① Use $\nabla_\mu X^\nu = \partial_\mu X^\nu + \Gamma^\nu{}_{\mu\rho} X^\rho$, $\nabla_\mu \omega_\nu = \partial_\mu \omega_\nu - \Gamma^\rho{}_{\mu\nu} \omega_\rho$ to show that

$$\nabla_\mu g_{\nu\rho} = \partial_\mu g_{\nu\rho} - C^\sigma_{\mu\nu} g_{\sigma\rho} - C^\sigma{}_{\mu\rho}\, g_{\nu\sigma}$$

② Using the properties of ∇_μ, show that the directional derivative has the properties:

$$D_{fV+gU} W^\mu = f D_V W^\mu + g D_U W^\mu$$

$$(D_V D_W - D_W D_V) f = (V^\nu \partial_\nu W^\mu - W^\nu \partial_\nu V^\mu)\, \partial_\mu f$$

③ Show that if $T^\mu{}_{\nu\rho}$ is parallel transported along an integral curve of a vector field V^μ, then

(α) if $T^\mu{}_{\nu\rho}(P)$ is known, then there is a unique parallel transported

$\tau_{tot}\, T^\mu{}_{\nu\rho}$ at each point of the curve

(β) If $F_{\mu\nu}$ and ω_μ are parallel transported, then $F_{\mu\nu} \omega^\mu \omega^\nu$ is constant along the curve

④ Calculate $\Gamma^{\mu}{}_{\nu\rho}$ for the Levi-Civita connections of the metric

$$ds^2 = d\theta^2 + \sin^2\theta \, d\varphi^2$$

$$ds^2 = -\rho^2 d\eta^2 + d\rho^2$$

$$ds^2 = -\cos\lambda \, dt^2 - \sin\lambda \left[dt\,dx + dx\,dt \right] + \cos\lambda \, dx^2 \quad , \quad t = \cot\lambda$$

Then, write down the equations that determine the parallel transport of a vector W^{μ}, along a curve with tangent vector $V^{\mu} = \frac{dx^{\mu}}{dt}$