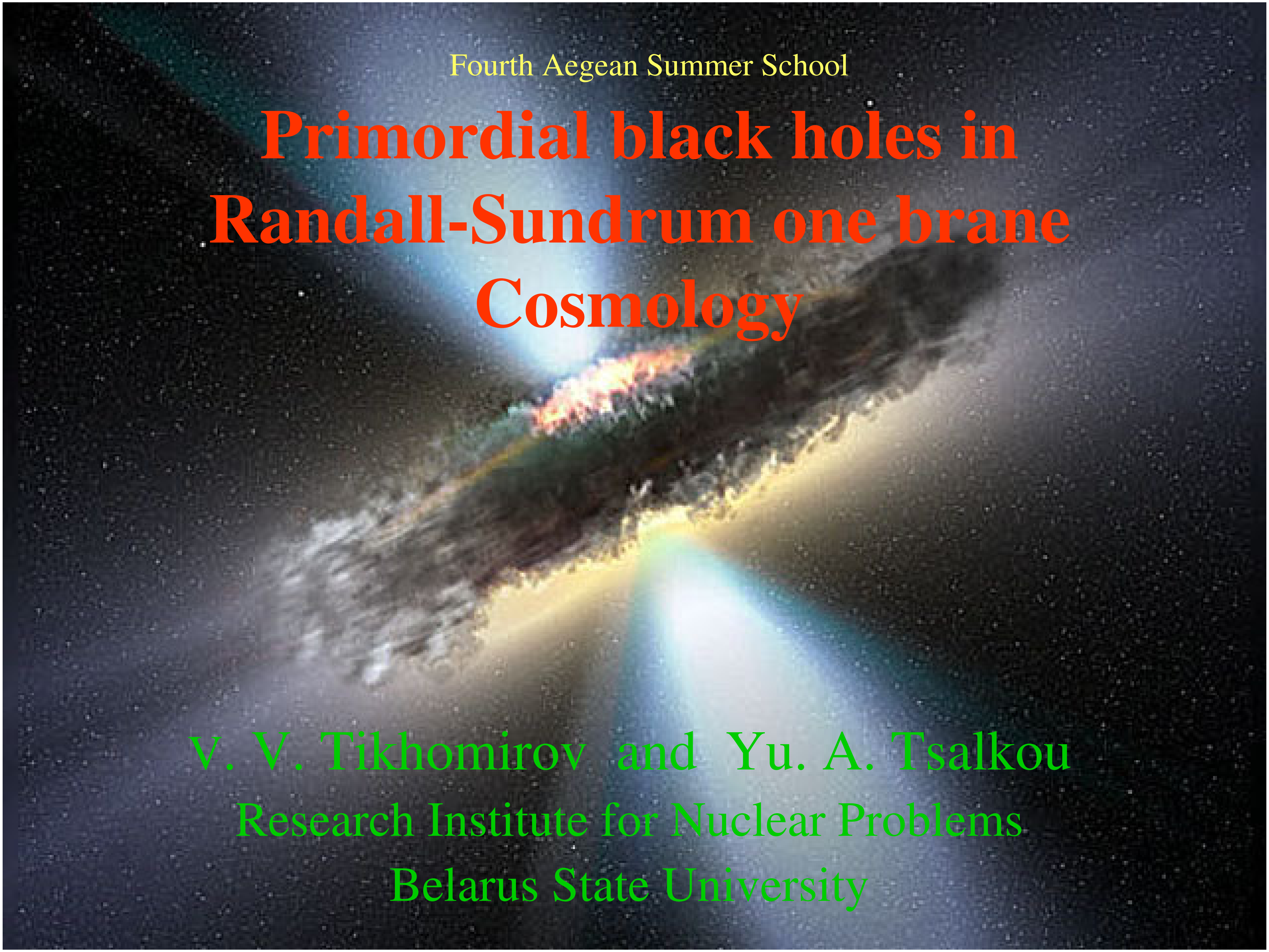

Fourth Aegean Summer School

Primordial black holes in Randall-Sundrum one brane Cosmology

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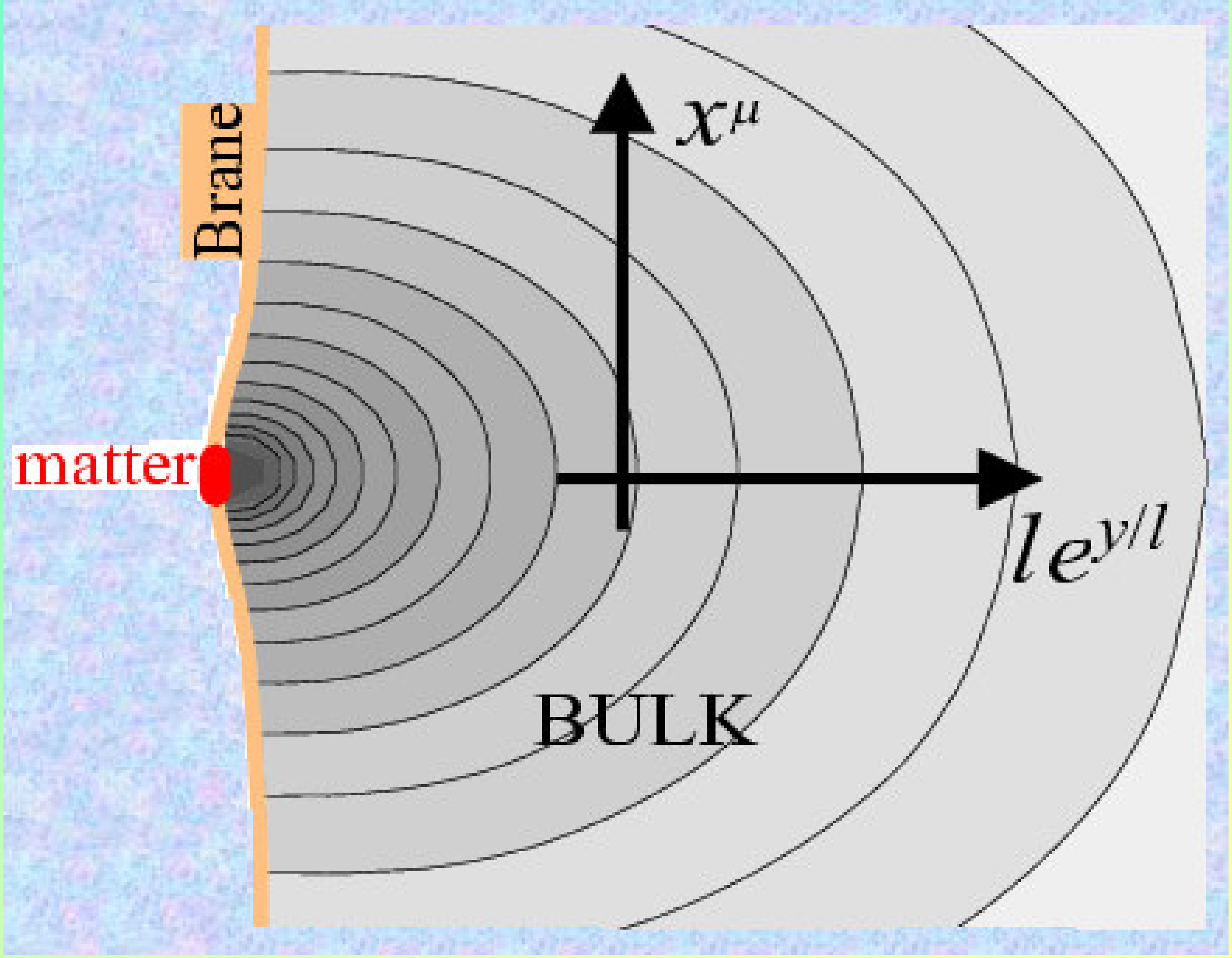
Research Institute for Nuclear Problems
Belarus State University

Plan

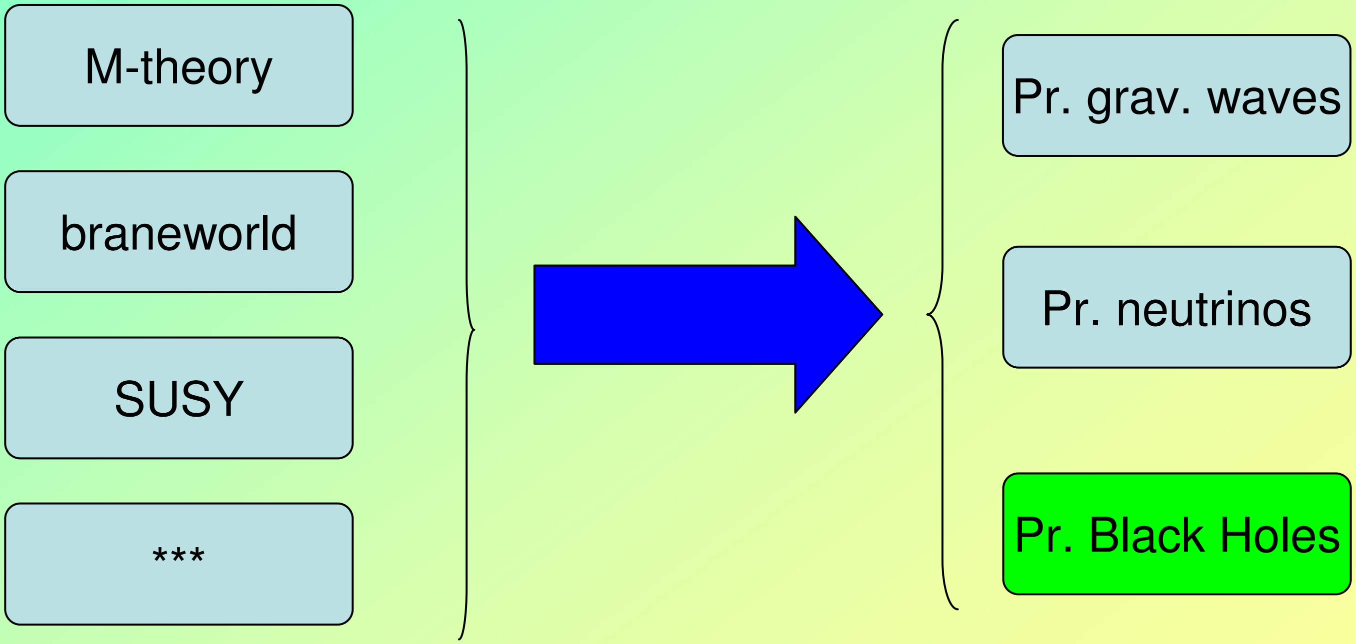
- **Introduction (RS2 and M-theory, gravity tests)**
- **Accretion on PBHs in RS2 braneworld**
    - collisionless approximation
    - PBH mass growth in RS2
    - new accretion efficiency
    - new PBH mass growth coefficient
- **New PBH constraints in RS2 braneworld**
    - – constraints in 4D case
    - – constraints from the total matter density
    - – constraints from the diffuse photon background
    - – constraints from the antiprotons excess
    - – constraints from He abundance
    - – constraints from D photo-disintegratin
    - – constraints from CMB spectrum distortion
    - – constraints from H ionization
- **Conclusion**

Gravitational field penetrates into the **bulk**



from Roy Maartens gr-qc/0312059

THEORY EXPERIMENT



M-theory

braneworld

SUSY

***

Pr. grav. waves

Pr. neutrinos

Pr. Black Holes

Randall Sundrum one brane (RS2) model

$$S=\frac{1}{16\pi G_5}\int d^5xG^{1/2}\left[{}^5R(G_{AB})-2\Lambda_5\right]+$$

$$\int d^4xg^{1/2}\left(\frac{1}{8\pi G_5}{}^4R(g_{\mu\nu})-\sigma+L_{matter}(g_{\mu\nu},\psi,\partial\psi)\right)$$

$\sigma=\dfrac{3}{4\pi G_5\, l}$ – brane tension (4D cosmological constant),

$\Lambda_5=-\dfrac{6}{l^2}$ – 5D cosmological constant,

l **– curvature radius,**

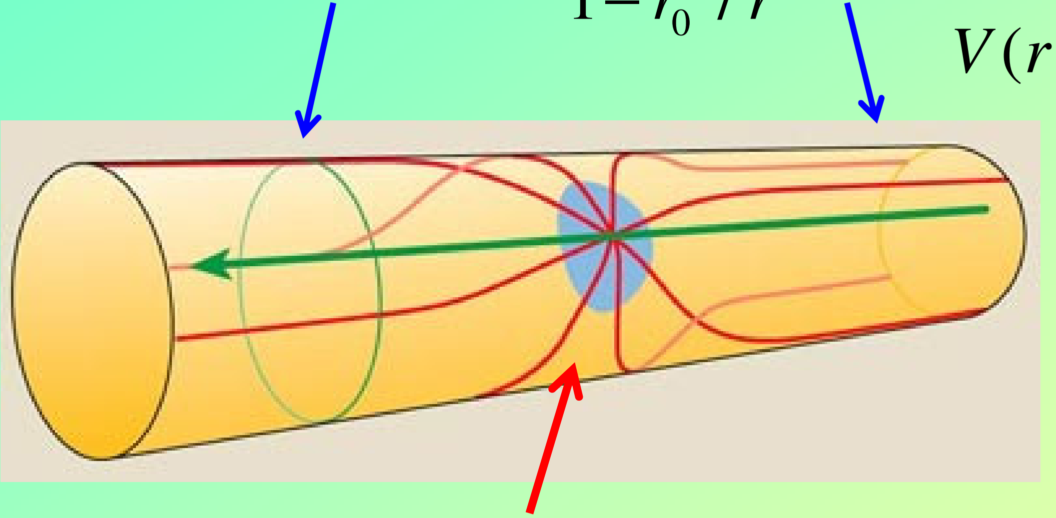
Λ_5 and σ are, thus, *fine tuned* $\Rightarrow$ effective $\Lambda_4=0$

Geometry modification at $r \sim l$

$$ds^2 = -(1 - r_0/r)dt^2 + \frac{1}{1 - r_0/r}dr^2 + r^2 d\Omega_2^2,$$

$$V(r) \approx \frac{GM}{r}\left(1 + \frac{2l^2}{3r^2}\right), \quad r \gg l$$

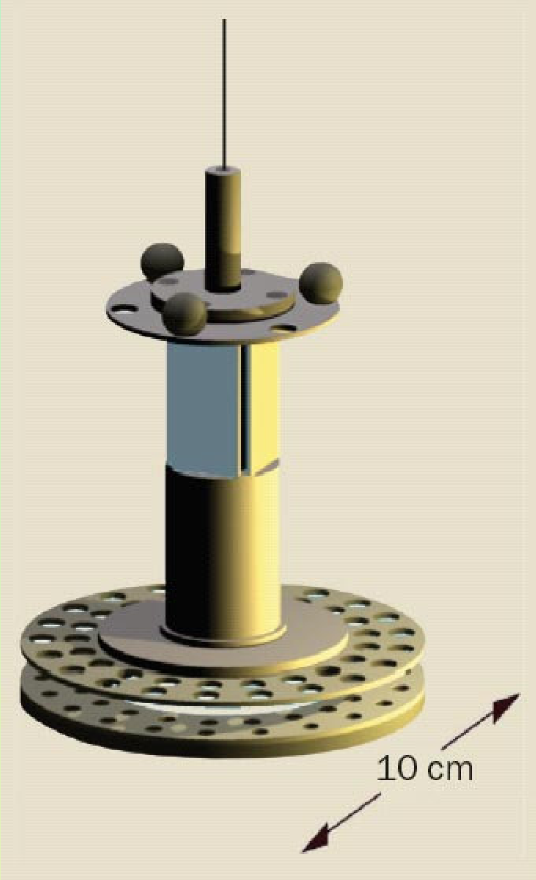
,



$$V(r) \approx \frac{GMl}{r^2}, \quad r \ll l,$$

$$ds^2 = -(1 - r_0^2/r^2)\,dt^2 + \frac{1}{1 - r_0^2/r^2}dr^2 + r^2 d\Omega_2^2, \qquad (1)$$

$$r_0 = \sqrt{\frac{8}{3\pi}}\sqrt{\frac{l}{l_4}}\sqrt{\frac{M}{M_4}}\,l_4$$ – 5D PBH ("Schwarzschild") **radius**



Torsion pendulum putts the inverse-square low to the test at r ~ 0.1 mm

upper ***l*** experimental bound: $l < 0.2\mu m \approx 10^{30} l_4$

S.J. Smullin et al. *Phys.Rev.D72:122001*,2005.

lower ***l*** bound is determined by Hawking evaporation:

$$t_{evap} = \frac{1}{g_{eff}} \left(\frac{l}{l_4}\right)\left(\frac{M_f}{M_4}\right)^2 t_4$$ – time of PBH evaporation

$$M_{\max} = M_4 \frac{l}{l_4}$$ – maximum 5D PBH mass

$$M_f < M_{\max}, \;\; t_{evap} \geq t_H \;\; \Rightarrow \;\; \frac{l}{l_4} > 10^{20}$$

$$l / l_4 = 10^{20} - 10^{30},$$

High-energy phase of RSII cosmological expansion

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\left(\rho + \frac{\rho^2}{2\lambda}\right) - \frac{k}{a^2}$$

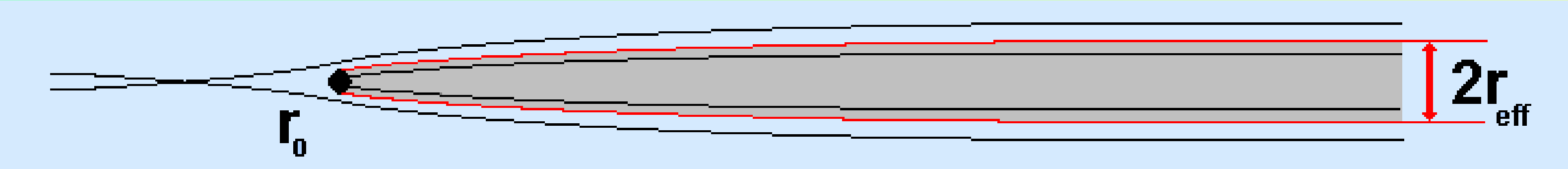
$$a(t) = a_0\left(\frac{t}{t_0}\right)^{1/4}, \qquad \rho = \frac{3M_4^2}{32 t_c t}, \qquad t < t_c = l/2$$

$$a(t) = a_0\left(\frac{t}{t_0^{1/2} t_c^{1/2}}\right)^{1/2}, \qquad \rho = \frac{3M_4^2}{32 t^2}, \qquad t > t_c = l/2$$

Accretion efficiency problem

Collisionless ultrarelativistic particles: $l_{sc} >> r_0$

$$l_{sc} \approx \frac{1}{n\sigma} \qquad \sigma \sim \frac{\alpha^2}{T^2}$$



accretion rate $$\frac{dM}{dt} = \pi r_{eff}^2 \rho(t)$$

4D case: $\rho = \frac{3M_4^2}{32t^2}, \quad r_{eff} = \frac{3\sqrt{3}}{2} r_0, \quad A = 1 - \frac{81}{32} fF > 0, \quad f \ll 1$

$$M(t) = M_i \frac{t}{At + (1-A)t_i} \rightarrow \frac{M_i}{A} \approx \boldsymbol{M_i}$$

B.J. Carr, S.W. Hawking, *MNRAS,* 168, 399 (1974);
B.J. Carr, *Astrophys.* J. 205, 1 (1975).

PBH mass growth is **negligible**

Frequent particle collision limit: $l_{sc} << r_0$

$$\frac{dM}{dt} = \mathbf{F}\pi r_{eff}^2 \rho(t)$$

F is *accretion efficiency*

4D case don't changes due to F is only coefficient, which didn't change a structure of equation

5D case: $$\rho = \frac{3M_4^2}{32t_c t}, \qquad r_{eff} = 2r_0$$

$$M(t) = M_i \left(\frac{t}{t_i}\right)^{\frac{2}{\pi}\mathbf{F}}$$

A.S. Majumdar, PRL 90(2003) 031303

considerable PBH mass growth is **possible**

$$\mathrm{F} > 1 \quad or \quad \mathrm{F} < 1\ ?$$

F < 1 ?

R. Guedens et al, *Phys. Rev.* D66, 083509 (2002);
D. Clancy et al, *Phys. Rev.* D68, 023507 (2003);
Y. Sendouda et al, *Phys. Rev.* D68, 103510 (2003);
Y. Sendouda et al, *Phys. Rev.* D71, 063512 (2005).

no, F > 1 !

V. V. Tikhomirov and Yu. A. Tsalkou, *Phys. Rev.* D72, 121301 (2005).

Cosmological background accretion in ***continuous*** approximation

$$\begin{cases} (\rho + P) u_{i;k} u^k = -P_{,i} - u_i P_{,k} u^k , \\ (\sigma u^k)_{;k} = 0. \end{cases}$$

$$\rho(T) = 3P(T) = \frac{\pi^2}{30} g_{eff} T^4, \qquad \sigma(T) = \frac{2\pi^2}{45} g_{eff} T^3$$

$$\begin{cases} uu' + \dfrac{r_0^2}{r^3} + \dfrac{T'}{T}\left(1 + u^2 - \dfrac{r_0^2}{r^2}\right) = 0, \\ 3\dfrac{T'}{T} + \dfrac{u'}{u} + \dfrac{2}{r} = 0 \end{cases}$$

At the "sonic" point:

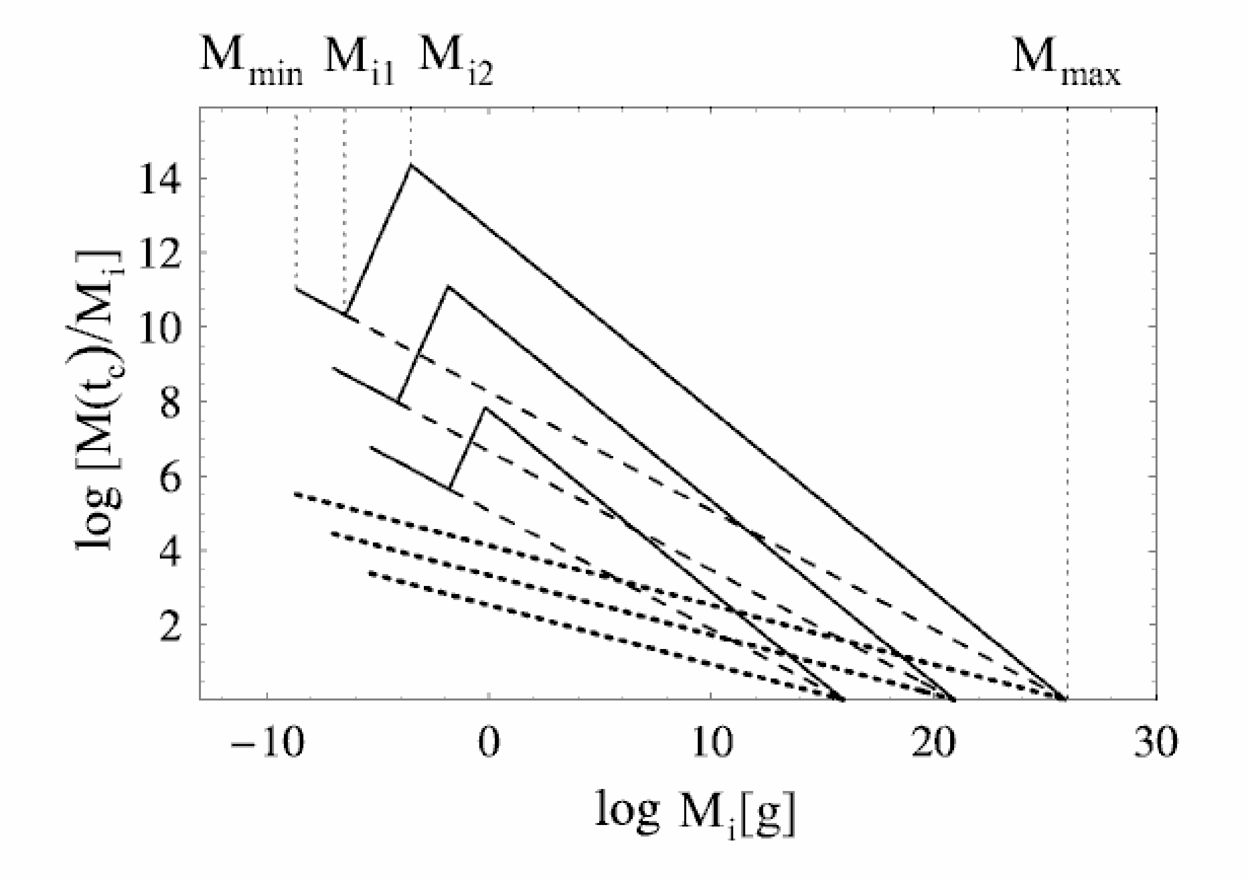
$$r = r_s = \sqrt{2}\, r_0, \quad u_s = u(r_s) = \frac{1}{2};\; T^2\left(1 + u^2 - \frac{r_0^2}{r^2}\right) = const = T_b{}^2$$

$$\frac{d\Sigma}{dt} = 4\pi r^2 \sigma(r) u(r) = 4\pi r_0^2 \sigma(T_b) \times \frac{8}{3\sqrt{3}}$$

$$F = \frac{8}{3\sqrt{3}} \approx 1.54 > 1$$

$$M(t) = M_i \left(\frac{t}{t_i} \right)^{\frac{2}{\pi}F} >>> 1$$

PBH mass growth can be **quite substantial:**

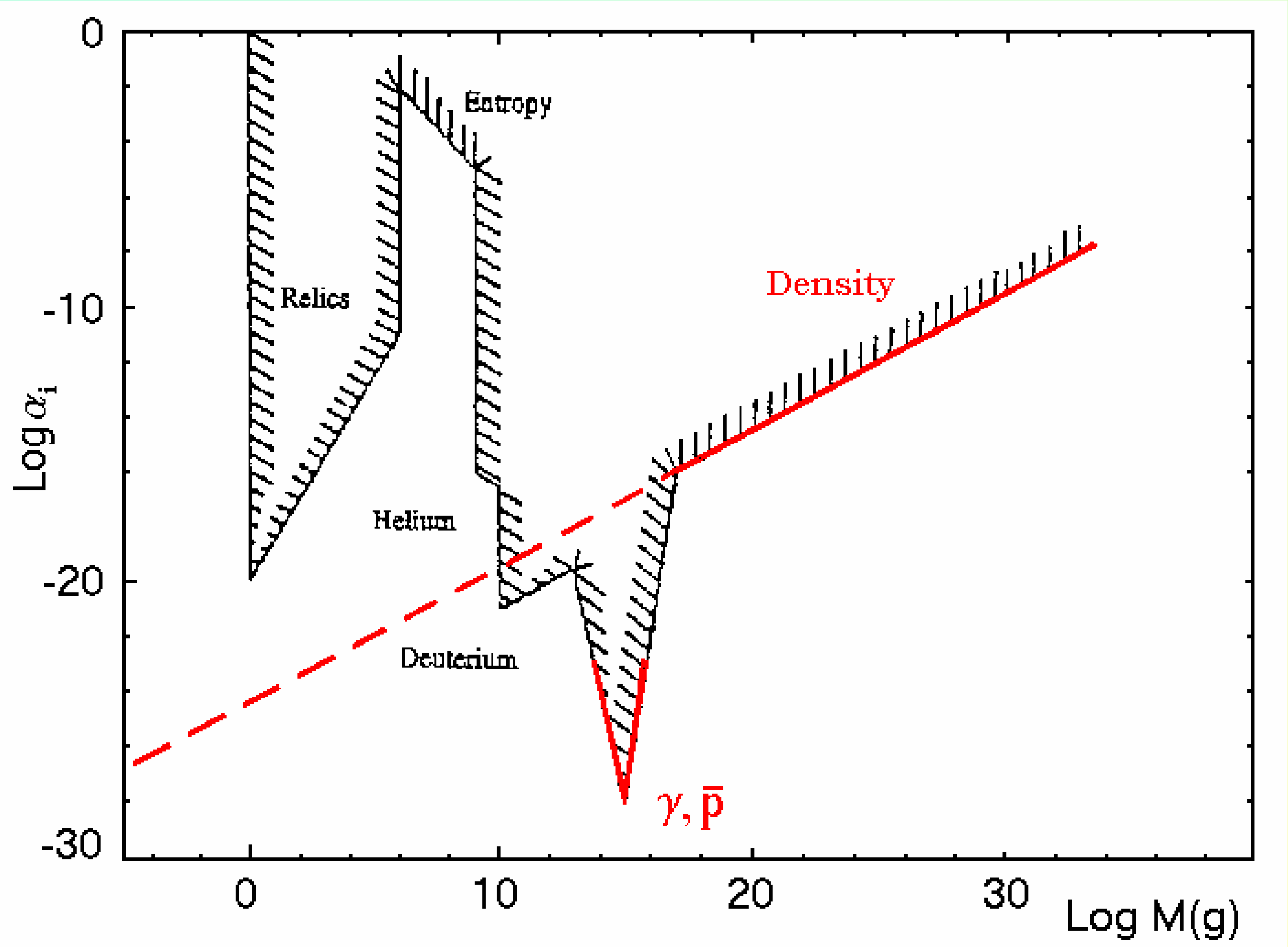


Mass growth coefficient for $l / l_4 = 10^{21}$, 10^{26} and 10^{31} for

F=1.54 (solid), *F=1 (dashed) and F=0.5 (dotted lines).*

Experimental PBH search

Constraints on 4D PBHs



Paper “Search for Gravitational Waves from **PBH** Binary Coalescence in the Galactic Halo”

arXiv:gr-qc/0505042 v1 10 May 2005

Search for Gravitational Waves from Primordial Black Hole Binary Coalescences in the Galactic Halo

(The LIGO Scientific Collaboration, http://www.ligo.org)

[1] Albert-Einstein-Institut, Max-Planck-Institut für Gravitationsphysik, D-14476 Golm, Germany

Constraints on initial PBH mass fraction by the **total matter density**

at a moment $t_i << t_c \equiv \frac{l}{2}$ PBHs form with masses $M_i = f\,8M_4^2 \frac{t_i^2}{t_c}$

$\alpha(M_i,t) \equiv \frac{\rho_{pbh,M_i}(t)}{\rho_{rad}(t)}$ – PBH mass fraction at moment

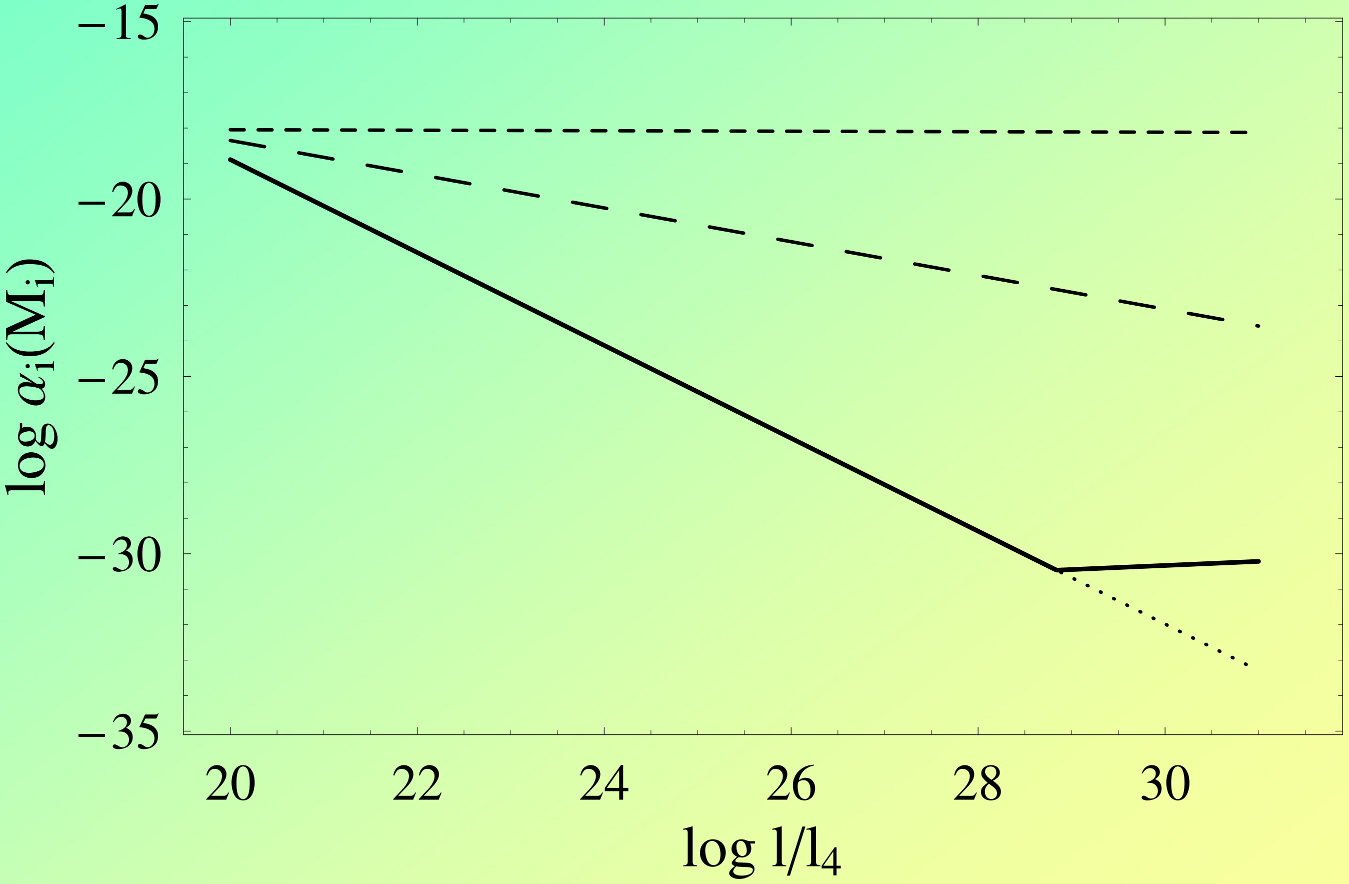
$\alpha_i \equiv \alpha(M_i,t_i)$ – initial mass fraction

$$\rho_{pbh,M_i}(t) \propto a(t)^{-3}, \quad \rho_{rad}(t) \propto a(t)^{-4} \quad \Rightarrow \quad \alpha(M_i,t) \propto M(t)a(t)$$

$\alpha(t_H) < \frac{0.3\rho_{cr}}{\rho_{rad}(t_H)} \quad \Rightarrow$ Dark Matter density limit on α_i

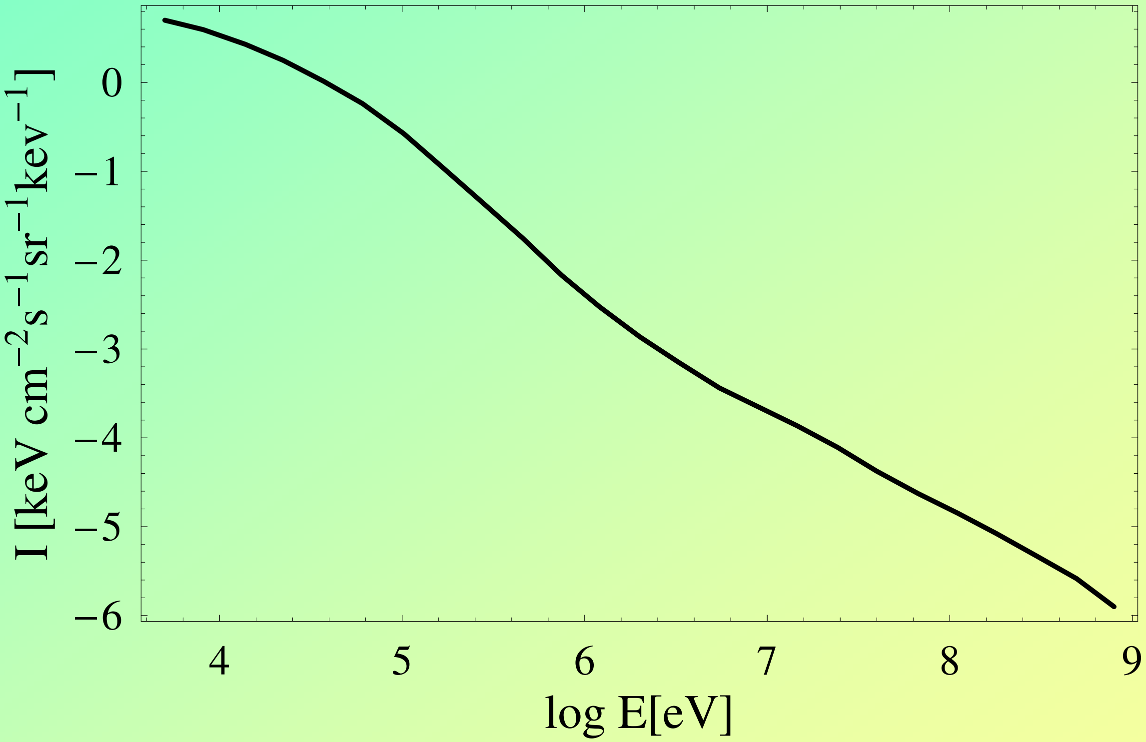
here $t_H \equiv t_{present} \approx 13.5 Gyr$

Constraints on initial PBH mass fraction by the **total matter density**



Dotted, long dashed and short dashed lines correspond to fixed accretion efficiencies F=1.54, 1 and 0.5, respectively.

Constraints on initial PBH mass fraction by the high-energy **diffuse photon background**



Diffuse photon background measure by HEAO-1 A2 (3-60keV), HEAO-1 A4 (80-450keV), SMM (0.45-1MeV), EGRET (~1MeV)

Constraints on initial PBH mass fraction by the high-energy **diffuse photon background**

Evolution of PBH number density:

$$n(M_i,t_i)=\alpha_i(M_i)\frac{\rho_{rad}(t_i)}{M_i}$$ – initial PBH number density

$$n(M_f,t_C)=\frac{a(t_i)^3}{a(t_C)^3}n(M_i,t_i)$$ – PBH number density after accretion

completion at $t=t_c\equiv\frac{l}{2},\quad M_f\equiv M(t_C)$ – final PBH mass

$$n(M_f,t)=\frac{a(t_C)^3}{a(t)^3}n(M_f,t_C)$$ – present PBH number density

$$t_{evap} = \frac{1}{g_{eff}}\left(\frac{l}{l_4}\right)\left(\frac{M_f}{M_4}\right)^2 t_4$$ – time of PBH evaporation

$$M_{\max} = M_4 \frac{l}{l_4}$$ – maximum 5D PBH mass

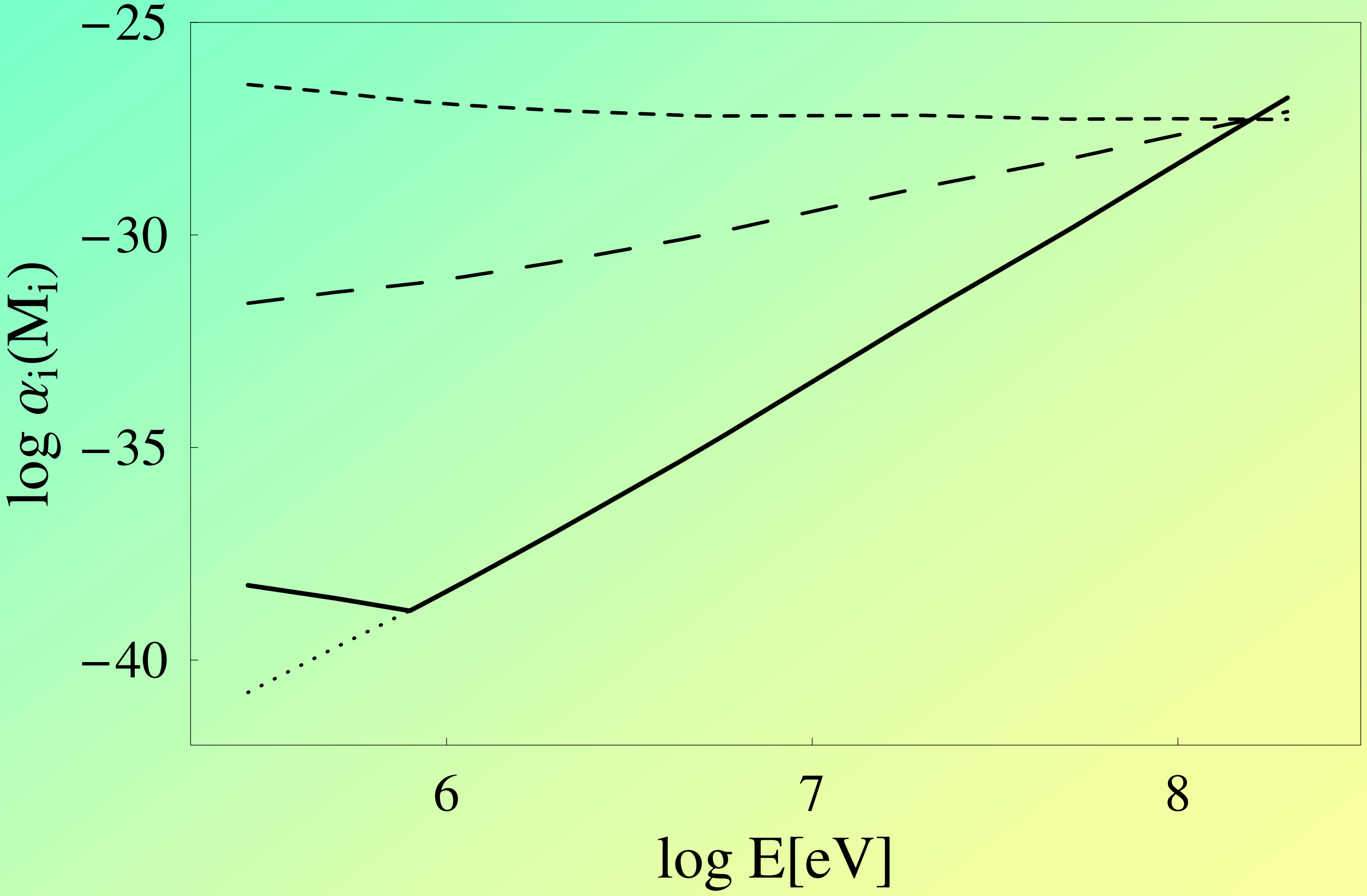
$$M_f < M_{\max}\,,\quad t_{evap} \geq t_H \quad \Rightarrow \quad \frac{l}{l_4} > 10^{\mathbf{20}}$$

$$M^* \approx 3\cdot 10^9 \left(\frac{l}{10^{31} l_4}\right)^{-1/2} g$$ – mass of PBHs evaporating at present

$$E_\gamma \sim 5T_{BH} = \frac{5}{2\pi r_0(M^*, l)} \approx 0.4\left(\frac{l}{10^{31} l_4}\right)^{-1/4} MeV$$ – typical energy of Hawking radiation

$$I(E_\gamma) \sim \frac{c}{4\pi}\frac{M^*}{E_\gamma} n(M^*, t_H)$$ – observed photon spectrum

Constraints on initial PBH mass fraction by the high-energy **diffuse photon background**



Dotted, long dashed and short dashed lines correspond to fixed accretion efficiencies F=1.54, 1 and 0.5, respectively.

Constraints on initial PBH mass fraction by **H ionization**

$$x \equiv \frac{p}{H} < 10^{-4}$$

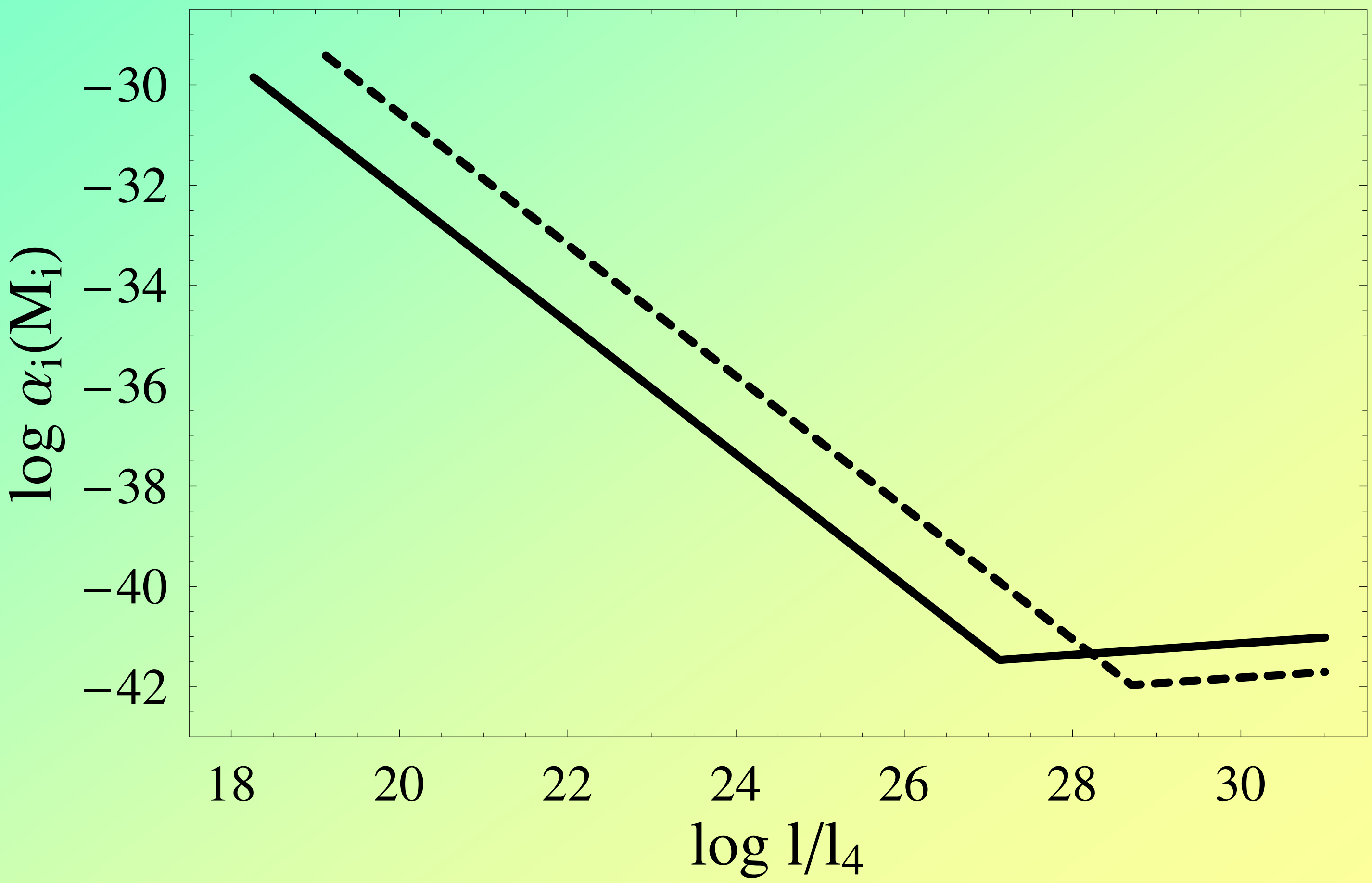
$$\alpha(t_{ev}) < 10^{-4} \frac{100eV}{0.7 \times m_p c^2} \frac{\Omega_b}{\Omega_{CMB}(t_{ev})}$$

$$\alpha(t_i) < 10^{-4} \frac{100}{0.3 \times 938 \times 10^6} \frac{0.3}{5.06 \times 10^{-5}(1+z_{ev})} \frac{m_i(t_{ev},l)}{m_{ev}(t_{ev},l)} \sqrt[4]{\frac{t_i(t_{ev},l)\cdot t_c}{t_{eq}^2}} \left(\frac{t_{eq}}{t_{ev}}\right)^{\frac{2}{3}}$$

P.D.Nael'skii, Pis'ma Astron. Zh. ***4****, 387 (1978) [Sov. Astron. Lett.* ***4****, 209 (1979)].*

Constraints on initial PBH mass fraction by **H ionization**

Constraints imposed by ionization at z = 1100 and 20



Constraints on initial PBH mass fraction α_i

at $l = l_{min}$, $l = l_{max}$ and maximum constraints at $l = l^*$

Source of constraint	*Logarithms of*					
	$\alpha_i(l_{max})$	l_{min}/l_4	$\alpha_i(l_{min})$	l^*/l_4	$\alpha_i(l^*)$	α_i *in 4D*
Total mass density	-30	20	-18	30	-31	-18
Diffuse photon background	-39	20	-27	30	**-39**	-27
Antiproton excess	-35	20	-28	30	-36	-29
D destruction at 400 sec	-27	15	-20	20	-28	-18
D destruction at 10^8 sec	-28	16	-20	24	-29	-19
D destruction at 10^{13} sec	-30	18	-19	27	-30	19
He destruction	-27	**14**	-17	19	-25	-17
SZ effect at $z = 2 \cdot 10^6$	-30	16	-21	24	-31	-21
SZ effect at $z = 1100$	-34	18	-22	27	-34	-21
H Ionization at $z = 1100$	-41	18	-29	27	**-41**	-28
H Ionization at $z = 20$	-41	19	-29	28	**-42**	-28

Unexampled constraints on PBH initial mass fraction exist In braneworld RS2 scenario

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Thanks for you attention