Gravitational lensing by rotating naked singularities for equatorial observer in the strong deflection limit

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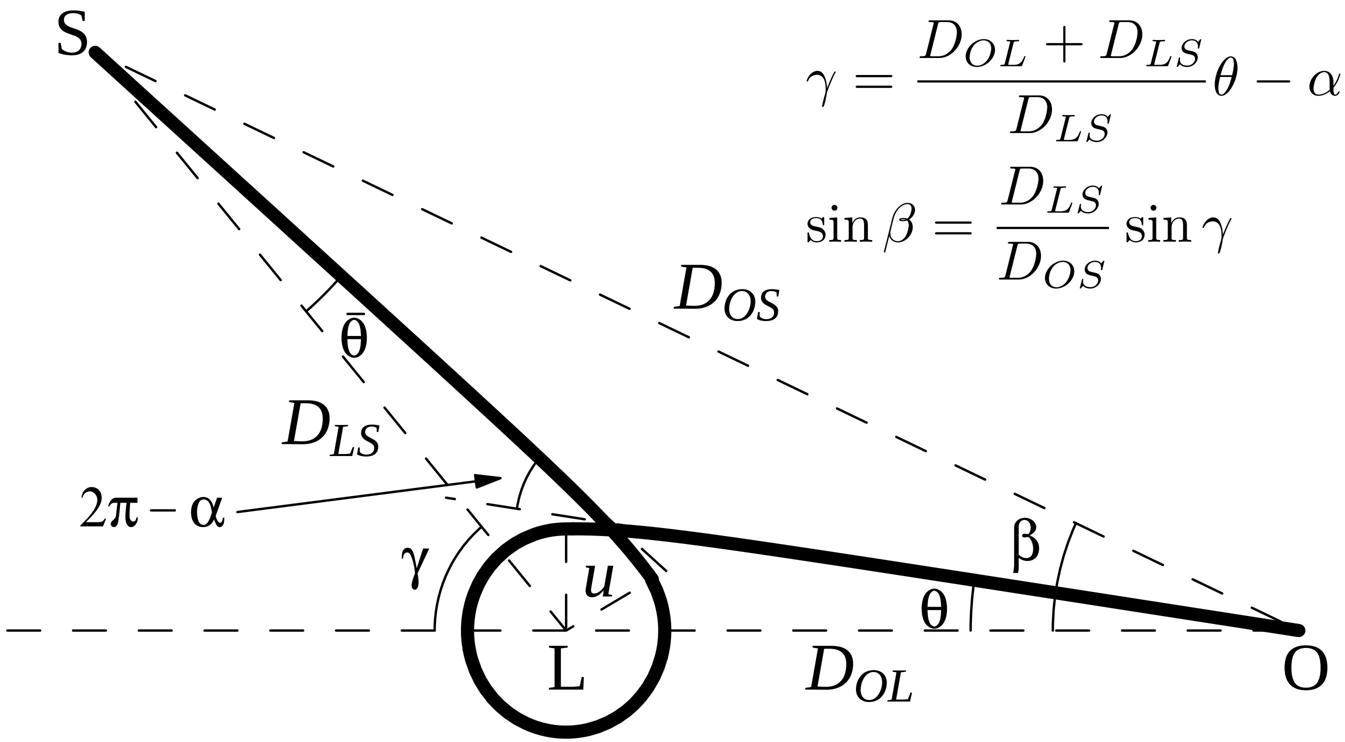
Concept for gravitational lensing in strong deflection limit

$$\gamma = \frac{D_{OL} + D_{LS}}{D_{LS}}\theta - \alpha(\theta) \quad \mod 2\pi$$

$$\sin\beta = \frac{D_{LS}}{D_{OS}}\sin\gamma$$



We consider a Kerr-like solution to the Einstein massless scalar equations ($R_{ij} = 8\pi\varphi_{,i}\varphi_{,j}$ with $\varphi_{,i}^{;i} = 0$, where R_{ij} is Ricci tensor and φ is the massless scalar field)

$$
\begin{aligned}
ds^2 &= \left(1 - \frac{2Mr}{\gamma\rho}\right)^{\gamma} (dt - wd\phi)^2 \\
&- \left(1 - \frac{2Mr}{\gamma\rho}\right)^{1-\gamma} \rho \left(\frac{dr^2}{\Delta} + d\theta^2 + \sin^2\theta d\phi^2\right) \\
&+ 2w(dt - wd\phi)d\phi,
\end{aligned}
$$

where

$$\gamma = \frac{M}{\sqrt{M^2 + q^2}}, \quad w = a\sin^2\theta, \quad \rho = r^2 + a^2\cos^2\theta,$$

$$\Delta = r^2 + a^2 - \frac{2Mr}{\gamma} \quad \textit{and} \quad \varphi = \frac{\sqrt{1-\gamma^2}}{4} \ln\left(1 - \frac{2Mr}{\gamma\rho}\right).$$

We consider a lensing in the equatorial plane. The reduce metric with condition $\theta = \frac{\pi}{2}$ is

$$ds^2 = -A(r)dt^2 + B(r)dr^2 + C(r)d\phi^2 - D(r)dtd\phi,$$

The Einstein deflection angle is

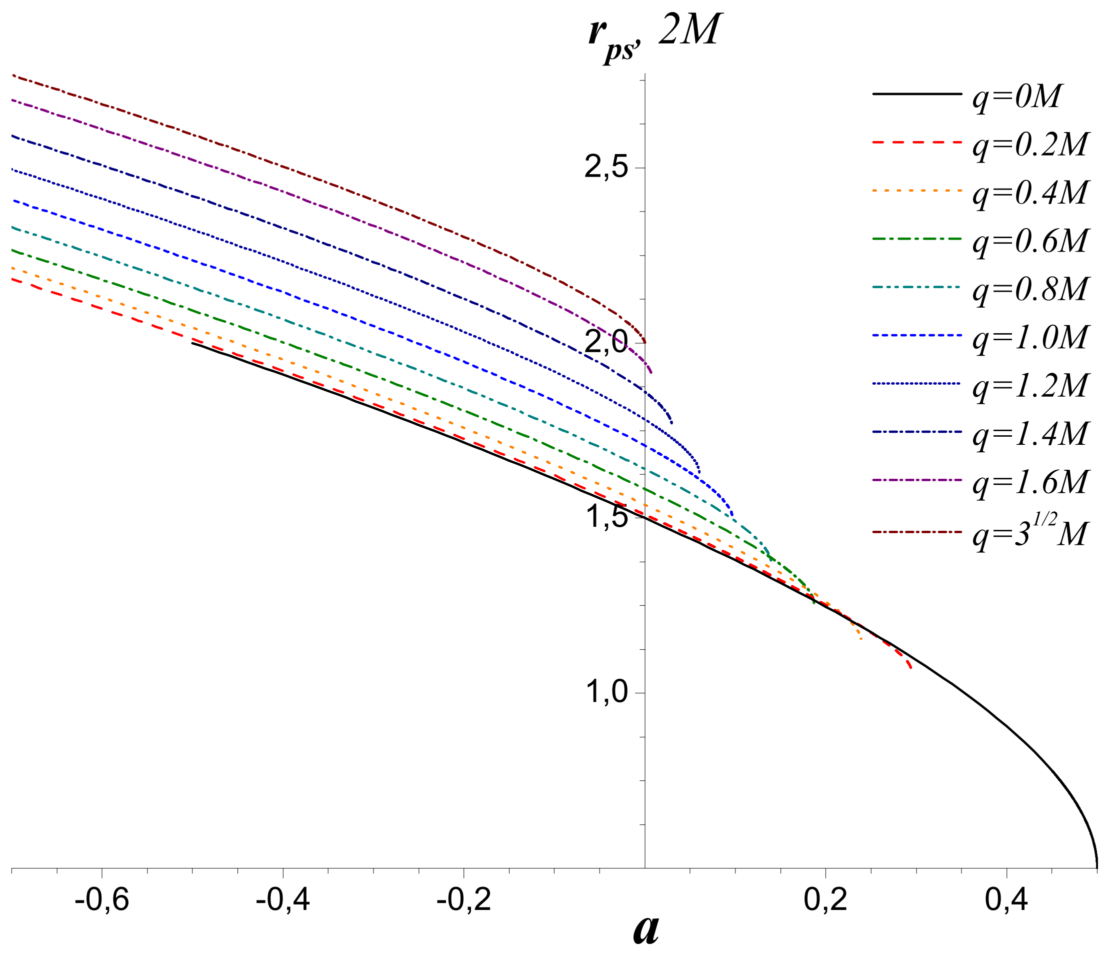
$$\alpha(x_0) = \phi_f(x_0) - \pi$$

$$\phi_f(x_0) = 2\int_{x_0}^{\infty} \frac{d\phi}{dx}dx$$

$$\frac{d\phi}{dx} = \pm \frac{\sqrt{B|A_0|}(D + 2JA)}{\sqrt{C}\sqrt{4AC + D^2}\sqrt{sgn(A_0)[A_0 - A\frac{C_0}{C} + \frac{J}{C}(AD_0 - A_0 D)]}}$$

where r_0 is the closest distance of approach of the light ray.

Photon sphere $\{r = r_{ps}\}$



In strong deflection limit the deflection angle is

$$\alpha(\theta) \quad = \quad -\bar{a}\ln\left(\frac{\theta D_{OL}}{J_{ps}}-1\right)+\bar{b}+O(J-J_{ps}),$$

where the coefficients of expansion are

$$J_{ps} = \frac{-D_{ps}+\sqrt{4A_{ps}C_{ps}+D_{ps}^2}}{2A_{ps}},$$

$$\bar{a} = \frac{R(0,x_{ps})}{2\sqrt{\beta_{ps}}},$$

$$\bar{b} = -\pi + b_R + \bar{a}\ln\left\{\frac{4\beta_{ps}C_{ps}}{J_{ps}|A_{ps}|(D_{ps}+2J_{ps}A_{ps})}\right\}$$

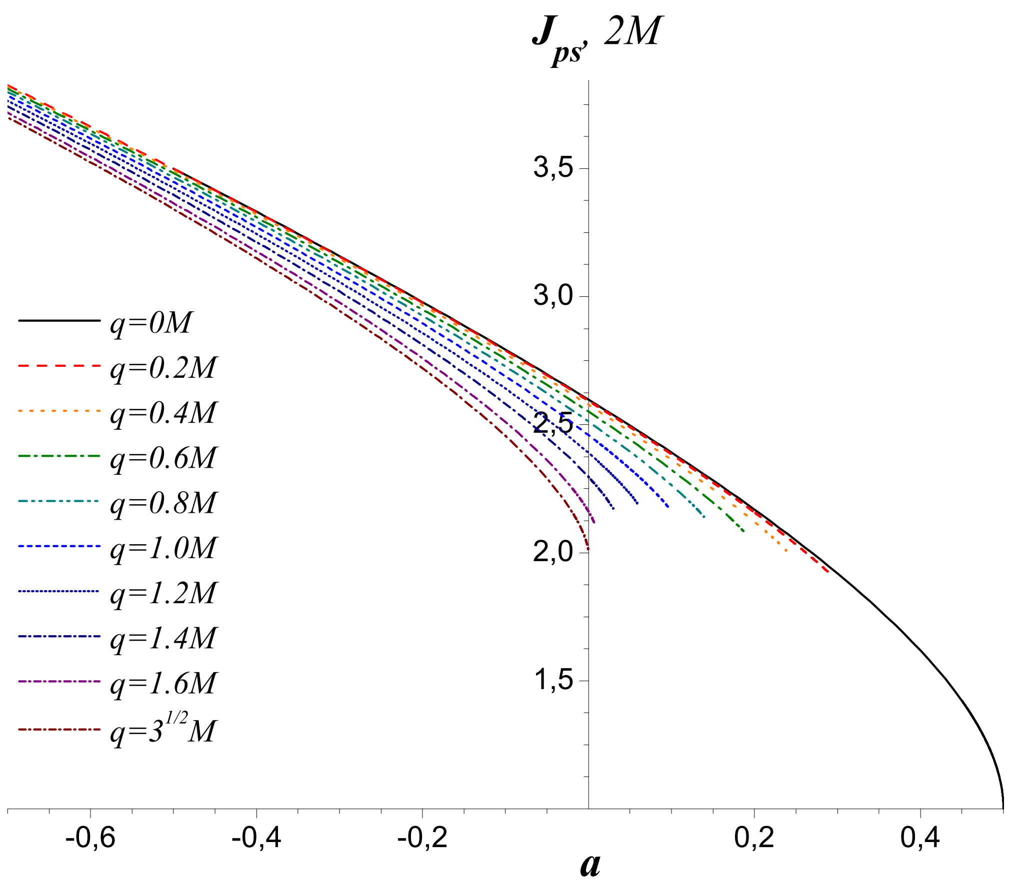


Figure 1: Strong deflection limit coefficient J_{ps} as a function of angular momentum a for scalar charge $q = 0M$ (from above) to scalar charge $q = \sqrt{3}M$ (below) by scalar charge $q = 0.2M$

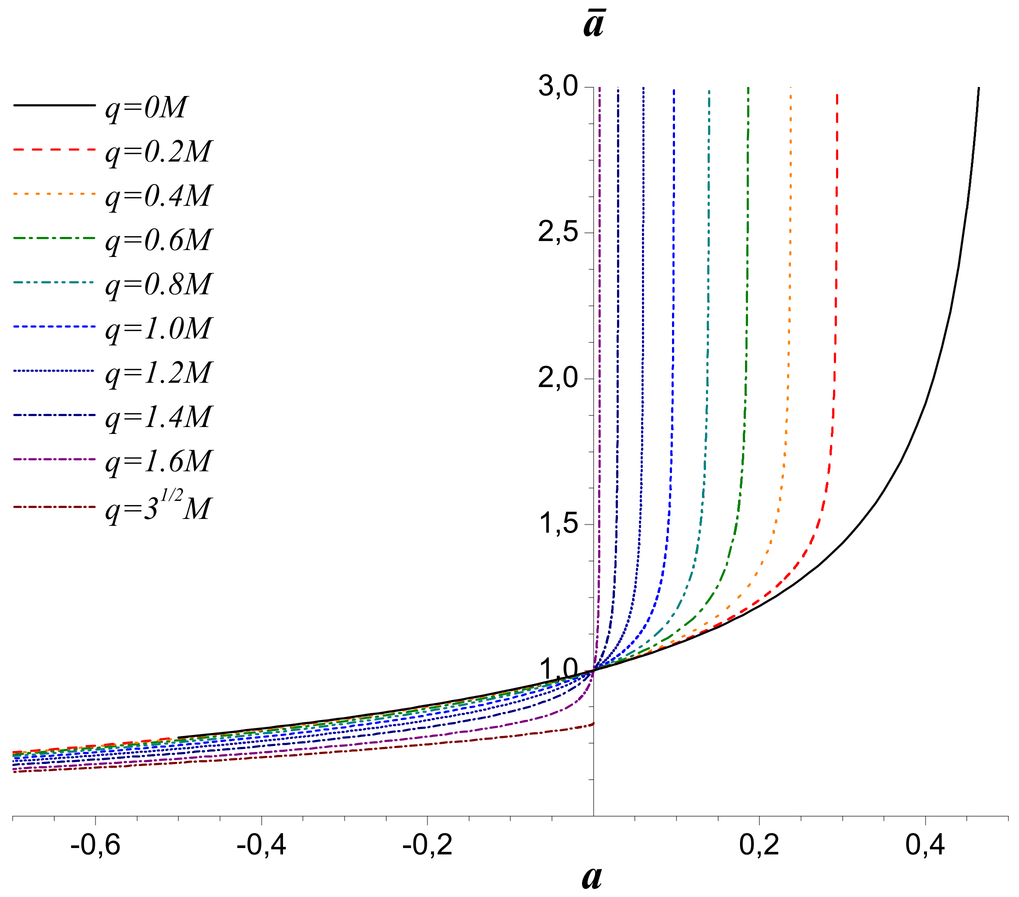


Figure 2: Strong deflection limit coefficient $\bar{a}$ as a function of angular momentum a for scalar charge $q = 0M$ to scalar charge $q = \sqrt{3}M$ by scalar charge $q = 0.2M$

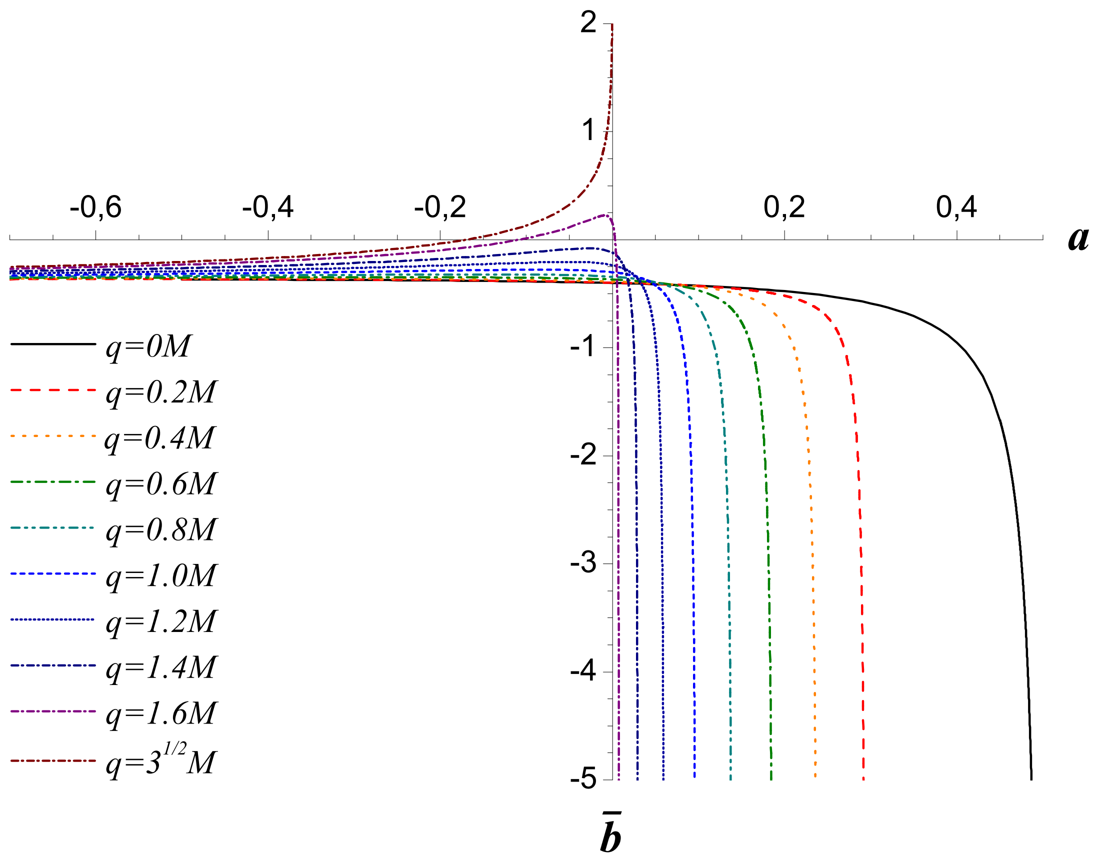


Figure 3: Strong deflection limit coefficient $\bar{b}$ as a function of angular momentum a for scalar charge $q = 0M$ to scalar charge $q = \sqrt{3}M$ by scalar charge $q = 0.2M$

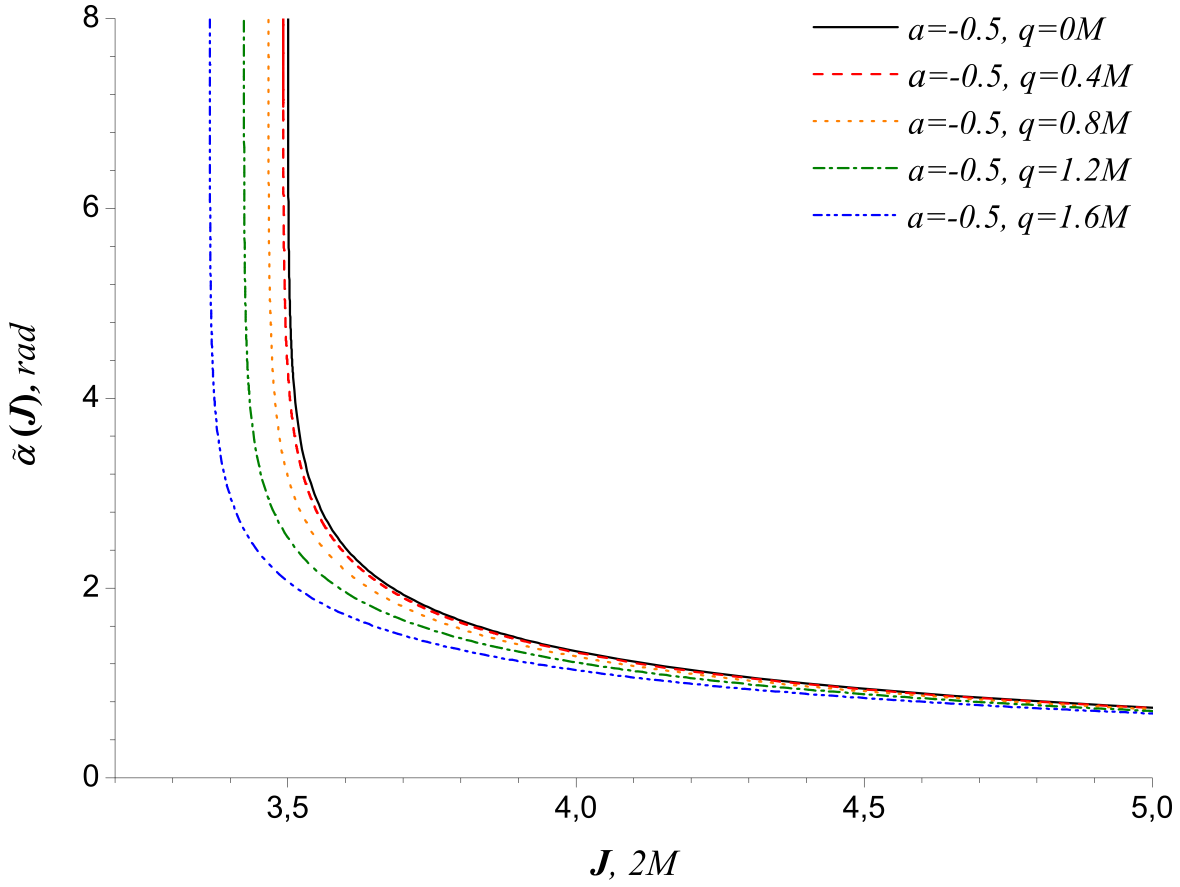


Figure 4: Deflection angle $\alpha(J)$ of the light ray as a function of impact parameter J for angular momentum $a = -0.5$ and scalar charge $q = 0M$ to scalar charge $q = 1.6M$ by scalar charge $q = 0.4M$

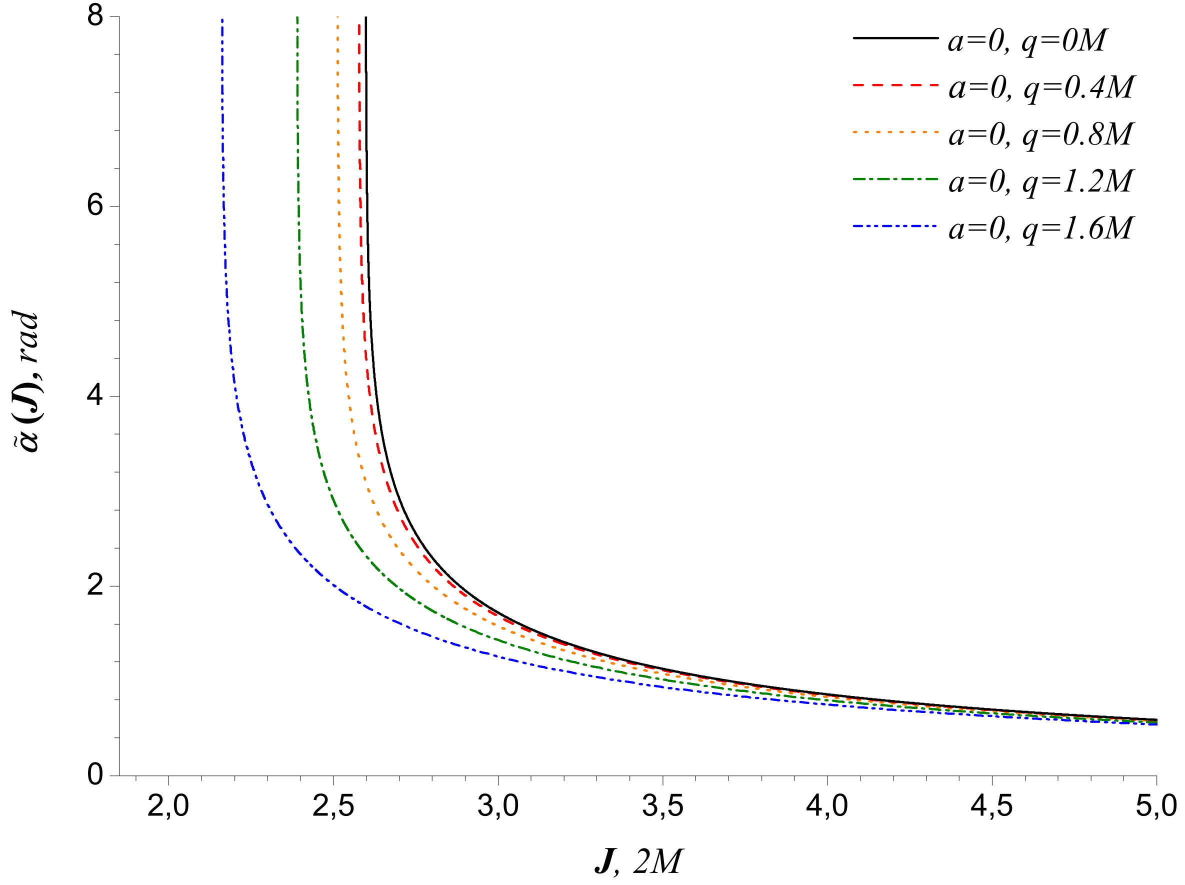


Figure 5: Deflection angle $\alpha(J)$ of the light ray as a function of impact parameter J for angular momentum $a = 0$ and scalar charge $q = 0M$ to scalar charge $q = 1.6M$ by scalar charge $q = 0.4M$

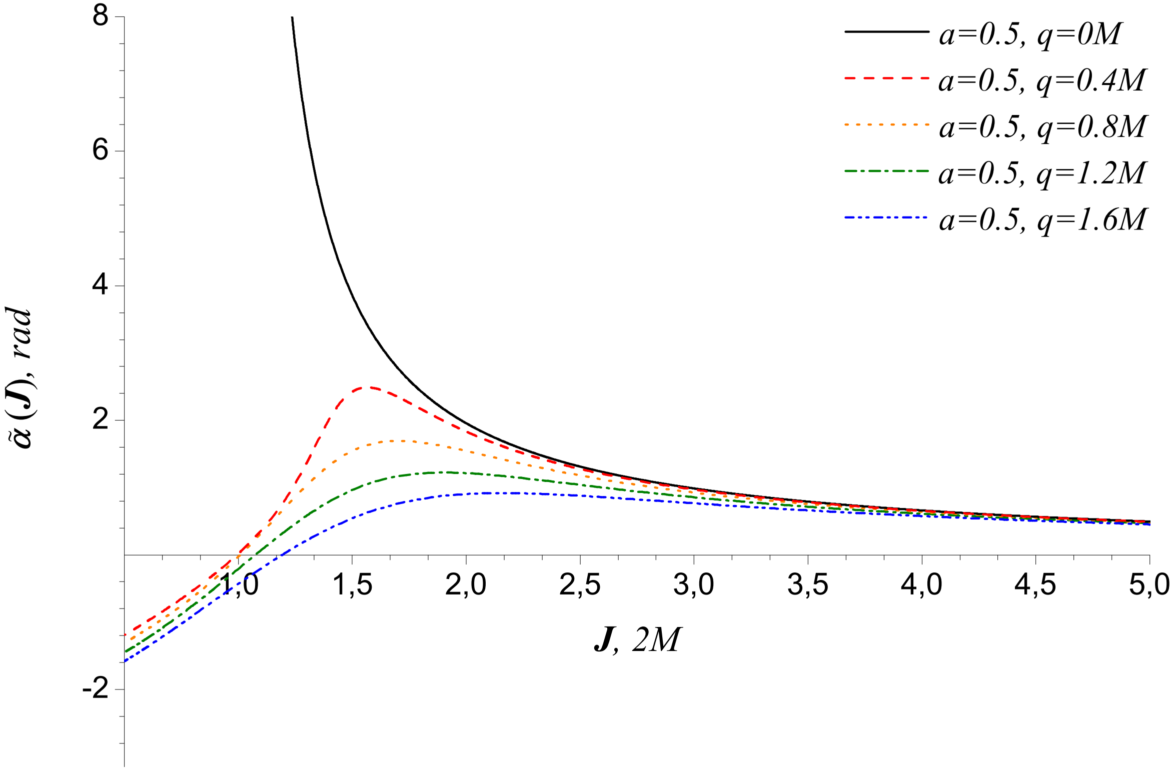


Figure 6: Deflection angle $\alpha(J)$ of the light ray as a function of impact parameter J for angular momentum $a = 0.5$ and scalar charge $q = 0M$ to scalar charge $q = 1.6M$ by scalar charge $q = 0.4M$

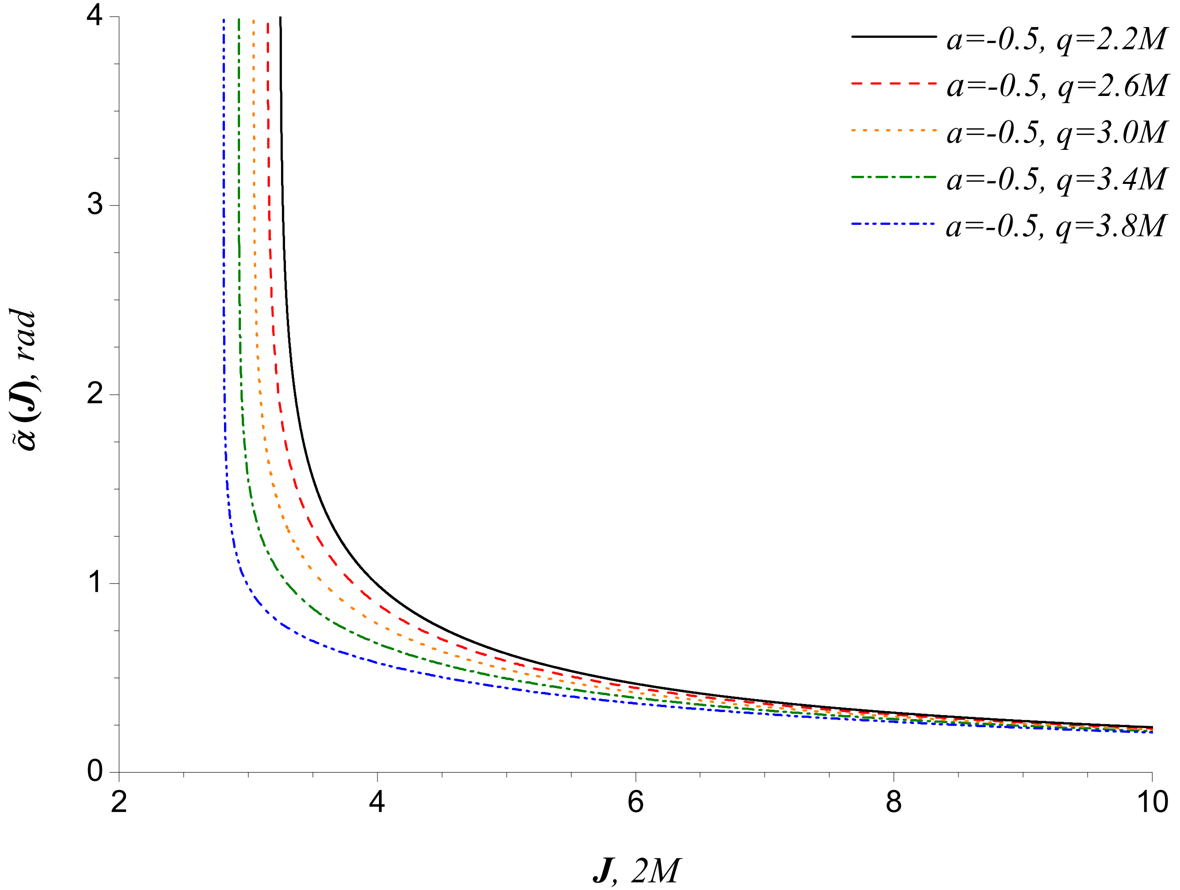


Figure 7: Deflection angle $\alpha(J)$ of the light ray as a function of impact parameter J for angular momentum $a = -0.5$ and scalar charge $q = 2.2M$ to scalar charge $q = 3.8M$ by scalar charge $q = 0.4M$

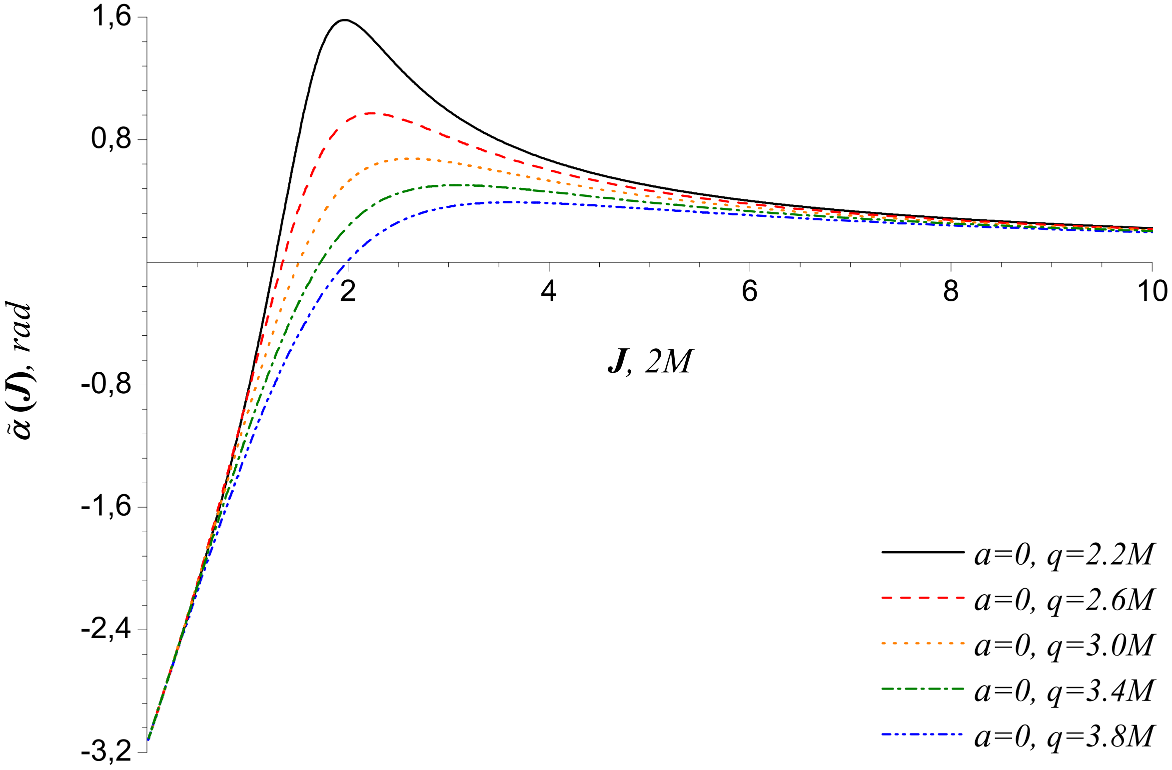


Figure 8: Deflection angle $\alpha(J)$ of the light ray as a function of impact parameter J for angular momentum $a = 0$ and scalar charge $q = 2.2M$ to scalar charge $q = 3.8M$ by scalar charge $q = 0.4M$

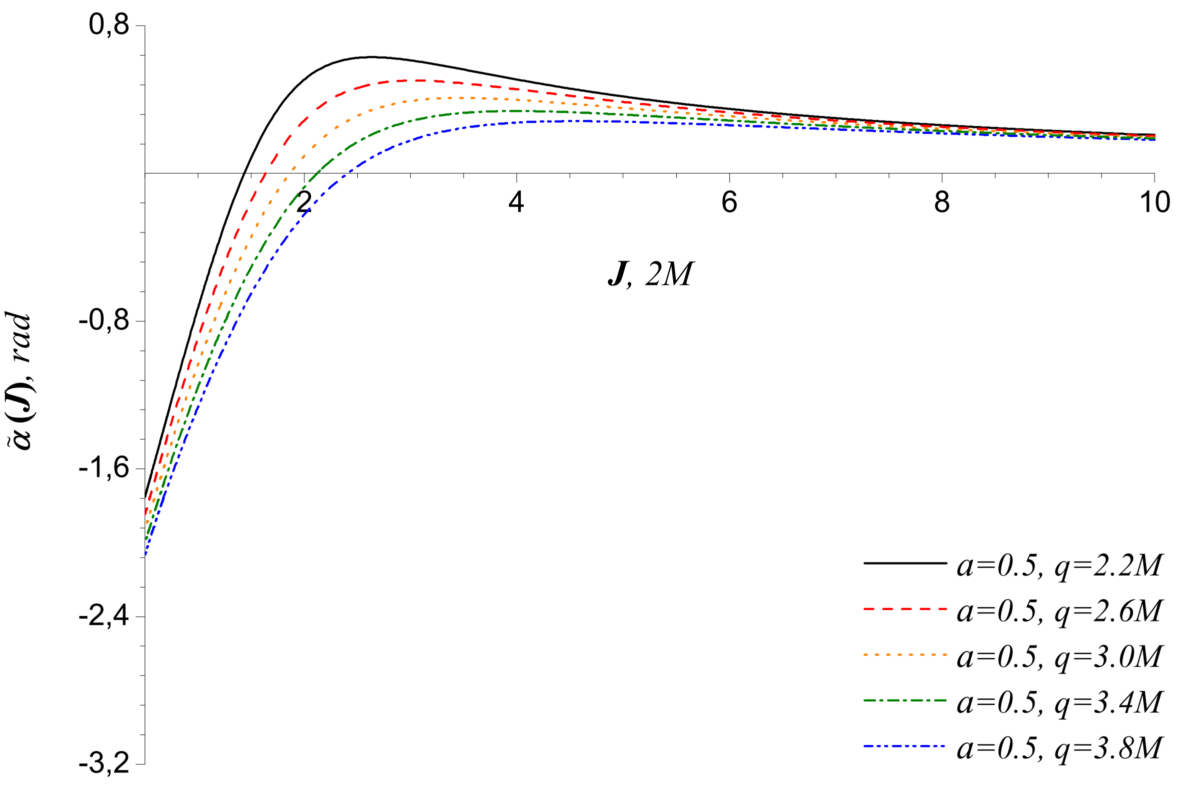


Figure 9: Deflection angle $\alpha(J)$ of the light ray as a function of impact parameter J for angular momentum $a = 0.5$ and scalar charge $q = 2.2M$ to scalar charge $q = 3.8M$ by scalar charge $q = 0.4M$

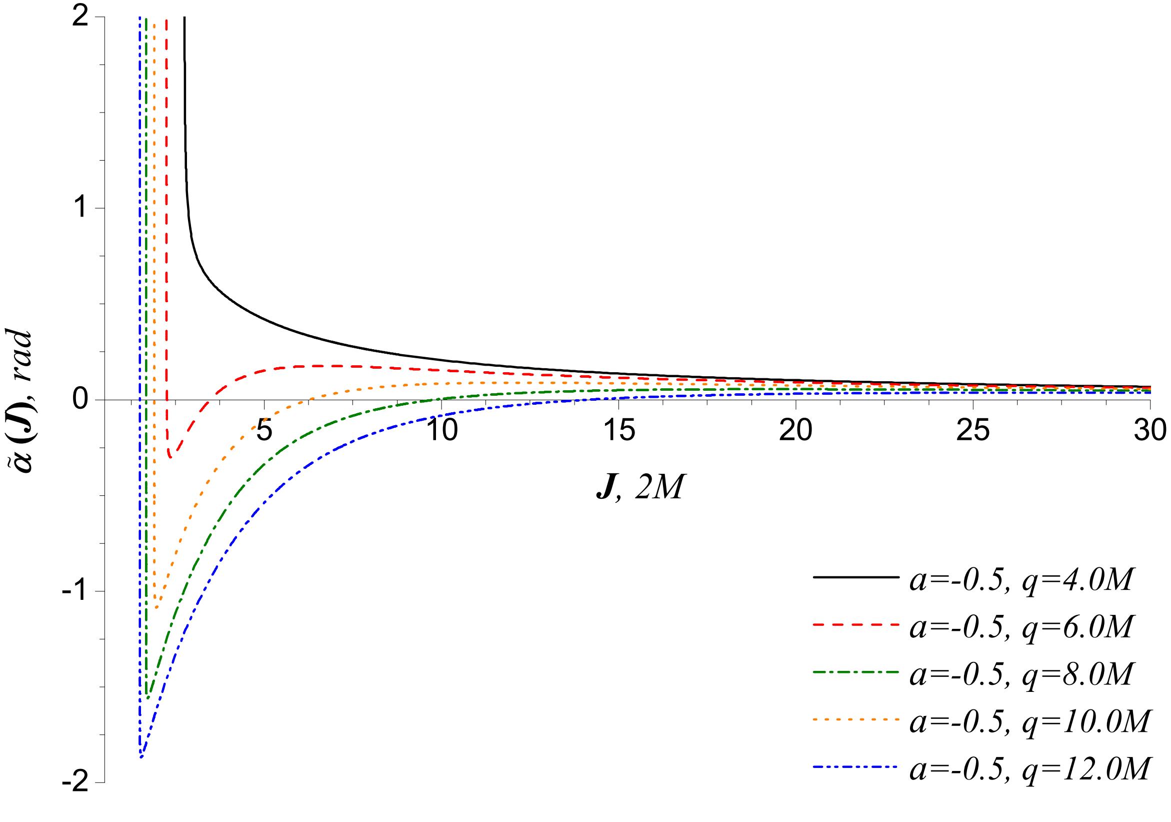


Figure 10: Deflection angle $\alpha(J)$ of the light ray as a function of impact parameter J for angular momentum $a = -0.5$ and scalar charge $q = 4.0M$ to scalar charge $q = 12.0M$ by scalar charge $q = 2.0M$

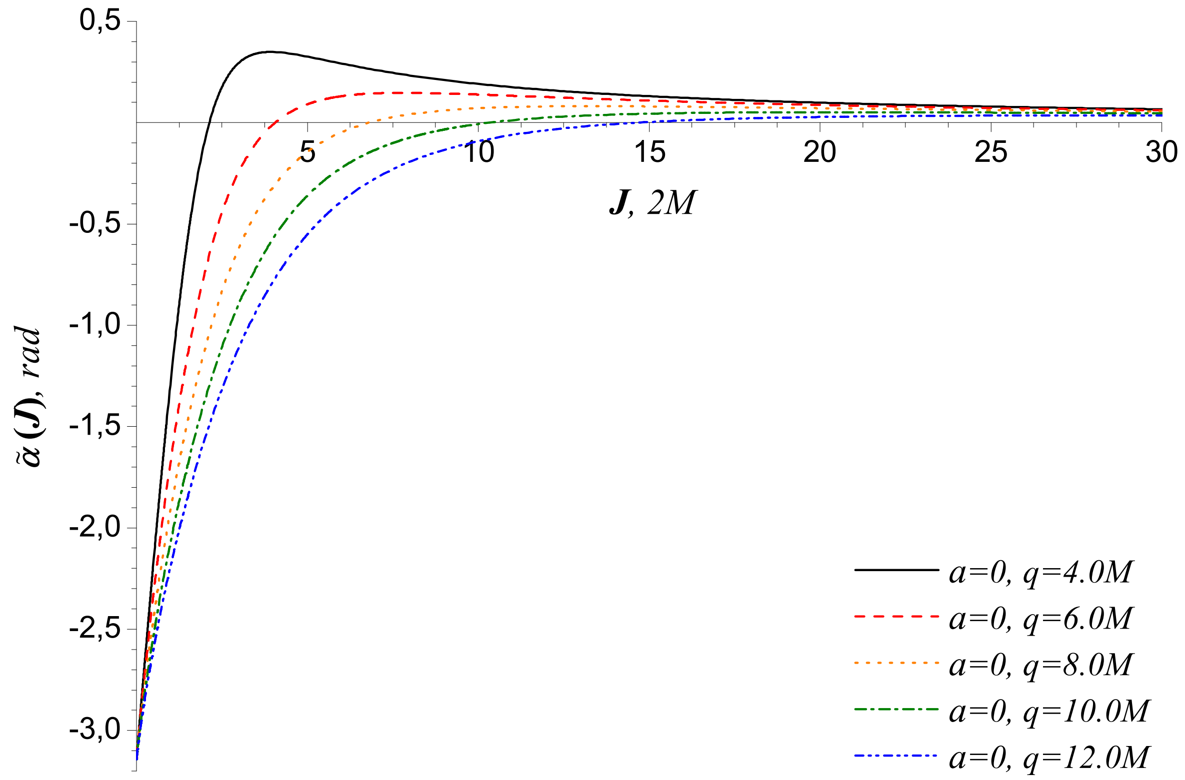


Figure 11: Deflection angle $\alpha(J)$ of the light ray as a function of impact parameter J for angular momentum $a = 0$ and scalar charge $q = 4.0M$ to scalar charge $q = 12.0M$ by scalar charge $q = 2.0M$

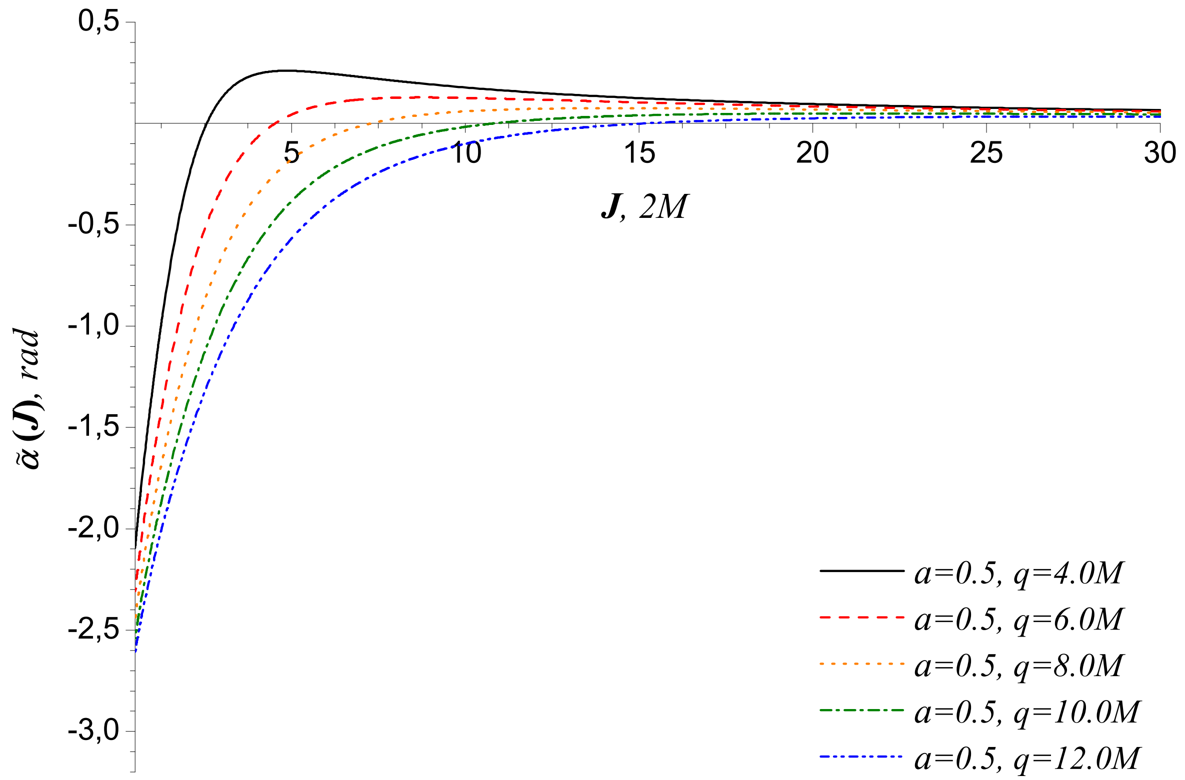


Figure 12: Deflection angle $\alpha(J)$ of the light ray as a function of impact parameter J for angular momentum $a = 0.5$ and scalar charge $q = 4.0M$ to scalar charge $q = 12.0M$ by scalar charge $q = 2.0M$

Observables

To obtain the coefficients $\bar{a}$ and $\bar{b}$, we separate the outermost image θ_1 from all the others $\theta_n|_{n\to\infty} \equiv \theta_\infty$ (innermost image) .

Our observables will thus be

$$s = \theta_1 - \theta_\infty = \theta_\infty e^{(\bar{b}-2\pi)/\bar{a}}, \qquad \tilde{r} = \frac{\mu_1}{\sum_{n=2}^{\infty} \mu_n} = e^{2\pi/\bar{a}}.$$

$$\bar{a} = \frac{2\pi}{\ln(\tilde{r})}, \qquad \bar{b} = \bar{a} \ln\left(\frac{rs}{\theta_\infty}\right).$$

$$J_{ps} = D_{OL}\theta_\infty.$$

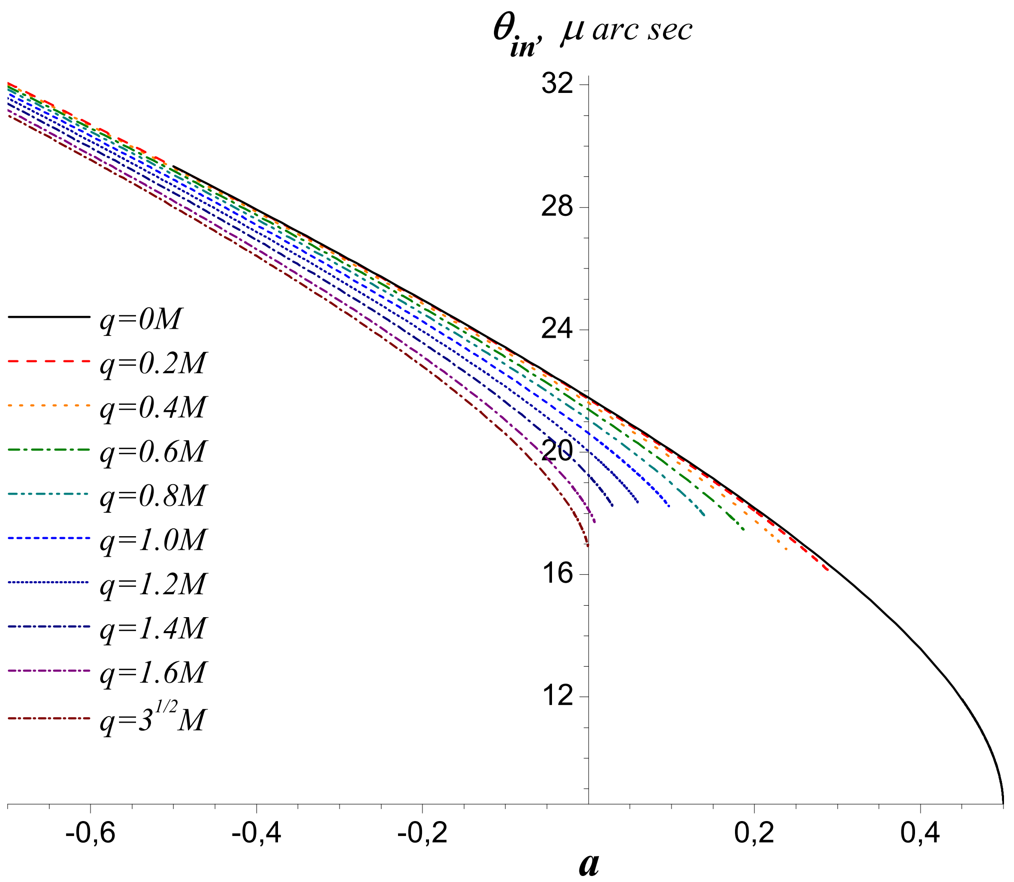


Figure 13: The inner relativistic images θ_∞ as a function of angular momentum a for scalar charge $q = 0M$ (from above) to scalar charge $q = \sqrt{3}M$ (below) by scalar charge $q = 0.2M$

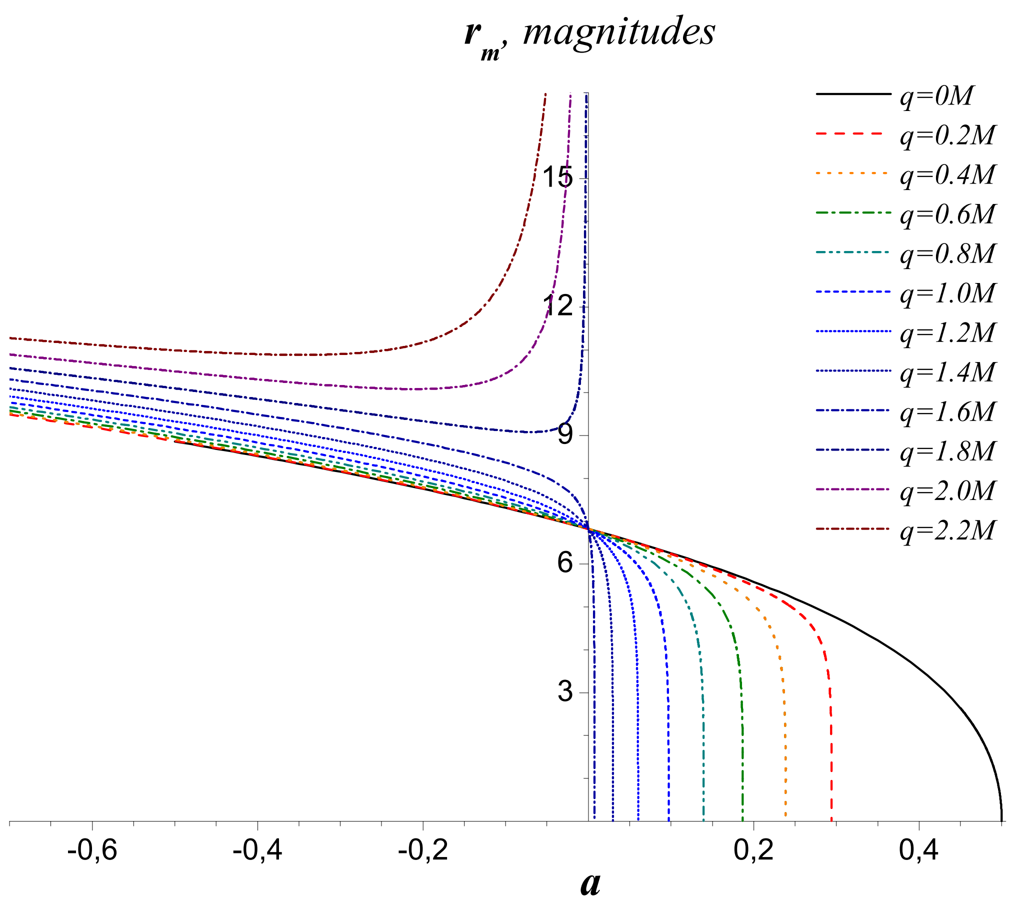


Figure 14: The ratio $\tilde{r}$ between the flux of the outermost and the flux of the all the others images for scalar charge $q = 0M$ to scalar charge $q = 2.2M$ by scalar charge $q = 0.2M$