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Regularized Codimension-2 Brane Cosmology II

In collaboration with

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MOTIVATION - OUTLINE

- We wish to study cosmology on a regularized brane,

✓ But: We need time dependent bulk solution – Difficult task!

⇒ Simplest scenario: Brane moving in static bulk (Mirage Cosmology) ,

✓ But: We need warping to have induced 4d cosmology.

I. Setup and Static Solutions of the warped model.
II. The Idea: Moving Brane.
III. Cosmology of the 4-Brane.
IV. Conclusions.

I. SETUP AND STATIC SOLUTION

E. Papantonopoulos, A. Papazoglou, V. Zamarias (2006)

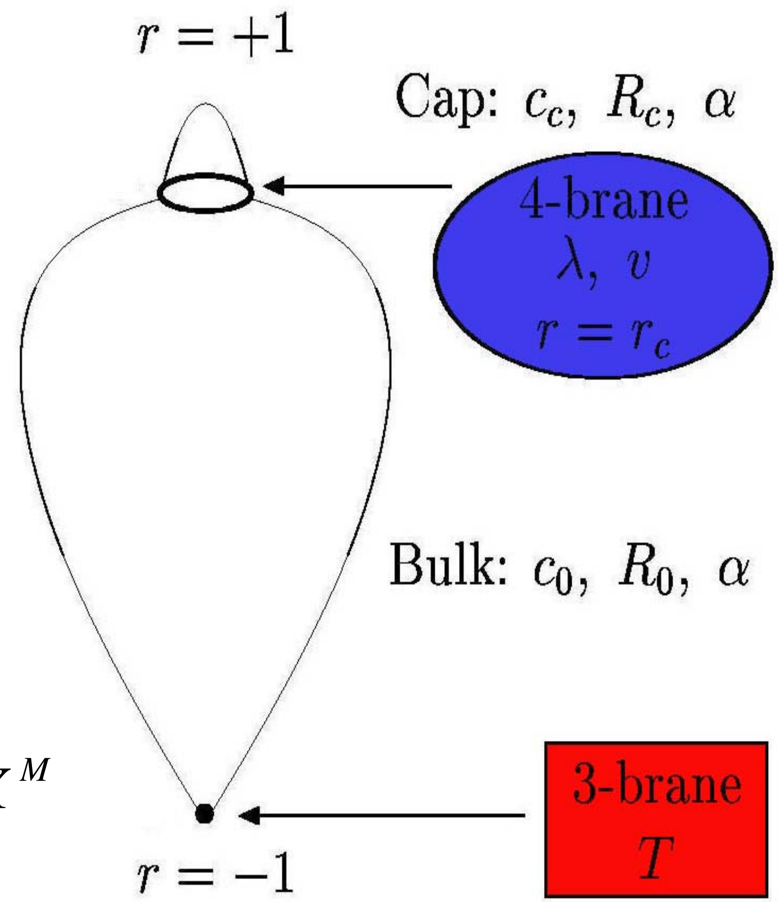
- Action:

$$S = \int d^6x \sqrt{-g}\left(\frac{M^4}{2}R - \Lambda_i - \frac{1}{4}F^2\right)$$

$$-\int d^5x \sqrt{-\gamma_+}\left(\lambda + \frac{v^2}{2}\left(\tilde{D}_{\hat{\mu}}\sigma\right)^2\right)$$

$$-\int d^4x \sqrt{-\gamma_-}\,T$$

with $\tilde{D}_{\hat{\mu}}\sigma = \partial_{\hat{\mu}}\sigma - e a_{\hat{\mu}}, \quad a_{\hat{\mu}} = A_M \partial_{\hat{\mu}} X^M$



- Metric Solution:

$$ds_6^2 = z(r,\alpha)^2 \eta_{\mu\nu} dx^\mu dx^\nu + R_i^2 \left[\frac{dr^2}{f(r,\alpha)} + c_i^2 f(r,\alpha) d\varphi^2 \right],$$

$$F_{r\varphi} = -c_i R_i M^2 S(\alpha) \frac{1}{z(r,\alpha)^4}$$

with $z(r,\alpha) = \frac{1}{2}\left[(1-\alpha) r + (1+\alpha)\right],$

$$f(r,\alpha) = \frac{1}{5(1-\alpha)^2}\left[-z(r)^2 + \frac{1-\alpha^8}{1-\alpha^3}\frac{1}{z(r)^3} - \alpha^3 \frac{1-\alpha^5}{1-\alpha^3}\frac{1}{z(r)^6} \right],$$

$$S(\alpha) = \sqrt{\frac{3}{5}\alpha^3 \frac{1-\alpha^5}{1-\alpha^3}}.$$

- ✓ The cap is smooth at $r = +1$ as long as

$$c_c = \frac{1}{X_+} \quad \text{with} \quad X_+ = \frac{20(1-\alpha)(1-\alpha^3)}{5+3\alpha^8-8\alpha^3}$$

- ✓ The conical singularity at $r = -1$ is supported by a codim-2 brane with tension

$$T = 2\pi M^4 (1 - c_o X_-) \quad \text{with} \quad X_- = \frac{3+5\alpha^8-8\alpha^5}{20\alpha^4(1-\alpha)(1-\alpha^3)}$$

- ✓ Restriction condition from quantum numbers

$$n = \frac{N}{2}\frac{2}{(1-\alpha^3)}\left[\frac{5(1-\alpha^3)}{8(1-\alpha^5)} - \alpha^3\right] \qquad \Rightarrow \alpha \ \text{ takes discrete values}$$

II. THE IDEA: MOVING BRANE

A. Kehagias, E. Kiritsis (hep-th/9910174)

❖ MIRAGE COSMOLOGY:

✓ Bulk fields.

✓ Probe Brane *MOVING* into the *STATIC* Bulk.

⇒ Induced metric of the brane is time dependent.

⇒ Observer on the brane feels a cosmological evolution.

⇒ Friedmann-like Equations.

E. Papantonopoulos, A. Papazoglou, V. Zamarias (2007)

- In general: inclusion of matter contribution $\Rightarrow$ evolution of ***both*** brane and bulk. But as a first step we want to avoid time dependent bulk solution,

$\Rightarrow$ Approximation: Bulk remains static while the brane matter merely makes the brane to move between the static bulk and the static cup with position $R(t)$. Here the brane is not merely a probe brane and we will use Junction Conditions (we include back-reaction of the brane energy density).

- Brane coordinates: $\sigma^{\hat{\mu}} = (t, x^i, \varphi)$.
- Brane embedding X^M: $X^i = x^i$, $X^r = R(t)$ and $X^\varphi = \varphi$,

  Outer bulk section: $X^0_{(out)} = t$,

  Inner cap section: $X^0_{(in)} = T(t)$.

- Continuity of the induced metric: $c_0 R_0 = c_c R_c$ (as in static case)

$$\dot{T}^2\left(1-\beta_+^2\frac{\dot{R}^2}{\dot{T}^2}\frac{R_0^2}{fz^2}\right)=\left(1-\dot{R}^2\frac{R_0^2}{fz^2}\right) \quad \text{with} \quad \beta_+ = X_+ c_0$$

➢ Induced metric on the brane:

$$ds_{(5)}^2 = -z^2\left(1-\dot{R}^2\frac{R_0^2}{fz^2}\right)dt^2 + z^2 d\vec{x}^2 + c_0^2 R_0^2 f\, d\varphi^2 .$$

Brane Proper Time

$$\dot{\tau}^2 = z^2\left(1-\dot{R}^2\frac{R_0^2}{fz^2}\right)$$

$$\Downarrow$$

$$a = z(R(\tau))$$

$$b = c_0 R_0 \sqrt{f(R(\tau))}$$

$$\boxed{ds_{(5)}^2 = -d\tau^2 + a^2(\tau)d\vec{x}^2 + b^2(\tau)\,d\varphi^2 .}$$

Need of warping

- Hubble parameters for the two scale factors:

$$H_a \equiv \frac{1}{a}\frac{da}{d\tau} = \frac{z'}{z}\frac{\dot{R}}{\sqrt{1-\dot{R}^2\frac{R_0^2}{fz^2}}}, \qquad H_b \equiv \frac{1}{b}\frac{db}{d\tau} = \frac{f'}{2fz}\frac{\dot{R}}{\sqrt{1-\dot{R}^2\frac{R_0^2}{fz^2}}}$$

$\Rightarrow$ The two Hubble rates are related: $\boxed{H_b = \frac{zf'}{2fz'}H_a}$

Close to the would-be conical singularity we have: $f' < 0$

$$\text{If} \quad H_a > 0 \;\Rightarrow\; H_b < 0$$

(If 4D space expands then internal space shrinks)

III. COSMOLOGY OF THE 4-BRANE

E. Papantonopoulos, A. Papazoglou, V. Zamarias (2007)

- Brane matter (Energy Momentum Tensor):

$$t_{\hat{\mu}}^{\hat{\nu}(br)} = -\frac{2}{\sqrt{-\gamma_+}} \frac{\delta S_{br}}{\delta \gamma_{+\nu}^{\mu}} = \mathrm{diag}\left(-\rho, P, P, P, \hat{P}\right)$$

with

$$\rho = \rho_0 + \rho_m = \lambda + \frac{v^2\left(n - eA_\varphi^+\right)^2}{2c_0^2 R_0^2 f(r_c)} + \rho_m$$

$$P = P_0 + P_m = -\rho_0 + P_m$$

$$\hat{P} = \hat{P}_0 + \hat{P}_m = -\lambda + \frac{v^2\left(n - eA_\varphi^+\right)^2}{2c_0^2 R_0^2 f(r_c)} + \hat{P}_m$$

- Coupling of the bulk gauge field to the brane matter :

$$\frac{\delta S_{br}}{\delta a^{\hat{\kappa}}} = \left(l, L, L, L, \hat{L}\right)$$

with

$$l = l_m$$

$$L = L_m$$

$$\hat{L} = \hat{L}_0 + \hat{L}_m = ev^2\left(n - eA_\varphi^+\right) + \hat{L}_m$$

- SO FAR:
- ✓ We put *matter* on the brane (perfect fluid), 6 parameters: $\rho, P, \hat{P}, l, L$ and $\hat{L}$.

⇒ The brane *MOVES* in the *static* bulk.

⇒ This movement *induces cosmology* on the brane with the need of *warping* in the bulk.

- ✓ Induced metric on the brane

$$ds^2_{(5)} = -d\tau^2 + a^2(\tau)d\vec{x}^2 + b^2(\tau)\,d\varphi^2 .$$

- ✓ Relations derived from the continuity of the induced metric.

$$c_0 R_0 = c_c R_c, \qquad \dot{T}^2\left(1-\beta_+^2\frac{\dot{R}^2}{\dot{T}^2}\frac{R_0^2}{fz^2}\right) = \left(1-\dot{R}^2\frac{R_0^2}{fz^2}\right)$$

- Junction conditions:

$$\left\{\hat{K}_{\hat{\mu}\hat{\nu}}\right\} = -\frac{1}{M^4} t^{(br)}_{\hat{\mu}\hat{\nu}}, \qquad \text{where } \{H\} = H^{in} + H^{out}$$

$$\left\{n_M F^M_N \partial_{\hat{\kappa}} X^N\right\} = -\frac{\delta S_{br}}{\delta a^{\hat{\kappa}}}.$$

❖ RESULTS:

- ✓ 2 Junction conditions for the gauge field (the (τ) and (i) components) give for the coupling: $l = L = 0$,
- ✓ The (φ) component of the gauge field junction and the (φφ) component of the metric junction determine $\hat{P}, \hat{L}$,
- ✓ The 2 remaining metric junction conditions give the Friedmann equation and the acceleration equation.

- The Friedmann Equation:

$$H_a^2 = C_1(a)\rho^2 + \frac{C_2(a)}{\rho^2} + C_3(a),$$

where $C_i\ (a)$ are expressed as known functions of f and z.

- Expansion around the static case $\rho_m^{(4)} << \rho_0$:

where the effective 4D $\rho_m^{(4)} = \int d\varphi \sqrt{g_{\varphi\varphi}}\rho_m = \frac{2\pi\beta_+}{X_+} R_0 \sqrt{f}\rho_m$

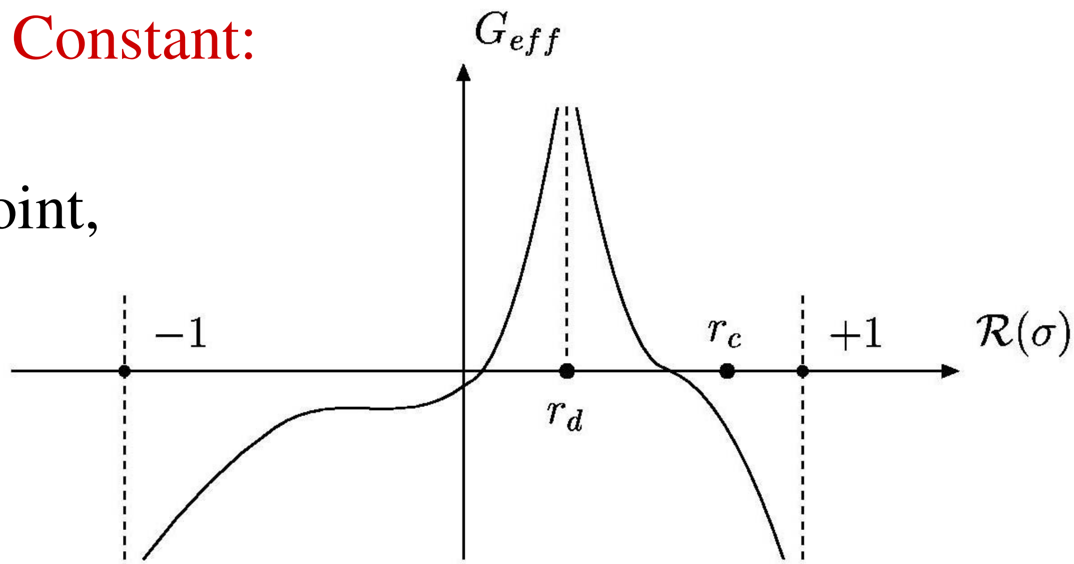
$$H_a^2 = \frac{8\pi}{3} G_{eff}(a)\rho_m^{(4)} + \Lambda_{eff}(a) + O(\rho_m^{(4)2})$$

Λ_{eff} : Mirage matter contribution

- The Effective Newton's Constant:

r_c : Static Equilibrium Point,

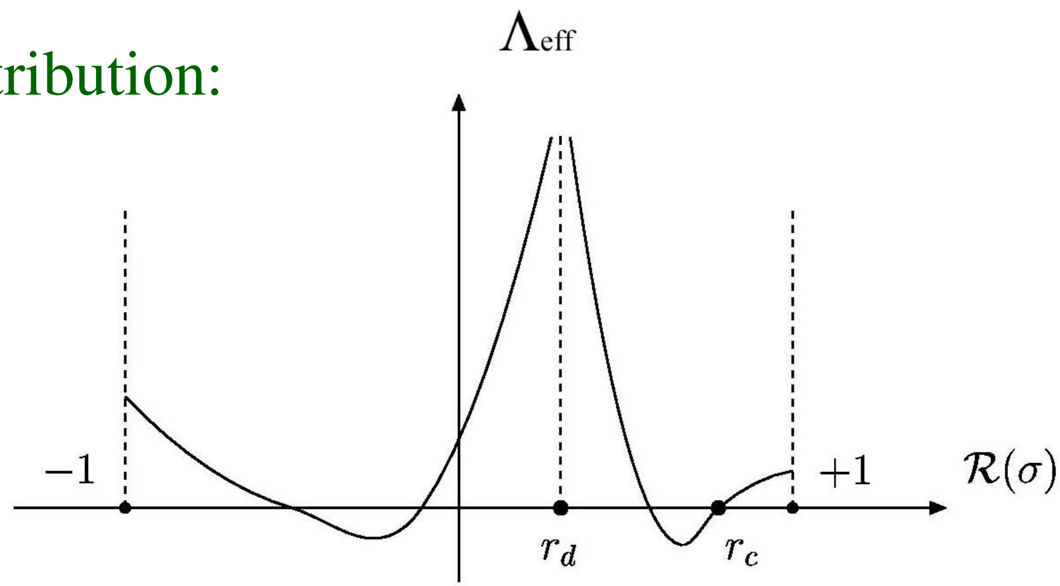
$$r_d: \quad z_d = \left(\frac{3(1-\alpha^8)}{8(1-\alpha^3)} \right)^{1/5} .$$



- The Mirage Matter Contribution:

At $r = r_c$: $\Lambda_{eff} = 0$

but $G_{eff} < 0$!!!



- Early Times Expansion of the Friedmann Equation: $\rho_m^{(4)} >> \rho_0$:

$$H_a^2 = C_4(a)\rho_m^2, \qquad \text{5D Freidmann Law}$$

- We can also derive the energy continuity equation:

$$\frac{d\rho_{tot}}{d\tau} + 3(\rho + P)H_a + (\rho + \hat{P})\frac{zf'}{2fz'}H_a = -W(a, H_a)$$

Significant Energy Flow
from brane to Bulk !!!

IV. CONCLUSIONS

- First step for getting cosmology in a regularized codimension-2 brane.

- We use the simplest scenario of a static bulk and a moving brane $\Rightarrow$ unsatisfactory cosmology!

- Need of ***time-dependent bulk solution***. (work in progress…)

  K. i. Maeda & H. Nishino, Phys. Lett. B 154 (1985) 358

  C. P. Burgess & al. [hep-th/0608083]

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  E. J. Copeland & O. Seto [0705.4169 hep-th]