Duality and Fluxes in String Compactifications

Andrei Micu

Physikalisches Institut der Universität Bonn

Based on `hep-th/0608171`
in collaboration with Jan Louis

Fourth Aegean Summer School - Mytilene 17–22 September 2007

Introduction

String theory – 10 dimensions vs 4 dimensional physics

Possible solution: compactify 6 dimensions on a manifold K.

General aspects of 4d physics determined by K

Example: unbroken 4d susy – need globaly defined spinor on K – simplest case $\rightarrow$ manifold with $SU(3)$ holonomy = Calabi–Yau manifold.

Topology – determines low energy field content

Given topology – CY manifolds come in families – parameterised by moduli

- small deformations of the CY metric
- massless scalar fields in 4d (flat directions)
- determine 4d couplings $\Rightarrow$ need to be fixed

Moduli stabilisation $\Longleftrightarrow$ Fluxes

Background value for field strengths F_p – harmonic on K (eom)

$$\int_{\gamma^\alpha} F_p = m^\alpha \quad \Leftrightarrow \quad F_p = m^\alpha \omega_\alpha \ ,$$

$\Rightarrow$ generate potentials for moduli

Flux + Dualities

- Mirror symmetry/T-duality smoothly relates RR fluxes in type II theories.
- Type IIB S-duality maps NS-NS into RR fluxes
- Mirror symmetry/T-duality of H flux $\longrightarrow$ deformation of the geometry $\longrightarrow$ manifolds with $SU(3) \times SU(3)$ structure
- Heterotic/type IIA duality takes gauge field fluxes to manifolds with $SU(3) \times SU(3)$ structure

$N = 2$ gauged supergravity

Multiplets

- gravity - graviton $g_{\mu\nu}$ and graviphoton A^0_μ
- vector multiplets - vector fields A^i_μ and complex scalar field X^i
- hypermultiplets - 4 real scalar fields ξ^A

Very constrained

Potential allowed only together with scalar manifold isometries gauging (charged scalar fields)

Fluxes in type II compactifications

Hodge diamond of Calabi–Yau

$$\begin{array}{ccccccc} & & & 1 & & & \\ & & 0 & & 0 & & \\ & 0 & & h^{(1,1)} & & 0 & \\ 1 & & h^{(2,1)} & & h^{(2,1)} & & 1 \\ & 0 & & h^{(1,1)} & & 0 & \\ & & 0 & & 0 & & \\ & & & 1 & & & \end{array}$$

$h^{(1,1)} = \dim H^{(1,1)}(Y)$

$h^{(2,1)} = \dim H^{(2,1)}(Y)$

left $\leftrightarrow$ right and up $\leftrightarrow$ down symmetric consequence of complex conjugation and Hodge duality.
Mirror symmetry exchanges $h^{(1,1)}$ and $h^{(2,1)}$.

Type IIA/CY$_3$: $N = 2$ sugra coupled to $h^{(1,1)}$ (Abelian) vector multiplets and $h^{(2,1)} + 1$ hypermultiplets.

Type IIB/CY$_3$: $N = 2$ sugra coupled to $h^{(2,1)}$ (Abelian) vector multiplets and $h^{(1,1)} + 1$ hypermultiplets.

Spectrum is invariant under mirror symmetry!

Flux	IIA	IIB
RR	$F_2 = m^i \omega_i; \quad F_0 = m_0$ $\underbrace{F_4 = e_i \tilde{\omega}^i; \quad F_6 = e_0 \mathcal{V}}_{2(h^{(1,1)}+1)}$	$F_1 = F_5 = 0$ $\underbrace{F_3 = p^A \alpha_A + q_A \beta^A}_{2(h^{(2,1)}+1)}$
NS-NS	$\underbrace{H_3 = \mu^A \alpha_A + \epsilon_A \beta^A}_{2(h^{(2,1)}+1)}$	$\underbrace{H_3 = \mu^A \alpha_A + \epsilon_A \beta^A}_{2(h^{(2,1)}+1)}$

Mirror symmetry $m^I \leftrightarrow p^A$ and $e_I \leftrightarrow q_A$.

What about ϵ_A and μ^A?

$\Rightarrow$ deformation of the geometry $\rightarrow$ manifolds with $SU(3) \times SU(3)$ structure.

Half-flat manifolds

Mirror symmetry: NS 3-form flux ($\mu = 0$) $\leftrightarrow$ half-flat manifold with $SU(3)$ structure $d\Omega \sim$ 4-form flux.

$SU(3)$ structure in 6 dimensions – invariant tensors: almost complex structure J and $(3,0)$ form Ω.

dJ and $d\Omega$ – intrinsic torsion

Half-flat (dual to NS flux):

$$\begin{aligned} d\omega_i &= \epsilon_i \beta^0 \\ d\alpha_0 &= \epsilon_i \tilde{\omega}^i \ , \quad d\alpha_a = d\beta^A = 0 \ , \end{aligned}$$

Special basis; breaks symplectic invariance!

Generalization

$$
\begin{aligned}
d\omega_i &= p_{iA}\beta^A - q_i^A\alpha_A \\
d\alpha_A &= p_{iA}\tilde{\omega}^i \\
d\beta^A &= q_i^A\tilde{\omega}^i
\end{aligned}
$$

Constraint (from $d^2\omega = 0$)

$$\langle(p_i, q_i); (p_j, q_j)\rangle = p_{iA}q_j^A - p_{jA}q_i^A = 0$$

Effect: p_{iA} and q_i^A charges for hyperscalars wrt all vector fields

Heterotic/$K3 \times T^2$ + flux

$N = 2$ sugra in 4d + SYM with gauge group G and $n_h \geq 20$ hypermultiplets

Crucial ingredient: Bianchi identity

$$dH = trR \wedge R - trF \wedge F$$

Take F_{inst} – solution $\rightarrow$ breaks gauge group to G

Coulomb branch: $G \rightarrow U(1)^{n_v}$, $\rightarrow n_v$ (Abelian) vectormultiplets

$$\int_{\gamma^\alpha} F^I_{flux} = m^{\alpha I} \quad \Leftrightarrow \quad F^I_{flux} = m^{\alpha I} \omega_\alpha \ ,$$

$m^{\alpha I}$ charges for hypermultiplets wrt all vector fields $\leftrightarrow$ mapped to q_I^A via heterotic/type IIA duality.

Conclusions

- RR fluxes respect mirror symmetry
- manifolds with $SU(3)(\times SU(3))$ structure – crucial for string dualities with NS-NS H-flux.
- Half-flat manifolds – dual to half of the NS-fluxes ($\mu = 0$)
- Full duality of H-flux involves manifolds with $SU(3) \times SU(3)$ structure (non-geometric fluxes)
- certain $SU(3) \times SU(3)$ structures – dual to heterotic gauge field fluxes