Precise calculation of the W boson pole mass beyond the Standard Model

Corfu 2022

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arXiv:2204.05285

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Motivation: new CDF result

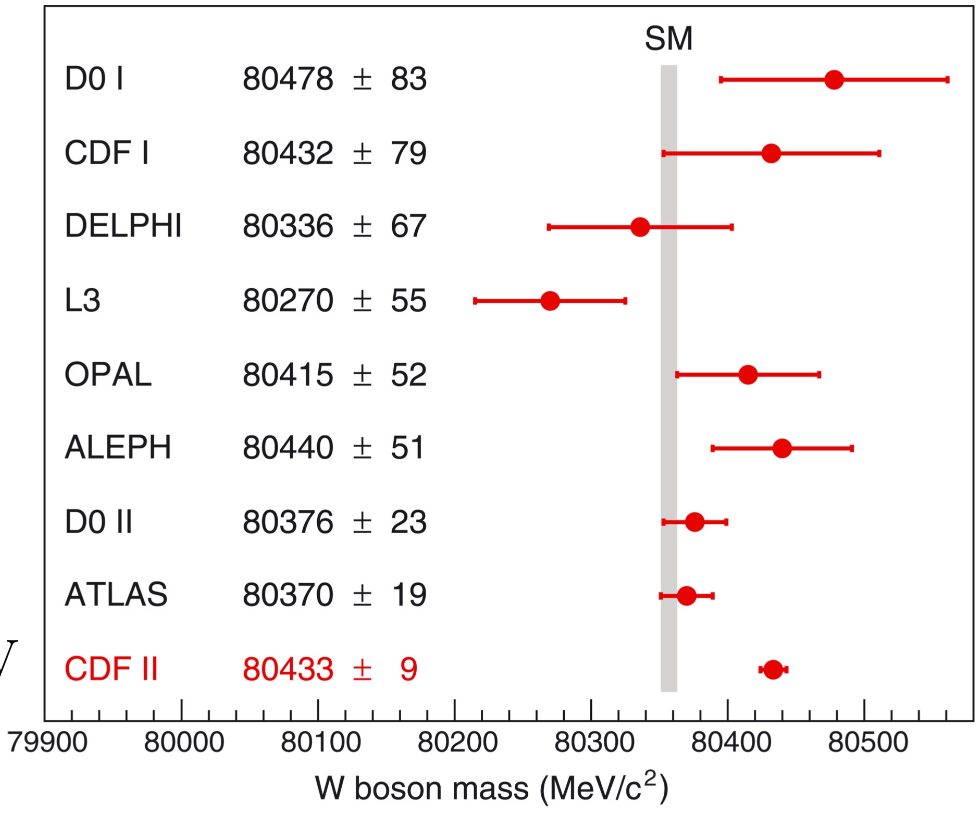
- CDF 2022 result

$$M_W^{\text{CDF}} = (80.4335 \pm 0.0094)\ \text{GeV}$$

- World average without new CDF result

$$M_W^{2021} = (80.379 \pm 0.012)\ \text{GeV}$$

- SM prediction:
  - $M_W^{\text{SM, OS}} = (80.355 \pm 0.006)\ \text{GeV}$
  - $M_W^{\text{SM},\overline{\text{MS}}} = (80.351 \pm 0.004)\ \text{GeV}$
  - $M_W^{\text{SM, fit}} = (80.3591 \pm 0.0052)\ \text{GeV}$



Side note: Why do we care about M_W at all?

- In principle there is nothing special about M_W as an observable.
- SM is a renormalizable theory. We need to fix 3 parameters: g_1, g_2, v so we only need 3 observables and we can choose them as we wish. You could also take M_W as an input.
- You cannot swipe the discrepancy under the carper --- if you take CDF result as an input than you'll see the problem somewhere else
- For practical reasons, we treat it as an output

$$\alpha = 7.2973525693(11)$$

$$M_Z = (91.1876 \pm 0.0021)\ \text{GeV}$$

$$G_F = 1.1663787(6) \times 10^5\ \text{GeV}^{-2}$$

$$M_W^{2021} = (80.379 \pm 0.012)\ \text{GeV}$$

How to compute m_W

- Usually something like this

$$m_W = \frac{1}{2} g_2 v + \text{quantum corrections}$$

- For technical and precision reasons the actual setup is more complicated. Start with QED-like EFT

$$\mathcal{L} = \mathcal{L}_{QED} - \frac{4G_F}{\sqrt{2}} (\bar{e}\gamma^\mu P_L \nu_e)(\bar{\nu}_\mu \gamma_\mu P_L \mu) + h.c.$$

experiment

$$\frac{1}{\tau_\mu} = \frac{G_F^2 m_\mu^5}{192\pi^3} F(\rho) \left[1 + H_1(\rho)\frac{\alpha(m_\mu)}{\pi} + H_2(\rho)\frac{\alpha^2(m_\mu)}{\pi^2}\right]$$

$$\rho = \frac{m_e^2}{m_\mu^2}$$

G_F (Fermi constant)

theory

$$G_F = G_F(\text{SM inputs}, \text{BSM params. inc. } m_W)$$

↓

$$M_W$$

Fermi constant in the SM

- Including quantum corrections, you can write the result as

$$\frac{G_\mu}{\sqrt{2}} = \frac{\pi\hat{\alpha}}{2\hat{s}_W^2 m_W^2} \frac{1}{1-\Delta\hat{r}_W}$$

  which also correctly resums leading 2-loop **SM** corrections
  [G. Degrassi, S. Fanchiotti and A. Sirlin, Nucl. Phys. B 351 (1991) 49]
- Solving for M_W

$$m_W^2 = \frac{1}{2} m_Z^2 \hat{\rho} \left[1 + \sqrt{1 - \frac{4\pi\hat{\alpha}}{\sqrt{2} G_\mu m_Z^2 \hat{\rho} (1-\Delta\hat{r}_W)}} \right]$$

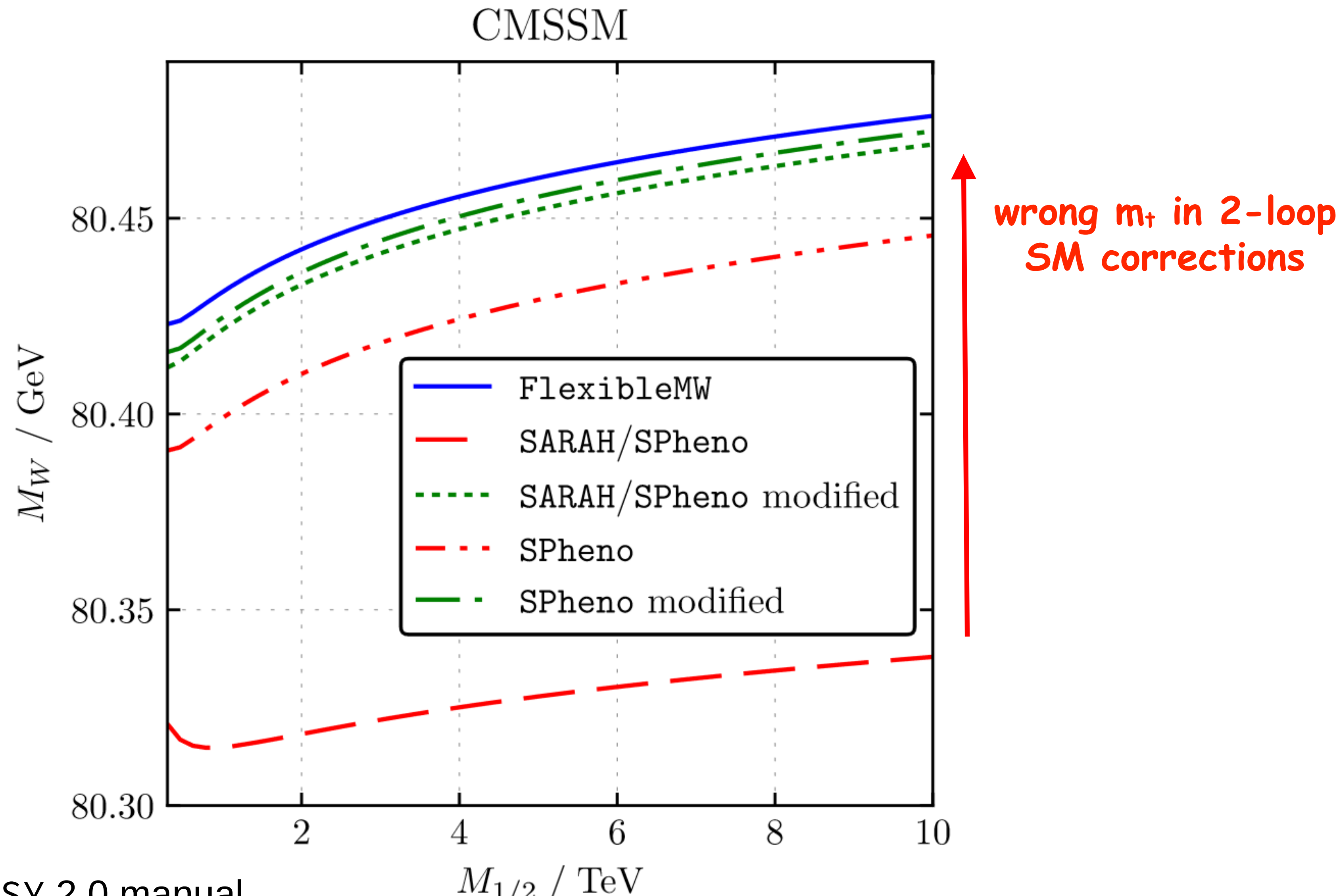
  where

$$\hat{\rho} = \frac{m_W^2}{m_Z^2 \hat{c}_W^2}$$

Spectrum generators

- For popular models that calculation have been done and packaged into computer programs
- Whenever you use a code that computes a particle spectrum from Lagrangian parameters for a given model you inevitably compute M_W along the way
- Examples:
  - MSSM: **FeynHiggs**, **Spheno**, **SoftSUSY**
  - Arbitrary models: **SARAH/Spheno**, **FlexibleSUSY**
- Historically, most SUSY spectrum generators follow the **BMPZ** paper [arXiv:9606211]
- Caveats:
  - When BMPZ wrote there work, no-one cared about heavy SUSY
  - The precision of evaluation of M_W might have been sufficient to compute SUSY spectrum, but maybe not enough to use it PEWO

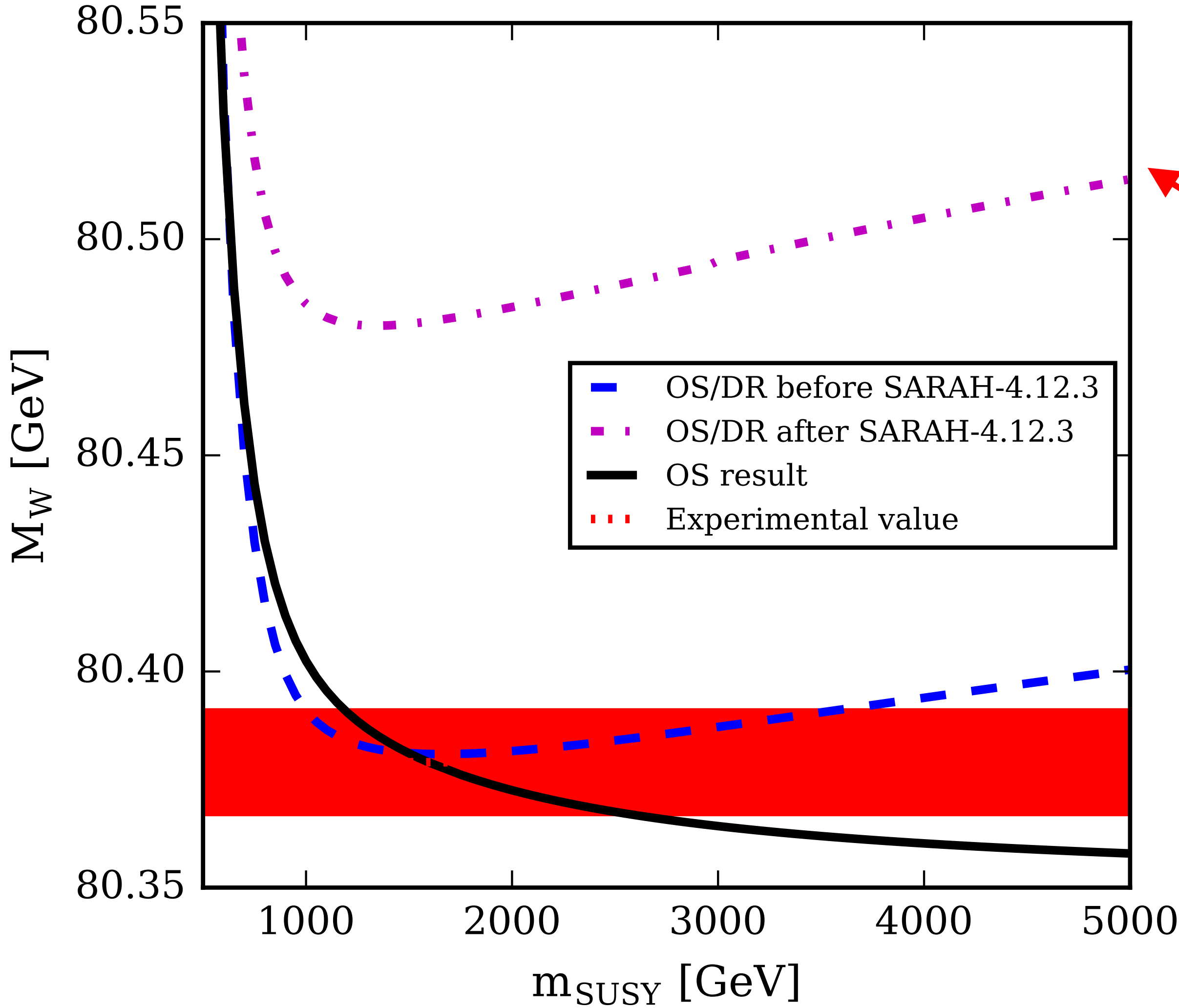
Curious case of M_W calculations in spectrum generators



FlexibleSUSY 2.0 manual [arXiv:1710.03760]

Importance of decoupling

Diessner & Weigelin [arXiv:1904.03634]



result of mixed OS/DR calculation grows with mass scale

M_W calculation with the decoupling property

- Treat SM exactly, keep BSM at exactly 1-loop

$$M_W^2 = (M_W^{\rm SM})^2\,(1+\Delta_W)$$

where

$$\Delta_W = \frac{s_W^2}{c_W^2 - s_W^2}\left[\frac{c_W^2}{s_W^2}\,(\Delta\hat{\rho}_{\rm tree} + \Delta\hat{\rho}_{\rm BSM}) - \Delta\hat{r}_{W,{\rm BSM}} - \Delta\alpha_{\rm em}^{\rm BSM}\right]$$

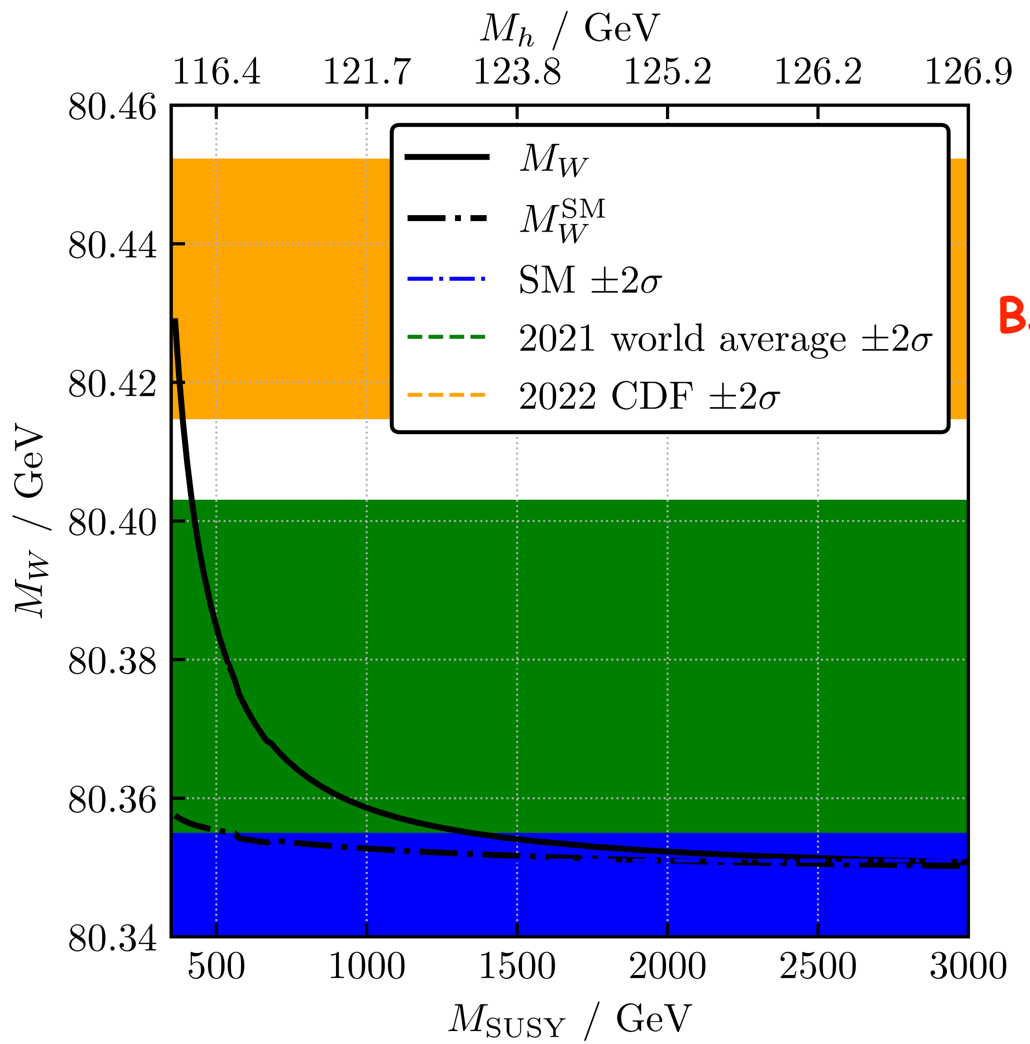
- $\Delta\hat{\rho}_{\rm tree}$, $\Delta\hat{\rho}_{\rm BSM}$, $\Delta\hat{r}_{W,{\rm BSM}}$ and $\Delta\alpha_{\rm em}^{\rm BSM}$ are strict 1-loop quantities --- e.g.

$$\Delta\hat{\rho}_{\rm BSM} = \frac{\hat{\rho}_{\rm BSM}}{\hat{\rho}_{\rm SM}} - 1 = \frac{1}{m_Z^2}\left[\Sigma_Z(m_Z^2) - \Sigma_Z^{\rm SM}(m_Z^2)\right] - \frac{1}{m_W^2}\left[\Sigma_W(m_W^2) - \Sigma_W^{\rm SM}(m_W^2)\right]$$

- The bracket [...] scales like $1/m^2$. The prefactor has still a non-decoupling property

$$s_W^2 c_W^2 = \frac{\pi\,\alpha_{\rm em}(M_Z)}{\sqrt{2}\,M_Z^2\,G_F\,\hat{\rho}_{\rm tree}\,(1-\Delta\hat{r})}$$

Example of decoupling (MSSM)



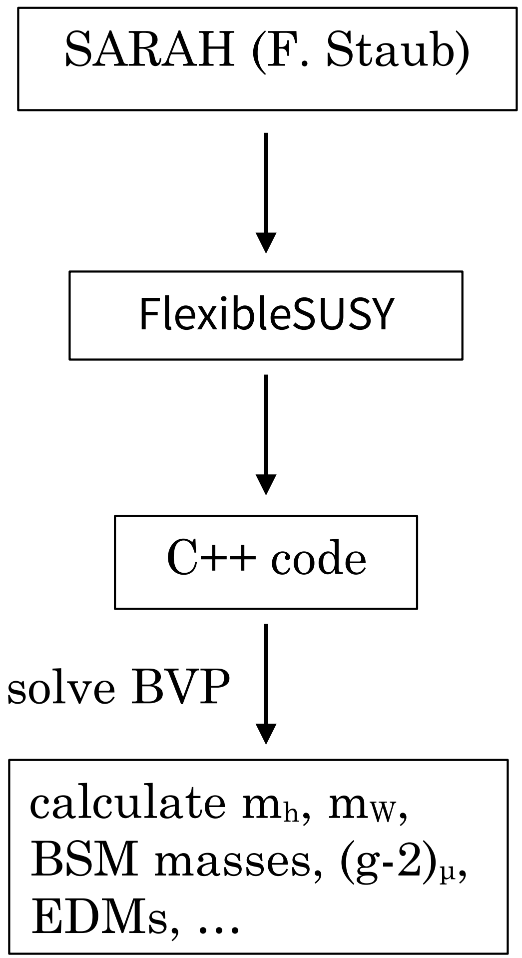
SM mW for a given Higgs boson mass

BSM prediction converges to SM the prediction with the same Higgs Boson mass

FlexibleSUSY in a nutshell

- FlexibleSUSY is a spectrum-generator generator. But what does it mean?
- There are codes like 2HDMC, SPheno, SOFTSUSY or SuSpect that calculate mass spectra and various observables for a predefined model (THDM in case of 2HDMC and MSSM/NMSSM in remaining cases).
- FlexibleSUSY creates a code analogue to such programs but for an arbitrary BSM model.
- Use known results for a generic QFT. Don't recalculate what you don't have to from the ground.
- Streamlining study of BSM phenomenology, reducing time needed to study a new model from years to weeks. No hand written code, less place for errors.

Program flow



- Analytic calculation: particle content + Lagrangian ⇒ tadpole equations, self-energies, mass matrices, RGEs, vertices etc.
- Creates code for numerical evaluation of various observables

Observables

- 1-loop masses and mixing matrices (higher order corrections are available for specific particles and/or in specific models)
- Hybrid fixed order/EFT calculation of Higgs mass ensuring high precision even for a heavy BSM physics
- muon g-2, lepton's EDMs, $l \to l'\gamma$, $b \to s\gamma$, scalar decays, electron g-2 (private/on request), other LFV observables ($l \to 3l'$, $l \to l'$ conversion in nuclei etc, private/on request)
- New calculation of M_W (this talk)
- `HiggsTools` (a successor to `HiggsBounds` and `HiggsSignals`) interface (still private)

R-symmetry

- R-symmetry is an additional symmetry of the SUSY algebra allowed by the Haag - Łopuszański - Sohnius theorem
- For N=1 SUSY it is a global $U(1)_R$ symmetry under which the SUSY generators are charged
- implies that the spinorial coordinates are also charged

$$Q_R(\theta) = 1,\ \theta \to e^{\imath\alpha}\theta$$

- Lagrangian invariance
  - Kähler potential K term is automatically invariant
  - R-charge of the superpotential W must be 2

$Q_R(d^2\theta)$=-2

$Q_R(\mathscr{L})$=0 $\longrightarrow$ $\mathcal{L} \ni \int d^2\theta\, W$

$Q_R(W)$=+2

  - soft-breaking terms must have R-charge 0

Low-energy R-symmetry realization

- Charges of component fields

$$\begin{array}{ccccccc} e^{\imath\alpha Q_R} & & e^{\imath\alpha Q_R} & & e^{\imath\alpha(Q_R-1)} & & \\ \Phi & = & \phi(y) & + & \sqrt{2}\theta\psi(y) & + & \theta\theta F(y) \end{array}$$

- "Natural" choice

Higgs	$Q_R = 1$	$Q_R = 1$	$Q_R = 0$
leptons and quarks	$Q_R = 0$	$Q_R = 0$	$Q_R = -1$

- Good: no barion and lepton number violating terms
- Bad: No Majorana masses for higgsinos and gauginos

One way to fix it: Dirac masses

Minimal R-Symmetric Supersymmetric Standardmodel (MRSSM)

Kribs et.al. arXiv:0712.2039

			$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	$U(1)_R$
Additional fields:	Singlet	$\hat{S}$	1	1	0	0
	Triplet	$\hat{T}$	1	3	0	0
	Octet	$\hat{O}$	8	1	0	0
	R-Higgses	$\hat{R}_u$	1	2	$-1/2$	2
		$\hat{R}_d$	1	2	$1/2$	2

$$\begin{aligned} W =& \mu_d \hat{R}_d \hat{H}_d + \mu_u \hat{R}_u \hat{H}_u \\ &+ \Lambda_d \hat{R}_d \hat{T} \hat{H}_d + \Lambda_u \hat{R}_u \hat{T} \hat{H}_u + \lambda_d \hat{S} \hat{R}_d \hat{H}_d + \lambda_u \hat{S} \hat{R}_u \hat{H}_u \\ &- Y_d \hat{d} \hat{q} \hat{H}_d - Y_e \hat{e} \hat{l} \hat{H}_d + Y_u \hat{u} \hat{q} \hat{H}_u \end{aligned}$$

MSSM vs. MRSSM

- MSSM superpotencial

$$\mu \hat{H}_u \hat{H}_d$$

$$-Y_d\, \hat{d}\, \hat{q}\, \hat{H}_d - Y_e\, \hat{e}\, \hat{l}\, \hat{H}_d + Y_u\, \hat{u}\, \hat{q}\, \hat{H}_u$$

- MSSM soft-SUSY breaking terms
  - B_μ - term
  - soft scalar masses
  - Majorana gaugino masses
  - A - terms

- MRSSM superpotencial

$$\mu_d\, \hat{R}_d\, \hat{H}_d + \mu_u\, \hat{R}_u\, \hat{H}_u$$

$$-Y_d\, \hat{d}\, \hat{q}\, \hat{H}_d - Y_e\, \hat{e}\, \hat{l}\, \hat{H}_d + Y_u\, \hat{u}\, \hat{q}\, \hat{H}_u$$

$$\Lambda_d\, \hat{R}_d\, \hat{T}\, \hat{H}_d + \Lambda_u\, \hat{R}_u\, \hat{T}\, \hat{H}_u + \lambda_d\, \hat{S}\, \hat{R}_d\, \hat{H}_d + \lambda_u\, \hat{S}\, \hat{R}_u\, \hat{H}_u$$

- MRSSM soft-SUSY breaking terms
  - B_μ - term (though no B_{μ_u}, B_{μ_d})
  - soft scalar masses
  - Dirac gaugino masses
  - no A-terms

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R-symmetry vs. matter parity

- Consider R-symmetric transformation of a generic supermultiplet

$$R : \Phi(x, \theta, \bar{\theta}) \to \Phi'(x, e^{\imath\varphi}\theta, e^{-\imath\varphi}\bar{\theta}) = e^{\imath\varphi R_\Phi}\Phi(x, \theta, \bar{\theta})$$

- In the MSSM one imposes the so-called matter parity

$$M_p = (-1)^{3(B-L)}$$

  - this is equivalent to R-pairity which is defined on components of a supermultiplet as $P_R = (-1)^{3(\mathrm{B}-\mathrm{L})+2s}$
  - This is also equivalent to R-symmetry $R = e^{\imath\varphi R_\Phi}$ with $\varphi = \pi$ and $R_\Phi = 3(B - L)$

- R-charges
  - MSSM: $R_\Phi = 0, 1$
  - MRSSM: $R_\Phi = 0, 1, 2$
- R-symmetry is more restrictive than matter parity

Particle content summary: MSSM vs. MRSSM

different number of physical state

completely new states

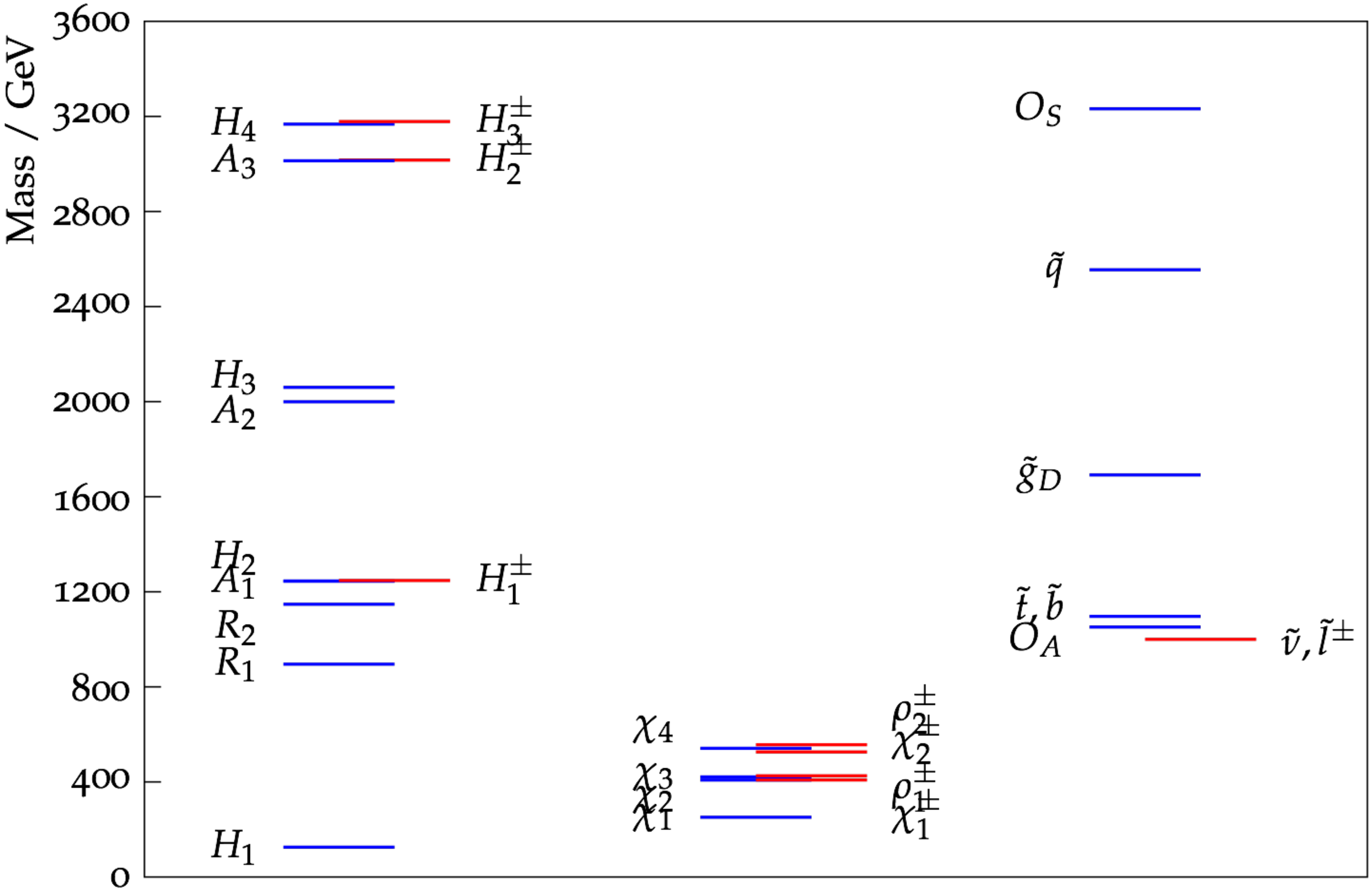
		Higgs			R-Higgs		
	CP-even	CP-odd	charged	charginos	neutral	charged	sgluon
MSSM	2	1	1	2	0	0	0
MRSSM	4	3	3	2+2	2	2	2

	neutralino	gluino
MSSM	4	1
MRSSM	4	1

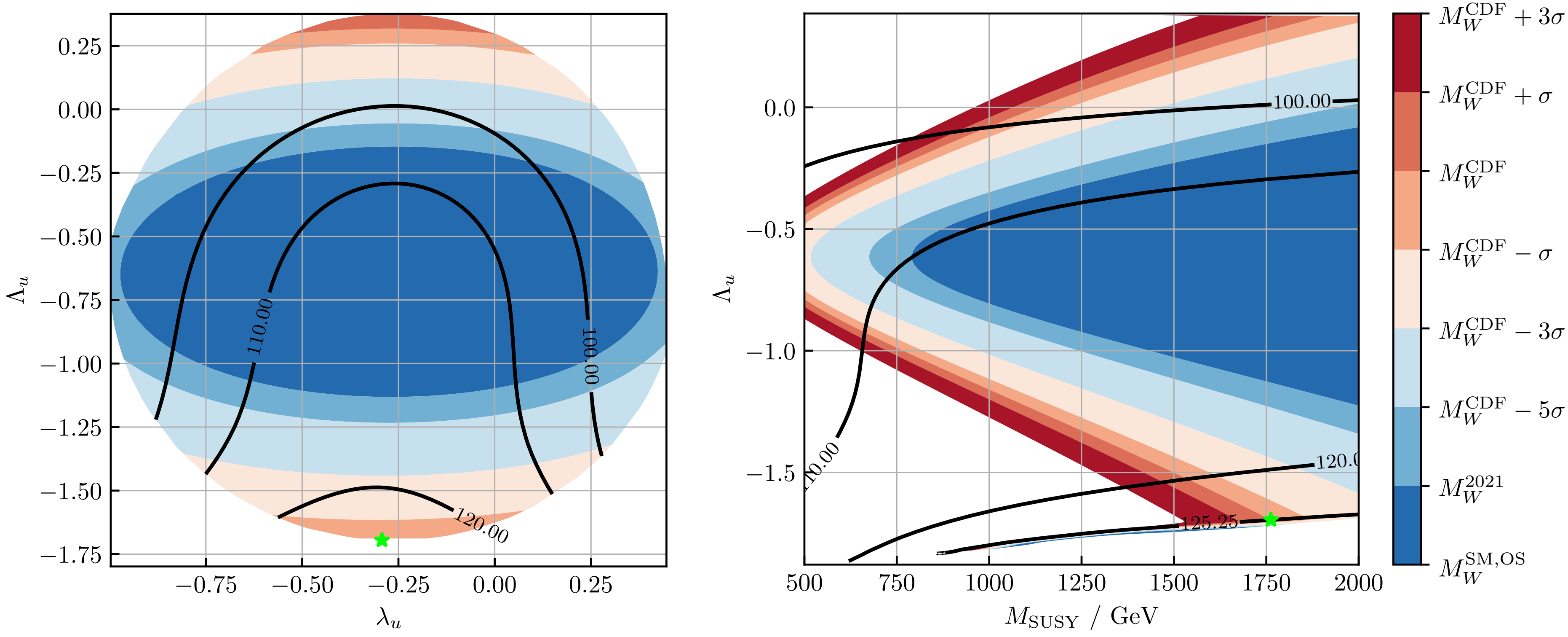
Majorana fermions

Dirac fermions

Exemplary mass spectrum

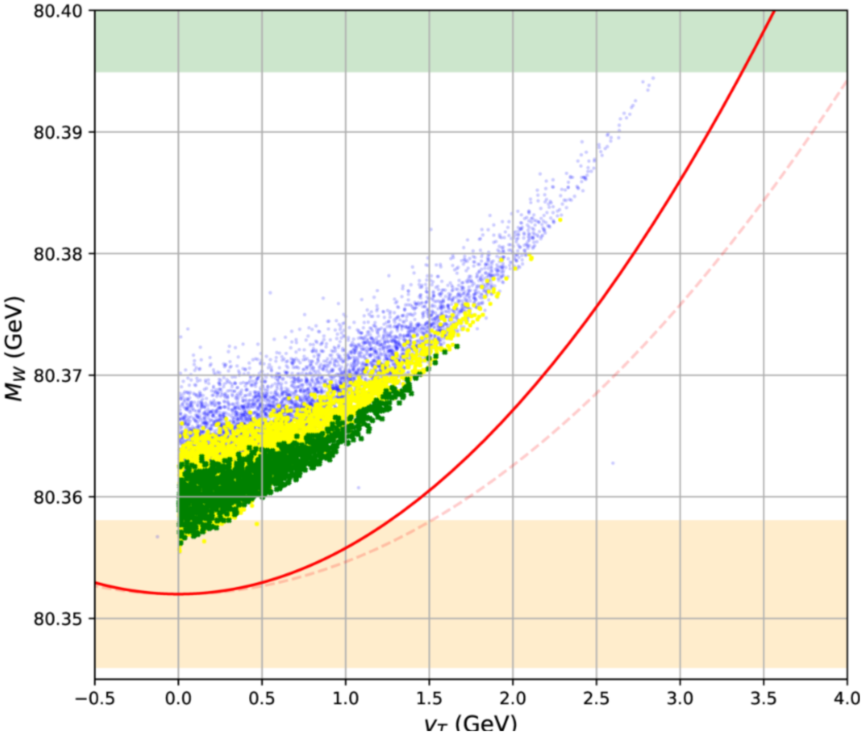


CDF 2022 excess in the MRSSM



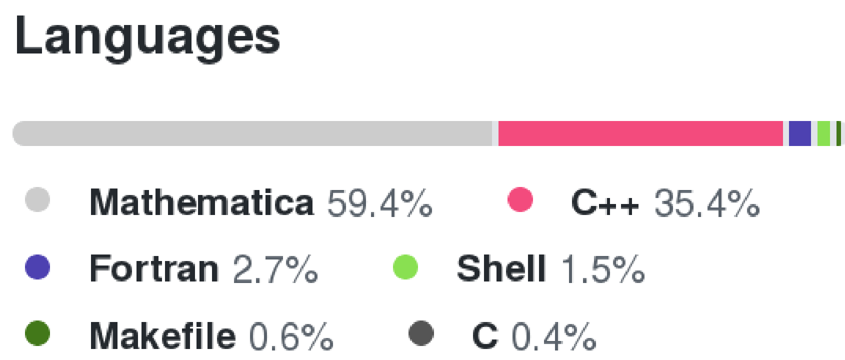
M_W calculation in SARAH/SPheno

- Recently Benakli, Goodsell, Ke and Slavich presented their own implementation of a decoupling calculation [arXiv:2208.05867] which is now part of SARAH 4.15.0
- Their approach follows closely ours



FlexibleSUSY development and support

- The code is written exploiting relatively modern features of C++ (C++14) with use of template metaprograming to reduce runtime overhead



- Development is done in public on github

- Large collection of unit tests, triggered by every commit to the main repository

- In case of any problems, please file an issue. We'll be happy to assist you.

Conclusions and outlook

- Fully automated, state-of-the-art prediction of W boson mass in an (almost) arbitrary BMS model
- You can get `FlexibleSUSY` from github (current version is *2.7.1*). Send me a message if you have any problems.

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Be quick to use it before the CDF excess goes away ;p