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The effects of SUSY on the emergent spacetime in the Lorentzian type IIB matrix model

Mitsuaki Hirasawa,[a,b,*] **Konstantinos N. Anagnostopoulos,**[c] **Takehiro Azuma,**[d] **Kohta Hatakeyama,**[e] **Jun Nishimura,**[f,g] **Stratos Papadoudis**[c] **and Asato Tsuchiya**[h]

[a] *Department of Physics "Giuseppe Occhialini", University of Milano-Bicocca, Piazza della Scienza, 3, I-20126 Milano, Italy*
[b] *Sezione di Milano Bicocca, Istituto Nazionale di Fisica Nucleare, Piazza della Scienza, 3, I-20126 Milano, Italy*
[c] *Physics Department, School of Applied Mathematical and Physical Sciences, National Technical University, Zografou Campus, GR-15780 Athens, Greece*
[d] *Institute for Fundamental Sciences, Setsunan University, 17-8 Ikeda Nakamachi, Neyagawa, Osaka, 572-8508, Japan*
[e] *Department of Physics, Kyoto University, Kyoto 606-8502, Japan*
[f] *KEK Theory Center, High Energy Accelerator Research Organization, 1-1 Oho, Tsukuba, Ibaraki 305-0801, Japan*
[g] *Graduate Institute for Advanced Studies, SOKENDAI, 1-1 Oho, Tsukuba, Ibaraki 305-0801, Japan*
[h] *Department of Physics, Shizuoka University, 836 Ohya, Suruga-ku, Shizuoka 422-8529, Japan*

E-mail: mitsuaki.hirasawa@mib.infn.it, konstant@mail.ntua.gr, azuma@mpg.setsunan.ac.jp, kohta.hatakeyama@gauge.scphys.kyoto-u.ac.jp, jnishi@post.kek.jp, sp10018@central.ntua.gr, tsuchiya.asato@shizuoka.ac.jp

The Lorentzian type IIB matrix model is a promising candidate for a nonperturbative formulation of superstring theory. Recently we performed complex Langevin simulations by adding a Lorentz invariant mass term as an IR regulator and found a (1+1)–dimensional expanding spacetime with a Lorentzian signature emerging dynamically at late times when the fermionic contribution is omitted. Here we find that this is merely an artifact of the Lorentz boosts by showing that the spontaneous breaking of rotational symmetry is eliminated if one chooses a Lorentz frame appropriately. On the other hand, when we include the fermionic contribution, we find some evidence suggesting the emergence of a smooth (3+1)–dimensional expanding Lorentzian spacetime.

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*Speaker

1. Introduction

Superstring theory is a promising candidate for a quantum theory of gravity, but it is consistently defined only in 10–dimensional spacetime. The standard model of particle physics, on the other hand, is defined in (3+1)–dimensional spacetime. By compactifying the extra dimensions, the effective spacetime in superstring theory at low energy becomes (3+1)–dimensional. However, there are infinitely many perturbative vacua including those having spacetime with different dimensions, and we cannot determine the vacuum that corresponds to our universe, at least at the perturbative level. Nonperturbative effects are thought to play an important role in lifting this degeneracy.

The type IIB matrix model [1] is a promising candidate for a nonperturbative formulation of superstring theory. The model is defined by dimensionally reducing 10–dimensional $\mathcal{N} = 1$ super Yang–Mills theory to zero dimensions. Spacetime emerges dynamically from the bosonic matrix degrees of freedom. The Euclidean version of this model has been studied analytically using the Gaussian expansion method [2–5] and also numerically using the complex Langevin method [6, 7], where the emergence of Euclidean 3–dimensional space has been observed. However, the relationship between the emergent space and our (3+1)–dimensional universe was unclear. On the other hand, the Lorentzian model has not been studied well due to the severe sign problem, which prevents us from applying conventional Monte Carlo methods[1]. Recently, numerical simulations have been performed using the complex Langevin method [12, 13] to overcome the sign problem [14–20]. The hope is that the dynamics of the model will result in the emergence of a (3+1)–dimensional expanding spacetime, where the extra dimensions are compactified via a spontaneous symmetry breaking (SSB) of the 9–dimensional rotational symmetry of space. See Refs. [21–24] for recent reviews on this model.

In our previous studies [18–20], we demonstrated that the Euclidean and the Lorentzian models are connected via analytic continuation. By adding a Lorentz invariant mass term in the action [25–38], which acts as an IR regulator, the Lorentzian model becomes inequivalent to the Euclidean model. In particular, we provided evidence for obtaining a smooth expanding spacetime. The signature of the metric changes dynamically, being Euclidean at early times and becoming Lorentzian at late times. While we found no evidence for a (3+1)–dimensional expanding spacetime, we reported that by omitting the fermionic contribution and tuning the model's parameters, a (1+1)–dimensional expanding spacetime emerges.

In this paper, we first show that this is merely an artifact resulting from the action of the Lorentz boost during the simulation due to the Lorentz symmetry of the model. By choosing a Lorentz frame that offers a natural definition of spacetime, the boost's effect is eliminated, and the 1–dimensional expansion disappears. Subsequently, we simulate the model by incorporating the dynamical effect of the fermions and present evidence suggesting the emergence of a smooth (3+1)–dimensional expanding spacetime, with six dimensions of space compactified via the SSB of the SO(9) rotational symmetry, in which supersymmetry (SUSY) plays a crucial role.

The rest of this paper is organized as follows. In Section 2, we explain the regularization of the Lorentzian model used in this work. In Section 3, we present the results obtained by performing a

[1]In Refs. [8–10], the Lorentzian model was studied by Monte Carlo methods using an approximation and the emergence of (3+1)–dimensional expanding spacetime was reported. However, it was found later [11] that the approximation amounts to replacing the complex weight e^{iS} by $e^{-\beta S}$ ($\beta > 0$), and that the emergent space is actually not continuous.

simulation of the bosonic model and point out that the appearance of the 1–dimensional expanding space is an artifact of the Lorentz boost. We also explain the method for removing this artifact. In Section 4, we show our results obtained by simulations of the model including the fermionic contribution. Section 5 is devoted to a summary and discussions.

2. Regularization of the Lorentzian type IIB matrix model

The Lorentzian type IIB matrix model is defined by the partition function

$$\begin{aligned} Z &= \int dA d\Psi \, e^{i(S_{\rm b}+S_{\rm f})} \ , \\ S_{\rm b} &= -\frac{N}{4}\mathrm{Tr}\left\{-2[A_0, A_i]^2 + [A_i, A_j]^2\right\} \ , \\ S_{\rm f} &= -\frac{N}{2}\mathrm{Tr}\left\{\Psi_\alpha (C\Gamma^\mu)_{\alpha\beta}[A_\mu, \Psi_\beta]\right\} \ , \end{aligned} \tag{1}$$

where A_μ and Ψ_α are bosonic and fermionic $N \times N$ Hermitian matrices, respectively. The indices μ and α run from 0 to 9 and 1 to 16, respectively, while the spatial index i runs from 1 to 9 only. The 16×16 matrices C and Γ^μ are the charge conjugation matrix and 10d Gamma matrices, respectively, after the Weyl projection. This model has $\mathcal{N} = 2$ SUSY, which is the maximal SUSY in 10d, implying that the model includes gravity. Furthermore, from the SUSY algebra, one can identify the constant shift $A_\mu \to A_\mu + c_\mu \mathbb{1}$ as the translation in this model. Thus, the eigenvalues of the matrices A_μ can be identified as the spacetime coordinates. This model also has SO(9,1) Lorentz symmetry, which should be partially broken at some point in time for the emergence of (3+1)–dimensional spacetime.

Since the partition function (1) of the Lorentzian model is not absolutely convergent, we need to regularize it. In this work, we use the Lorentz invariant mass term

$$S_\gamma = -\frac{1}{2}N\gamma \mathrm{Tr}\left(A_\mu\right)^2 = \frac{1}{2}N\gamma\left\{\mathrm{Tr}\,(A_0)^2 - \mathrm{Tr}\,(A_i)^2\right\} \tag{2}$$

as an IR regulator, where γ is a mass parameter and the $\gamma \to 0$ limit should be taken after taking the large–N limit. The model with this regulator has been studied in various contexts [25–38]. In particular, Ref. [32] reported the emergence of expanding spacetime by solving the classical equation of motion, although the dimensionality of space is not determined at the classical level.

3. The (1+1)–dimensional expanding spacetime as an artifact of the Lorentz boost

In this section, we discuss the emergence of the (1+1)–dimensional expanding spacetime observed in the bosonic model, and show that this is merely an artifact of the Lorentz boost.

In the simulation, we "gauge–fix" the SU(N) symmetry so that the matrix A_0 is diagonalized as $A_0 = \mathrm{diag}(\alpha_1, \alpha_2, \cdots, \alpha_N)$, where $\alpha_1 < \alpha_2 < \cdots < \alpha_N$. In order to see the structure of the spatial matrices A_i in that basis, we define a quantity

$$\mathcal{A}_{pq} = \frac{1}{9}\sum_{i=1}^{9}\left|(A_i)_{pq}\right|^2 \ , \tag{3}$$

which is shown later in Fig. 4. As in this case, the obtained spatial matrices have a band–diagonal structure in general when the emergent space shows a clear expanding behavior. Using the band–width n, we define the time by

$$t_a = \sum_{i=1}^{a} |\bar{\alpha}_i - \bar{\alpha}_{i-1}| \,, \tag{4}$$

where $\bar{\alpha}_i$ is an average of α's in the i–th block with size n defined as

$$\bar{\alpha}_i = \frac{1}{n} \sum_{\nu=1}^{n} \langle \alpha_{i+\nu} \rangle \ . \tag{5}$$

We also define the $n \times n$ block matrices in the spatial matrices as

$$\left(\bar{A}_i\right)_{kl}(t_a) = (A_i)_{(k+a-1)(l+a-1)} \ , \tag{6}$$

which are interpreted as representing the state of the universe at t_a. In what follows, we omit the index a of t_a for simplicity and shift the time so that the results are symmetric around $t = 0$. As an order parameter of the SSB of the SO(9) symmetry, we define the "moment of inertia tensor" as

$$T_{ij}(t) = \frac{1}{n} \mathrm{tr} \left(\bar{A}_i(t) \bar{A}_j(t) \right) \ , \tag{7}$$

where "tr" is used for traces of $n \times n$ block matrices, discriminating it from "Tr" used for traces of $N \times N$ matrices. If the SO(9) symmetry is not spontaneously broken, the nine eigenvalues $\lambda_i(t)$ of the tensor $T(t)$ become degenerate in the large–N limit. In all the simulations in this work[2], we use $N = 96$, $\gamma = 4$, and the block size is chosen to be $n = 12$.

In Fig. 1, we plot the eigenvalues α_i of the matrix A_0 (Left) and the eigenvalues $\lambda_i(t)$ of $T_{ij}(t)$ (Right). The eigenvalues α_i are distributed on a curve which is almost parallel to the real axis for large $|\alpha_i|$, indicating the emergence of real time at late times[3]. We also see that one out of nine eigenvalues of $T_{ij}(t)$ grows with t, which suggests the emergence of an expanding (1+1)–dimensional spacetime at late times for the chosen parameters.

However, this is an artifact of Lorentz boosts as we see below. In Fig. 2 (Left), we plot the trace of each spatial block matrix in the SO(9) basis which diagonalizes the moment of inertia tensor $T(t)$. We find that one of them grows linearly in time, which indicates that the obtained configurations are Lorentz boosted. Therefore, we need to remove the effects of the Lorentz boost to obtain the proper information of the emergent spacetime.

For that purpose, we choose a Lorentz frame by minimizing the quantity

$$\mathcal{T} = \mathrm{Tr} \left(A_0^\dagger A_0 \right) \ , \tag{8}$$

with respect to Lorentz transformations on each sampled configuration. This can be achieved by performing the (1+1)–dimensional Lorentz transformation

$$\begin{pmatrix} A_0' \\ A_i' \end{pmatrix} = \begin{pmatrix} \cosh\sigma & \sinh\sigma \\ \sinh\sigma & \cosh\sigma \end{pmatrix} \begin{pmatrix} A_0 \\ A_i \end{pmatrix} \tag{9}$$

[2]In order to stabilize the complex Langevin simulations, we have introduced a parameter η as described in Ref. [21], which is taken to be $\eta = 0.005$ in this work. This procedure is similar to the so-called dynamical stabilization, which has been used in complex Langevin simulations of finite density QCD [39, 40].

[3]We have also confirmed that the quantity $\langle \mathrm{tr}\bar{A}_i(t)^2 \rangle$ with the spatial matrices $A_i(t)$ defined by (6) is close to real, indicating the emergence of real space at late times. This applies to all the cases discussed in this paper.

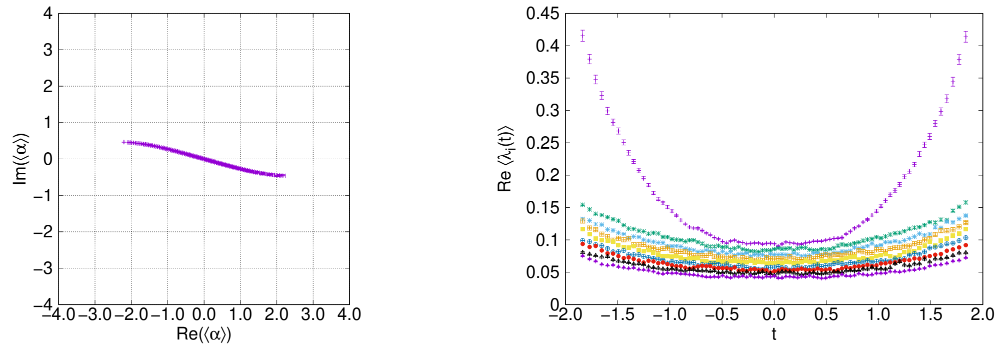

Figure 1: (Left) The eigenvalues α_i of the matrix A_0 are plotted in the complex plane for the bosonic model with $N = 96$ and $\gamma = 4$. (Right) The real part of the eigenvalues $\lambda_i(t)$ of $T_{ij}(t)$ are plotted against time for the bosonic model with $N = 96$ and $\gamma = 4$, where we find that one out of nine eigenvalues starts to grow at some point in time.

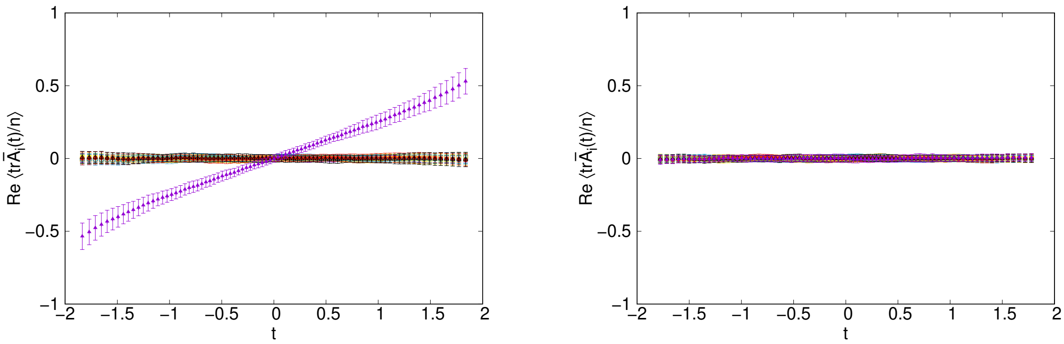

Figure 2: (Left) The real part of the trace of each spatial matrix is plotted against time for the bosonic model with $N = 96$ and $\gamma = 4$. Different colors of the data points correspond to a different index i for the spatial matrices $\bar{A}_i(t)$. (Right) The real part of the trace of each spatial matrix after the Lorentz transformation is plotted against time.

iteratively in such a way that the quantity (8) is minimized with respect to σ at each step, where $i = 1, 2, \ldots, 9$ and σ is a real parameter.

Let us discuss how σ is determined at each step. Plugging A'_0 in Eq. (8), we get

$$\mathcal{T}' = \cosh^2\sigma \,\mathrm{Tr}\left(A_0^\dagger A_0\right) + \sinh^2\sigma \,\mathrm{Tr}\left(A_i^\dagger A_i\right) + 2\cosh\sigma \sinh\sigma \,\mathrm{Re}\,\mathrm{Tr}\left(A_0^\dagger A_i\right) . \tag{10}$$

Thus the problem reduces to the minimization of

$$f(x) = a\cosh x + b\sinh x , \tag{11}$$

where we have defined $x = 2\sigma$ and

$$\begin{aligned} a &= \mathrm{Tr}\left(A_0^\dagger A_0\right) + \mathrm{Tr}\left(A_i^\dagger A_i\right) , \\ b &= 2\,\mathrm{Re}\,\mathrm{Tr}\left(A_0^\dagger A_i\right) . \end{aligned} \tag{12}$$

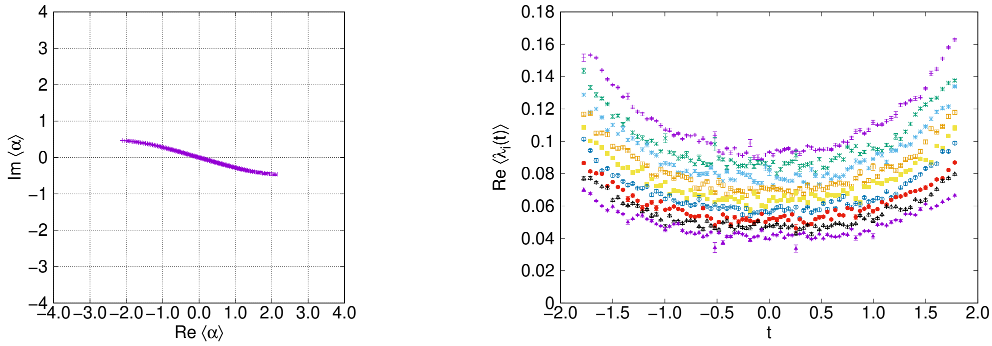

Figure 3: (Left) The eigenvalues α_i of the matrix A_0 after the Lorentz transformation are plotted in the complex plane for the bosonic model with $N = 96$ and $\gamma = 4$. (Right) The real part of the eigenvalues $\lambda_i(t)$ of $T_{ij}(t)$ after the Lorentz transformation are plotted against time for the bosonic model with $N = 96$ and $\gamma = 4$. The growth of one out of nine eigenvalues has disappeared.

We find that the minimum $\sqrt{a^2 - b^2}$ is obtained at $x = \tanh^{-1}\left(-\frac{b}{a}\right)$, where $\left|\frac{b}{a}\right| < 1$ as one can prove from the inequality

$$\mathrm{Tr}\,(A_0 \pm A_i)^\dagger\,(A_0 \pm A_i) \geq 0\,. \tag{13}$$

Note that the matrix A_0 is no longer diagonal after the Lorentz transformation. Therefore, we redefine α_i by diagonalizing A_0 as

$$A_0 \to P^{-1} A_0 P \equiv \tilde{A}_0\,, \tag{14}$$

where P is a general complex matrix and $\tilde{A}_0 = \mathrm{diag}(\alpha_1, \alpha_2, \cdots, \alpha_N)$ is a complex diagonal matrix with the ordering

$$\mathrm{Re}\,\alpha_1 < \mathrm{Re}\,\alpha_2 < \cdots < \mathrm{Re}\,\alpha_N\,. \tag{15}$$

Accordingly, we transform the spatial matrices A_i as

$$A_i \to P^{-1} A_i P\,. \tag{16}$$

In Fig. 2 (Right), we plot the trace of each spatial matrix after the Lorentz transformation. By comparing this plot with Fig. 2 (Left), we find that the linear growth of the trace of the block matrices has disappeared.

In Fig. 3, we plot the eigenvalues α_i of the matrix A_0 (Left) and the eigenvalues $\lambda_i(t)$ of $T_{ij}(t)$ (Right) after the Lorentz transformation, which should be compared with Fig. 1. While the eigenvalue distribution of α_i is not affected significantly by the Lorentz transformation, the nine eigenvalues of $T_{ij}(t)$ come quite close to each other after the Lorentz transformation, indicating that the 1–dimensional expansion is indeed an artifact of the Lorentz boost.

4. The effect of SUSY

When we include the fermionic contribution, the existence of near–zero eigenvalues of the Dirac operator makes the complex Langevin simulations untrustable. This problem is known as the

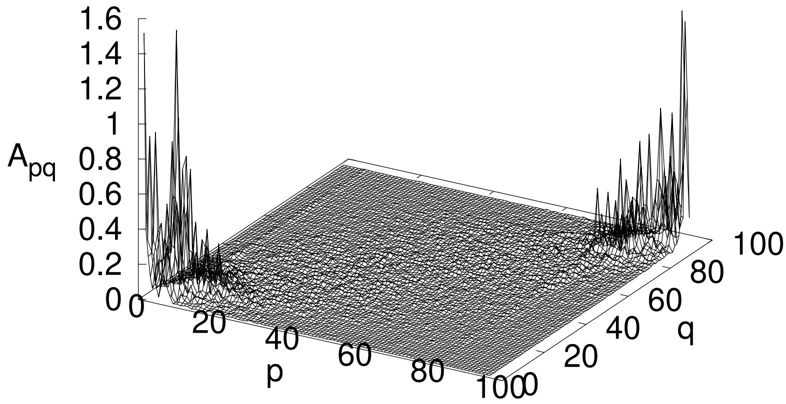

Figure 4: The structure of the spatial matrices is shown by $\mathcal{A}_{pq}$ defined in (3) for the model including the fermionic contribution with $N = 96$, $\gamma = 4$, $m_{\rm f} = 3.5$, $\tilde{d} = 5$ and $\xi = 16$.

singular drift problem [41, 42]. In order to overcome this problem, we add a fermionic mass term

$$S_{m_{\rm f}} = iNm_{\rm f}{\rm Tr}\left[\bar{\Psi}_\alpha\left(\Gamma_7\Gamma_8^\dagger\Gamma_9\right)_{\alpha\beta}\Psi_\beta\right], \tag{17}$$

where $m_{\rm f}$ is a mass parameter. We eventually need to make the $m_{\rm f} \to 0$ extrapolation to retrieve the original model. Note that, at $m_{\rm f} = \infty$, the fermionic degrees of freedom decouple and the model becomes equivalent to the bosonic model.

We find that the complex Langevin method works only for $m_{\rm f} \gtrsim 3.5$ with the present matrix size $N = 96$. The results for $m_{\rm f} = 3.5$, however, are still qualitatively the same as those of the bosonic model. In order to enhance the effect of SUSY without decreasing $m_{\rm f}$, we attempt to suppress the bosonic fluctuations by modifying the Lorentz invariant mass term as

$$S_\gamma = \frac{1}{2}N\gamma\left\{{\rm Tr}\,(A_0)^2 - \sum_{i=1}^{\tilde{d}}{\rm Tr}\,(A_i)^2 - \xi\sum_{j=\tilde{d}+1}^{9}{\rm Tr}\,\left(A_j\right)^2\right\}, \tag{18}$$

where ξ (≥ 1) is an additional parameter, which is introduced to suppress the fluctuations of $(9-\tilde{d})$ bosonic matrices. Note that this term breaks the Lorentz symmetry[4] from SO(9,1) to SO($\tilde{d}$,1) for $\xi > 1$. We show our results for this modified model with $N = 96$, $\gamma = 4$, $m_{\rm f} = 3.5$, $\tilde{d} = 5$ and $\xi = 16$ after the Lorentz transformation that removes the artifact of the Lorentz boost.

In Fig. 4, we plot the quantity $\mathcal{A}_{pq}$ defined in (3), where we see a band–diagonal structure of the spatial matrices. In Fig. 5 (Left), we plot the eigenvalues of A_0. We find that the distribution of

[4]The idea of introducing a mass term of this kind is inspired by a BMN–type deformation [43] of the type IIB matrix model, which preserves SUSY [44]. See also Ref. [45] for complex Langevin simulations of the Euclidean model with this SUSY deformation.

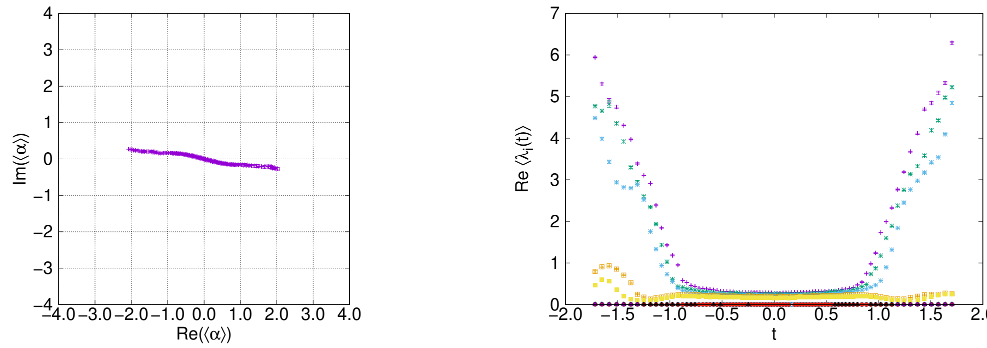

Figure 5: (Left) The eigenvalues α_i of the matrix A_0 are plotted in the complex plane for the model including the fermionic contribution with $N = 96$, $\gamma = 4$, $m_\mathrm{f} = 3.5$, $\tilde{d} = 5$ and $\xi = 16$. (Right) The real part of the eigenvalues $\lambda_i(t)$ of $T_{ij}(t)$ are plotted against time for the same model. Three out of nine eigenvalues start to grow at some point in time, which clearly indicates the emergence of an expanding 3–dimensional space.

the eigenvalues becomes parallel to the real axis at late times, which implies that the time becomes real in that region. In Fig. 5 (Right), we plot the eigenvalues $\lambda_i(t)$ of $T_{ij}(t)$. While the SO($\tilde{d}$) spatial rotational symmetry seems to be preserved at early times, it is broken spontaneously to SO(3) at some point in time. After this SSB, only three eigenvalues start to grow. This result suggests that the expanding (3+1)–dimensional spacetime appears at late times in the presence of SUSY.

5. Summary

We have performed first–principle calculations of the Lorentzian type IIB matrix model using the Lorentz invariant mass term as an IR regulator. From the simulation of the bosonic model, we found that the Lorentz boosts may cause severe artifacts in the emergent spacetime structure. We performed a Lorentz transformation on the sampled configurations to remove these artifacts. We then found that the SSB of SO(9) does not occur in the bosonic model. Hence, the fermionic contribution is expected to be crucial for the emergence of (3+1)–dimensional spacetime.

When we include the fermionic contribution, we have to modify the model by adding the fermionic mass term (17) in order to make the complex Langevin method work. We find that, at $m_\mathrm{f} = 3.5$, which is the minimal value that we were able to achieve for the present matrix size $N = 96$, the results are qualitatively the same as those of the bosonic model.

In order to enhance the effect of SUSY without decreasing m_f further, we reduced the quantum fluctuations of the bosonic matrices by modifying the Lorentz invariant mass term as in Eq. (18) with the parameter ξ. We performed simulations in the $\tilde{d} = 5$ case and found that the SO($\tilde{d}$) spatial rotational symmetry is spontaneously broken, and (3+1)–dimensional expanding spacetime appears at some point in time. Note that these results are obtained after the Lorentz transformation that removes the artifact of the Lorentz boosts. Recently it has been proposed [46] that the Lorentz symmetry should be "gauge–fixed" in defining the Lorentzian type IIB matrix model. Then the configurations will not get Lorentz boosted during the simulation.

In order to investigate whether the (3+1)–dimensional spacetime emerges in the original model, we need to take the limits of $m_\mathrm{f} \to 0$, $\xi \to 1$, $N \to \infty$ and $\gamma \to 0$, eventually. Performing these extrapolations is an important future direction.

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