating BH
- Solve the equations
- Compute the probability of absorption from the BH
- Use it to study Hawking radiation emission

➡ Unfortunately the equations of motion of SM fields around a rotating BH are quite complex to be solved exactly. For that reason we will follow an **approximate method**, consisting of:

- Solve the equations of motion near the BH's horizon , in the limit $r \rightarrow r_H$
- Solve the equations of motion far away from the BH's horizon , in the limit $r >> r_H$
- Stretch and match the two solutions in the intermediate zone

The Equations of motion for **BRANE** fields

- The radial part R of the Eq. of motion for a general s-spin field on the brane can be written in terms of partial waves expansion

$$\Delta^{-s}\frac{d}{dr}\left(\Delta^{s+1}\frac{dR}{dr}\right)+\left(\frac{K^2-isK\Delta'}{\Delta}+4is\omega r+s(\Delta''-2)\delta_{s,|s|}-\Lambda_{sj}\right)R=0$$

with $K=(r^2+a^2)\omega-am$, $\Delta=r^2+a^2-\frac{\mu}{r^{n-1}}$

m, j the quantum numbers

and Λ_{sj} a link between the radial and angular part of the equations

The Matching Technique

- Take the $r \rightarrow r_H$ limit on e.o.m. (near horizon limit). The solution near the horizon can be expressed in terms of Hypergeometric Functions
- Take the $r \gg r_H$ limit on e.o.m. (far field limit). Solution in terms of Bessel ($s=0$) or Kummer ($s \neq 0$)
- Stretch NH solution for large r , FF solution for small r, and find out that we can make the streched solutions match in the low energy-rotation regime
- That way we have constructed a smooth solution valid for all r

Boundary condition

We must also take into account the BH's most famous property! Once a particle passes the horizon it can never escape!

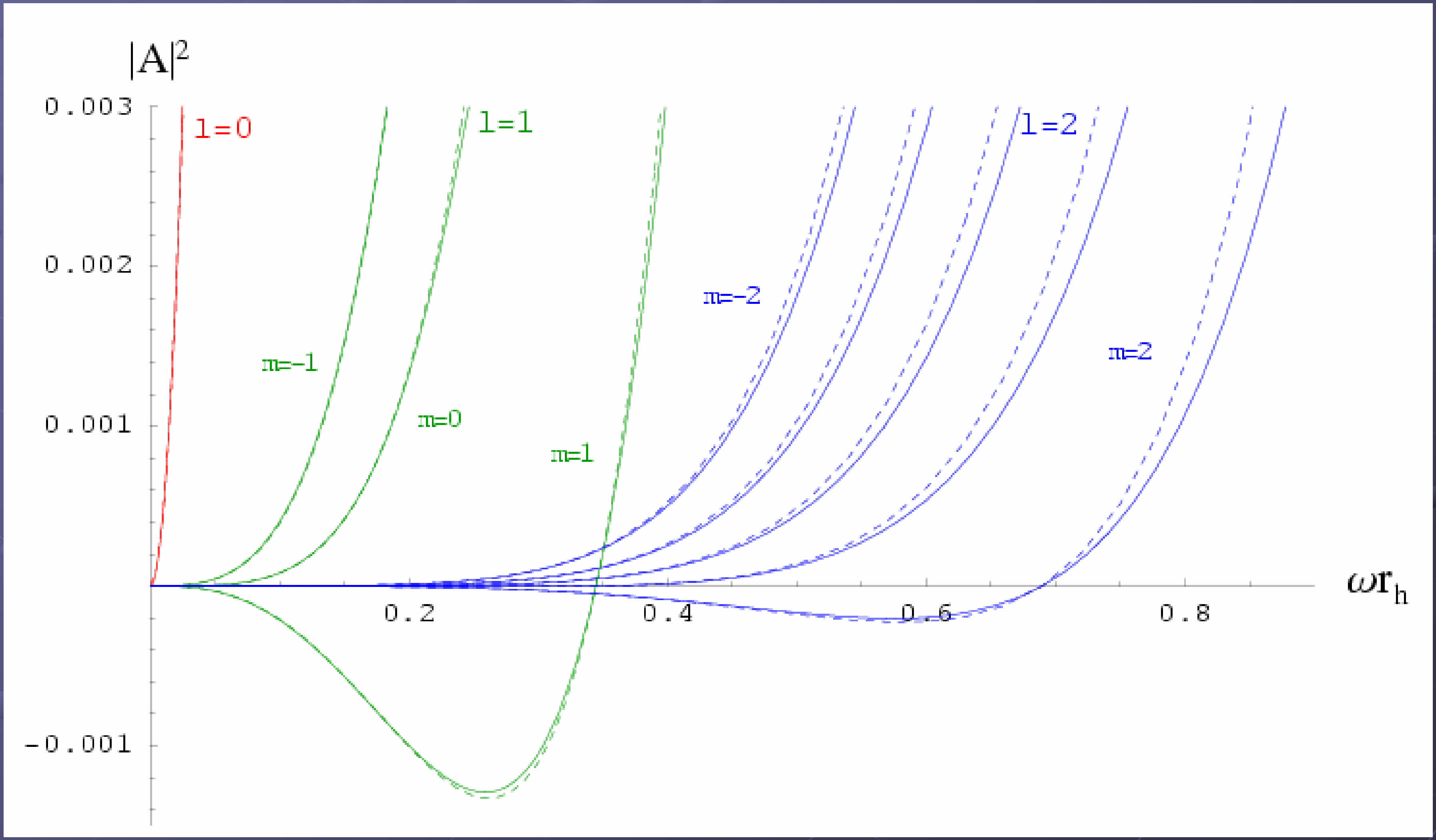
In the near horizon solution we impose the boundary condition that for $r \rightarrow r_H$ the solution is purely ingoing .

Computing the absorption Probability

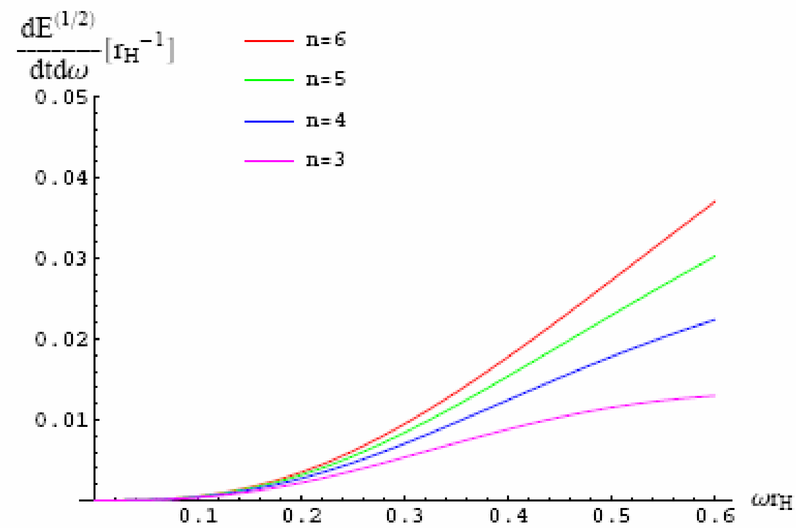
- We want to compute the probability a particle of given energy to be absorbed by the black hole
- Expand the solution we have constructed, for $r \to \infty$ in terms of ingoing and outgoing spherical waves
- Compute the ratio of their amplitudes
- Use the ratio to compute the abs. probability

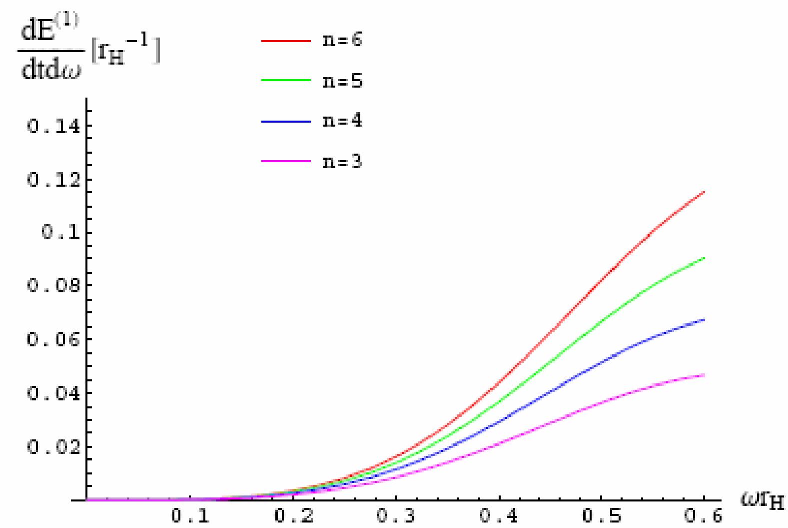
- The method is approximate, valid for low particle energy, low rotation of the black hole
- **However** it gives very good results even in the intermediate energy and rotation regime

Comparison with the exact numerical result , n=2, $a_*=a/r_H=0.4$ for **brane scalar**

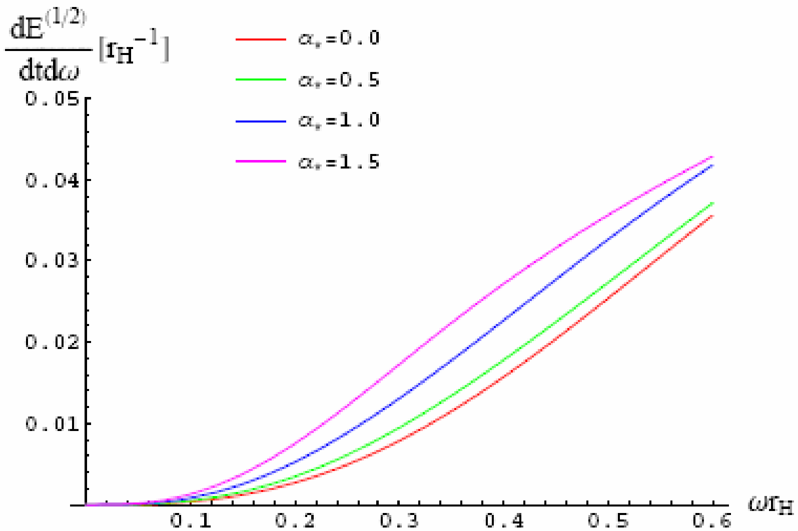


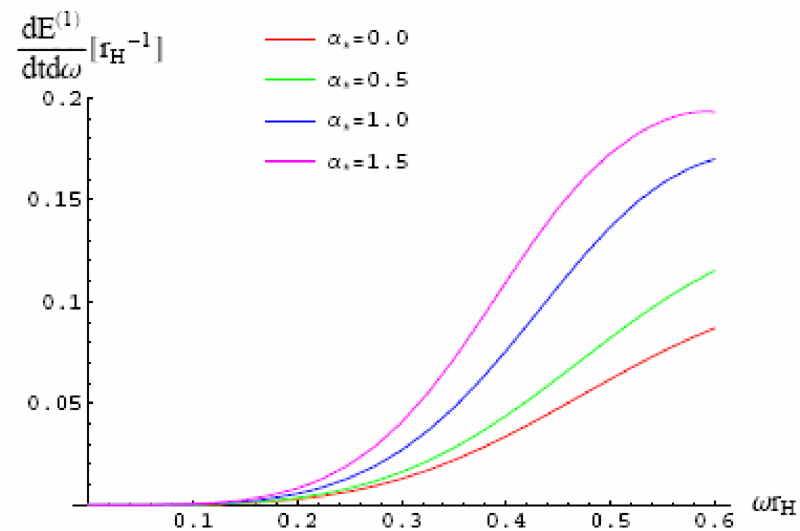
Fermion-Boson energy emission $a_*=0.5$





Fermion-Boson energy emission n=6





Brane/Bulk Scalar energy emission for various n, a_*

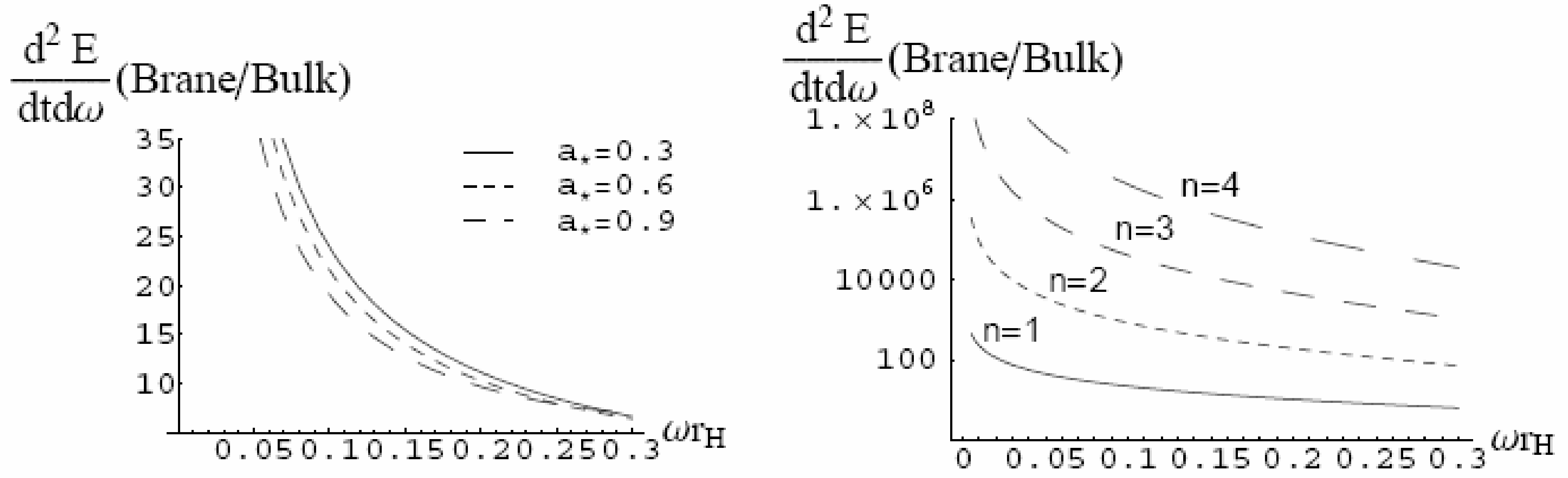


Figure 5: Brane-to-bulk ratio of the energy emission rates for scalar fields **(a)** for $n = 5$ and various a_*, and **(b)** for $a = 0.5$ and various n

Results

- **First partial wave** : the dominant one in all cases by several orders of magnitude
- The derived analytic expression for the absorption probability, valid for low energy and low rotation, proved accurate even in the **intermediate regime**.
- Increase of **n , a$_*$** results in an enhancement on the emitted energy on the brane, but in an decrease of emission on the bulk. However in higher energy regime the latter is expected to change.
- **Gauge boson** emission is found to be dominant over fermion and scalar emission on the brane.
- **Brane scalar** emission is dominant over the **bulk scalar** emission in the low energy regime, however further numerical studies on the bulk emission case are needed in order to make a final conclusion over the whole energy regime

Conclusions

- We analytically studied Hawking radiation both on the ***brane*** and in the ***bulk*** from a (4+n) dimensional rotating black hole, in the low energy and low rotation regime.
- The equations of motion were analytically solved using ***a matching technique***
- The equation solutions were found in the “near horizon” and “far field” regime and were stretched and matched in the intermediate zone, allowing us to compute the ***absorption coefficient***, a quantity which determines the Hawking radiation emission.

- We were then able to examine BH's Hawking radiation in terms of particle's quantum numbers and space-time properties, **namely a, the rotation parameter of the black hole, and n, the number of extra dimensions** that might exist in nature.