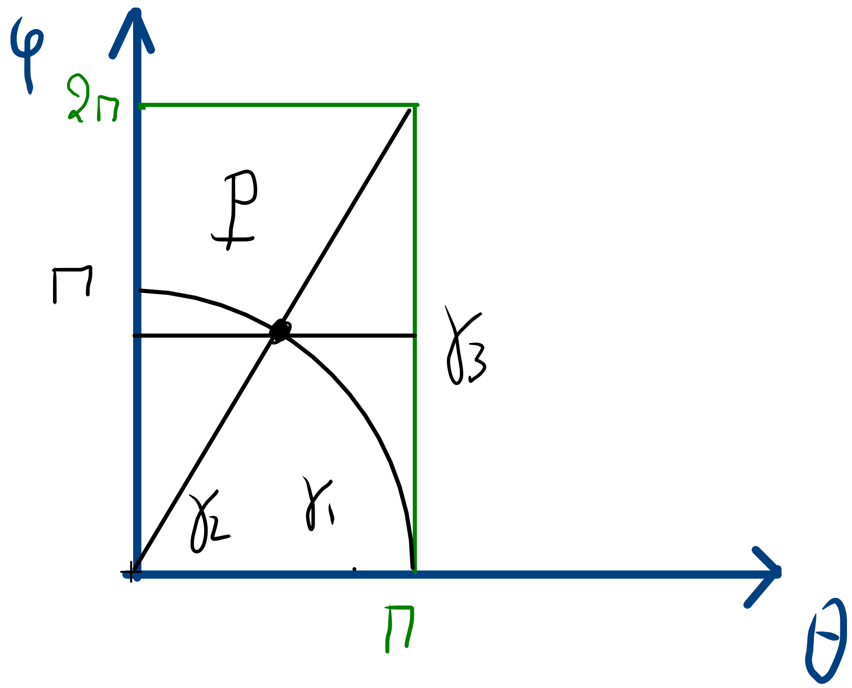

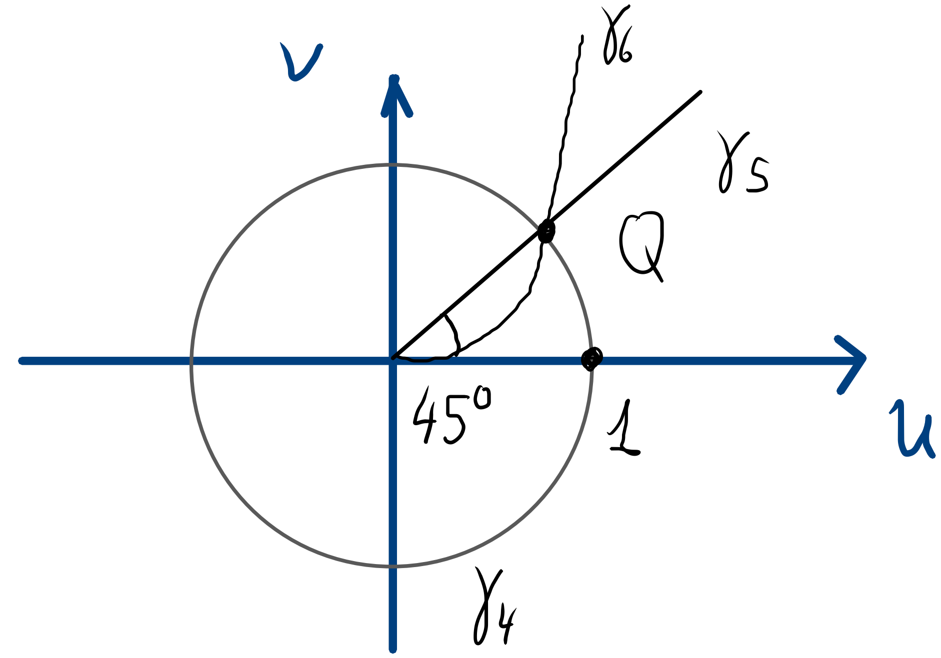

① Consider the sphere $x^2+y^2+z^2=1$ and the coordinate systems (θ,φ) and (u,v) defined in the previous lecture.

(α) Write the equations $x^\mu(t)$ for the curves $\gamma_1,\dots,\gamma_6$ in the coordinate systems that they are defined. Use a parameter t of your choice

(β) Compute the components $V^\mu_{(i)}$ of the vectors of the curves γ_i at P or Q in the defining coordinate systems

(γ) Compute the components of $V_{(6)}$ at Q in the (θ,φ) and (s,t) coordinate systems

(δ) Compute $V_{(i)}(f)$, where $f: S^2 \to \mathbb{R}$ s.t.

$$f(x,y,z) = (x^2+y^2)\cos z$$

2 Consider $\mathbb{R}^3$ and the coordinate systems

$$(x^\mu) = (x,y,z) \qquad (x^{\mu'}) = (\chi,\theta,\varphi), \text{ where}$$

$$x = \sinh\chi \sin\theta \cos\varphi$$

$$y = \sinh\chi \sin\theta \sin\varphi$$

$$z = \cosh\chi \cos\theta$$

(α) Compute the transformation matrix $\left(\frac{\partial x^{\mu}}{\partial x^{\mu'}} \right)$

(β) Consider the coordinate basis vector fields

$$e_x = \frac{\partial}{\partial x} \qquad e_\theta = \frac{\partial}{\partial \theta} \qquad e_\varphi = \frac{\partial}{\partial \varphi}$$

Compute their components in the x^{μ} coordinate system