Spontaneous Symmetry Breaking in the Bulk as a Localization Mechanism on the Brane

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Orfeu Bertolami and Carla Carvalho, arXiv:0705.1923 [hep-th]

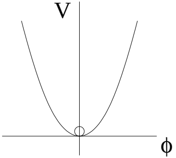
Motivation (... Why not?)

Motivated by the dimensional asymmetry characteristic of braneworlds:

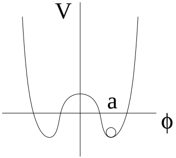
- Populate the bulk with matter fields and couple them non-minimally to gravity.
- Break Lorentz symmetry in the bulk and study consequences in the brane:
  - Reverberate [G.D. Starkman, D. Stojkovic, M. Trodden, Phys.Rev.Lett.87, 231303 (2001)]: Must have a bulk with the same symmetries as those of the brane, since perturbations in the bulk propagate into perturbations in the brane.
  - Phenomenology still to be worked out...
- Use spontaneous symmetry breaking to relate the mass on the brane with a bulk mechanism to look for signatures of extra dimensions. A localization mechanism on the brane?

Spontaneous symmetry breaking (or a clever, little man's view)

Symmetry of the system (Lagrangian) is not shared by the ground state (vacuum) solution: degenerate vacua.
For a scalar field ϕ, let $V(\phi) = \mu^2(\phi^2/2) + \lambda(\phi^4/4)$.
If $\mu^2 > 0$, then $\langle\phi\rangle = 0$.

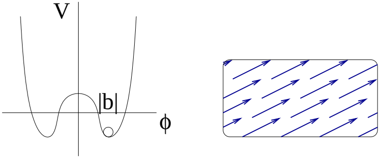


If $\mu^2 < 0$, then $V(\phi) = (\lambda/4)(\phi^2 - a^2)^2 - (\lambda/4)a^4$, with $a^2 = -(\mu^2/\lambda)$ and $\langle\phi\rangle = \pm a$.



Lorentz symmetry breaking

For a vector field B_μ, let $V(\mathbf{B}) = B_\mu B^\mu - b^2$. Then $\langle B_\mu \rangle = \pm b_\mu$.



The vacuum contains an intrinsic direction defined by $\langle B_\mu \rangle$, thus violating rotation invariance and consequently violating Lorentz invariance.

[O. Bertolami, C.C., Phys.Rev.D74, 084020 (2006)]
[O. Bertolami, C.C., gr-qc/0612129]

Bulk scalar field with non-minimal coupling

Consider a bulk scalar field Φ non-minimally coupled to gravity

$$\mathcal{L} = \frac{R}{\kappa_{(5)}^2} - 2\Lambda_{(5)} + \xi\Phi^2 R - \frac{g^{\mu\nu}}{2}(\nabla_\mu\Phi)(\nabla_\nu\Phi) - V(\phi^2),$$

where $\kappa_{(5)}^2 = 8\pi G_N = 1/M_P^3$ is the 5-dimensional gravitational coupling constant, $\Lambda = \Lambda_{(5)} + \Lambda_{(4)}$ is the 5-dimensional cosmological constant with $\Lambda_{(4)} = \sigma\delta(N)$.

Why the non-minimal coupling: the simplest interaction with a canonical kinetical term, reduces to Brans-Dicke up to a field transformation for a vanishing vev.

Bulk equations of motion

In the gravitational sector,

$$\left(\frac{1}{\kappa_{(5)}^2}+\xi\Phi^2\right)G_{\mu\nu}+\Lambda g_{\mu\nu}-\xi\Sigma_{\mu\nu}^{(\Phi)}=\frac{1}{2}T_{\mu\nu}^{(\Phi)},$$

with

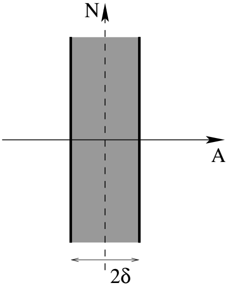
$$\begin{aligned}\Sigma_{\mu\nu}^{(\Phi)} &= \nabla_\mu\nabla_\nu\Phi^2-g_{\mu\nu}g^{\alpha\beta}\nabla_\alpha\nabla_\beta\Phi^2,\\ T_{\mu\nu}^{(\Phi)} &= (\nabla_\mu\Phi)(\nabla_\nu\Phi)+g_{\mu\nu}\left[-\frac{1}{2}g^{\alpha\beta}(\nabla_\alpha\Phi)(\nabla_\beta\Phi)-V(\Phi^2)\right].\end{aligned}$$

In the scalar field sector,

$$g^{\mu\nu}\nabla_\mu\nabla_\nu\Phi-\frac{\partial V}{\partial\Phi}+2\xi\Phi R=0.$$

Boundary conditions

We regard the brane as a Z_2–symmetric shell of thickness 2δ in the limit $\delta \to 0$.



- The Z_2–symmetry establishes the continuity conditions of the fields across the brane,
- Derivatives of quantities discontinuous across the brane generate singular distributions on the brane. Integration of these terms in N relates the induced geometry with the localization of the induced stress-energy.

The junction condition from the ϕ equation:

$$\int dN(\Phi \mathrm{eqn}) \Rightarrow \nabla_N \Phi - 4\xi K \Phi = 0.$$

The junction condition from the (AB) Einstein equation:

$$\left(\frac{1}{\kappa_{(5)}^2} + \xi\phi^2\right)(-K_{AB} + g_{AB}K) = -g_{AB}\left(\sigma + \xi\nabla_N\phi^2\right).$$

Two separable conditions on the boundary ($N = 0$):

$$\begin{aligned} \nabla_N \Phi &= -4\xi\Phi\,\sigma\frac{d/(d-1)}{\lambda/\kappa_{(5)}^2 + \xi\Phi^2(1 + 8\xi/(d-1))}, \\ K_{AB} &= -g_{AB}\,\sigma\frac{1/(d-1)}{\lambda/\kappa_{(5)}^2 + \xi\Phi^2(1 + 8\xi/(d-1))}. \end{aligned}$$

We obtain for the propagation of ϕ on the brane that

$$g^{AB}\nabla_A\nabla_B\phi - \frac{\partial V}{\partial\phi} + 2\xi\phi\left[R^{(ind)} - K_{AB}K_{BA} + (1+8\xi)\,K^2\right] = 0.$$

From the (NN) Einstein equation we extract the induced intrinsic curvature $R^{(ind)}$

$$\begin{aligned}
&\left(\frac{1}{\kappa_{(5)}^2} + \xi\phi^2\,(1+4\xi)\right)R^{(ind)} = \\
= \quad & \left(\frac{1}{4} + 2\xi\right)(\nabla_C\phi)^2 + \frac{1}{2}V + 2\xi\phi\frac{\partial V}{\partial\phi} + \Lambda_{(5)} \\
- \quad & K_{CD}K_{CD}\left(\frac{1}{\kappa_{(5)}^2} + \xi\phi^2\,(1-4\xi)\right) + K^2\left(\frac{1}{\kappa_{(5)}^2} + \xi\phi^2\left(1-32\xi^2\right)\right),
\end{aligned}$$

Induced equations on the brane (β')

from the (AN) Einstein equation we find the conservation of the induced stress-energy tensor $\mathcal{T}_{AB}$

$$\begin{aligned} G_{AN} &= K_{AB;B} - K_{;A} \\ &= -\nabla_B \left(\int_{-\delta}^{+\delta} dN\, G_{AB} \right) = -\kappa_{(5)}^2 \nabla_B \mathcal{T}_{AB} = 0, \end{aligned}$$

and from the (AB) Einstein equation

$$\begin{aligned} G_{AB}^{(ind)} &= \left(\frac{1}{\kappa_{(5)}^2} + \xi\phi^2 \right)^{-1} \left[\left(\frac{1}{2} + 2\xi \right) (\nabla_A \phi)(\nabla_B \phi) + 2\xi\phi \nabla_A \nabla_B \phi \right] \\ &- g_{AB} \left[R^{(ind)} + K^2 \frac{-d^2 + d + 4}{2d^2} \right]. \end{aligned}$$

Read off of the induced Einstein equation the Λ_{eff}, comprising all the terms proportional to g_{AB} when all the contributions from the matter fields are evaluated at the vev's.

- For $\langle\phi\rangle = 0$,

$$\Lambda_{eff}\frac{1}{\kappa_{(5)}^2} = \Lambda_{(5)} + K^2\frac{1}{\kappa_{(5)}^2}\frac{d^2-d+4}{2d^2};$$

- When $\langle\phi\rangle \neq 0$, with $\nabla_A\langle\phi\rangle = 0$,

$$\begin{aligned}\Lambda_{eff}\left(\frac{1}{\kappa_{(5)}^2} + \xi\langle\phi\rangle^2(1+4\xi)\right) &= \Lambda_{(5)} + K^2\frac{1}{\kappa_{(5)}^2}\frac{d^2-d+4}{2d^2}\\ &+ \frac{1}{2}V(\langle\phi\rangle^2) + 2\xi\langle\phi\rangle\left.\frac{\partial V}{\partial\phi}\right|_{\langle\phi\rangle}\\ &+ \xi\langle\phi\rangle^2K^2\left[\frac{d^2-d+4}{2d^2} + 4\xi\left(\frac{-d^2+3d+4}{2d^2} - 8\xi\right)\right].\end{aligned}$$

Effective potential

The evolution of ϕ on the brane

$$g^{AB}\nabla_A\nabla_B\phi - \frac{\partial V_{eff}}{\partial\phi} + \left(\frac{1}{4}+2\xi\right)(\nabla_C\phi)^2\,\frac{2\xi\phi}{1/\kappa^2_{(5)}+\xi\phi^2(1+4\xi)} = 0$$

subject to the effective potential induced on the brane

$$\begin{aligned}
-\frac{\partial V_{eff}}{\partial\phi} &= \frac{1}{1/\kappa^2_{(5)}+\xi\phi^2(1+4\xi)}\left\{-\frac{\partial V}{\partial\phi}\left(\frac{1}{\kappa^2_{(5)}}+\xi\phi^2\right)\right. \\
&+ 2\xi\phi\left[\frac{1}{2}V+\Lambda_{(5)}\right] \\
&+ \left. 2\xi\phi K^2\left[\frac{1}{\kappa^2_{(5)}}\left(-\frac{2}{d}+2+8\xi\right)+\xi\phi^2\left(-\frac{2}{d}+2+12\xi\right)\right]\right\}
\end{aligned}$$

where $V(\phi^2)$ is the bulk potential, assumed to have a Higgs type form $V(\phi)=\mu^2_{(5)}(\phi^2/2)+\lambda_{(5)}(\phi^4/4)$ with $\lambda_{(5)}>0$.

Conditions of the parameters

Integrate to find V_{eff}. Expect that there exists a hierarchy of scales depending on whether $\langle\phi\rangle$ is related to the Standard Model or the grand unified theory scale: $|\xi\phi^2| \ll 1/\kappa^2_{(5)}$. Then

$$V_{eff} = \mu^2_{eff}(\phi^2/2) + \lambda_{eff}(\phi^4/4) + O[\phi^6\sigma^2\kappa^2_{(5)}(\xi\kappa^2_{(5)})^3],$$

up to sub-dominant logarithmic terms in ξ, where

$$\begin{aligned} \mu^2_{eff} &\sim \mu^2_{(5)} + \lambda_{(5)}\frac{1}{\xi\kappa^2_{(5)}} - \sigma^2\xi\kappa^4_{(5)}, \\ \lambda_{eff} &\sim \lambda_{(5)} + \sigma^2\xi^2\kappa^6_{(5)}. \end{aligned}$$

The parameters of the potential will influence the magnitude of its minimum and thus the mass of ϕ measured on the brane.

$$\begin{aligned} \lambda_{eff} > 0 &\Rightarrow \lambda_{(5)} > -\sigma^2\xi^2\kappa^6_{(5)} \\ \mu^2_{eff} < 0 &\Rightarrow \mu^2_{(5)} < -2\sigma^2\xi\kappa^4_{(5)}. \end{aligned}$$

A note on the dimensionality of ϕ

The bulk scalar field ϕ, being a five-dimensional field, has dimensions $[\phi] = M^{3/2}$. Accordingly, $[\mu_{(5)}] = M$ and $[\lambda_{(5)}] = M^{-1}$. In order to recover characteristically four-dimensional quantities, we define the four-dimensional scalar field Φ as the rescaling of ϕ by an appropriate mass scale M_ϕ.

[An observation: In the mode expansion of a bulk field, this mass can be identified with the mode function dependent on the direction N evaluated at the position of the brane in the bulk.]

For $\phi = M_\phi^{\frac{1}{2}}\Phi$, the induced equation of motion for Φ on the brane becomes

$$\Box\Phi - \frac{1}{M_\phi}\frac{\partial V_{eff}}{\partial \Phi} + \left(\frac{1}{4} + 2\xi\right)(\nabla_C\Phi)^2 \frac{2\xi M_\phi \Phi}{1/\kappa_{(5)}^2 + \xi M_\phi \Phi^2(1+4\xi)} = 0.$$

The effective scalar field Φ

The parameters of the effective potential will scale as

$$\frac{1}{M_\phi} V_{eff}(\Phi^2) = \mu_{eff}^2 \Phi^2 + \lambda_{eff} M_\phi \Phi^4 + M_\phi^2 O[\Phi^6]$$

with

$$\begin{aligned} \mu_{eff}^2 &\sim \mu^2 - 2\sigma^2 \xi \kappa_{(5)}^4 \ , \\ M_\phi \lambda_{eff} &\sim \lambda + M_\phi \xi^2 \sigma^2 \kappa_{(5)}^6 \ , \end{aligned}$$

where $\mu = \mu_{(5)}$ and $\lambda = M_\phi \lambda_{(5)}$. For $\xi > 0$ we have two possible mechanisms for the generation of a non-vanishing vev:

- the canonical way, via the potential associated with the scalar field,
- the braneworld way, via the interaction of the scalar field with the brane tension.

Constraints from the standard model: back to reality

- For the latter to be viable in the context of the SM, then $\langle \Phi \rangle = 246$ GeV, as contrained by the interaction with the boson vectors, and

$$\left| \frac{\mu_{eff}^2}{M_\phi \lambda_{eff}} \right| \sim \frac{1}{\xi M_\phi} \frac{1}{\kappa_{(5)}^2}$$

  must be of order TeV^2.
- In order to recover the four-dimensional gravitational coupling constant in the induced Einstein equation, we find from the ϕ contribution that $M_{Pl(4)}^{-2} = \kappa_{(5)}^2 M_\phi$. It follows that

$$M_{Pl}^3 \equiv \frac{1}{\kappa_{(5)}^2} \sim M_{Pl(4)}^2 M_\phi .$$

Then,

$$\left| \frac{\mu_{eff}^2}{M_\phi \lambda_{eff}} \right| \sim \frac{1}{\xi} M_{Pl(4)}^2 \gg TeV^2 .$$

Results

Since $\langle \Phi^2 \rangle \sim \left| \frac{\mu_{eff}^2}{M_\phi \lambda_{eff}} \right|$, then $\langle \Phi \rangle \sim M_{Pl(4)}$. The brane mediated mechanism of SSB is rendered unviable by the phenomenological hierarchy between the SM typical energy scale of TeV and the Planck scale of the induced dynamics of Φ on the brane. Moreover, for $\Lambda_{eff} = 0$, then $\sigma^2 \sim M_{Pl(4)}^2 M_\phi \left[-\Lambda_{(5)} - M_\phi V(\langle \Phi \rangle^2) \right]$.

- $\left| \mu_{eff}^2 \right|$ dominates: the scalar field becomes a short range field about the brane $|\mu_{eff}| \sim M_{Pl(4)}$ and therefore strongly localized therein $\delta \sim 1/\left|\mu_{eff}\right|$; then $M_\phi \lambda_{eff} \sim \xi$ and for
  - $M_\phi \sim M_{Pl(4)}$, then $\sigma \sim (10^{16})^4 TeV^4$ and $\Lambda_5 \sim (10^{16})^5 TeV^5$;
  - $M_\phi \sim TeV$, then $\sigma \sim (10^{16})^3 TeV^4$ and $\Lambda_5 \sim (10^{16})^4 TeV^5$.
- $1/M_\phi \lambda_{eff}$ dominates: for $|\mu_{eff}| \sim TeV$ and
  - $M_\phi \sim TeV$, then $\sigma \sim (10^{16})^2 TeV^4$ and $\Lambda_5 \sim (10^{16})^2 TeV^5$;
  - $M_\phi \sim 10^{-16} TeV$, then $\sigma \sim 10^{16} TeV^4$ and $\Lambda_5 \sim 10^{16} TeV^5$;

Conclusions

- Populated the bulk a real scalar field, non-minimally coupled to gravity.
- Upon spontaneous symmetry breaking, found the induced mass of the scalar field to be of order $M_{Pl(4)}$, thus strongly localized about the locus of the brane.
- Localization is achieved but at a scale way off the standard model scale, hence no sign of a spontaneous symmetry breaking mechanism in the bulk would be manifest in the induced masses on the brane.
- For $\xi = 0$, $V_{eff} = V + 3\sigma^2\kappa^2_{(5)}d^2/[(d+1)(d-1)]$: σ contributes to V_{eff} but no longer to the mechanism of spontaneous symmetry breaking or to the mixing of the discontinuity in **K** with that in the normal derivative of the bulk field.
- Matter localized on the brane seems to interact with bulk matter fields through gravity only when a non-minimal coupling exists.

Localization

Bulk potential point of view:

- use finiteness of integration of the action along the normal direction:
  [B. Bajc. G. Gabadadze, Phys.Lett.B474 (2000)]
  [I. Oda, Phys.Lett.B571 (2000)];
- study the dynamics along the direction normal to the surface of the brane:
  [K. Farakos, P. Pasipoularides, Phys.Lett.B621 (2005)];

Effective potential point of view:

- analyse the constraints from the effective mass:
  [A. Kehagias, K. Tamvakis, Phys.Lett.B628 (2005)];
- consider the inverse of the mass inside the brane as a measure of the confinement to the brane:
  [M. Laine, H.B. Meyer, K. Rummukainen, M. Shaposhnikov, JHEP 0404 (2004)] ;