(1) Show that for $r < 2M$

$$ds^2 = \frac{32M^3}{r} e^{-r/2M} \left[-dV^2 + dU^2 \right] + r^2 d\Omega^2$$

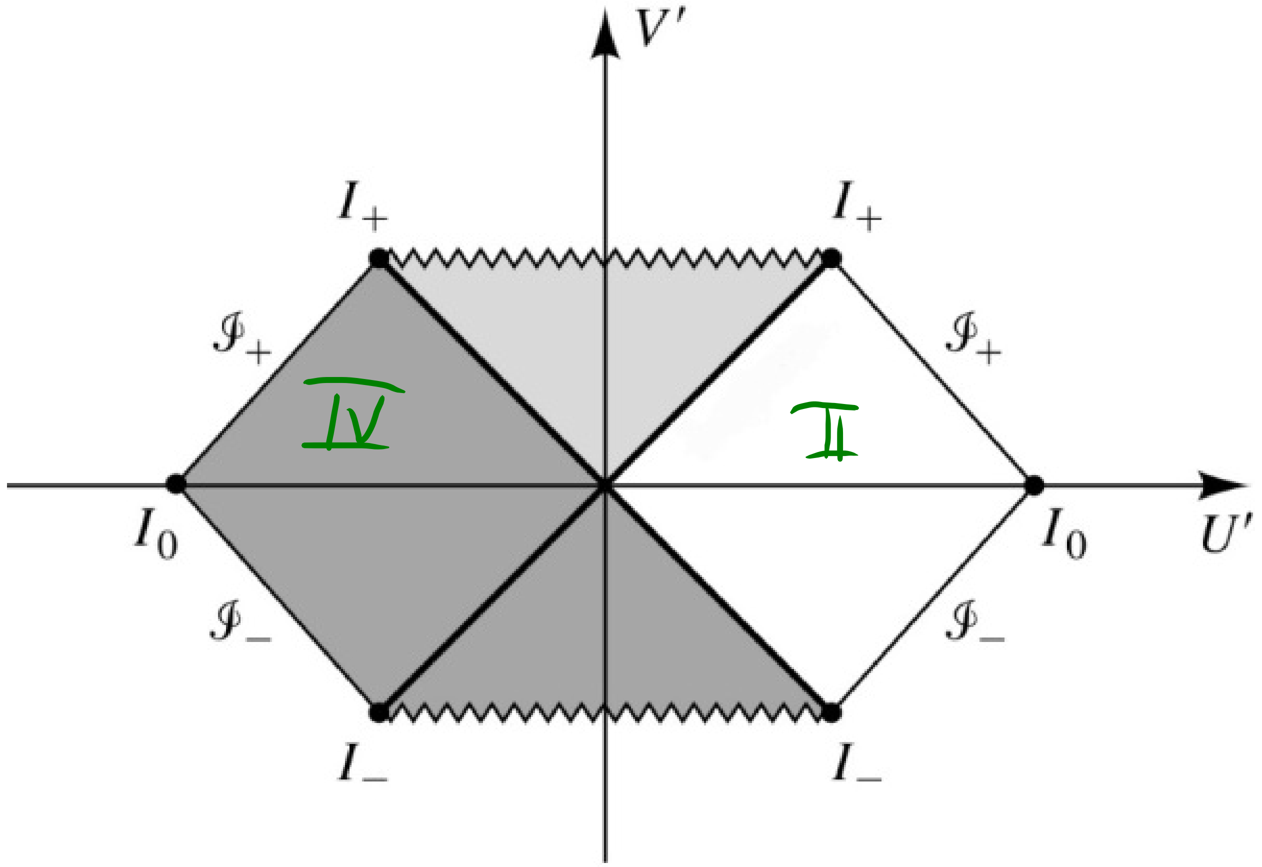
(2) Starting from $ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2 d\Omega^2$, show that

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dv^2 + 2\, dv\, dr + r^2 d\Omega^2$$

$$= -\left(1 - \frac{2M}{r}\right) d\tilde{t}^2 + \left(1 + \frac{2M}{r}\right) dr^2 + \frac{4M}{r} d\tilde{t}\, dr + r^2 d\Omega^2$$

(3) Explain why in a Kruskal diagram, a massive particle escaping from an event in region II to infinity on a radial trajectory, must follow a curve, which on the (U,V) diagram asymptotes the $U=V$ line.

(4) Show that all massive particles moving on $d\theta = d\varphi = 0$ worldlines from $t = -\infty$ to $t = +\infty$ in region II, must follow worldlines that start from I_- and end on I_+



(5) Show that all photons moving on $d\theta = d\varphi = 0$ trajectories coming from infinity into region IV and then escaping back to infinity, move on world lines that start from $\mathcal{I}_-$ and end on $\mathcal{I}_+$ of region IV

(6) 13. [C] (a) An observer falls feet first into a Schwarzschild black hole looking down at her feet. Is there ever a moment when she cannot see her feet? For instance, can she see her feet when her head is crossing the horizon? If so, what radius does she see them at? Does she see her feet hit the singularity at $r = 0$ assuming she remains intact until her head reaches that radius? Analyze these questions with an Eddington–Finkelstein or Kruskal diagram.

(b) *Is it dark inside a black hole?* An observer outside sees a star collapsing to a black hole become dark. But would it be dark inside a black hole assuming a collapsing star continues to radiate at a steady rate as measured by an observer on its surface?

Hartle 12.13