① Consider the sphere $x^2+y^2+z^2=1$, and the coordinate systems (θ,φ) and (u,v) defined in Lecture 01.

(α) If $\omega = d\theta + \sin\varphi \, d\varphi$, compute its components in the (u,v) system

$\sigma = \dfrac{dv}{1-v}$, " " " (θ,φ) "

(β) Consider the functions $S^2 \to \mathbb{R}$ defined by

$$f(x,y,z) = x \cdot y \qquad g(x,y,z) = x e^{-z}$$

Compute df and dg in the two coordinate systems. Show that their components are related by the known transformation rule for 1-forms

(γ) Compute du and dv as linear combinations of $d\theta$ and $d\varphi$

② Show that $T^{(k,\ell)}_{p}M$ is $n^{k+\ell}$-dimensional

③ If S, T, F are $(k_1,\ell_1), (k_2,\ell_2), (k_3,\ell_3)$ tensors, then show that

(α) $S \otimes T$ is a tensor (a linear function of its arguments)

(β) $(\alpha S + \beta F) \otimes T = \alpha S \otimes T + \beta F \otimes T \qquad \alpha, \beta \in \mathbb{R}$

(γ) compute the components of $S \otimes T$ in the $\{\partial_\mu\}, \{dx^\mu\}$ bases

④ If S is a $(3,2)$ tensor, compute the transformation of its components when $\{\partial_\mu, dx^\mu\} \to \{\partial_{\mu'}, dx^{\mu'}\}$

⑤ Compute the components of $A_{[\mu|\nu|\rho]}$, $S_{[\alpha\beta|\gamma|\delta]}$, $T^{(\mu\nu)\rho}{}_{(\alpha|\beta|\gamma)}$ in terms of $A_{\mu\nu\rho}$, $S_{\alpha\beta\gamma\delta}$, $T^{\mu\nu\rho}{}_{\alpha\beta\gamma}$ respectively