The emergence of spacetime in IIB string theory

A study of the complex Langevin method applied on the IKKT model and variants

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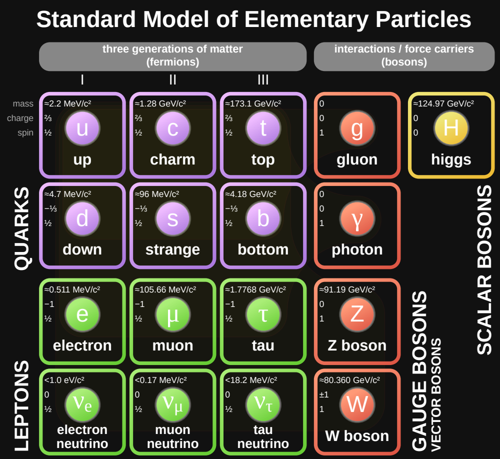
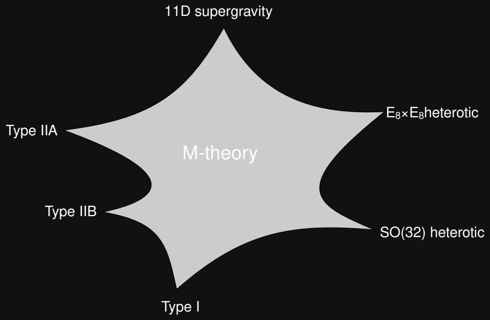
Introduction



- In the Standard model all interactions are unified in one description but gravity:
- A theory of quantum gravity has several problems:
    - Its classigal analog (General Relativity) is different from the other classical field theories.
    - It is much weaker than other interactions, probing prohibitevely high energy physics.

Introduction



- String theory offers a solution:
  - Provides a unified description of all interactions.
  - Different string theories are connected via dualites
- String theory has its own problems
  - The timespace dimensions are 10 contrary to 4.
  - Still probing prohibitevely high energy physics.

Outline

Outline

1. Complex Langevin Method (CLM)
   - Monte Carlo
   - The method
   - Application to (computational) physics
2. Type IIB superstring theory
   - The IKKT matrix model
   - The Euclidean IKKT model
   - Lorentzian IKKT model and variants

Baseline

Assume a functional $O : V \longrightarrow \mathbb{R}$ on a measurable real vector space V and a probability measure $w : \mathcal{B}(V) \longrightarrow \mathbb{R}_+$ on the Borel sets $\mathcal{B}(V)$.[1]

Expectation value:

$$\langle O \rangle = \int O dw$$

Sample configurations in V and measure O in real time T, with probability w to approximate the integral as:

$$\langle O \rangle = \sum_{\tau \in T} O_\tau$$

[1] Usually induced from a measure $\mu : \mathcal{F} \longrightarrow \mathbb{R}_+$ on a σ–algebra $\mathcal{F}$ of a sample space Ω via a random variable (measurable function) $\Omega \longrightarrow V$.

Problems

Sign problem

A set function $w : \mathcal{B}(V) \longrightarrow \mathbb{C}$ is ill–defined as a measure.[2]

Overlap problem

However, the phase–quenched one $|w| : \mathcal{B}(V) \longrightarrow \mathbb{R}_+$ can be(come) a measure:[3]

$$\langle O \rangle_0 = \int O d|w|$$

Re–weighting suggests calculating two well–defined integrals:

$$\langle O \rangle = \frac{\langle O \exp \imath \arg w \rangle_0}{\langle \exp \imath \arg w \rangle_0}$$

[2] $\mathbb{C}$ is neither well–ordered nor bounded from below like $\mathbb{R}_+$.

[3] $\Re w : \mathcal{B}(V) \longrightarrow \mathbb{R}$ is not, as it is still unbounded from below.

Solutions

Monte Carlo simulations rely on probability weight w for fast convergence.

If w is complex, two mainstream approaches are popular in the bibliography:

- Make $\arg w$ static via deforming the integration contours, (Lefshetz Thimbles)[4]
- Map the theory to a (thermalized) stochastic process, with which effective configuration space sampling may be possible. (Complex Langevin)[5]

[4] Alexandru, Basar, Bedaque, Ridgway, and Warrington 2016

[5] Aarts, James, Seiler, and Stamatescu 2011; Aarts, Seiler, and Stamatescu 2010; Klauder 1984; Parisi 1983

Outline

$\mathbb{R}$ Langevin

Assume stochastic process $\phi : T \times V \longrightarrow \mathbb{R}$ is generated by stochastic differential equation

$$d\phi = v d\tau + \sigma d\eta,$$

via standard Wiener (gaussian noise) process $\eta : T \times V \longrightarrow \mathbb{R}$, with drift

$$-v = \nabla \log w = \nabla \log |w| \qquad ,$$

constant noise variance σ^2, and initial configuration ϕ_0.

This process has well defined probability density given by a corresponding Fokker–Planck equation, equivalent to a corresponding time observable functional O time evolution

$$\frac{d}{d\tau}\rho = -\mathcal{L}^\dagger \rho \text{ and } \frac{d}{d\tau}\langle O \rangle = \mathcal{L}\langle O \rangle, \ \mathcal{L}\_ = \nabla \cdot (\nabla\_ + v\_).$$

The Fokker–Planck hamiltonian $\mathcal{L}$ spectrum needs to be bounded from below by 0.

$\mathbb{C}$ Langevin

Assume stochastic process $\phi : T \times V \longrightarrow \mathbb{R}$ is generated by stochastic differential equation

$$d\phi = \upsilon d\tau + \sigma d\eta,$$

via standard Wiener (gaussian noise) process $\eta : T \times V \longrightarrow \mathbb{R}$, with drift now complex

$$-\upsilon = \nabla \log w = \nabla \log |w| + \imath \nabla \arg w,$$

constant noise variance σ^2, and initial configuration ϕ_0.

This process is ill–defined, because the complex drift υ imposes a complex differential $d\phi$, while ϕ is a real stochastic process.

$$\frac{d}{d\tau}\rho = -\mathcal{L}^\dagger \rho \text{ and } \frac{d}{d\tau}\langle O \rangle = \mathcal{L}\langle O \rangle, \; \mathcal{L}\_ = \nabla \cdot (\nabla\_ + \upsilon\_).$$

The Fokker–Planck hamiltonian $\mathcal{L}$ becomes complex too.

$\mathbb{C}$ Langevin complexified

Assume stochastic process $\phi : T \times V \longrightarrow \mathbb{C}$ is generated by stochastic differential equation

$$d\phi = v d\tau + \sigma d\eta,$$

via standard Wiener (gaussian noise) process $\eta : T \times V \longrightarrow \mathbb{R}$, with drift complexified

$$-v = \nabla \log w \qquad ,$$

constant noise variance σ^2, and initial configuration ϕ_0.

This process is well–defined. Part of CLM conjecture states that complexified observables evolve according to the original Fokker–Planck hamiltonian

$$\frac{d}{d\tau}\rho = -\mathcal{L}^\dagger \rho \text{ and } \frac{d}{d\tau}\langle O \rangle = \mathcal{L}\langle O \rangle, \mathcal{L}\_ = \nabla \cdot (\nabla\_ + v\_).$$

The complexification of the degrees of freedom affects ∇.

Complex Langevin validity

Complex Langevin (strong validity conditions)

$\forall w : \mathcal{B}(V) \longrightarrow \mathbb{C}$ theory and $\forall O : V \longrightarrow \mathbb{R}$ observable:

- $\exists O : V \longrightarrow \mathbb{C}$ admits a holomporphic analytic continuation by complexfication of degrees of freedom,
- The distribution of $|v| : V \longrightarrow \mathbb{R}_+$ falls off at $|v| \to \infty$ superexponentially.[6]

Complex Langevin (strong) theorem

$\forall O : V \longrightarrow \mathbb{R}$ satisfying validity conditions, at equilibrium:

$$\langle O \rangle \overbrace{=}^{\text{stochastic quantization}} \int O d\rho \overbrace{=}^{\text{complex langevin}} \int O d\rho = \langle O \rangle$$

[6] Nagata, Jun Nishimura, and Shimasaki 2016a

Outline

Langevin discretization

Langevin process with time sampling $\mathbb{N} \longrightarrow T$ and $\Delta\phi_n = \phi_n - \phi_{n-1}$, $\forall n \in \mathbb{N}$:

$$d\phi = \upsilon d\tau + \sigma d\eta \longrightarrow \Delta\phi_n = \upsilon_n \Delta\tau_n + \eta_n \sigma \sqrt{\Delta\tau_n}$$

Expectation over the corresponding Fokker–Planck probability ρ become sums over the thermalized Langevin process:

$$\int O d\rho \longrightarrow \sum_{n\in\mathbb{N}} O_n$$

Checking drift norm $|\upsilon|$ histogram for large norm fall-off.

Connection with physics

Action S

The weight density w stems from an action functional $S : V \longrightarrow \mathbb{K}$ as $w \propto \exp(-S)$.

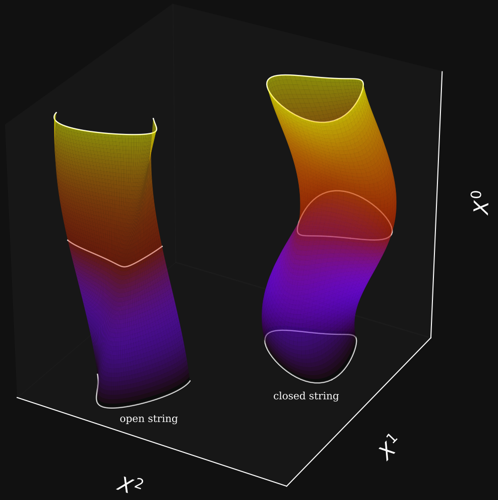
The drift is then $v \propto -\nabla S$.

The *sign problem* then becomes a *complex action problem*, as:

$$|w| = \exp(-\Re S) \text{ and } \arg w = -\Im S$$

Outline

The fundamental strings



String theory implies that:

- worldlines $X : T \longrightarrow \mathcal{X}$ become worldsheets $X : T \times \Sigma \longrightarrow \mathcal{X}$,
  - point particles become strings with extra degrees of freedom,
  - particle states become string modes,
  - interactions as worldline intersections become worldsheet topologies.

Supersting theory assumes $\dim \mathcal{X} = 10$ timespace dimensions.

Type IIB superstring action and the IKKT matrix model

Type IIB is an $\mathcal{N} = 2$ supersymmetric theory of closed strings with identical chirality.

Type IIB Green–Schwartz action in the Schild gauge [7]:

$$g^2 S = g^2(S_{\text{boson}} + S_{\text{fermion}})$$
$$= \frac{1}{4}\langle\sqrt{-\det h}\{X^\mu|X^\nu\}\{X_\mu|X_\nu\}\rangle_{T\times\Sigma} - \frac{1}{2}\langle\sqrt{-\det h}\bar{\psi}_\alpha \Gamma^\mu{}_{\alpha\beta}\{X_\mu|\psi_\beta\}\rangle_{T\times\Sigma}$$

IKKT matrix model:[8]

$$-N^{-1}S = -N^{-1}(S_{\text{boson}} + S_{\text{fermion}})$$
$$= \frac{1}{4}\operatorname{tr}_{T\times\Sigma}([A^\mu|A^\nu][A_\mu|A_\nu]) + \frac{1}{2}\operatorname{tr}_{T\times\Sigma}(\bar{\psi}_\alpha \Gamma^\mu{}_{\alpha\beta}[A_\mu|\psi_\beta])$$

[7] Schild 1977
[8] Ishibashi, Kawai, Kitazawa, and Tsuchiya 1997

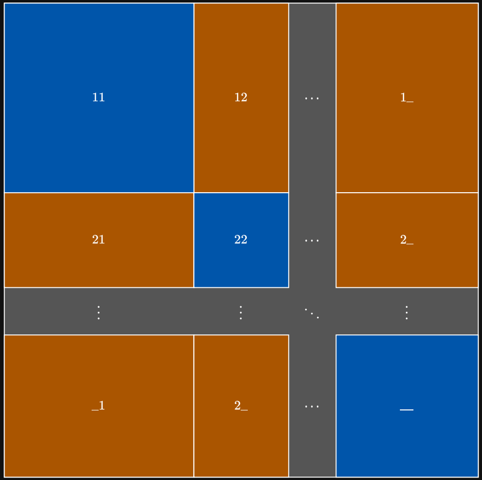
Type IIB superstring and IKKT matrix model correspondence

Type IIB string theory	IKKT N–size matrix model
∂	0
$X \in \mathcal{X}^{T \times \Sigma}$	$A \in \mathrm{M}_N \mathbb{C}^{\dim \mathcal{X}}$ traceless hermitian
$\psi \in \mathcal{U}^{T \times \Sigma}$	$\psi \in \mathrm{M}_N \mathbb{C}^{\dim \mathcal{U}}$ traceless hermitian
g^{-2}	N
$\imath\{\_\|\_\}$	$[\_\|\_]$
$\langle \sqrt{-\det h}\,\_ \rangle_{T \times \Sigma}$	$\mathrm{tr}_{T \times \Sigma}\,\_$

IKKT is also equivalent to zero–volume $\mathcal{N} = 1$ super Yang–Mills theory.

The model can be defined defined for $\dim \mathcal{X} = 4$, $\dim \mathcal{X} = 6$, $\dim \mathcal{X} = 10$.

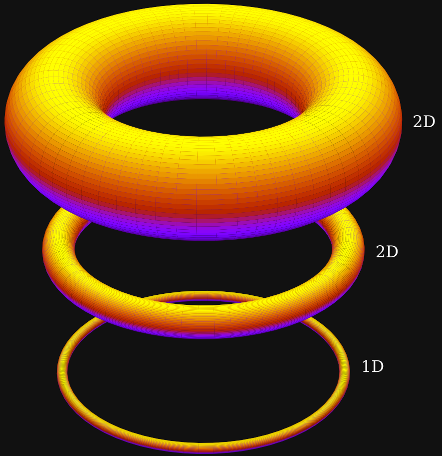
Second quantization of strings



- Long distance D-brane interactions in type IIB string theory reproduced by 1–loop calculations in IKKT.[a]
- Light cone IIB string field theory derived from Schwinger–Dyson equations for the Wilson loops, identified with string creation/annihilation operators.[b]
- Conjectured to be a nonperturbative definition of the unique superstring theory behind string dualities:
  - reproducible perturbative expansion of type IIB to all orders,
  - timespace coordinates identification with matrix eigenvalues.

[a] Ishibashi, Kawai, Kitazawa, and Tsuchiya 1997
[b] Aoki, Iso, Kawai, Kitazawa, Tsuchiya, and Tada

Compactification of extra dimensions



- These identifications allow the study of
  - the dynamical emergence of timespace
  - the dynamical compactification of extra dimensions
  - the time evolution of space as an expansion of the universe
- Requirements
  - homogenous and infinite time at $N \to \infty$
  - 3–dimensional expanding with timespace at $t \to \infty$
  - real timespace with Lorentzian metric signature

Outline

Wick rotation

Lorentzian | Euclidean

$A_0 \longrightarrow \imath A_{10}$

$\mathit{\Gamma}_0 \longrightarrow \imath \mathit{\Gamma}_{10}$

$\eta \longrightarrow$ metric signature change $\longrightarrow \delta$

Dynamical compactification is realised as a rotational symmetry breaking:

$$\text{for } D < \dim \mathcal{X},\ \mathrm{SO}_{\dim \mathcal{X}} \longrightarrow \mathrm{SO}_D$$

Model well defined in $\dim \mathcal{X} = 4$, 6 and 10 dimensions.

Corresponding models studied under the analytical/approximate Gaussian Expansion Method.[9]

[9] Aoyama, Jun Nishimura, and Toshiyuki Okubo 2011; J. Nishimura, T. Okubo, and F. Sugino 2005; Jun Nishimura, Toshiyuki Okubo, and Fumihiko Sugino 2011

Langevin dynamics

Integrating fermions out of expectation integrals, a bosonic–only effective action stems:

$$S_{\text{effective}} = S_{\text{boson}} - \log \operatorname{pf} \mathcal{M}$$

$\mathcal{M}$ is the fermion matrix for which $(\mathcal{M}\psi)_\alpha = \mathit{\Gamma}_{\mu\alpha\beta}[A_\mu|\psi_\beta]$.

Bosonic drift:

$$\upsilon_{\text{boson}}|_\mu = -\frac{1}{2} N \operatorname{tr}[A_\nu|[A_\nu|A_\mu]]$$

Fermionic drift (singular):

$$\upsilon_{\text{fermion}}|_\mu = \frac{1}{2} \operatorname{tr}\left(\frac{\partial \mathcal{M}}{\partial A_\mu^\top} \mathcal{M}^{-1}\right)$$

Solving problems

Complexification in the bosonic degrees of freedom

A_μ traceless hermitian $\longrightarrow$ A_μ traceless

Running away in the imaginary direction

The action has gauge symmetries, which can bring A_μ as close to hermitian as possible.[10]

Singular fermion drift

A deformation is added to shift the spectrum of $\mathcal{M}$ away from the origin:[11]

$$\Delta S_{\text{fermion}} = \frac{1}{2} N m_{\text{fermion}} \operatorname{tr}_{T\times\Sigma}(\overline{\psi}_\alpha \gamma_{\alpha\beta}(\mathit{\Gamma})\psi_\beta)$$

[10] Nagata, Jun Nishimura, and Shimasaki 2016b

[11] Anagnostopoulos, Azuma, Ito, Jun Nishimura, Toshiyuki Okubo, and Papadoudis 2020; Anagnostopoulos, Azuma, Ito, Jun Nishimura, and Papadoudis 2018; Ito and Jun Nishimura 2016

Spontaneous symmetry breaking

Normally, the ordered eigenvalues provide the length scales for each direction, but after complexification they become non–holomorphic observables!

Use the diagonal elements $T_{\mu\mu}$ of the moment of inertia instead and manually break the rotational symmetry $\mathrm{SO}_{\dim\mathcal{X}}$ with a deformation of the form

$$\Delta S_{\text{boson}} = \frac{1}{2} N^2 \varepsilon \sum_{\mu} m_{\text{boson}}|_{\mu} T_{\mu\mu},\, T_{\mu\mu} = N^{-1} \operatorname{tr}_{T\times\Sigma}(A_\mu A_\mu),$$

where anisotropy is controlled by the mass vector m_{boson}.[12]

$$\rho_\mu = \frac{\langle T_{\mu\mu}\rangle}{\sum_\mu \langle T_{\mu\mu}\rangle},$$

with $\lim_{m_{\text{fermion}}\to 0} \lim_{\varepsilon\to 0} \lim_{N^{-1}\to 0} \rho_\mu$ extrapolated on CLM valid runs.

[12] Anagnostopoulos, Azuma, Ito, Jun Nishimura, Toshiyuki Okubo, and Papadoudis 2020; Anagnostopoulos, Azuma, Ito, Jun Nishimura, and Papadoudis 2018

Simulation

- Choose an initial configuration $A(0)$.
- Simulate the Complex Langevin stochastic process with adaptive timestep:

$$\Delta A(\tau_n) = \upsilon(A(\tau_n))\Delta\tau_n + \eta(\tau_n)\sigma\sqrt{\Delta\tau_n}.$$

  - Evaluate $\Delta\tau_n$ based on $|\upsilon(A(\tau_{n-1}))|$.
  - At each timestep τ_n, gauge–cool the current configuration $A(\tau_n)$ before the next update.
  - Evaluate fermion drift $\upsilon_{\text{fermion}}$ by noisy estimators

$$\operatorname{tr}\left(\frac{\partial\mathcal{M}}{\partial A_\mu^\top}\mathcal{M}^{-1}\right) = \left\langle \eta^* \frac{\partial}{\partial A_\mu^\top}\mathcal{M}\chi \right\rangle, \chi = \mathcal{M}^{-1}\eta \text{ as a solution of } \mathcal{M}^\dagger\mathcal{M}\chi = \mathcal{M}^\dagger\eta$$

  solved with the conjugate–gradient method, with a computation shorthand for $\mathcal{M}$

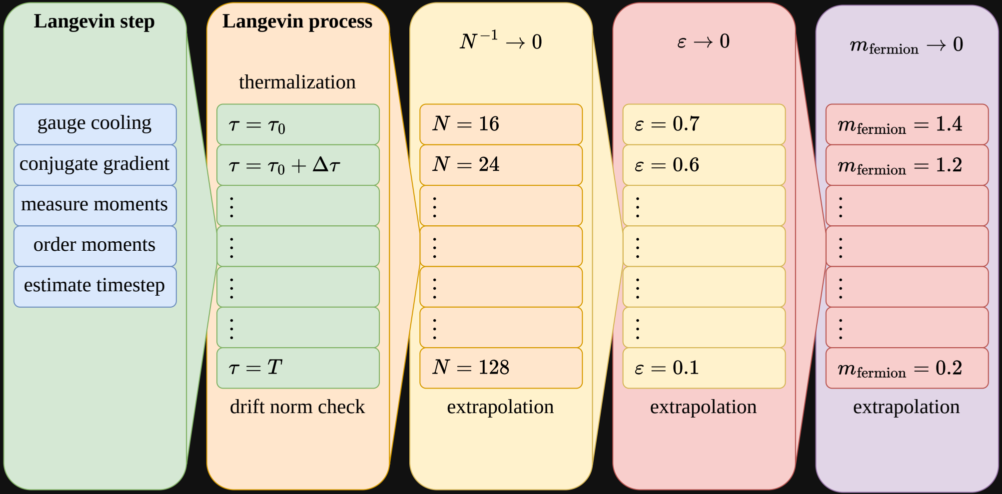
$$(\mathcal{M}\psi)_\alpha = \Gamma_{\mu\alpha\beta}[A_\mu|\psi_\beta]$$

  - Wait for the stochastic process to thermalize adequately.
  - Measure $T_{\mu\mu}$ diagonal values and order them by $\Re T_{\mu\mu}$, $\forall\mu \in \mathbb{N}_{\dim\mathcal{X}}$.

Simulations

- At the end of each run, discard it or not based on the histogram of the drift norm $|v|$.
- Estimate $\langle T_{\mu\mu} \rangle$, $\forall \mu \in \mathbb{N}_{\dim \mathcal{X}}$.
- Repeat simulations for several N sizes to extrapolate at $N^{-1} \to 0$.
- Repeat batches of simulations for several values of ε to get $\varepsilon \to 0$.
- Repeat batches of batches of simulations for several values of m_{fermion}.

Simulation recipe



Resources

ARIS[13]

6072731 core hours
23277 core hours/week

Fugaku[14]

3737446 node hours
35814 node hours/week

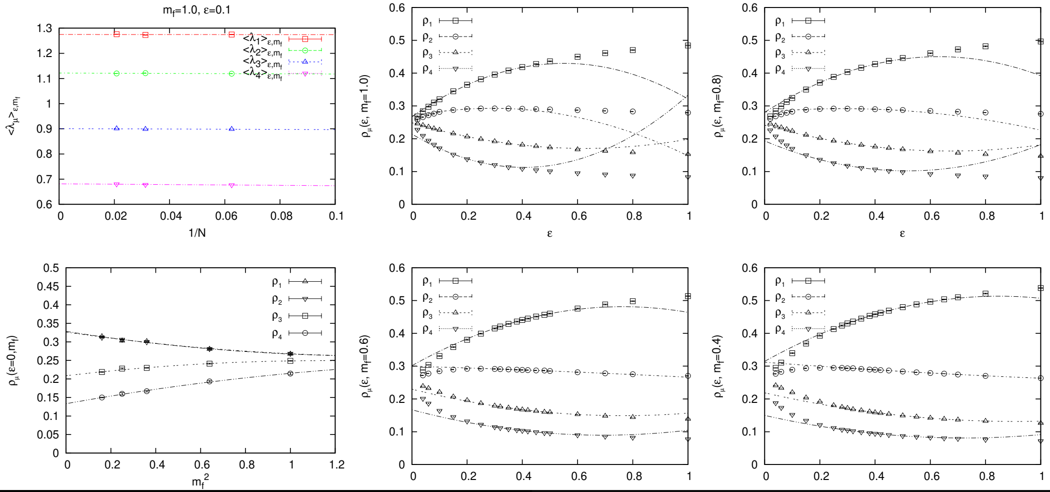
Oakbridge[15]

306624 node hours
2938 node hours/week

[13] **https://www.hpc.grnet.gr/**
[14] **https://www.fujitsu.com/global/about/innovation/fugaku/**
[15] **https://www.cc.u-tokyo.ac.jp/en/supercomputer/obcx/service/**

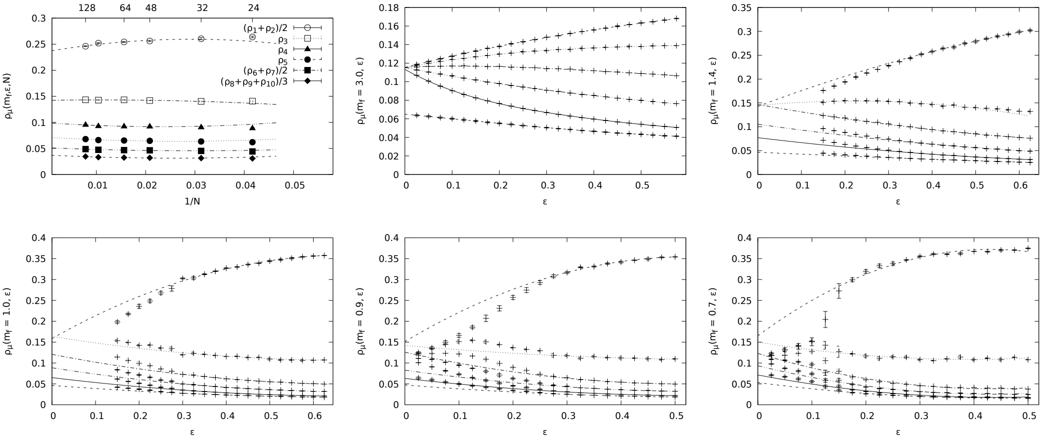
$\dim \mathcal{X} = 6$

Anagnostopoulos, Azuma, Ito, Jun Nishimura, and Papadoudis 2018



$\dim \mathcal{X} = 10$

Anagnostopoulos, Azuma, Ito, Jun Nishimura, Toshiyuki Okubo, and Papadoudis 2020



Comments

Approximating analytic calculations using the Gaussian Expansion Method have shown:

- $\mathrm{SO}_6 \longrightarrow \mathrm{SO}_3$,[16] for $\dim \mathcal{X} = 6$
- $\mathrm{SO}_{10} \longrightarrow \mathrm{SO}_3$,[17] for $\dim \mathcal{X} = 10$

Both these results are confirmed via the Complex Langevin Method!

Difficulties with the $\mathrm{SO}_{10} \longrightarrow \mathrm{SO}_3$ result:

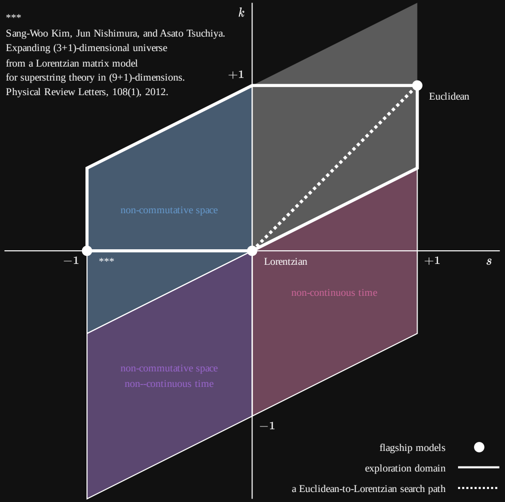
- Well, SO_3 is not SO_4!
- Also, all dimensions are of finite extent.

Are these artifacts of the Euclidean model? Lets go study the Lorentzian model!

[16] Aoyama, Jun Nishimura, and Toshiyuki Okubo 2011
[17] Jun Nishimura, Toshiyuki Okubo, and Fumihiko Sugino 2011

Outline

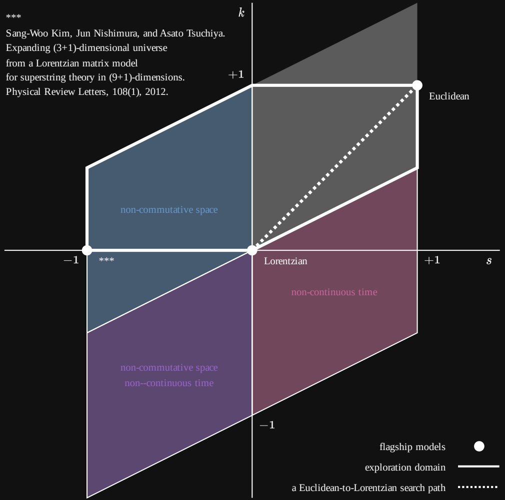
Euclidean IKKT



$$N^{-1} S_{\text{boson}} = \frac{1}{4}\operatorname{tr}(F_{ij}F_{ij}) + \frac{1}{2}\operatorname{tr}(F_{0i}F_{0i})$$

with $F_{\mu\nu} = \imath[A_\mu | A_\nu]$

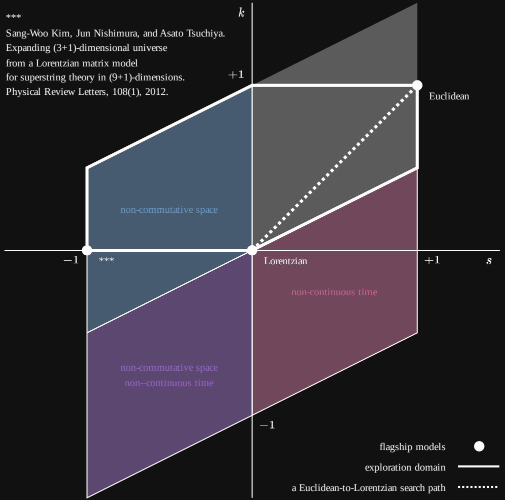
Lorentzian IKKT



$$N^{-1}S_{\text{boson}} = \frac{1}{4}\operatorname{tr}(F_{ij}F_{ij}) - \frac{1}{2}\operatorname{tr}(F_{0i}F_{0i})$$

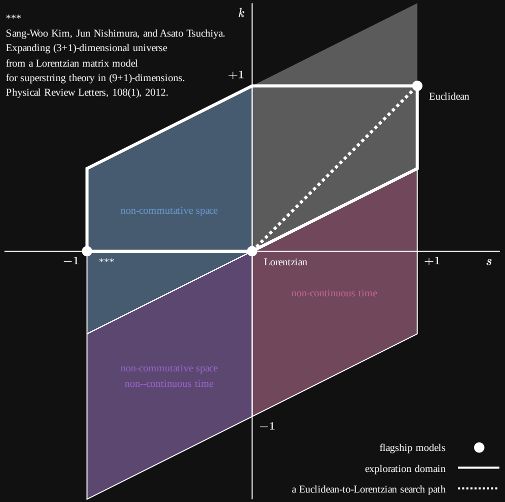
$$\text{with } F_{\mu\nu} = \imath[A_\mu|A_\nu]$$

Lorentzian IKKT theory space



$$
\begin{aligned}
N^{-1}S \\
&= -\imath \exp\left(\imath s \frac{\pi}{2}\right)\left(\frac{1}{4}\operatorname{tr}(F_{ij}F_{ij})\right. \\
&\quad \left. - \exp(-\imath k \pi)\frac{1}{2}\operatorname{tr}(F_{0i}F_{0i})\right) \\
&\quad \text{with } F_{\mu\nu} = \imath[A_\mu|A_\nu]
\end{aligned}
$$

Lorentzian IKKT theory space



- The Lorentzian IKKT model is equivalent to the Euclidean model by a contour deformation for $s = k = u$:

$$A_0 \to \exp\left(-\frac{3}{8}\imath u\right) A_0$$

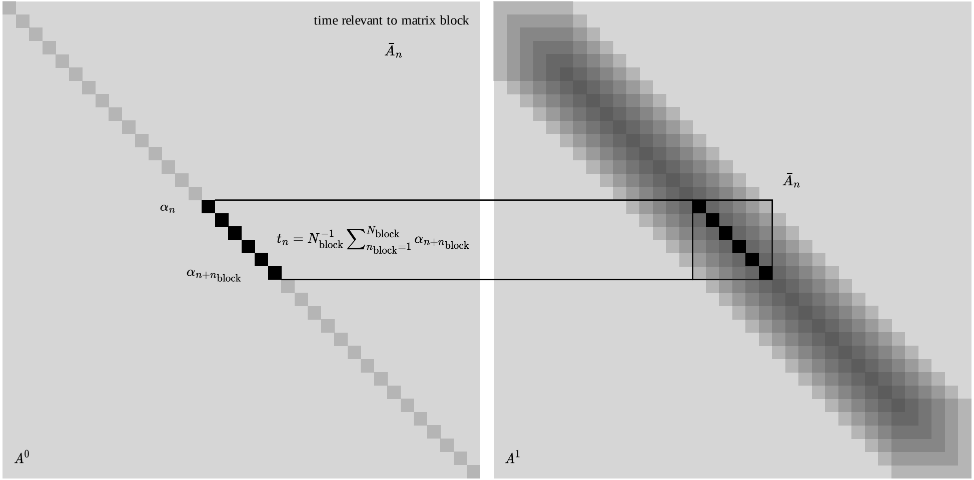
$$A_i \to \exp\left(+\frac{1}{8}\imath u\right) A_i$$

- Wick rotation in target space:

$$A_0 \to \exp\left(-\frac{1}{2}\imath u\right) A_0$$

- $\mathcal{M}$ is real but still singular, making the $m_{\text{fermion}} > 0$ deformation still necessary!

Lorentzian IKKT time modelling



Lorentzian–Euclidean IKKT equivalence

Corresponding moment of inertia expectations:

$$\langle T_{00}\rangle_{\text{Lorentzian}} = \langle T_{00}\rangle_{\text{Euclidean}} \exp\left(-\imath\frac{3}{4}\pi\right) \text{ and } \langle T_{ii}\rangle_{\text{Lorentzian}} = \langle T_{ii}\rangle_{\text{Euclidean}} \exp\left(\imath\frac{1}{4}\pi\right)$$

where:

$$T_{\mu\nu} = N^{-1}\,\mathrm{tr}\, A_\mu A_\nu$$

The time characteristic slope (phase):

$$\langle \alpha\rangle_{\text{Lorentzian}} = \langle \alpha\rangle_{\text{Euclidean}} \exp\left(-\imath\frac{3}{8}\pi\right)$$

Lorentzian IKKT observables

The ordered eigenvalues λ_i of the spatial moment of inertia T_{ij} are non–holomorphic.

The condition is strong! CLM is still valid, so long an alternative holomorphic computation exists.

It exists! Via Vieta's formula and trace expressions for the characteristic polynomial.

Ordering of the complex(ified) eigenvalues is done via $\Re\lambda_i$.

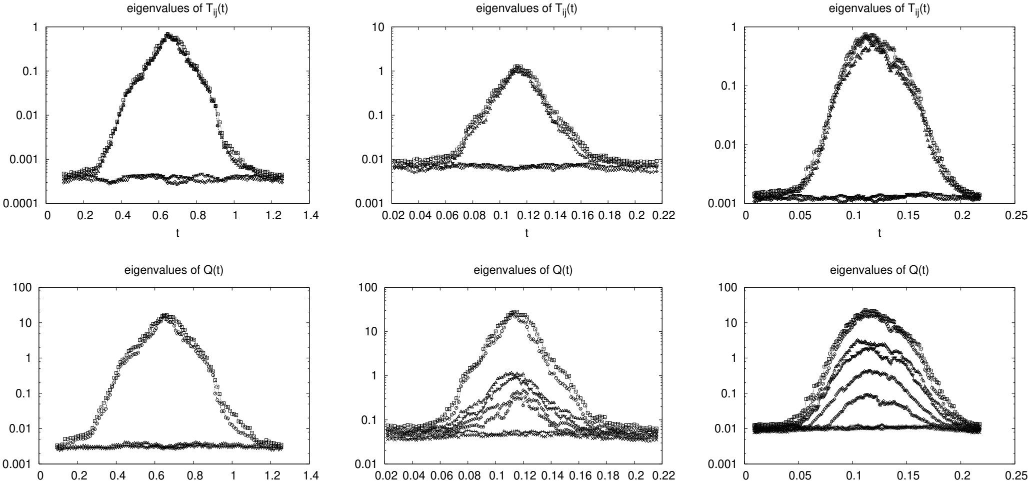
Actual observables are block–averaged like time α:

$$\bar{T}_{ij}(t) = n^{-1}\,\mathrm{tr}_{\mathrm{block}}(\bar{A}_i(t)\bar{A}_j(t)) \text{ and } Q_{ij} = \sum_{i=1}^{\dim\mathcal{X}-1} \bar{A}_i(t)\bar{A}_i(t) \text{ with } t = n^{-1}\,\mathrm{tr}_{\mathrm{block}}\,\alpha$$

and $\bar{A}_i$ denotes the block–submatrices.

From singular to smoother space

Jun Nishimura and Tsuchiya 2019



Lorentz-invariant mass term

Introduce a Lorentz–invariant mass term as a regularization:[18]

$$S_{\text{mass}} = -\imath \frac{1}{2} N\gamma \exp\left(\imath s \frac{\pi}{4}\right)\left(\exp\left(-\imath k\pi\right) \operatorname{tr}(A_0 A_0) - \operatorname{tr}(A_i A_i)\right)$$

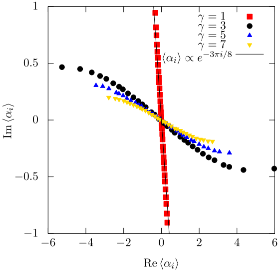
- $\gamma \leq 0$ respects Lorentzian–Euclidean equivalence with no *expanding* classical solutions.
- $\gamma > 0$ has interesting classical solutions with *expanding* space![19]

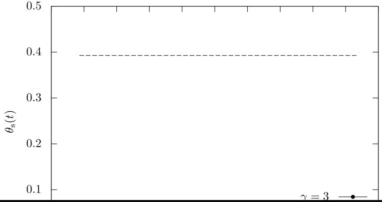
[18] Steinacker 2018
[19] Hatakeyama, Matsumoto, Jun Nishimura, Tsuchiya, and Yosprakob 2020

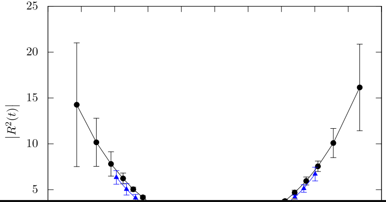
γ phase transition

Jun Nishimura 2022

Anagnostopoulos, Azuma, Hatakeyama, Hirasawa, Ito, Jun Nishimura, Papadoudis, and Tsuchiya 2023

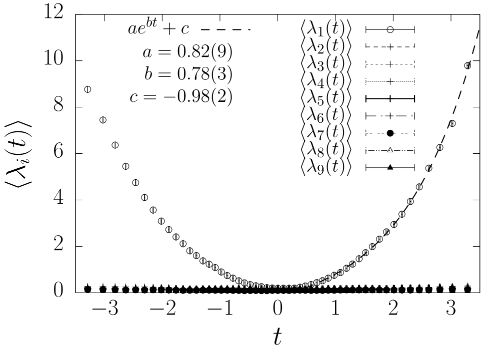




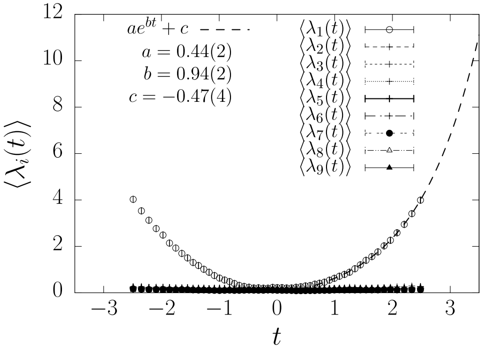


Towards smaller γ

Hirasawa, Anagnostopoulos, Azuma, Hatakeyama, Jun Nishimura, Papadoudis, and Tsuchiya 2022
Hirasawa, Anagnostopoulos, Azuma, Hatakeyama, Jun Nishimura, Papadoudis, and Tsuchiya 2023



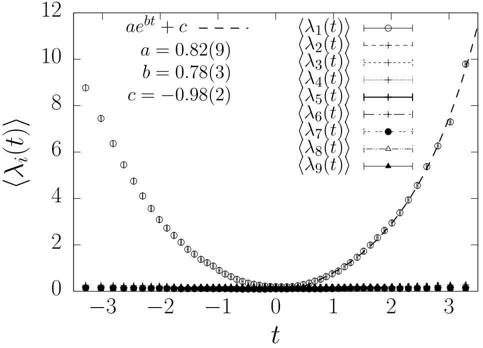
$\gamma = 2.6$ and $m_{\text{fermion}} = 10$



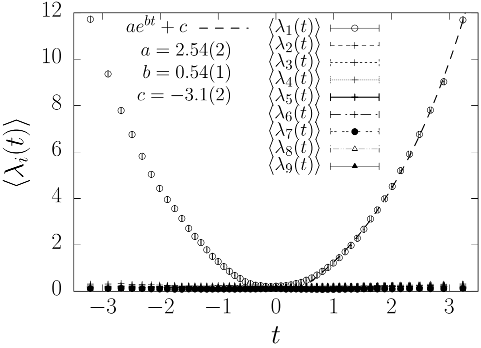
$\gamma = 4.$ and $m_{\text{fermion}} = 10$

Towards smaller m_{fermion}

Hirasawa, Anagnostopoulos, Azuma, Hatakeyama, Jun Nishimura, Papadoudis, and Tsuchiya 2022
Hirasawa, Anagnostopoulos, Azuma, Hatakeyama, Jun Nishimura, Papadoudis, and Tsuchiya 2023



$\gamma = 2.6$ and $m_{\text{fermion}} = 10$



$\gamma = 2.6$ and $m_{\text{fermion}} = 5$

Conclusions

- Euclidean IKKT matri$X \cdot X = h_{ab} X^a X^b$x model exhibits $\mathrm{SO}_{10} \longrightarrow \mathrm{SO}_3$ spontaneous symmetry breaking:
  - Consistent with the GEM result.
  - Finite spacetime dimensions.
- Lorentzian IKKT matrix model equivalent to Euclidean after all:
  - Not, when adding a Lorentz–invariant mass term with $\gamma > 0$.
  - Shows signs of expanding 1 dimensional space (bosonic phase) at $\gamma \geq 2.6$ and $m_{\text{fermion}} \geq 5.0$.
- What is next?
  - 1 dimensional configurations zero pf $\mathcal{M}$, so m_{fermion} needs to decrease further.
  - Apply techniques that allow reducing m_{fermion} further.

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