Quantum Geometry and String Compactifications

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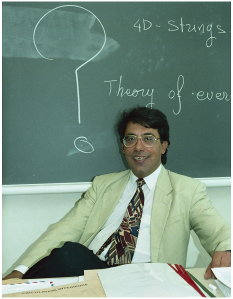
Kounnas Memorial Day, Corfu 2022



Remembering Costas

- I did my PhD under the supervision of Costas between 2001 and 2004.
- Before that I did not know him however I *certainly knew* his formidable voice which was resonating in the corridors of École normale
- As a shy student, I was actually rather impressed by Costas, in particular by some of his expressions that that I would get to know well, like:

  "I hope that your computation is correct, otherwise I will kill you!"

  which was his way to express how he cared about his students, in unique Costas' style.
- Shortly after the beginning of my PhD however, he brought me to this very place, a funding moment in my career, where I discovered the Costas that we all knew:

- How passionnate Costas was about physics
- How much Costas enjoyed having good time with his dear friends

Thanks to Costas, I immediately felt "at home" in our physicists' community.

As an advisor, Costas was certainly not stingy with his time. I remember one Friday evening at 6pm, while I was crossing the corridor to go back home for the week-end, out of the blue I heard Costas' deep voice:

"Dan come here, I will teach you Kähler geometry!"

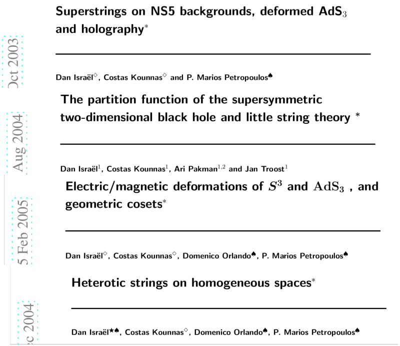
And this is what he did, until very late in the night. And, more generally, notwithstanding how busy he could have been, as soon as I came to his office and started talking phyics, he forgot about everything else.

Costas had a very central role in making me the physicist I am today and I am very indebted to him for that. Some important lessons were:

- Be passionate about what you're doing
- Follow your own path and not the latest fashion
- Follow your physical intuition
- Don't be afraid to cut the gordian knot.
- If you dont' agree with your conversation partner don't hesitate to tell him frankly (even your own advisor, which led to heated discussions!)

The subject of my PhD under Costas supervision was the study of string theory in AdS_3 and related space-times using worldsheet CFT techniques.

- Together Marios we derived the partition function of the $SL(2,\mathbb{R})$ WZW model, a long standing problem, and its marginal deformations. We showed that the null deformation $J^- \bar{J}^-$ is a new decoupling limit interpolating between Little string theory and $D1/D5$ holography.
- With Jan Troost, Ari Pakman and Domenico Orlando we studied more general holographic backgrounds of NS5-branes and F1 strings. We also clarified the interplay between GSO projection and target space geometry.
- These string vacua are very relevant today for holography of $T\bar{T}$ and $J\bar{T}$ deformations.


Oct 2003

Superstrings on NS5 backgrounds, deformed AdS_3 and holography*

Dan Israël◇, Costas Kounnas◇ and P. Marios Petropoulos♠

Aug 2004

The partition function of the supersymmetric two-dimensional black hole and little string theory *

Dan Israël[1], Costas Kounnas[1], Ari Pakman[1,2] and Jan Troost[1]

5 Feb 2005

Electric/magnetic deformations of S^3 and AdS_3 , and geometric cosets*

Dan Israël◇, Costas Kounnas◇, Domenico Orlando♠, P. Marios Petropoulos♠

c 2004

Heterotic strings on homogeneous spaces*

Dan Israël⋆♠, Costas Kounnas◇, Domenico Orlando♠, P. Marios Petropoulos♠

It was a very intense period of my career and of my life, thanks to Costas enthousiam, creativity, vast scientific culture and generosity. Later on he did his best to help for my career and advise me about physics.

It had been a privilege to be one of Costas students, collaborators and friends.

Quantum geometry in the works of Costas

Costas made many profound contributions to string theory, supergravity, QFT and cosmology. I'll focus today on two recuring themes of his work:

- Free-fermion models, where the internal geometry is replaced by a set of free 2d fermions with intricate boundary conditions (see Ignatios' talk)
- Stringy geometry when the supergravity picture is not valid, in particular when T-duality and windings play a crucial role (see Elias' talk).

And the questions I will attempt to partly address are :

- Is there a quantum (non-)geometry framework encompassing free-fermion constructions among more general ones?
- String compactifications on Calabi-Yau manifolds have a quantum symmetry, *mirror symmetry*. To which extent can it play a role the way T-duality does?

➥ No-scale supergravity, one of Costas famous achievements, will be another key ingredient.

Non-geometric Calabi-Yau backgrounds

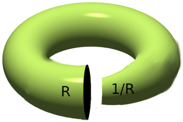
Generalized Scherk-Schwarz reductions

- String theory on compact manifolds: moduli space of vacua

$$\boxed{\mathcal{M} = O(\Gamma)\backslash G/H} \qquad O(\Gamma) \subset G \text{ isometry group of a charge lattice } \Gamma$$

- $O(\Gamma)$ contains "stringy" symmetries as T-dualities
- Could appear in transition functions ➥ T-folds, U-folds,... *(Dabholkar, Hull '02)*
- Fibration over S^1 with (non-geometric) monodromy twist:

$$\phi(x^\mu, y) = e^{\frac{Ny}{2\pi R}} \phi(x^\mu) \; , \; M = e^N \in O(\Gamma)$$



- M of finite order ➥ critical points with Minkowski vacuum
- If M breaks (partially) supersymmetry ➥ Spontaneous SUSY breaking à la Scherk-Schwarz *(Kounnas, Ferrara, Porrati, Zwirner 88)*
- At low-energies, described by gauged supergravity of the no-scale type, a recuring theme in Costas' work

$\mathcal{N}=2$ vacua from type IIA on $K3\times T^2$ *(Hull,D.I., Sarti '17)*

- Type IIA superstrings on $K3 \hookrightarrow \mathcal{M}_6 \to T^2$ fibrations with monodromy twists

Low-energy limit of type IIA on $K3\times T^2$

- $\mathcal{N}=4$ SUGRA in four dimensions
- Field content: SUGRA multiplet $(g_{\mu\nu}, \psi^i_\mu, A^{1,\dots,6}_\mu, \chi^i, \tau)$
  22 vector multiplets $(A^a_\mu, \lambda^a_i, \mathcal{M})$
- Scalars $\mathcal{M}$, τ take value in the coset $\frac{O(6,22)}{O(6)\times O(22)} \times \frac{SL(2)}{O(2)}$

- Moduli space of K3 compactifications $O(\Gamma_{4,20})\backslash O(4,20)/O(4)\times O(20)$

  ➥ Consider monodromies $\boxed{\mathcal{M}\in O(\Gamma_{4,20})\subset O(4,20)}$

- Goal: $\mathcal{N}=4\to\mathcal{N}=2$ *spontaneous SUSY breaking*

Gauged supergravity analysis

- $K3 \times T^2$ with monodromy twists $M_i = e^{N_i} \in O(\Gamma_{4,20})$ along T^2

  ➥ structure constants $\boxed{t_{iI}{}^J = N_{iI}{}^J}$ of $\mathcal{N} = 4$ gauged supergravity
- Encode potential and SUSY breaking mass terms

Vacua with spontaneous SUSY breaking $\mathcal{N} = 4 \ \rightarrow \mathcal{N} = 2$

- Gravitini transform in $(\mathbf{2}, \mathbf{1}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{2}, \mathbf{1})$ of $\{SU(2) \times SU(2) \cong SO(4)\} \times SO(20) \subset O(4, 20)$
- Minkowski vacua from *elliptic* monodromies in $\{SO(4) \times SO(20)\} \cap O(\Gamma_{4,20}) \subset O(4, 20)$
- Half-SUSY vacua from monodromies in $\{SU(2) \times SO(20)\} \cap O(\Gamma_{4,20}) \subset O(\Gamma_{20}) \subset O(4, 20)$

- Such solutions, if any, are necessarily non-geometric (as $K3$ diffeos in $O(3, 19) \subset O(4, 20)$) ➥ mirror-folds?
- Their construction relies on recent works on mirror symmetry of $K3$ surfaces

Non-linear sigma models on K3 and mirrored automorphisms

K3 compactifications: elementary facts

K3-surfaces

- K3 surface X: Kähler 2-fold with a nowhere vanishing holomorphic 2-form Ω
- Inner product on $H^2(X,\mathbb{Z}) : \langle \alpha, \beta \rangle = \int \alpha \wedge \beta \in \mathbb{Z}$
- $H^2(X,\mathbb{Z})$ isomorphic to unique even, unimodular lattice of signature $(3,19)$:
$$\Gamma_{3,19} \cong E_8 \oplus E_8 \oplus U \oplus U \oplus U \;, \qquad U = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$
- Lattice of total cohomology $H^\star(X,\mathbb{Z})$: $\boxed{\Gamma_{4,20} \cong E_8 \oplus E_8 \oplus U \oplus U \oplus U \oplus U}$

Moduli space of Ricci-flat metrics on K3

- Choice of space-like oriented 3-plane $\Sigma = (\Omega, J) \subset \mathbb{R}^{3,19} \cong H^2(X,\mathbb{R})$
- $\boxed{\mathcal{M}_{\text{KE}} \cong O(\Gamma_{3,19}) \backslash\, O(3,19) \,/\, O(3) \times O(19) \;\times \mathbb{R}_+}$

Moduli space of non-linear sigma-models on $K3$

- Choice of metric & B-field $\leftrightarrow$ choice of space-like oriented 4-plane $\Pi \subset \mathbb{R}^{4,20}$
- $\boxed{\mathcal{M}_\sigma \cong O(\Gamma_{4,20}) \backslash\, O(4,20) \,/\, O(4) \times O(20)}$ *(Seiberg, Aspinwall-Morrison)*
- $O(\Gamma_{4,20})$ contains "non-geometric symmetries" as mirror symmetry

Lattice-polarized mirror symmetry

- Picard lattice $S(X) = H^2(X,\mathbb{Z}) \cap H^{1,1}(X) \subset \Gamma_{3,19}$, of signature $(1,\rho-1)$

Polarized K3 surfaces

- Lattice M of signature $(1,r-1)$ with primitive embedding in $S(X)$
  ➥ M-polarized surface (X,M)

Lattice-polarized mirror symmetry *(Dolgachev, Nikulin)*

M-polarized surface (X,M) and $\tilde{M}$-polarized surface $(\tilde{X},\tilde{M})$ LP-mirror if

$$\boxed{\Gamma^{3,19} \cap M^{\perp} = U \oplus \tilde{M}}$$

Reminicent of the complex structure/Kähler moduli exchange of CY$_3$ mirror symmetry.

➥ Relation with physicist's mirror symmetry (Greene-Plesser) $\bar{Q}_R \mapsto -\bar{Q}_R$?

Automorphisms of K3 surfaces and mirror symmetry

- Non-symplectic order p automorphism σ_p: $\sigma_p^\star(\Omega) = e^{\frac{2i\pi}{p}}\Omega$
- Invariant sublattice of $\Gamma_{3,19}$: $S(\sigma_p) \subseteq S(X)$
- Orthogonal complement $T(\sigma_p) = S(\sigma_p)^\perp \cap \Gamma_{3,19}$

Non-symplectic automorphisms and mirror symmetry

- *p-cyclic* K3 surface X: $\boxed{W = w^p + f(x,y,z)} \circlearrowleft \sigma_p : w \mapsto e^{\frac{2i\pi}{p}} w$
- Greene-Plesser mirror $\tilde{X}$: $\boxed{\tilde{W} = \tilde{w}^p + \tilde{f}(\tilde{x},\tilde{y},\tilde{z})/G} \circlearrowleft \tilde{\sigma}_p : \tilde{w} \mapsto e^{\frac{2i\pi}{p}} \tilde{w}$
- Theorem (Artebani et al., Comparin et al., Bott et al.): *The $S(\sigma_p)$-polarized surface X and the $S(\tilde{\sigma}_p)$-polarized surface $\tilde{X}$ are lattice-polarized mirrors.*

Corollary

$T(\tilde{\sigma}_p)$ is the orthogonal complement of $T(\sigma_p)$ in $\Gamma_{4,20}$: $\boxed{T(\tilde{\sigma}_p) \cong T(\sigma_p)^\perp \cap \Gamma_{4,20}}$

$$\Gamma_{4,20} \otimes \mathbb{R} \cong \Big(T(\sigma_p) \oplus T(\tilde{\sigma}_p)\Big) \otimes \mathbb{R}$$

Mirrored K3 automorphisms *(Hull, DI, Sarti' 17)*

Lattice definition

- Let X be a p-cyclic K3 surface, and $\tilde{X}$ its mirror.
- Theorem: one can extend the diagonal action of $(\sigma_p, \tilde{\sigma}_p)$ on $T(\sigma_p) \oplus T(\tilde{\sigma}_p)$ to an action on the whole lattice $\Gamma_{4,20}$.
- This defines a lattice isometry in $O(\Gamma_{4,20})$ associated with the action of a NLSM automorphism $\hat{\sigma}_p$, that we name *mirrored automorphism*.

Intrinsic definition

- Denoting by μ the BH/LP mirror involution, $\boxed{\hat{\sigma}_p := \mu \circ \tilde{\sigma}_p \circ \mu \circ \sigma_p}$

★In other words, a mirrored automorphism is the "gluing" of a Calabi-Yau automorphism and of a automorphism of the mirror Calabi-Yau.

Interesting fact for string compactifications

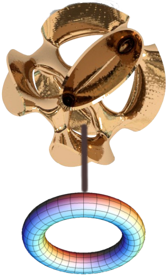
- $T(\sigma_p)$ and $T(\tilde{\sigma}_p)$ of signatures $(2, r)$ and $(2, 20-r)$.
- Action of $\hat{\sigma}_p$ ➥ diagonal space-like $O(2) \times O(2) \subset O(4, 20)$ of order p

Compactifications with mirrored automorphisms twists

K3 fibrations with non-geometric monodromies

Reduction with monodromy twists

- Generalized Scherk-Schwarz reduction of type IIA on $K3 \times T^2$
- Monodromies $\leftrightarrow$ mirrored automorphism $\hat{\sigma}_p \in O(\Gamma_{4,20})$ around each circle
- Leave one gravitino invariant ➥ $\mathcal{N} = 2$ SUSY



★Minima of the effective SUGRA potential: $\mathcal{N} = 2$ Minkowski vacua

➥ non-geometric worldsheet models?

Asymmetric gepner model orbifolds for K3 surfaces

Landau–Ginzburg orbifold for K3 surfaces

- $(2,2)$ QFT in 2d : K3 compactification in small-volume limit
- LG model $\boxed{W = Z_1^{p_1} + Z_2^{p_2} + Z_3^{p_3} + Z_4^{p_4}}$, $K = \text{lcm}(p_1, \ldots, p_4)$
- GSO projection: diagonal $\mathbb{Z}_K$ orbifold $Z_\ell \mapsto e^{2i\pi/p_\ell} Z_\ell$
  ➥ fields in twisted sectors $\gamma = 0, \ldots, K-1$
- Quantum symmetry of LG orbifold: $\sigma_K^{\Omega} : \phi_\gamma \mapsto e^{2i\pi\gamma/K} \phi_\gamma$
- IR fixed point: $\mathcal{N} = (4,4)$ SCFT with $c = \bar{c} = 6$ ➥ Gepner model

A simple class of asymmetric $K3$ Gepner models *(Intriligator&Vafa'90,DI'15)*

- $\boxed{\sigma_{p_1} : Z_1 \mapsto e^{2i\pi/p_1} Z_1}$ orbifold ➥ field $(Z_1^{n_1} \cdots)$ has charge $Q_{p_1} \equiv \frac{n_1}{p_1}$ mod 1

★ Project w.r.t. shifted $\mathbb{Z}_{p_1}$ orbifold charge: $\hat{Q}_{p_1} = Q_{p_1} + \frac{\gamma}{p_1}$ ➥ discrete torsion

- Interpretation: order p subgroup of the quantum symmetry group

  $\boxed{\sigma_{p_1}^{\Omega} := (\sigma_K^{\Omega})^{K/p_1}}$ ➥ γ-tw. sector field has charge $Q_{p_1}^{\Omega} \equiv \frac{\gamma}{p_1} \mod 1$

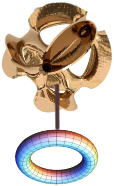
- Discrete torsion restores half of space-time SUSY, from left-movers only

Correspondence

- Consider a freely acting orbifold of $K3 \times T^2$ where $K3$ is a Gepner model :
  - shift on S^1 with σ_{p_1} on the Gepner model
  - shift on $\tilde{S}^1$ with σ_{p_2} on the Gepner model

  } both with discrete torsion
- σ_p and $\sigma_p^{\Omega} := (\sigma^{\Omega})^{K/p}$ exchanged by mirror symmetry $(\bar{Q}_R \mapsto -\bar{Q}_R)$
- $K3$ orbifold with discrete torsion: projection $Q_{p_1} + Q_{p_1}^{\Omega} \in \mathbb{Z}$

  ➥ Corresponds to the diagonal action of $(\sigma_{p_1}, \tilde{\sigma}_{p_1})$



★Therefore we have identified the $\mathcal{N} = 2$ Minkowski minima of compactifications with mirrored automorphisms twists

Main features

Supersymmetry breaking

- All space-time supercharges from left-movers ➥ non-geometric
- No massless Ramond-Ramond states

Low-energy 4d theory

- $\mathcal{N}=2$ vacua of $\mathcal{N}=4$ gauged SUGRA
- Axio-dilaton and torus moduli in vector multiplets ➥ $\mathcal{N}=2$ STU SUGRA
- Surviving K3 moduli (if any): hypermultiplets

Heterotic dual *(Gautier, Hull, Israel '19)*

- Non-perturbative heterotic dual: asymmetric toroidal orbifold
- Modular invariance: winding shift ↔ non-perturbative in type II (NS5 charge)

Moduli space *(Gautier, Israel '20)*

- Heterotic perturbative corrections to the vector multiplet prepotential
- Hypermultiplets moduli space is α' and g_s exact

Conclusions

- Non-geometric compactifications of superstring theory may be the most generic ones yet poorly understood
- New symmetries of CY compactifications: mirrored automorphisms
- Non-geometric compactifications based on CY geometries ➥ "mirrorfolds"
- New type II/heterotic $\mathcal{N}=2$ dual pairs and their quantum moduli space
- Direct relation to free-fermion constructions of Ignatios, Costas and Costas, some of them corresponding to "small levels" Gepner models
- Described at low-energy by gauged supergravity with spontaneous SUSY breaking, one of Costas' recurring subjects of research
- Some open questions and work in progress:
  1. Insights on NS5-brane winding shifts in the type IIA frame
  2. CY_3-based constructions ➥ $\mathcal{N}=1$ type II vacua without RR fluxes
  3. Non-Abelian gauge groups from non-perturbative effects
- For this and other projects I will certainly continue to be influenced by Costas' legacy.