Renormalization group and cMERA

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gauge theory and related physical models

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Emergence of space-time

Large N reduction

matrix models for noncritical strings

matrix models for superstring theory

AdS/CFT correspondence (gauge/gravity correspondence)

Emergence of space-time seems natural in quantum gravity,
since the space-time itself fluctuates in quantum gravity.

AdS/CFT correspondence

Maldacena (1997)

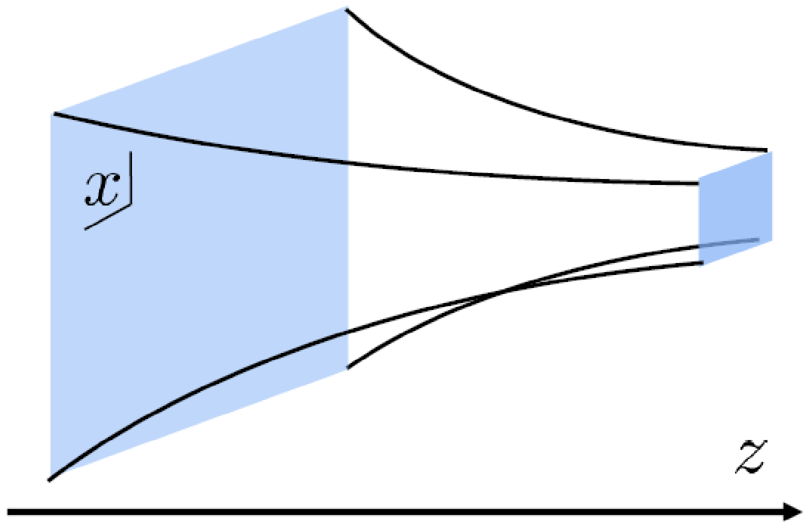
CFT in (d+1)-dim. ⟺ (Quantum) gravity on AdS_{d+2}

large N and strong coupling ⟺ classical gravity

$z = \epsilon$

UV

ϵ : lattice spacing



$z \gg 1$

IR

$$ds^2 = \frac{dz^2 + (dx^\mu)^2}{z^2}$$

$$x^\mu \to \rho x^\mu$$

$$z \to \rho z$$

z : scale for renormalization group

Susskind, Witten (1998), de Boer, Verlinde, Verlinde (2000)

Emergence of bulk geometry

In AdS/CFT, bulk geometry emerges from d.o.f. of quantum field theory

Can we derive (quantum) gravity directly from d.o.f. of quantum field theory?

Quantum entanglement is expected to give an answer or a hint to this question

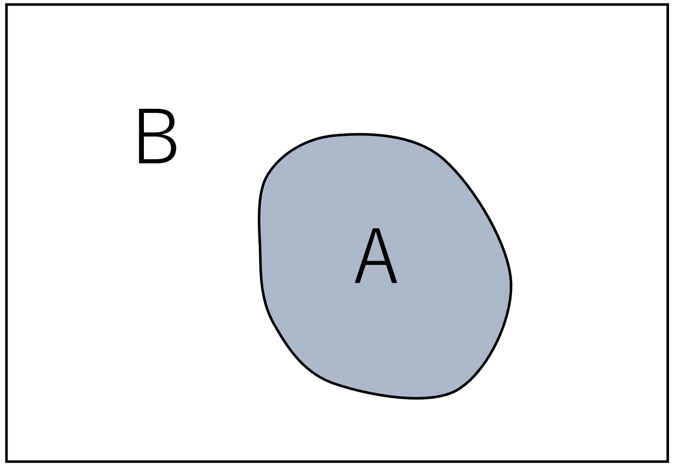
Entanglement entropy (EE)

- Definition of EE

$$\mathcal{H} = \mathcal{H}_{\rm A} \otimes \mathcal{H}_{\rm B}$$

$$\rho_{\rm A} = {\rm Tr}_{\rm B}\ \rho_{\rm tot}$$

$$S_{\rm A} = -{\rm Tr}_{\rm A}(\rho_{\rm A} \log \rho_{\rm A})$$



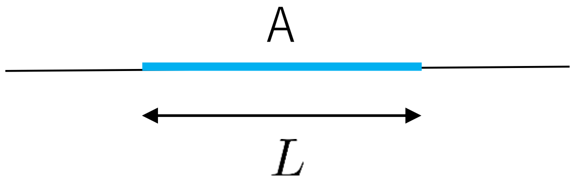
- EE in quantum field theory

d : space dimensionality

$d \geq 2$ $\quad S_{\rm A} \propto \frac{|\partial {\rm A}|}{\epsilon^{d-1}}$

$|\partial {\rm A}|$: area of boundary $\quad \epsilon$: UV cutoff

$d = 1$ $\quad S_{\rm A} = \frac{c}{3} \log L$



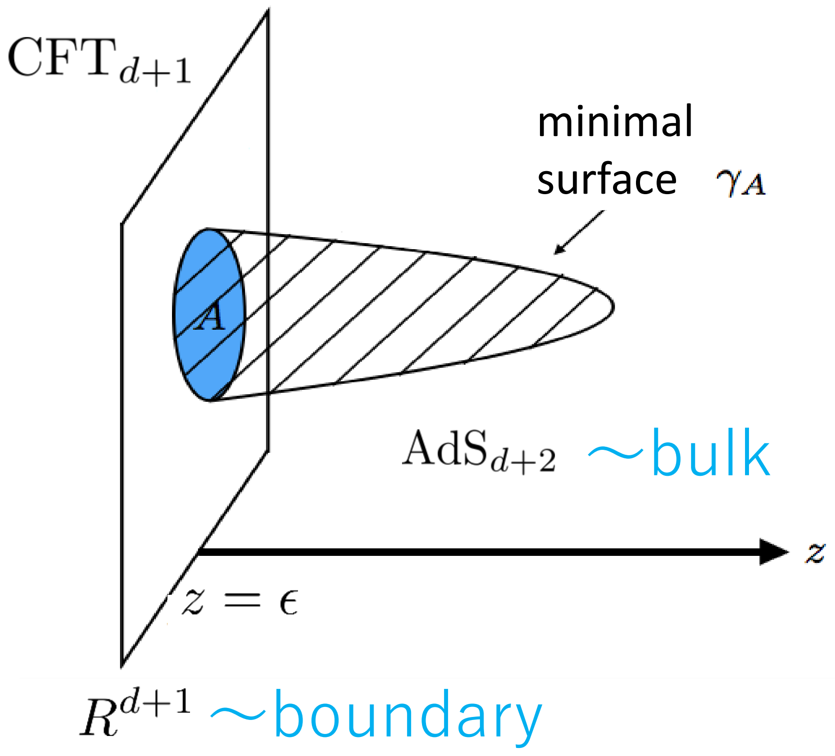
Quantum entanglement and geometry

- Ryu-Takayanagi (RT) formula in AdS/CFT (2006)

$$S_{\mathrm{A}} = \frac{\mathrm{Min}(\mathrm{area}(\gamma_{\mathrm{A}}))}{4G_{\mathrm{N}}}$$

S_{A} : EE for the region A in CFT on the boundary

G_{N} : Newton constant



Quantum entanglement and emergence of space-time

- Understanding the relationship between emergence of space-time and quantum entanglement allows us to gain insights into the construction of quantum gravity.
- The cMERA, a continuum counterpart of the MERA, is expected to realize the emergence of space-time through quantum entanglement.
- While it is obtained successfully based on the variational method (VM) in free field theory, it is quite nontrivial to construct in interacting field theory, which is relevant in the context of AdS/CFT correspondence.
- Here we propose an approach based on the renormalization group to the cMERA in interacting field theory.

Contents

Matrix product state

By using tensor network, we can efficiently construct a trial wave function in quantum system

Ex.) spin chain consisting of n spins in 1d

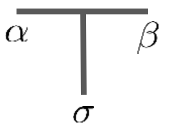
$$|\Psi\rangle = \sum_{\sigma_1,\sigma_2,\ldots,\sigma_n} \Psi(\sigma_1,\sigma_2,\ldots,\sigma_n)\,|\sigma_1,\sigma_2,\ldots,\sigma_n\rangle$$

d.o.f. s^n

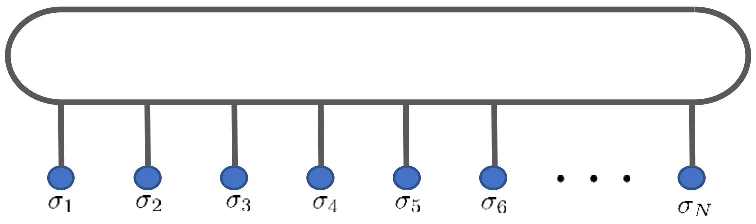
matrix product state

$$\Psi(\sigma_1,\sigma_2,\ldots,\sigma_n) = \mathrm{tr}[M^{(1)}(\sigma_1)M^{(2)}(\sigma_2)\ldots M^{(n)}(\sigma_n)]$$

$M^{(i)}_{\alpha\beta}(\sigma)$: $\chi \times \chi$ matrix

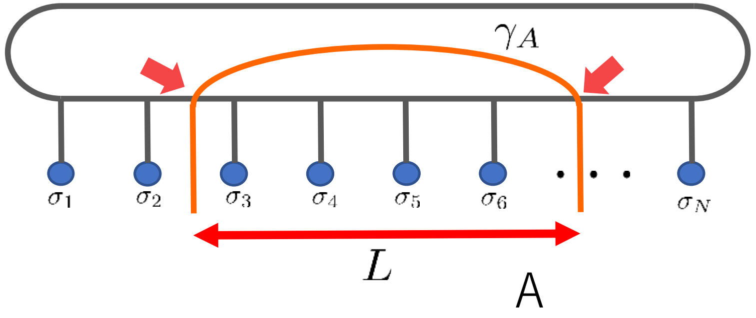


d.o.f. $ns\chi^2$



determined by VA

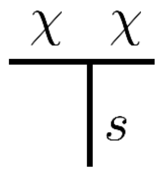
EE in matrix product state



γ_A : curve connecting two ends of A

$\#\mathrm{bonds}(\gamma_\mathrm{A})$: # of bonds crossed by γ_A

χ : bond dimension



bond represents correlation b/w tensors

➡ each bond contributes to EE at most by $\log \chi$

➡ $S_\mathrm{A} \leq \underbrace{\min(\#\mathrm{bonds}(\gamma_\mathrm{A}))}_{=2} \log \chi$

➡ $S_A \leq 2 \log \chi$ Cf.) EE in quantum critical system in 1d $S = \frac{c}{3} \log L$

➡ matrix product state does not reproduce EE in quantum critical system in 1d

Tree-structure tensor network

Tree-structure tensor network (TTN)

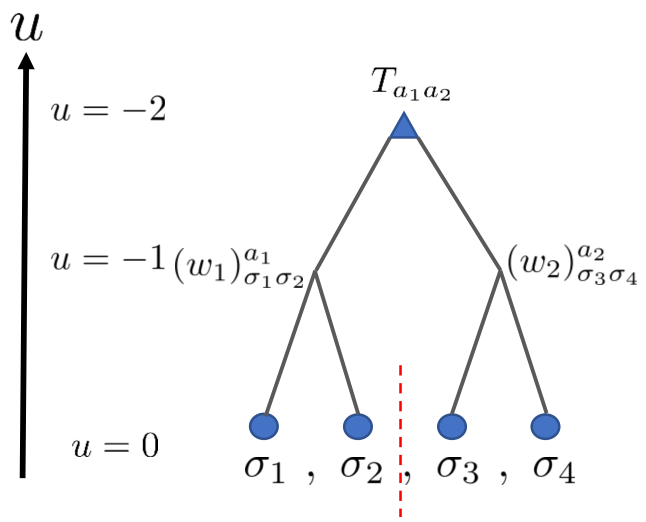
Ex.) spin-chain consisting of 4 spins in 1d

$$|\Psi_1\rangle = \sum_{\sigma_1,\sigma_2,\sigma_3,\sigma_4} \sum_{a_1,a_2} T_{a_1 a_2} (w_1)^{a_1}_{\sigma_1\sigma_2} (w_2)^{a_2}_{\sigma_3\sigma_4} |\sigma_1,\sigma_2,\sigma_3,\sigma_4\rangle$$

$$= \sum_{a_1,a_2} T_{a_1 a_2} |a_1\rangle \otimes |a_2\rangle$$

tensors T, w_1 and w_2 called isometry
merge two spins into one spin

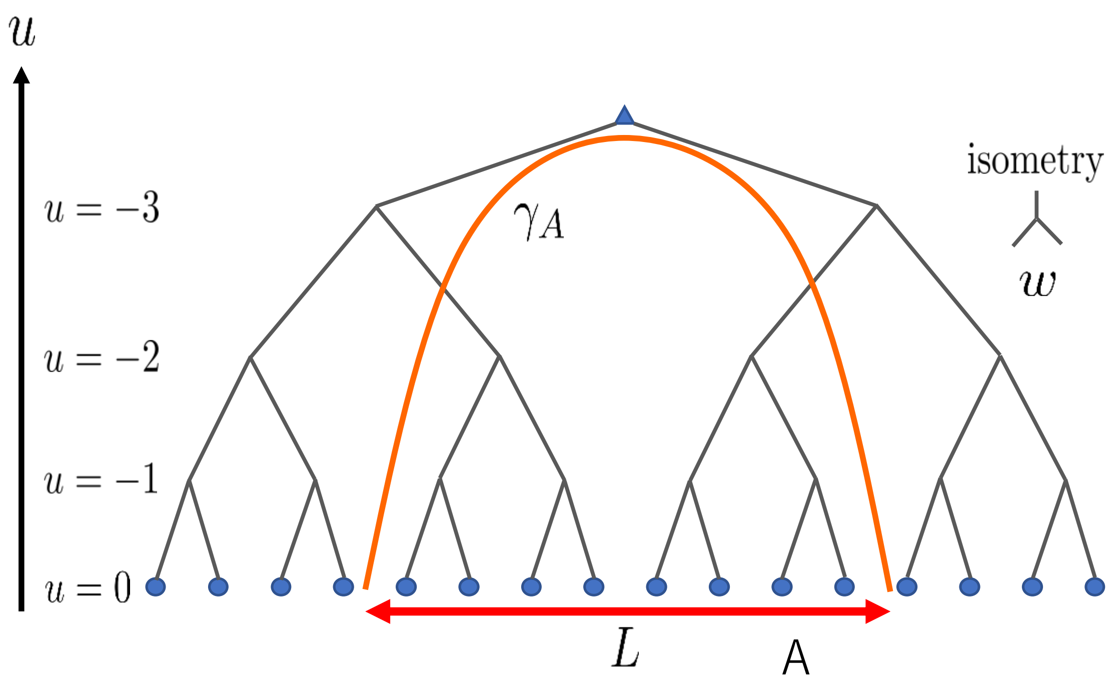
tensors are determined by VM



If layers parametrized by u are regarded as scale,
TTN looks like RG in real space, but not exactly RG

correlation b/w spin 2 and spin 3 is not taken into account

EE in TTN



$\min(\#\text{bonds}(\gamma_A)) = 2$ also in this case

➡ $S_A \leq 2\log\chi$

➡ TTN does not reproduce
EE in quantum critical system in 1d

MERA

[Vidal(2007), Evenbly, Vidal(2009)]

Multi-scale **E**ntanglement **R**enormalization **A**nsatz

Ex.) spin-chain consisting of 4 spins in 1d

$$|\Psi\rangle = \sum_{\sigma_1,\sigma_2,\sigma_3,\sigma_4} \sum_{a_1,a_2} \sum_{b_1,b_2} T_{a_1 a_2} (w_1)^{a_1}_{\sigma_1 b_1} (w_2)^{a_2}_{b_2 \sigma_4} u^{b_1 b_2}_{\sigma_2 \sigma_3} |\sigma_1, \sigma_2, \sigma_3, \sigma_4\rangle$$

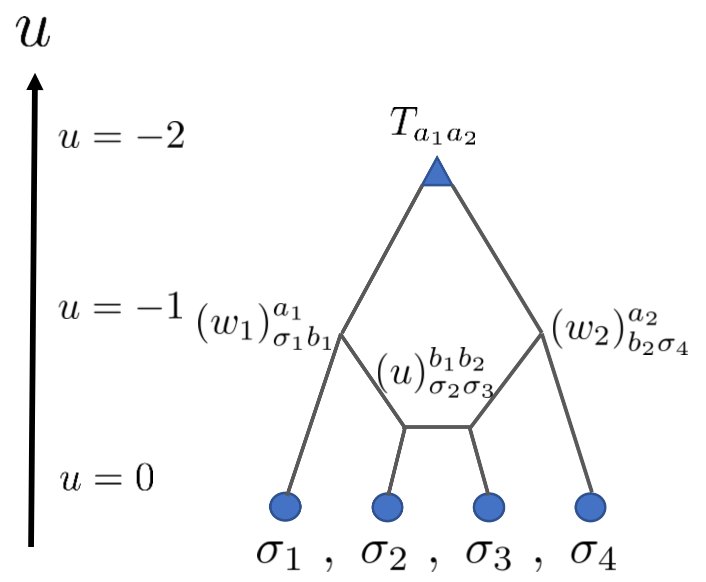
unitary matrix

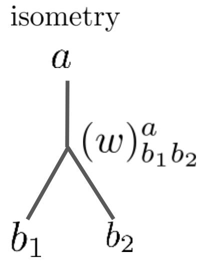
$(u)^{a_1 a_2}_{b_1 b_2}$: create (or erase) correlation between nearest neighbors

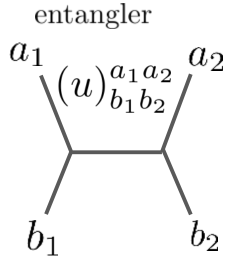
(dis)entangler

↓ entangler

↑ disentangler







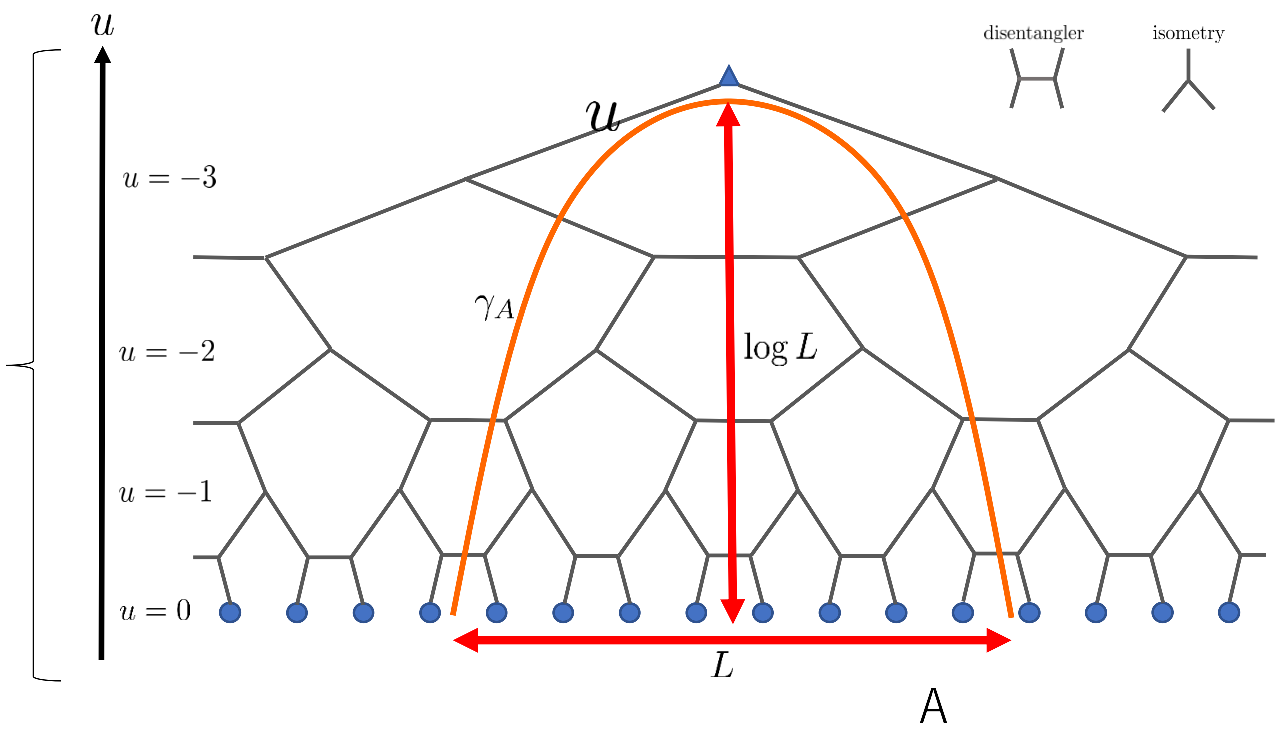
isometry and entangler are determined by VA

MERA and AdS/CFT

Swingle (2009)

discretized

AdS_2



$S_{\mathrm{A}} \leq \min(\#\mathrm{bonds}(\gamma_{\mathrm{A}})) \log \chi$
$= 2\log L$

$S_A \leq (2\log\chi)\log L$

Cf.) EE in quantum critical system in 1d

$$S = \frac{c}{3}\log L$$

reproduce EE in 1d quantum critical system

Motivated by RT formula, we regard $\#\mathrm{bonds}(\gamma_{\mathrm{A}})$ as discrete length

$$ds^2 = du^2 + \frac{e^{2u}}{\epsilon^2}dx^2 \qquad ds^2 = \frac{dz^2+dx^2}{z^2}$$

ϵ : lattice spacing $\quad z = \epsilon e^{-u} \quad$ AdS_2 (constant time slice of AdS_3)

MERA and AdS/CFT (cont'd)

- quantum critical system has scale invariance

  ➡ entangler $(u)^{a_1 a_2}_{b_1 b_2}$ are constant

  ➡ $$ds^2 = du^2 + \frac{e^{2u}}{\epsilon^2}dx^2$$

- in non-critical system, $(u)^{a_1 a_2}_{b_1 b_2}$ depend on u

  ➡ $$ds^2 = g_{uu}(u)du^2 + \frac{e^{2u}}{\epsilon^2}dx^2$$

$g_{uu}(u)$: strength of entanglement at the scale u

cMERA

Haegeman, Osborne, Verschelde, Verstraete(2011)
Nozaki, Ryu, Takayanagi(2012)

- taking the continuum limit of the tensor network is quite nontrivial
- continuum MERA should be important for extracting geometry
- trial function for ground state at UV   ground state at IR

$$|\Psi_0\rangle = U(0,-\infty)|\Omega\rangle \qquad \hat{L}|\Omega\rangle = 0$$

- entanglement renormalization transformation

$$U(u_1,u_2) = \mathrm{P}\exp\left(-i\int_{u_1}^{u_2}(\hat{K}(u)+\hat{L})du\right)$$

$\hat{K}(u)$: entangler

$\hat{L}$: dilation operator $\quad t\to\rho^2 t,\quad x^i\to\rho x^i$

- In free field theory, $\hat{K}(u)$ can be determined by the variational method

➡ $|\Psi_0\rangle$ agrees with the ground state

Quantum information metric

- Hilbert-Schmidt distance

$$D_{\mathrm{HS}}(\rho_A, \rho_B) = \frac{1}{2}\mathrm{Tr}(\rho_A - \rho_B)^2$$

- Quantum information metric

$$D_{\mathrm{HS}}(\rho_\lambda, \rho_{\lambda+\delta\lambda}) = G_{\lambda\lambda}(\delta\lambda)^2 + \mathcal{O}((\delta\lambda)^3)$$

- For pure states $\rho_A = |\Psi_A\rangle\langle\Psi_A|$ and $\rho_B = |\Psi_B\rangle\langle\Psi_B|$,

$$D_{\mathrm{HS}}(\Psi_A, \Psi_B) = 1 - |\langle\Psi_A|\Psi_B\rangle|^2$$

$$G_{\lambda\lambda}(\delta\lambda)^2 = 1 - |\langle\Psi(\lambda)|\Psi(\lambda+\delta\lambda)\rangle|^2$$

Quantum information metric in cMERA

$$G_{uu}du^2 = \mathcal{N}^{-1}(1 - |\langle\Psi(u)|e^{i\hat{L}du}|\Psi(u+du)\rangle|^2)$$

$$|\Psi(u)\rangle = U(u, -\infty)|\Omega\rangle$$

$\hat{L}$: dilatation operator

we subtract contribution of dilatation to extract only contribution of entangler

$$G_{uu} = \mathcal{N}^{-1}(\langle\Psi_0(u)|\hat{K}(u)^2|\Psi_0(u)\rangle - \langle\Psi_0(u)|\hat{K}(u)|\Psi_0(u)\rangle^2)$$

measure local quantum entanglement

➡ should be related to geometry at u

conjecture $G_{uu} \sim g_{uu}$

Free scalar field theory

- In free scalar field theory

$$G_{uu}(u) = \frac{e^{4u}}{4(e^{2u} + m^2/\Lambda^2)^2} \quad \xrightarrow{m^2/\Lambda^2 \to 0} \quad G_{uu}(u) \to \frac{1}{4}$$

AdS metric

Nozaki, Ryu, Takayanagi(2012)

- In the case where the region A is a half space ($x^1 > 0$), by calculating EE based on $|\Psi_0(u)\rangle$ and comparing the result with the RT formula, one obtains $G_{uu} \sim g_{uu}$

Fernandez, Melgarejo, Molina, Vilaplana (2021)

- However, this is an observation for fee field (the RT formula is not applicable)

- Indeed, classical geometry corresponds to the strong coupling regime in field theory

- Thus, it is important to find the entangler $\hat{K}(u)$ in interacting field theory

Contents

Renormalization group approach

- How to assume $\hat{K}(u)$ in VA for interacting field theory is not clear
- We construct cMERA based on the renormalization group (RG) rather than VA
- Cf.) VA for interacting scalar field theory

Fernandez-Melgarejo, Molina-Vilaplana (2020)

We make a comment later

- Cf.) RG approach

$O(N)$ vector free field theory — Fliss, Leigh, Parrikar (2018)

interacting scalar field theory — Cotler, Mozaffar, Mollabashi, Naseh (2018)

different from ours
our method is much simpler

RG approach to cMERA (cont'd)

1. We obtain the ground state at scale u, $|\Psi_0(u)\rangle$, based on RG

2. By calculating $\partial_u|\Psi_0(u)\rangle$, we obtain the entangler $\hat{K}(u)$

$$|\Psi_0(u)\rangle = \mathrm{P}\exp\left[-i\int_{u_{\mathrm{IR}}}^{u} du'(\hat{K}(u')+\hat{L})\right]|\Psi_0(u_{\mathrm{IR}})\rangle$$

$$\Rightarrow \quad \partial_u|\Psi_0(u)\rangle = -i(\hat{K}(u)+\hat{L})|\Psi_0(u)\rangle$$

RG

1. perturbation theory ⇒ insights into nonperturbative cMERA
   know what type of terms appear generally

2. nonperturbative method ～exact renormalization group
   (functional renormalization group)

Contents

Perturbation theory

- equation for cMERA

$$\partial_u|\Psi_0(u)\rangle = -i(\hat{K}(u)+\hat{L})|\Psi_0(u)\rangle = \hat{X}|\Psi_0(u)\rangle$$

$$\hat{X} = -i(\hat{K}(u)+\hat{L})$$ anti-Hermitian operator

- perturbative expansion

$$|\Psi_0(u)\rangle = |\Psi_0^{(0)}(u)\rangle + \alpha|\Psi_0^{(1)}(u)\rangle + \cdots$$

$$\hat{X}(u) = \hat{X}_0 + \alpha\hat{X}_1 + \cdots$$

0th order in α $\quad \partial_u|\Psi_0^{(0)}\rangle = \hat{X}_0|\Psi_0^{(0)}\rangle$ → obtain $\hat{X}_0$ and substitute it

1st order in α $\quad \partial_u|\Psi_0^{(1)}\rangle = \hat{X}_1|\Psi_0^{(0)}\rangle + \hat{X}_0|\Psi_0^{(1)}\rangle$

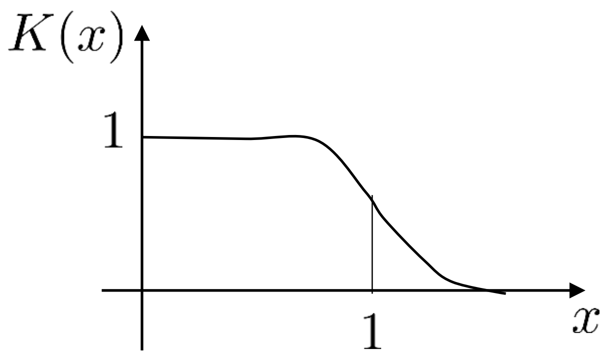
→ obtain $\hat{X}_1$

(d+1)-dimensional free scalar field

- Lagrangian

$$L = \int d^d x \left[-\frac{1}{2}\partial_\mu \hat{\phi}(t,x)\partial^\mu \hat{\phi}(t,x) - \frac{1}{2}m^2\hat{\phi}^2(x) \right]$$

cutoff function



- introduction of effective cutoff Λ

$$L_\Lambda = \int_p K^{-1}\left(\frac{p^2}{\Lambda^2}\right) \left[\frac{1}{2}\dot{\hat{\phi}}(t,p)\dot{\hat{\phi}}(t,-p) - \frac{1}{2}(p^2+m^2)\hat{\phi}(t,p)\hat{\phi}(t,-p) \right]$$

$$\int_p \equiv \int \frac{d^d p}{(2\pi)^d}$$

- canonical conjugate momenta and canonical commutation relation

$$\hat{\pi}(p) \equiv \frac{\delta L_\Lambda}{\delta \dot{\hat{\phi}}(-p)} = K^{-1}(p^2/\Lambda^2)\dot{\hat{\phi}}(p)$$

$$[\hat{\phi}(p), \hat{\pi}(q)] = i(2\pi)^d \delta(p+q)$$

$$[\hat{\phi}(p), \hat{\phi}(q)] = [\hat{\pi}(p), \hat{\pi}(q)] = 0$$

(d+1)-dimensional free scalar field (cont'd)

- Hamiltonian

$$H_\Lambda = \int_p \frac{1}{2} K\left(\frac{p^2}{\Lambda^2}\right)\left[\hat{\pi}(p)\hat{\pi}(-p) + K^{-2}\left(\frac{p^2}{\Lambda^2}\right)(p^2+m^2)\hat{\phi}(p)\hat{\phi}(-p)\right]$$

- rescaling

$$\hat{\phi}(k) \to e^{-\frac{d+1}{2}u}\hat{\phi}(e^{-u}k), \quad \hat{\pi}(k) \to e^{-\frac{d-1}{2}u}\hat{\pi}(e^{-u}k)$$

- rescaled Hamiltonian

$$H(u) = \int_p \frac{1}{2} K_p \left[\hat{\pi}(p)\hat{\pi}(-p) + K_p^{-2}\omega_p^2\hat{\phi}(p)\hat{\phi}(-p)\right]$$

$\Lambda = \Lambda_0 e^u$ Λ_0 : bare cutoff UV limit $u=0$ IR limit $u=-\infty$

$K_p = K(p^2/\Lambda_0^2)$

$\omega_p = \sqrt{p^2 + e^{-2u}m^2}$ u-dependence appears only here

(d+1)-dimensional free scalar field (cont'd)

- creation-annihilation operators

$$\hat{a}(k) = \frac{1}{\sqrt{2}}\left(\sqrt{K_k^{-1}\omega_k}\hat{\phi}(k) + \frac{i}{\sqrt{K_k^{-1}\omega_k}}\hat{\pi}(k)\right)$$

$$\hat{a}^\dagger(k) = \frac{1}{\sqrt{2}}\left(\sqrt{K_k^{-1}\omega_k}\hat{\phi}(-k) - \frac{i}{\sqrt{K_k^{-1}\omega_k}}\hat{\pi}(-k)\right)$$

$$[\hat{a}(p), \hat{a}^\dagger(q)] = (2\pi)^d \delta(p-q)$$

- Hamiltonian

$$H(u) = \int_p \frac{1}{2} K_p \left[\hat{\pi}(p)\hat{\pi}(-p) + K_p^{-2}\omega_p^2 \hat{\phi}(p)\hat{\phi}(-p)\right] = \int_p \omega_p \left(\hat{a}^\dagger(p)\hat{a}(p) + \frac{V}{2}\right)$$

V volume of space

no cutoff dependence

(d+1)-dimensional free scalar field (cont'd)

- vacuum（ground state）

$$\hat{a}(p)|\Psi_0(u)\rangle = 0$$

- m-particle state

$$|\Psi_m(k_1,\cdots,k_m;u)\rangle = \frac{1}{\sqrt{m!}}\prod_{i=1}^{m}\hat{a}^\dagger(k_i)|\Psi_0(u)\rangle$$

- energy eigenvalue

$$H(u)|\Psi_m(k_1,\cdots,k_m;u)\rangle = \left(\sum_{i=1}^{m}\omega_{p_i} + \int_p \frac{V}{2}\omega_p\right)|\Psi_m(k_1,\cdots,k_m;u)\rangle$$

vacuum energy　　　no cutoff dependence

Vacuum wave functional for free scalar field

- eigenstate of $\hat{\phi}$

$$\hat{\phi}(p)|\phi\rangle = \phi(p)|\phi\rangle$$

- wave functional for vacuum (ground state)

$$\Psi_0[\phi; u] \equiv \langle\phi|\Psi_0(u)\rangle$$

$$0 = \langle\phi|\hat{a}(p)|\Psi_0(u)\rangle = \frac{1}{\sqrt{2}}\left\{\sqrt{K_p^{-1}\omega_p}\phi(p) + \frac{1}{\sqrt{K_p^{-1}\omega_p}}\frac{\delta}{\delta\phi(-p)}\right\}\Psi_0[\phi; u]$$

$$\Rightarrow \quad \Psi_0[\phi; u] = \exp\left[-\frac{1}{2}\int_p \phi(p)K_p^{-1}\omega_p\phi(-p) + \frac{V}{4}\int_p \ln\left(2K_p^{-1}\omega_p\right)\right]$$

normalization $1 = \langle\Psi_0(u)|\Psi_0(u)\rangle = \int \mathcal{D}\phi|\Psi_0[\phi](u)|^2$

Ground state in interacting theory

- Hamiltonian

$$H(u) = H_0(u) + \alpha H_{int}(u)$$

$$H_0(u) = \int_p K_p \frac{1}{2}\left(\hat{\pi}(p)\hat{\pi}(-p) + K_p^{-1}\omega_p^2\hat{\phi}(p)\hat{\phi}(-p)\right)$$

$$H_{\rm int}(u) = \boxed{\frac{\delta m^2(u)}{2}\int_p \hat{\phi}(p)\hat{\phi}(-p)} + \frac{\lambda(u)}{4!}\int_{p_1,\cdots,p_4}\prod_{i=1}^{4}\hat{\phi}(p_i)(2\pi)^d\delta\left(\sum p_i\right)$$

mass counter term

- perturbative expansion

$$|\Psi_0(u)\rangle = |\Psi_0^{(0)}(u)\rangle + \alpha|\Psi_0^{(1)}(u)\rangle + \mathcal{O}(\alpha^2)$$

$$E_0(u) = E_0^{(0)}(u) + \alpha E_0^{(1)}(u) + \mathcal{O}(\alpha^2)$$

Ground state in interacting theory (cont'd)

- Formula for the first order in perturbative expansion

$$|\Psi_0^{(1)}(u)\rangle = \sum_{n\neq 0}\int_{k_1,\ldots,k_n} \frac{\langle \Psi_n^{(0)}(k_1,\ldots,k_n;u)|H_{\rm int}(u)|\Psi_0^{(0)}(u)\rangle}{E_0^{(0)}(u)-E_n^{(0)}(k_1,\cdots,k_n;u)}|\Psi_n^{(0)}(k_1,\ldots,k_n;u)\rangle$$

- We obtain

$$\begin{aligned}|\Psi_0^{(1)}(u)\rangle = &-\frac{\lambda}{4!}\int_{k_1,\ldots,k_4}\frac{(2\pi)^d\delta(k_1+\cdots+k_4)}{\omega_{k_1}+\cdots+\omega_{k_4}}\prod_{i=1}^{4}\sqrt{\frac{K_{k_i}}{2\omega_{k_i}}}\hat{a}^\dagger(k_i)|\Psi_0^{(0)}\rangle \\ &-\left(\frac{\delta m^2}{2}+\frac{\lambda}{4!}\int_p\frac{6K_p}{2\omega_p}\right)\int_k\frac{1}{2\omega_k}\frac{K_k}{2\omega_k}\hat{a}^\dagger(k)\hat{a}^\dagger(-k)|\Psi_0^{(0)}\rangle\end{aligned}$$

$$K_p = K(p^2/\Lambda_0^2)$$

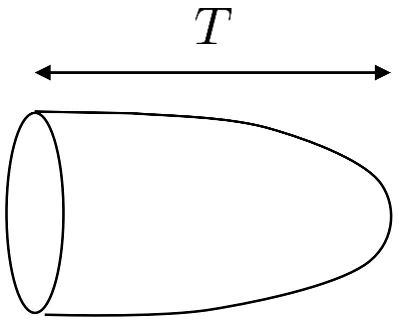
$$\omega_p = \sqrt{p^2+e^{-2u}m^2}$$

Wave functional through path-integral

- path-integral representation of ground state

$$\Psi_0[\phi] = \lim_{T\to\infty} \int_{\varphi(0,p)=\phi(p)} D\varphi \; e^{-\int_{-T}^{0} d\tau L} = \lim_{T\to\infty} \langle \phi | e^{-TH} | \Phi \rangle \sim \langle \phi | \Psi_0 \rangle$$

$\varphi(0,p) = \phi(p)$



- Lagrangian

$$L(u) = L_0(u) + L_{\rm int}(u)$$

$$L_0(u) = \int_p K_p^{-1} \left[\frac{1}{2} \partial_\tau \varphi(\tau,p) \partial_\tau \varphi(\tau,-p) + \frac{1}{2} \omega_p^2 \varphi(\tau,p) \varphi(\tau,-p) \right]$$

- Expand around a classical solution in L_0

$$\varphi(\tau,p) = \varphi_{\rm c}(\tau,p) + \chi(\tau,p)$$

classical solution: $\varphi_{\rm c}(\tau,p)$ — quantum fluctuation: $\chi(\tau,p)$

b.c. $\varphi_{\rm c}(0,p) = \phi(p)$, $\chi(0,p) = 0$

$$\varphi_{\rm c}(\tau,p) = \frac{e^{\omega_p(\tau+T)} - e^{-\omega_p(\tau+T)}}{e^{\omega_p T} - e^{-\omega_p T}} \phi(p)$$

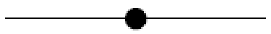
$$\chi(\tau,p) = \sum_{n=1}^{\infty} \sin\left(\frac{\pi n}{T} \tau \right) \chi_n(p)$$

$$\langle \chi(\tau,p) \chi(\tau',p') \rangle \xrightarrow{T\to\infty} \frac{K_p}{2\omega_p} \left(e^{-\omega_p|\tau-\tau'|} - e^{\omega_p(\tau+\tau')} \right) (2\pi)^d \delta(p+p')$$

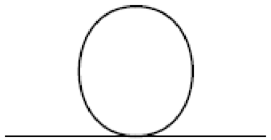
first order in perturbative expansion

$$\Psi_0 = e^I \Psi_0^{(0)} \qquad I = 1 + \alpha I^{(1)} + \alpha^2 I^{(2)} + \cdots$$

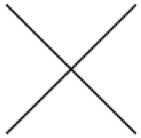
$I^{(1)}$:

(121) = 

$$= -\frac{\delta m^2}{4}\int_p \varphi(p)\varphi(-p)\frac{1}{\omega_p}$$

(122) = 

$$= -\frac{\lambda}{16}\int_{p_1 p_2} \varphi(p_1)\varphi(-p_1)\,\frac{K_2}{\omega_1(\omega_1+\omega_2)}$$

(141) = 

$$= -\frac{\lambda}{24}\int_{p_1\cdots p_4} \varphi_1\cdots\varphi_4\,\frac{\tilde{\delta}(p_1+p_2+p_3+p_4)}{\omega_1+\omega_2+\omega_3+\omega_4}$$

agree with the results obtained in canonical formulation

Second order in perturbative expansion

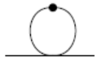
$I^{(2)}$

(221) = 

$$= \frac{(\delta m^2)^2}{16} \int_p \phi(p)\phi(-p)\frac{K_p}{\omega_p^3}$$

(222) = 

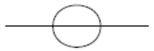
$$= \frac{\delta m^2 \lambda}{32} \int_{p_1 p_2} \phi(p_1)\phi(-p_1)\frac{K_1 K_2 (2\omega_1 + \omega_2)}{\omega_1^3(\omega_1+\omega_2)^2}$$

(223) = 

$$= \frac{\delta m^2 \lambda}{32} \int_{p_1 p_2} \phi(p_1)\phi(-p_1)\frac{K_2^2}{\omega_1\omega_2(\omega_1+\omega_2)^2}$$

(224) = 

$$= \frac{\lambda^2}{256} \int_{p_1 p_2 p_3} \phi(p_1)\phi(-p_1) K_1 K_2 K_3 \, \frac{5\omega_1^3 + (6\omega_1^2 + \omega_2\omega_3)(\omega_2+\omega_3) + \omega_1(2\omega_2^2 + 5\omega_2\omega_3 + 2\omega_3^2)}{\omega_1^3(\omega_1+\omega_2)^2(\omega_1+\omega_3)^2(\omega_1+\omega_2+\omega_3)}$$

(225) = 

$$= \frac{\lambda^2}{96} \int_{p_1 p_2 p_3} \phi(p_1)\phi(-p_1) K_2 K_3 K_4$$

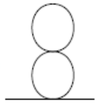
$$\cdot \frac{1}{\omega_1(\omega_1+\omega_2)(\omega_1+\omega_3)(\omega_1+\omega_4)(\omega_1+\omega_2+\omega_3)(\omega_1+\omega_2+\omega_4)(\omega_1+\omega_3+\omega_4)(\omega_1+\omega_2+\omega_3+\omega_4)^2}$$

$$\cdot [6\omega_1^4 + 12\omega_1^3(\omega_2+\omega_3+\omega_4) + (\omega_2+\omega_3)(\omega_3+\omega_4)(\omega_2+\omega_4)(\omega_2+\omega_3+\omega_4)$$

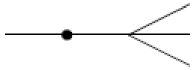
$$+ 2\omega_1^2\{4\omega_2^2 + 4\omega_3^2 + 4\omega_4^2 + 9\omega_3\omega_4 + 9\omega_2(\omega_3+\omega_4)\}$$

$$+ 2\omega_1\{\omega_2^3 + \omega_3^3 + \omega_4^3 + 4(\omega_2^2 + \omega_3\omega_4)(\omega_3+\omega_4) + \omega_2(4\omega_3^2 + 9\omega_3\omega_4 + 4\omega_4^2)\}]$$

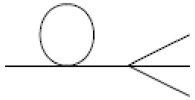
$(p_4 = p_1 - p_2 - p_3)$

(226) = 

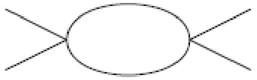
$$= \frac{\lambda^2}{128} \int_{p_1 p_2 p_3} \phi(p_1)\phi(-p_1) K_2^2 K_3 \frac{\omega_1^2 + (\omega_2+\omega_3)^2 + \omega_1(2\omega_2 + 3\omega_3)}{\omega_1\omega_3(\omega_1+\omega_2)(\omega_2+\omega_3)(\omega_1+\omega_3)^2(\omega_1+\omega_2+\omega_3)}$$

(241) = 

$$= \frac{\delta m^2 \lambda}{12} \int_{p_1 \cdots p_4} \phi_1 \cdots \phi_4 \, \frac{K_1 \tilde{\delta}(p_1+p_2+p_3+p_4)}{\omega_1(\omega_1+\omega_2+\omega_3+\omega_4)^2}$$

(242) = 

$$= \frac{\lambda^2}{48} \int_{p_1 \cdots p_5} \phi_1 \cdots \phi_4 K_1 K_5 \, \frac{(3\omega_1+\omega_2+\omega_3+\omega_4+2\omega_5)\tilde{\delta}(p_1+p_2+p_3+p_4)}{\omega_1(\omega_1+\omega_5)(\omega_1+\omega_2+\omega_3+\omega_4)^2(\omega_1+\omega_2+\omega_3+\omega_4+\omega_5)}$$

(243) = 

$$= \frac{\lambda^2}{8} \int_{p_1 \cdots p_5} \phi_1 \cdots \phi_4 K_5 K_6$$

$$\cdot \frac{(\omega_1+\omega_2+\omega_3+\omega_4+\omega_5+\omega_6)\tilde{\delta}(p_1+p_2+p_3+p_4)}{(\omega_1+\omega_2+\omega_3+\omega_4)(\omega_1+\omega_2+\omega_5+\omega_6)}$$

$$\cdot \frac{1}{(\omega_3+\omega_4+\omega_5+\omega_6)(\omega_1+\omega_2+\omega_3+\omega_4+2\omega_5)(\omega_1+\omega_2+\omega_3+\omega_4+2\omega_6)}$$

$(p_6 = -p_1 - p_2 + p_5)$

(261) = 

$$= \frac{\lambda^2}{72} \int_{p_1 \cdots p_6} \phi_1 \cdots \phi_6 K_7 \, \frac{\tilde{\delta}(p_1+p_2+p_3+p_4+p_5+p_6)}{(\omega_1+\omega_2+\omega_3+\omega_7)(\omega_4+\omega_5+\omega_6+\omega_7)(\omega_1+\omega_2+\omega_3+\omega_4+\omega_5+\omega_6)}$$

$(p_7 = p_4 + p_5 + p_6)$

Obtain $\hat{X}_0$

$$\partial_u \Psi_0^{(0)}[\phi] = \partial_u \exp\left[-\frac{1}{2}\int_p \phi(p)K_p^{-1}\omega_p\phi(-p) + \frac{V}{4}\int_p \ln\left[2K_p^{-1}\omega_p\right]\right]$$

$$= \int_p \left\{-\frac{1}{2}\phi(p)K_p^{-1}\partial_u\sqrt{p^2+e^{-2u}m^2}\phi(-p) + \frac{V}{4}\partial_u\sqrt{p^2+e^{-2u}m^2}\right\}\Psi_0^{(0)}[\phi]$$

$$= m^2e^{-2u}\int_k \frac{1}{4\omega_k^2}\left\{a^\dagger(k)a^\dagger(-k) + a^\dagger(k)a(k) + a(-k)a^\dagger(-k) + a(k)a(-k) - V\right\}\Psi_0^{(0)}[\phi]$$

$$= m^2e^{-2u}\int_k \frac{1}{4\omega_k^2}a^\dagger(k)a^\dagger(-k)\Psi_0^{(0)}[\phi]$$

$$\partial_u|\Psi_0^{(0)}\rangle = \hat{X}_0|\Psi_0^{(0)}\rangle$$

$$\hat{X}_0 = m^2e^{-2u}\int_k \frac{1}{4\omega_k^2}\{\hat{a}^\dagger(k)\hat{a}^\dagger(-k) - \hat{a}(k)\hat{a}(-k)\} = -im^2e^{-2u}\int_k \frac{1}{4\omega_k^2}\{\hat{\phi}(k)\hat{\pi}(-k) + \hat{\pi}(k)\hat{\phi}(-k)\}$$

$$\hat{a}(k)|\Psi_0^{(0)}(u)\rangle = 0$$

obtained as anti-Hermitian operator

Obtain $\hat{X}_1$

$$\hat{X}_1 = -im^2 e^{-2u}\frac{\lambda}{4!}\int_{k_1,\dots,k_4}\left\{\left(\frac{1}{\omega_{k_1}+\cdots+\omega_{k_4}}\right)^2\left(\frac{1}{\omega_{k_1}}+\cdots+\frac{1}{\omega_{k_4}}\right)+\frac{1}{\omega_{k_1}+\cdots+\omega_{k_4}}\left(\frac{1}{2\omega_{k_1}^2}+\cdots\frac{1}{2\omega_{k_4}^2}\right)\right\}$$

$$\times\frac{1}{4}\tilde{\delta}(k_1+\cdots+k_4)\left\{\left(-\hat{\phi}(k_1)\hat{\phi}(k_2)\hat{\phi}(k_3)\frac{K_{k_4}}{2\omega_{k_4}}\hat{\pi}(k_4)+\frac{K_{k_1}}{2\omega_{k_1}}\hat{\pi}(k_1)\frac{K_{k_2}}{2\omega_{k_2}}\hat{\pi}(k_2)\frac{K_{k_3}}{2\omega_{k_3}}\hat{\pi}(k_3)2\hat{\phi}(k_4)\right)\right.$$

$+$ (3 terms exchange of 1,2,3,4) $\}$

does not exist in the ansatz used in the variational method

$$+2im^2e^{-2u}\left\{\frac{\delta m^2}{2}+\frac{\lambda}{4!}\int_p\frac{6K_p}{2\omega_p}\right\}\int_k\frac{1}{2\omega_k^3}\frac{K_k}{2\omega_k}\left\{\hat{\phi}(k)\hat{\pi}(-k)+\hat{\pi}(k)\hat{\phi}(-k)\right\}$$

Fernandez-Melgarejo, Molina-Vilaplana (2020)

$$+i\left\{\frac{\partial_u\delta m^2}{2}+\frac{\lambda}{4!}\int_p\frac{6K_p}{2\omega_p^2\omega_p}m^2e^{-2u}\right\}\int_k\frac{1}{2\omega_k}\frac{K_k}{2\omega_k}\left\{\hat{\phi}(k)\hat{\pi}(-k)+\hat{\pi}(k)\hat{\phi}(-k)\right\}$$

obtained as anti-Hermitian operator

$$\partial_u\delta m^2 = -\frac{\lambda}{4}\int_p\frac{2\frac{p^2}{\Lambda_0^2}K_p'}{\omega_p}-2\delta m^2$$

Contents

Toward nonperturbative cMERA

- In the spirit of the exact RG (functional RG), we derive a nonperturbative functional differential equation for the ground state wave functional

$$\Psi_0[\phi] = \int_{\varphi(0,p)=\phi(p)} D\varphi \; e^{-\int_{-\infty}^{0} d\tau L}$$

$$0 = \frac{\partial}{\partial u} \int \mathcal{D}\phi \; \Psi^*[\phi]\Psi[\phi]$$

$$\Rightarrow \quad \partial_u \Psi_0[\phi](u) = -\int_p \frac{\dot{K}_k}{4\omega_k} \left[\frac{\delta^2 \Psi_0[\phi](u)}{\delta\phi(k)\delta\phi(-k)} + \frac{1}{\Psi_0[\phi](u)} \frac{\delta\Psi_0[\phi](u)}{\delta\phi(k)} \frac{\delta\Psi_0[\phi](u)}{\delta\phi(-k)} \right]$$

- We confirmed that the vacuum wave functional that we obtain perturbatively satisfies this equation to the first order.
  We are examining the second order

- We expect to find the nonperturbative entangler by solving this equation nonperturbatively

  We can infer from perturbative results what type of terms appear in the solution

Contents

Conclusion

- We proposed a method based on RG to construct cMERA in interacting field theory

$$|\Psi_0(u_1)\rangle = U(u_1, u_2)|\Psi(u_2)\rangle \qquad U(u_1,u_2) = \mathrm{P}\exp\left(-i\int_{u_1}^{u_2}(\hat{K}(u)+\hat{L})du\right)$$

$$\partial_u|\Psi_0(u)\rangle = -i(\hat{K}(u)+\hat{L})|\Psi_0(u)\rangle$$

first obtain $|\Psi_0(u)\rangle$ based on RG and determine $\hat{K}(u)$ using the above equation

- We construct cMERA perturbatively to gain insight into nonperturbative cMERA

e.g. what type of terms appear generally in $\hat{K}(u)$

our $\hat{K}(u)$ includes a type of terms that are not included in the trial ansatz for $\hat{K}(u)$ in VA by Fernandez-Melgarejo and Molina-Vilaplana (2020)

- We also discussed a nonperturbative method based on the exact RG

Outlook

- calculate EE and find relationship between entangler and geometry in interacting field theory
- develop nonperturbative cMERA further
- extension to gauge field theory