

Studies of the D0-brane matrix models at low temperatures

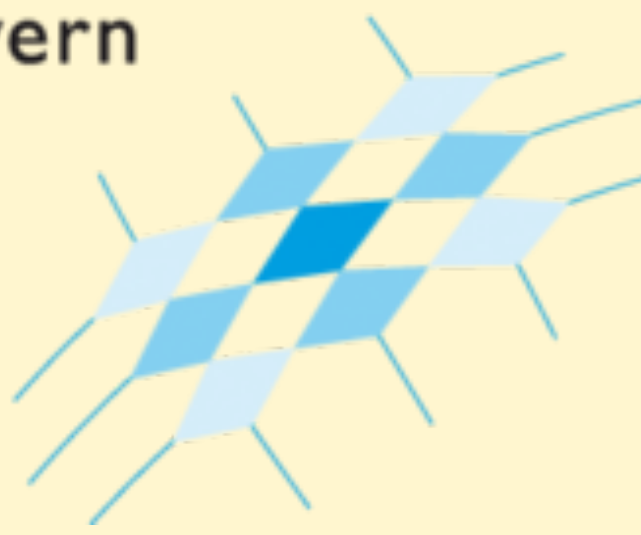
Stratos Pateloudis

University of Regensburg, Germany

Workshop on Noncommutative and generalized geometry in
string theory,
gauge theory and related physical models

Kerkyra: 19/09/22

Elitenetzwerk
Bayern



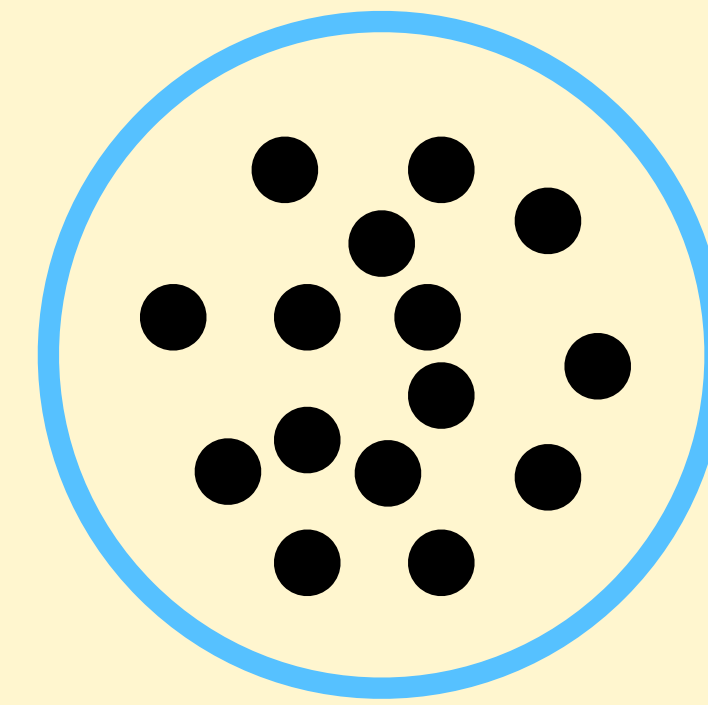
Based on: 2110.01312, 2205.06098 & work in progress

With: Bergner, Bodendorfer, Hanada, Rinaldi, Schäfer, Vranas, Watanabe (MCSMC)

Why low temperatures?

Plan of the talk

- Definition of the models
- Holography
- Relation with gravity
- Confinement in D0-matrix model
- Simulations, tests at low temperatures
- Comparison with eternal energy of the black zero brane
- Role of gauge constraint?



D0-matrix model (BFSS)

$$S = \frac{1}{2g_{YM}^2} \int dt \text{Tr} \left\{ (D_t X_M)^2 + [X_M, X_N]^2 + i\bar{\psi}^\alpha D_t \psi^\beta + \bar{\psi}^\alpha \gamma_{\alpha\beta}^M [X_M, \psi^\beta] \right\}$$

$X_{N \times N}$: $N \times N$ bosonic hermitian matrices with $M = 1, \dots, 9$

$$D_t : D_t \mathcal{O} = \partial_t \mathcal{O} - i[A_t, \mathcal{O}]$$

$\psi_{N \times N}$: $N \times N$ fermionic hermitian matrices with $\alpha = 1, \dots, 16$

$$\lambda = g_{YM}^2 N = [\text{energy}]^3$$

- Dimensional reduction of 4D $\mathcal{N} = 4$ / 10D $\mathcal{N} = 1$
- Matrix regularisation of 11D supermembrane De Wit-Hoppe-Nicolai, 1988
- Matrix model of M-theory (BFSS) Banks-Fischler-Shenker-Susskind, 1996
- Dual to type IIA black 0-brane near 't Hooft limit Itzhaki-Maldacena-Sonnenschein-Yankielowicz, 1998

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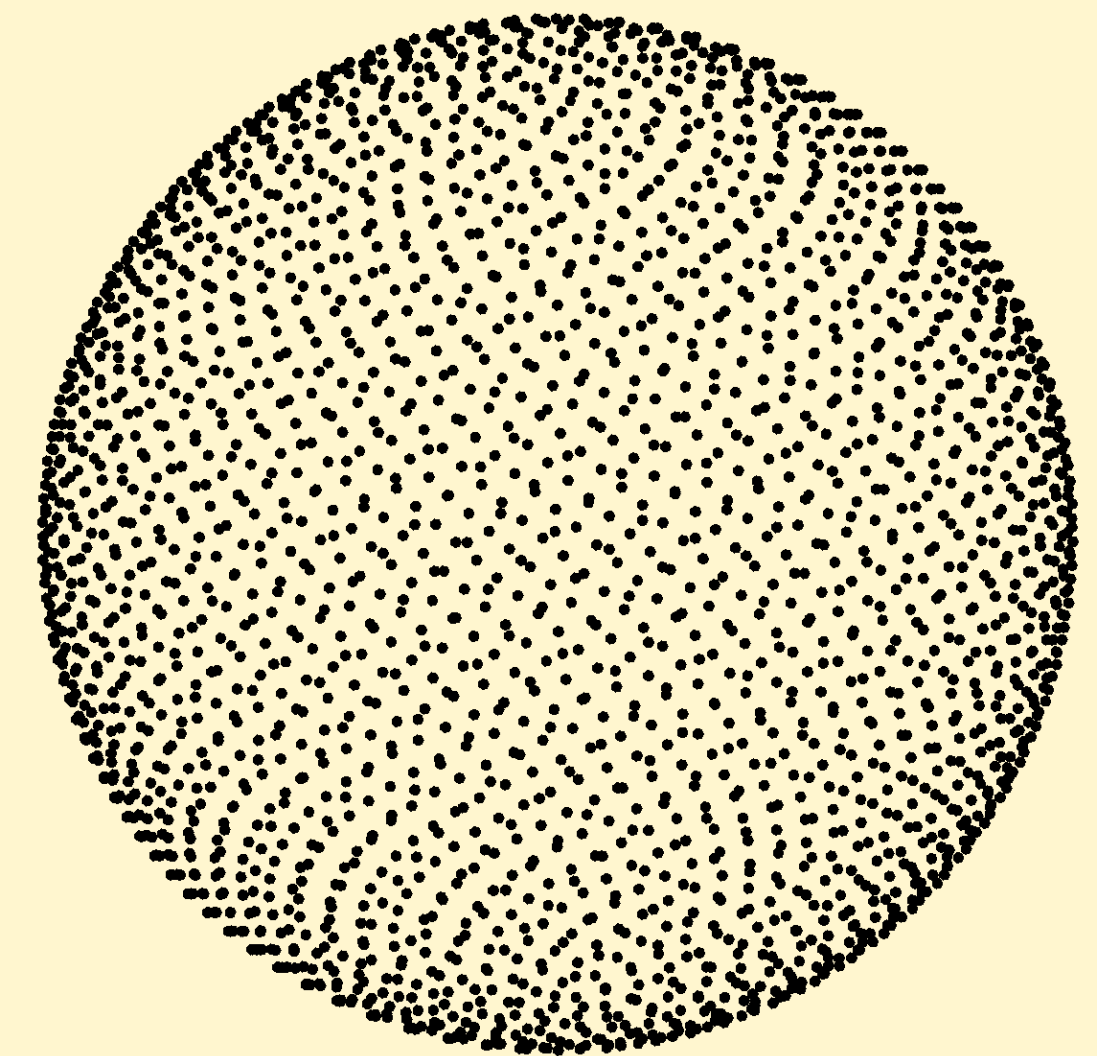
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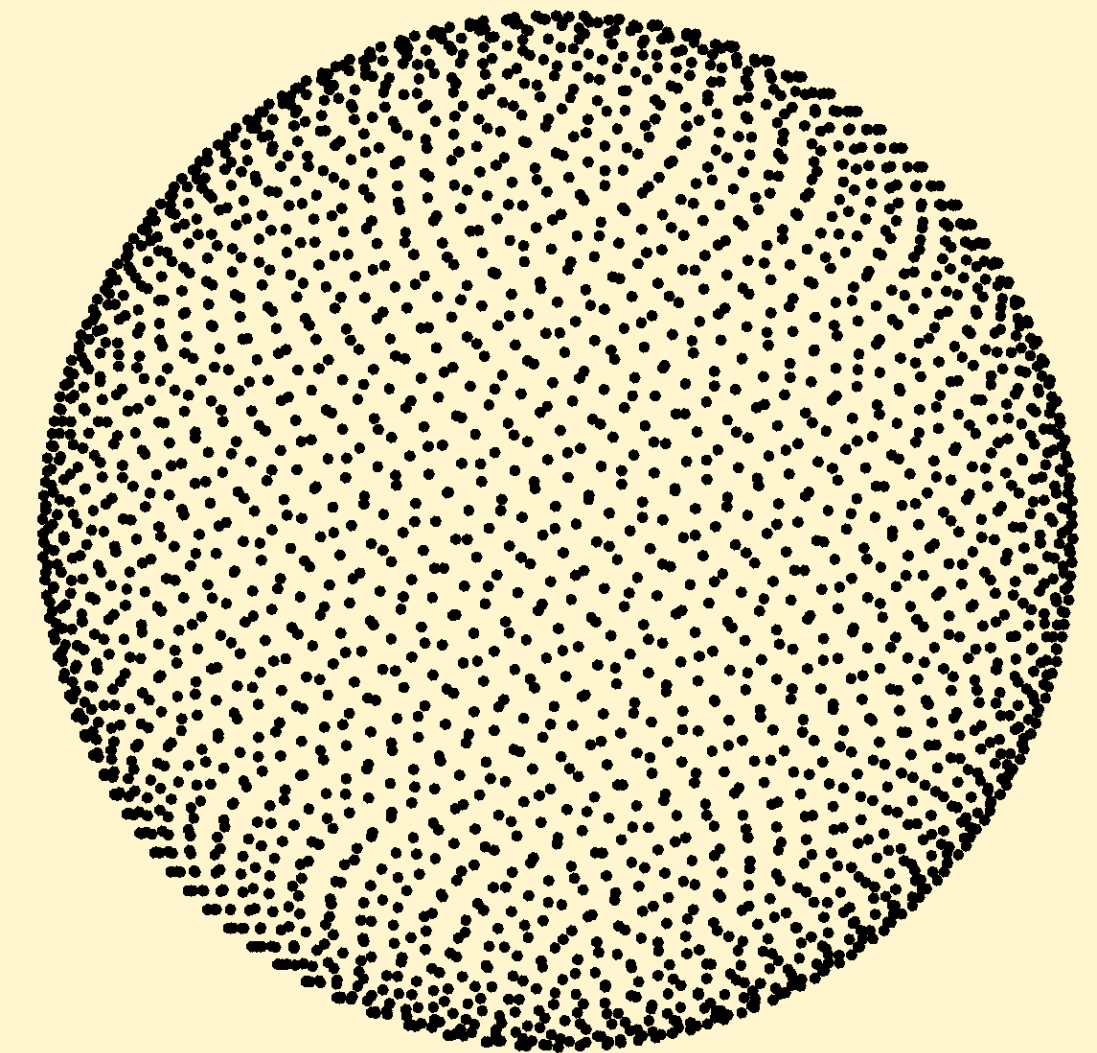
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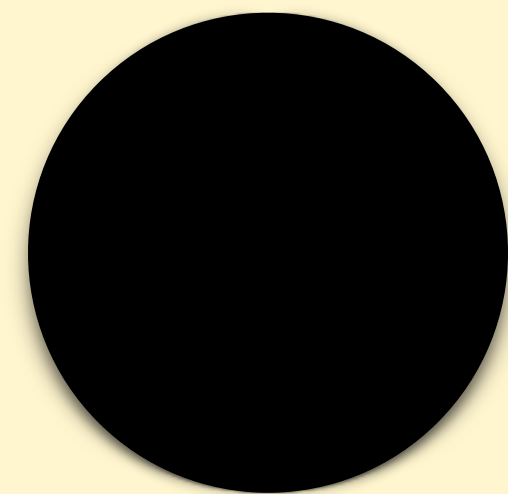
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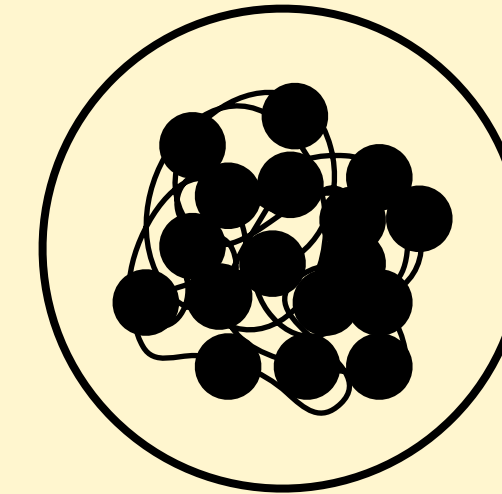
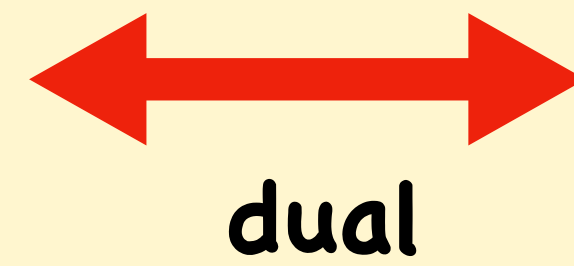
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Gauge/gravity duality in string theory



Black p -brane
in IIA/IIB string



$(p+1)$ -d $U(N)$ SYM
(D_p -branes + strings)

In this talk $p=0$

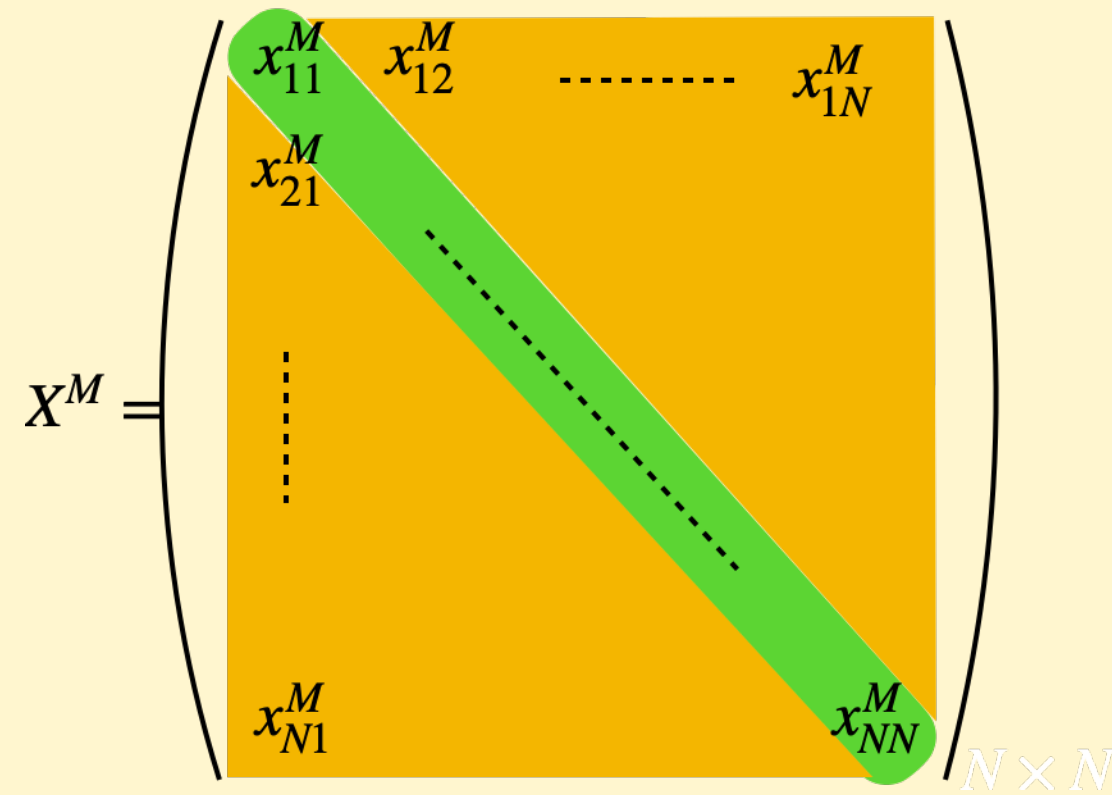
For $p=-1$ \longrightarrow IKKT model

Anagnostopoulos, Azuma, Ito, Kim, Nishimura, Okubo, Papadoudis, Tsuchiya,...

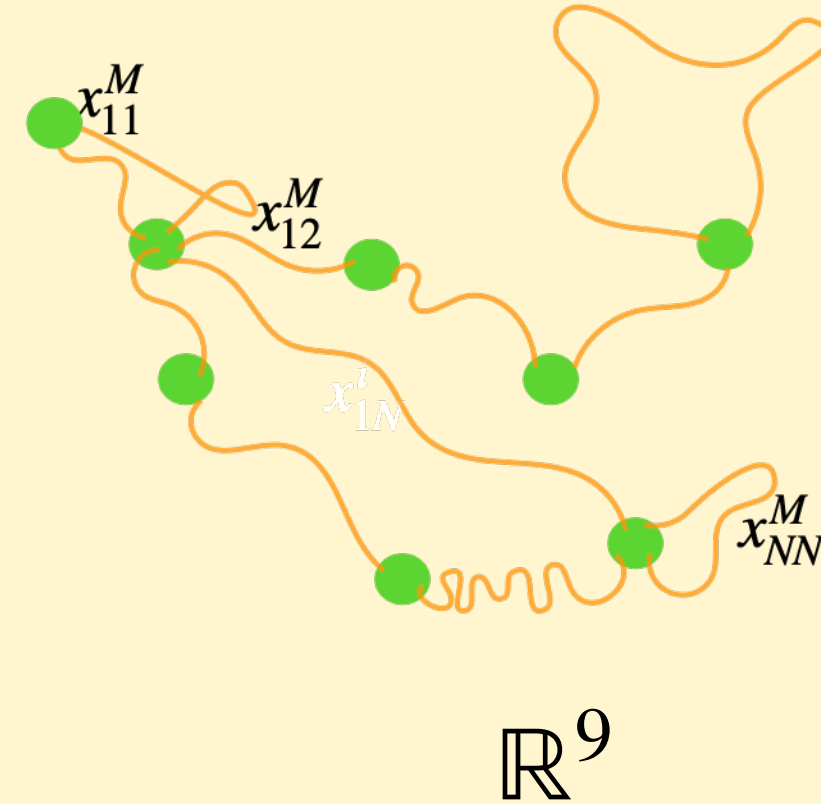
The curious case of $p=0$

$$\lambda = g_{YM}^2 N = [\text{energy}]^3 \quad \Rightarrow \quad g_{\text{eff}} = \frac{\lambda}{E^3}$$

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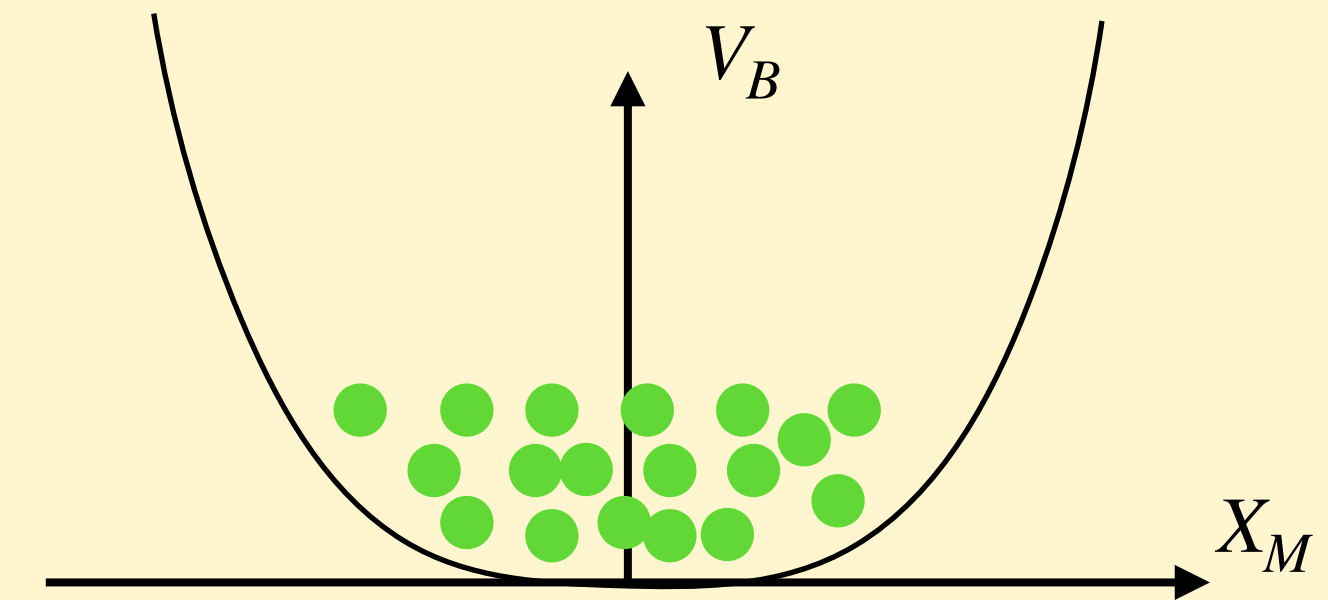


Witten 1995



$$M = 1, \dots, 9$$

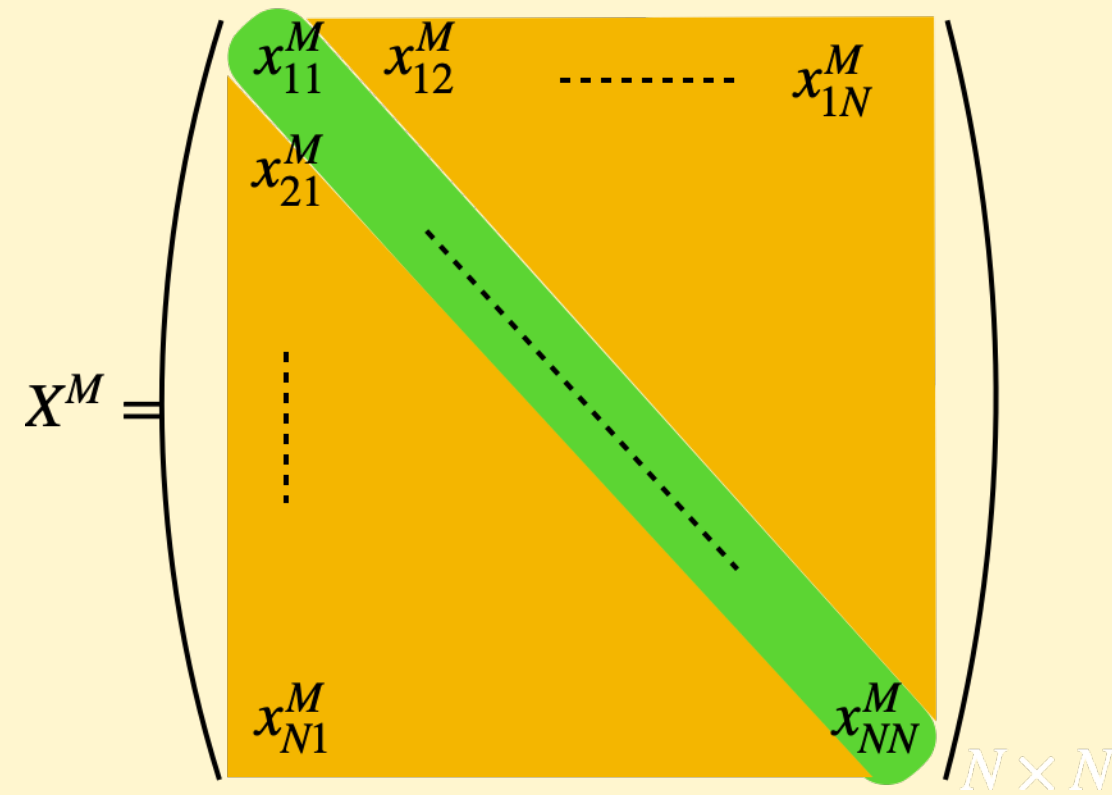
$$V_B = [X_M, X_N]^2 \sim X_M^4$$



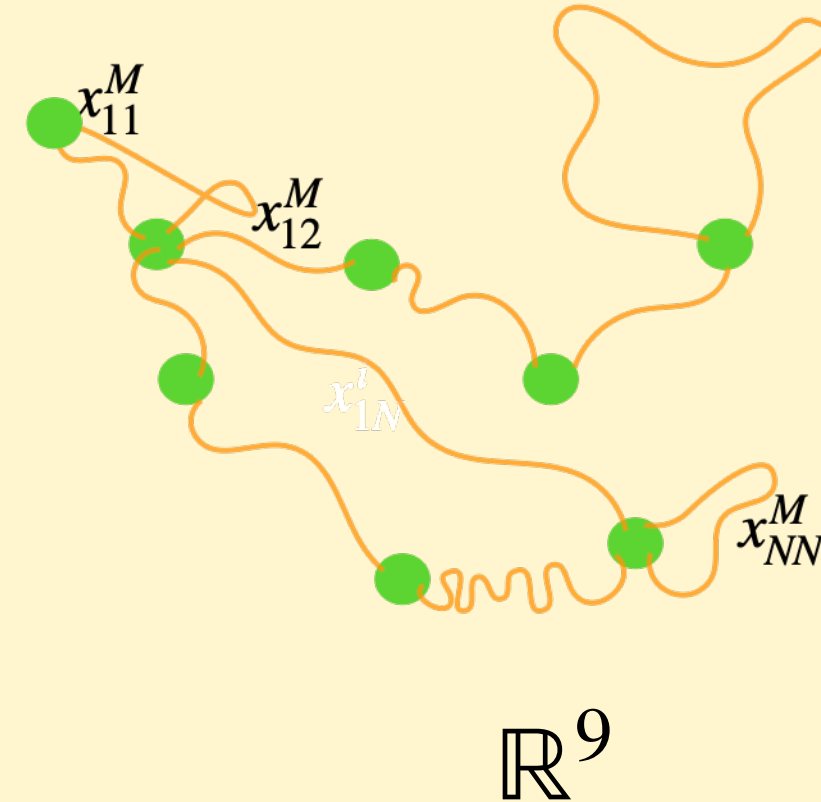
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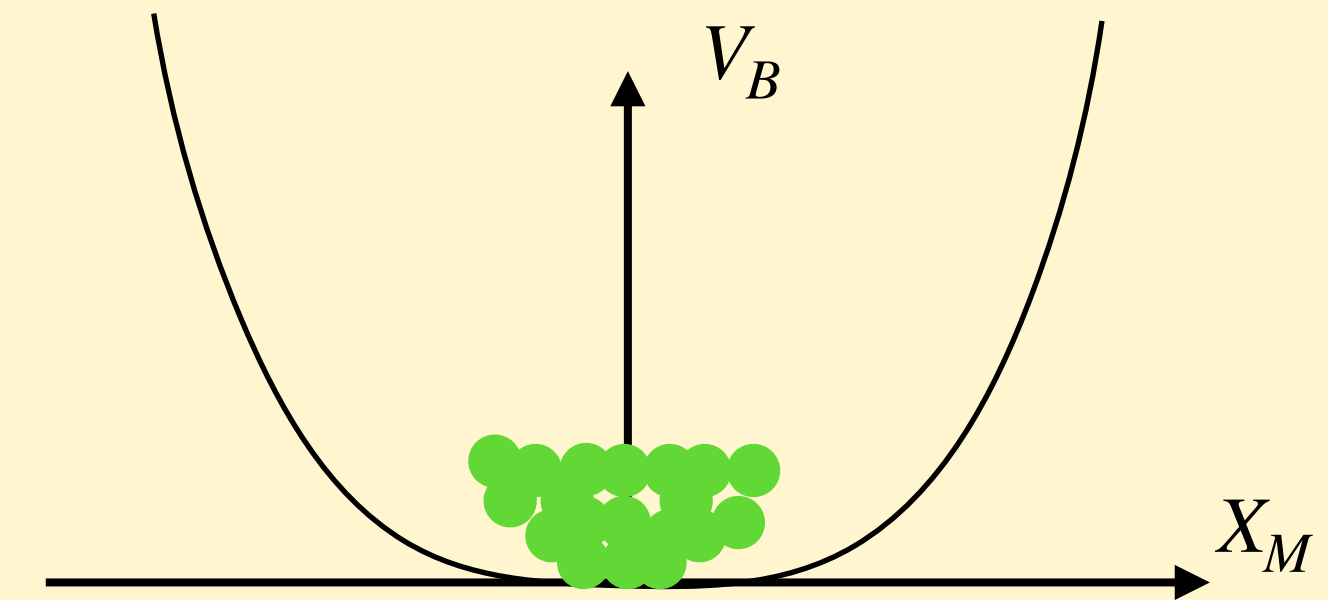


Witten 1995



$$M = 1, \dots, 9$$

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What does this correspond to in the gravity side?

Deformation of the D0-matrix model

The BMN model

Berestein, Maldacena, Nastase, 2002

$$S_{BMN} = S_b + S_f + \Delta S_b + \Delta S_f$$

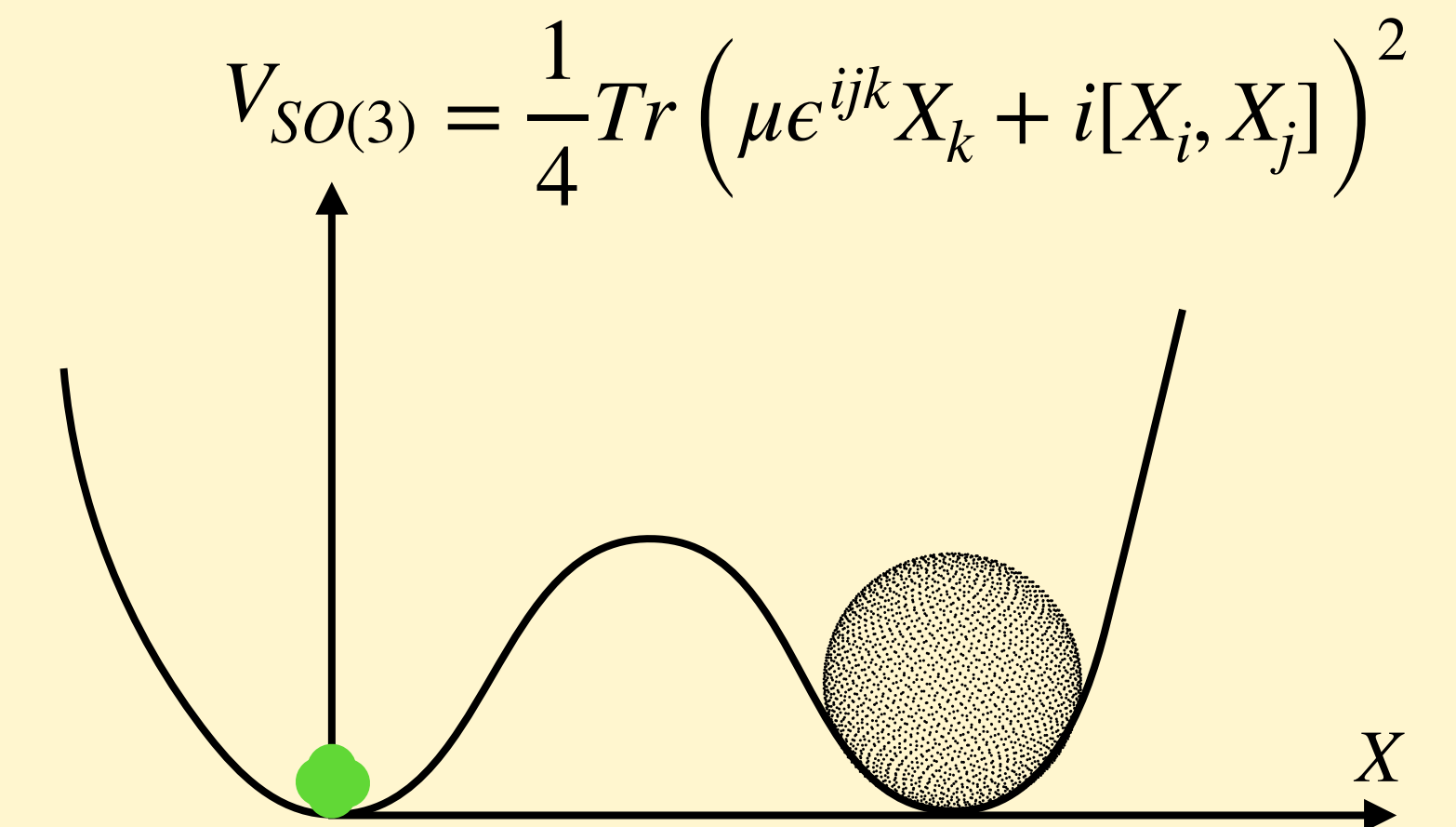
BFSS

$$S_b = \frac{N}{\lambda} \int_0^\beta dt \text{Tr} \left\{ \frac{1}{2} \sum_{I=1}^9 (D_t X_I)^2 - \frac{1}{4} \sum_{I,J=1}^9 [X_I, X_J]^2 \right\},$$

$$S_f = \frac{N}{\lambda} \int_0^\beta dt \text{Tr} \left\{ i\bar{\psi} \gamma^{10} D_t \psi - \sum_{I=1}^9 \bar{\psi} \gamma^I [X_I, \psi] \right\},$$

$$\Delta S_b = \frac{N}{\lambda} \int_0^\beta dt \text{Tr} \left\{ \frac{\mu^2}{2} \sum_{i=1}^3 X_i^2 + \frac{\mu^2}{8} \sum_{a=4}^9 X_a^2 + i\mu \sum_{i,j,k=1}^3 \epsilon^{ijk} X_i X_j X_k \right\},$$

$$\Delta S_f = \frac{3i\mu N}{4\lambda} \int_0^\beta dt \text{Tr} (\bar{\psi} \gamma^{123} \psi),$$



- Mass terms for bosons, fermions
- $SO(9) \rightarrow SO(3) \times SO(6)$
- $SU(2)$ vacua, i.e. fuzzy spheres

The curious case of $p=0$

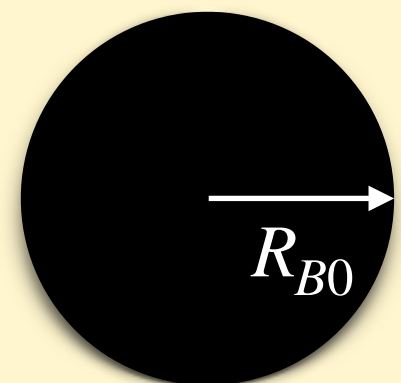
Strong coupling \longleftrightarrow Low energies

Black zero-brane in IIA SUGRA

$$g_{eff} = \frac{\lambda}{E^3}$$

$$\frac{ds^2}{\alpha'} = H(r)^{-\frac{1}{2}} f(r) dt^2 + H(r)^{\frac{1}{2}} \left(\frac{dr^2}{f(r)} + r^2 d\Omega_8^2 \right)$$

$$H(r) = \frac{240\pi^5 \lambda}{r^7}, \quad f(r) = 1 - \left(\frac{r_0}{r} \right)^7$$



$$E = 7.41 N^2 \lambda^{-3/5} T^{14/5}, \quad S = 11.52 N^2 \lambda^{-3/5} T^{9/5}$$

$$\frac{R_{B0}^2}{\alpha'} \sim g_{eff}^{\frac{1}{2}} \sim \sqrt{\frac{\lambda}{E^3}}$$

$$e\phi \Big|_{horizon} \sim \frac{g_{eff}^{7/4}}{N}$$

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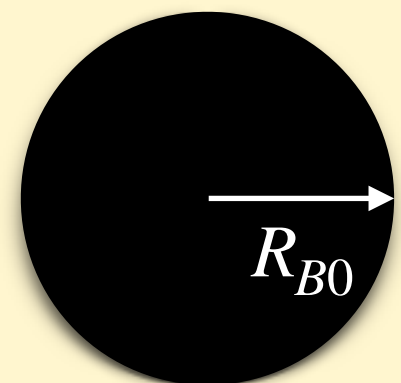
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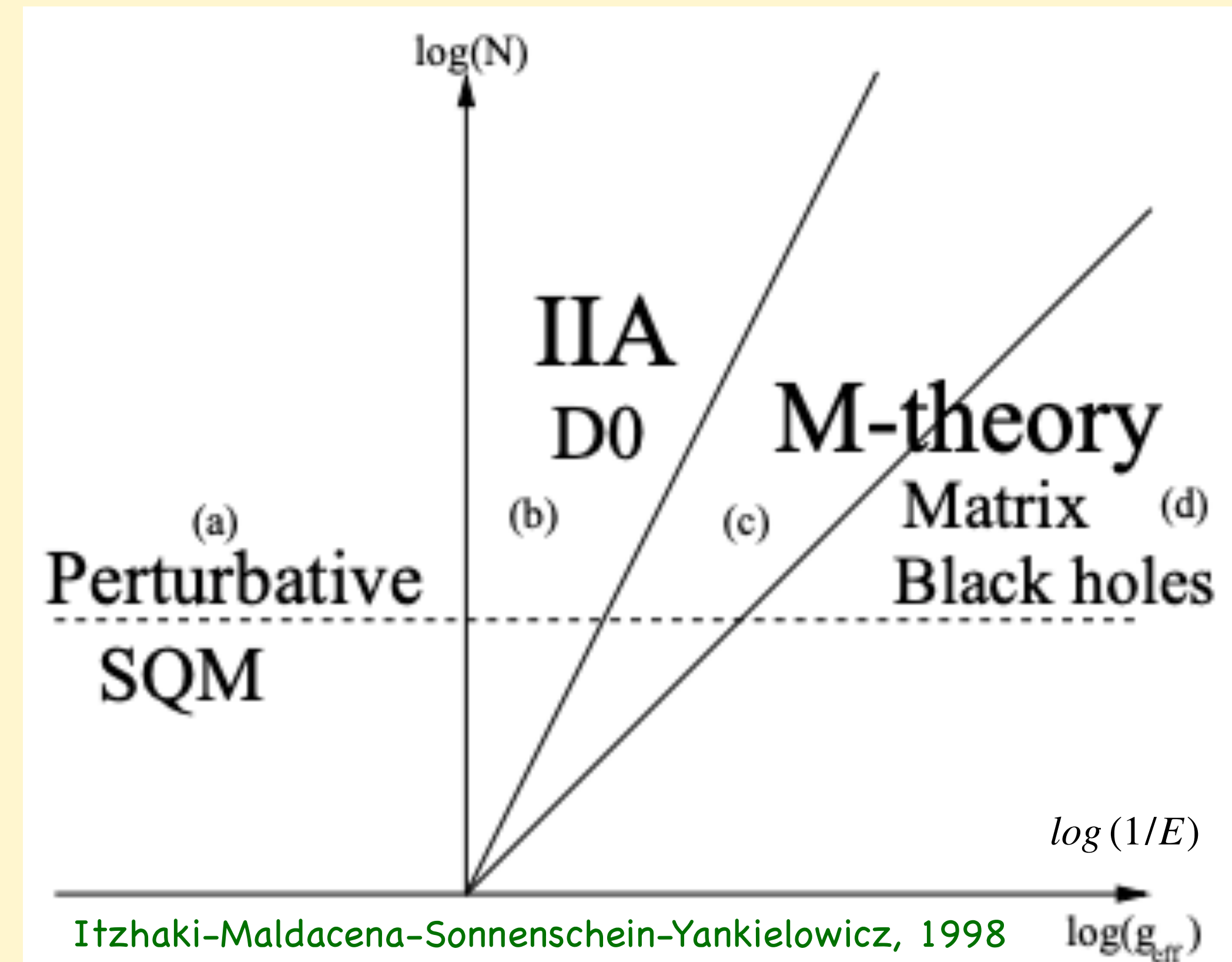
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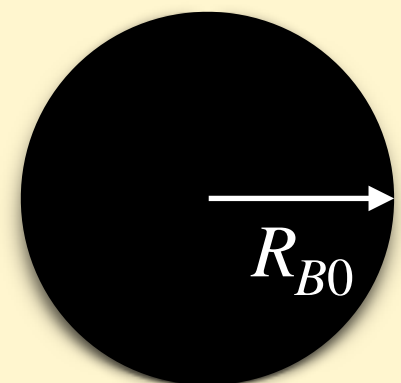
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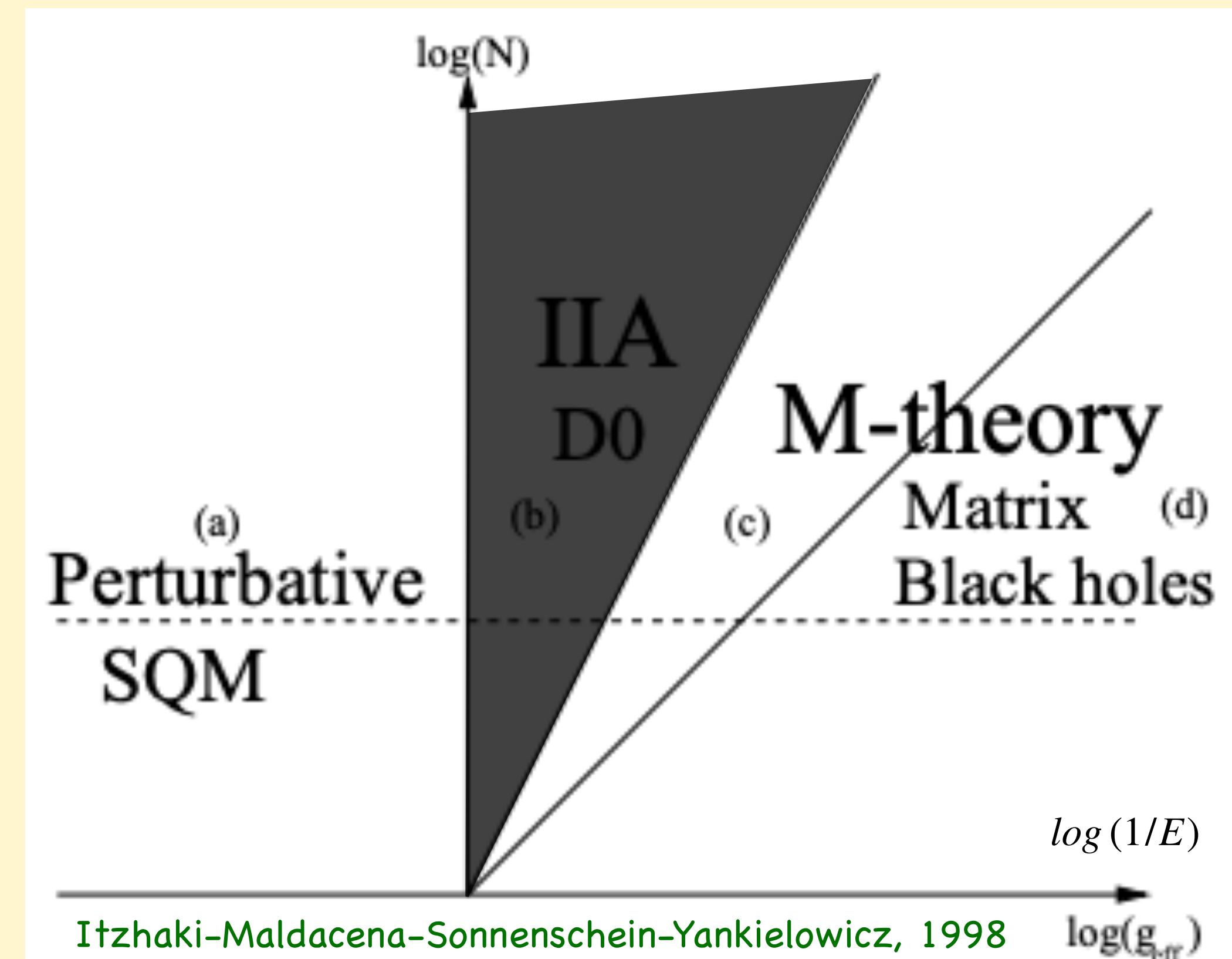
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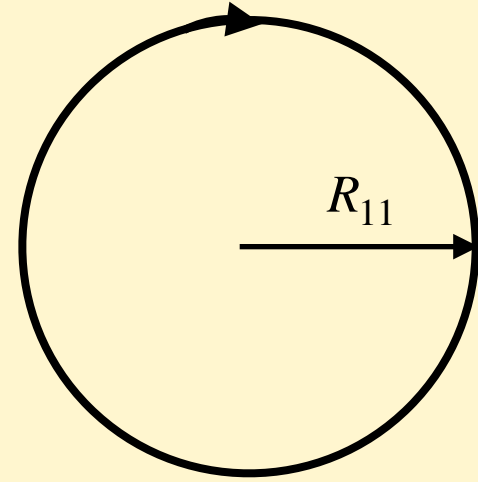
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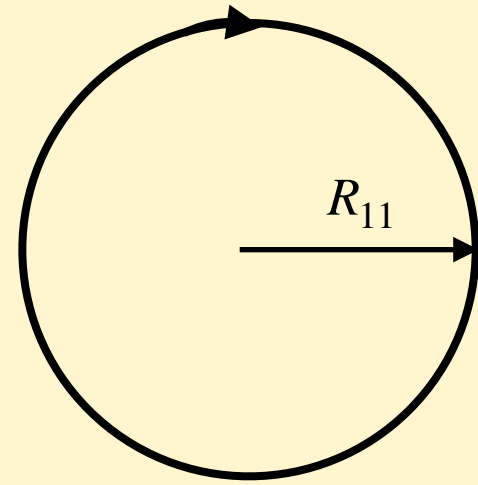


Type IIA string theory is defined as M-theory compactified on a circle S^1

$$R_{11} \sim 2\pi g_s l_s = 2\pi l_s e^\phi \simeq l_s \frac{g_{\text{eff}}^{7/4}}{N}, \quad g_{\text{eff}} = \frac{\lambda}{E^3} \quad \text{Strong coupling/low energies corresponds to the M-theory region}$$

To probe M-theory region $E \ll 1$ ($E = 7.41 N^2 \lambda^{-3/5} T^{14/5}$) \longrightarrow Low temperatures

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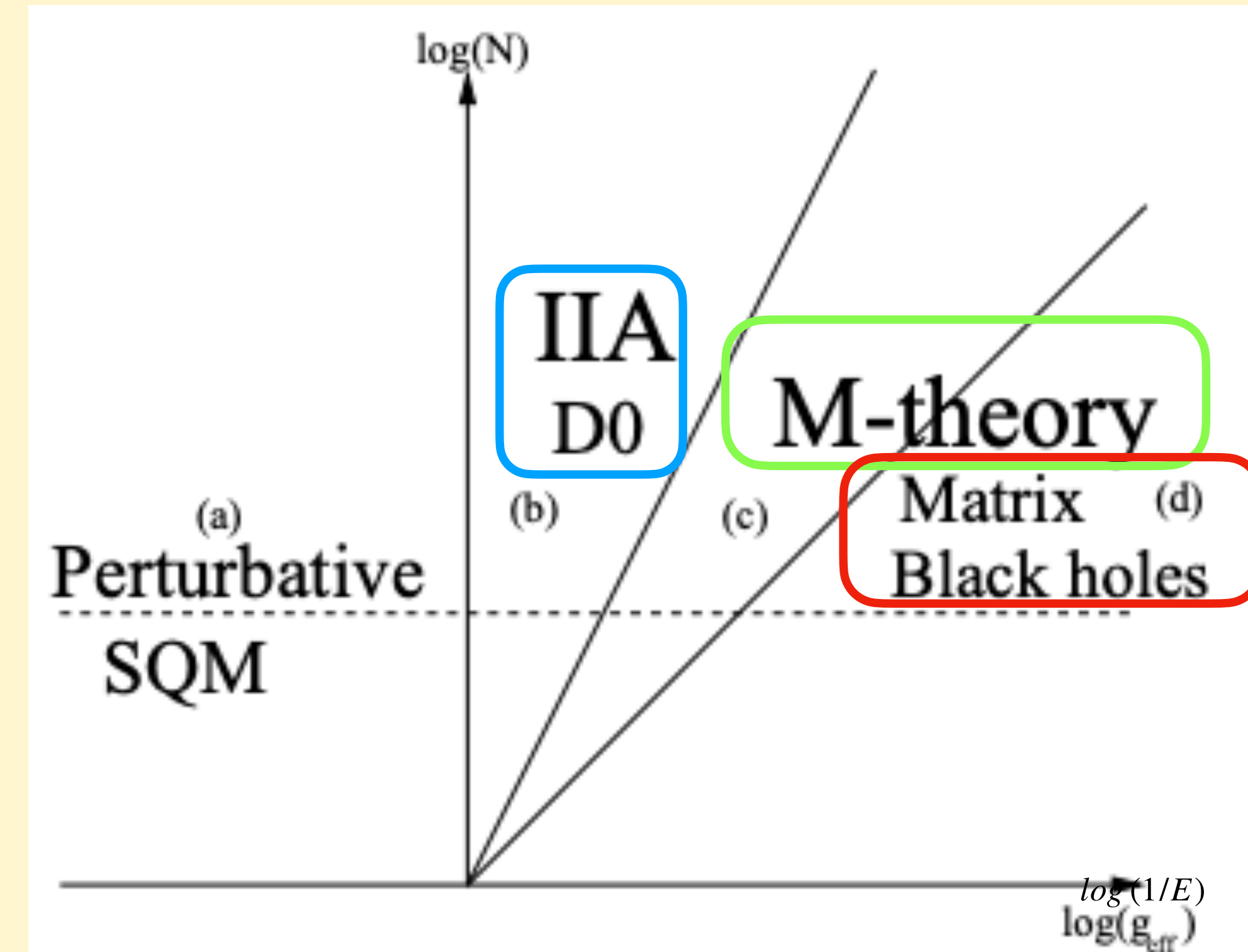
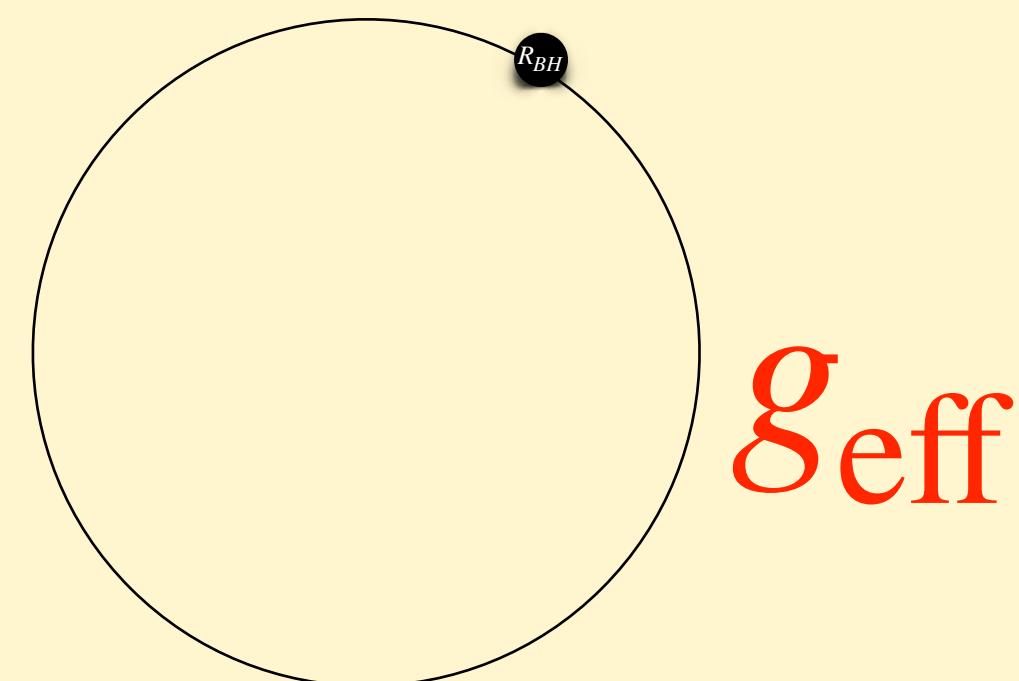
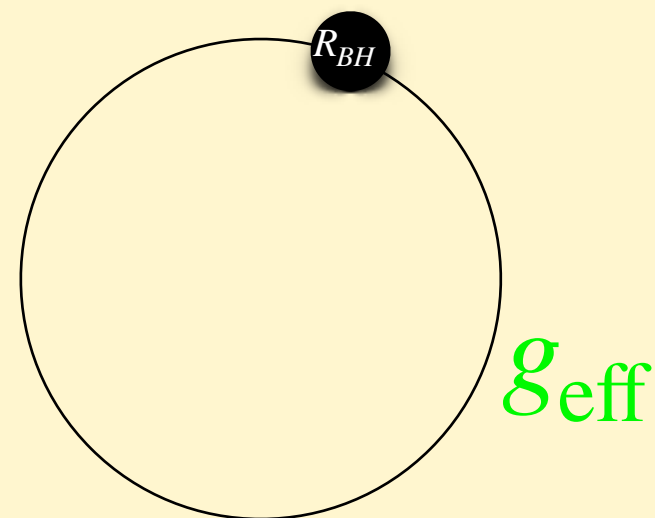
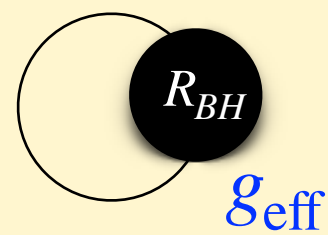


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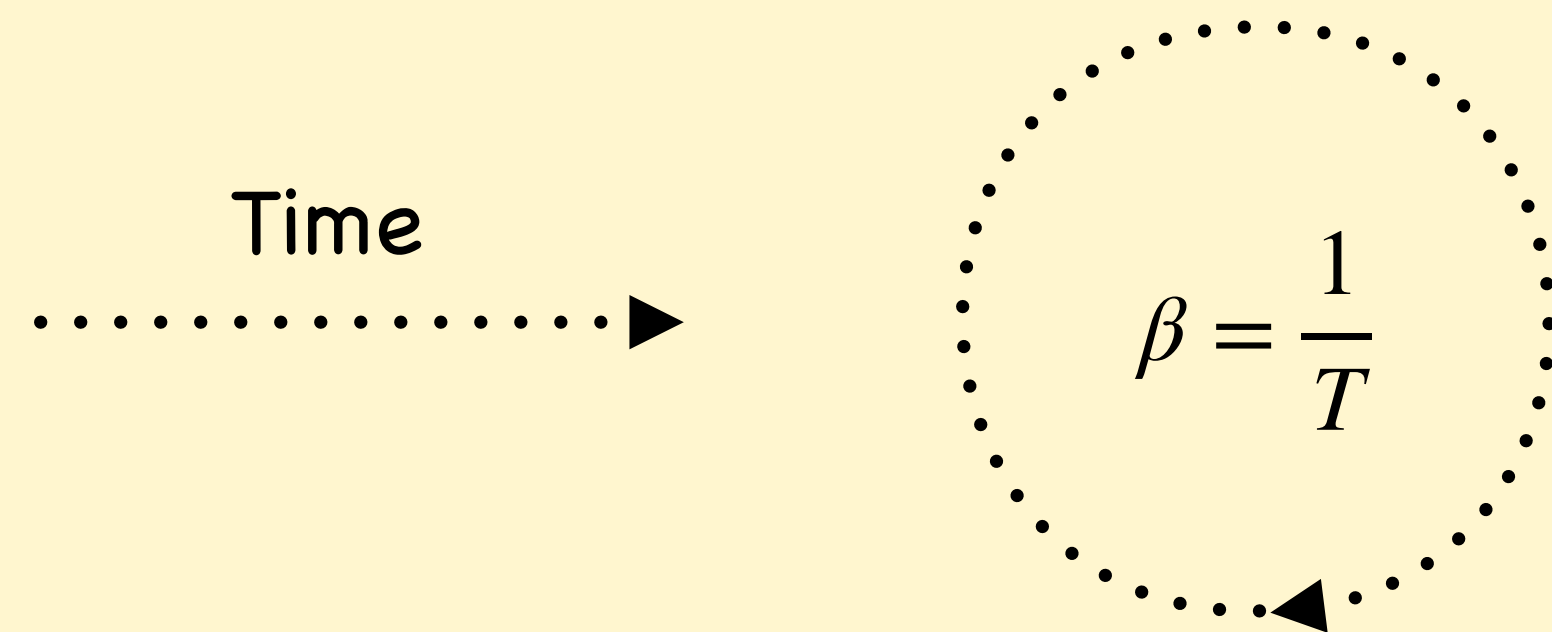
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Switch to simulations

Model on the lattice

- We can do Monte Carlo simulations
- Borrow techniques from lattice QCD
- (0+1)-d matrix quantum mechanics

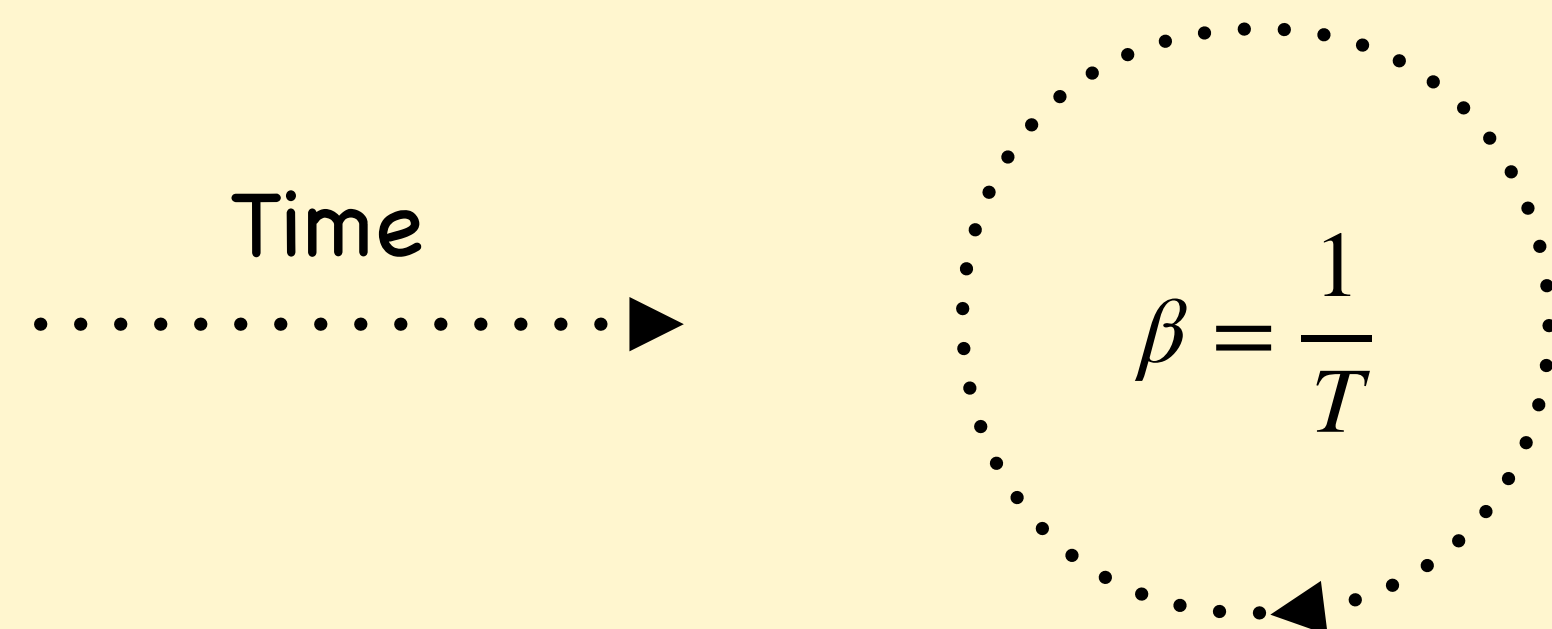


○ Parameters

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T	\longrightarrow	Temperature

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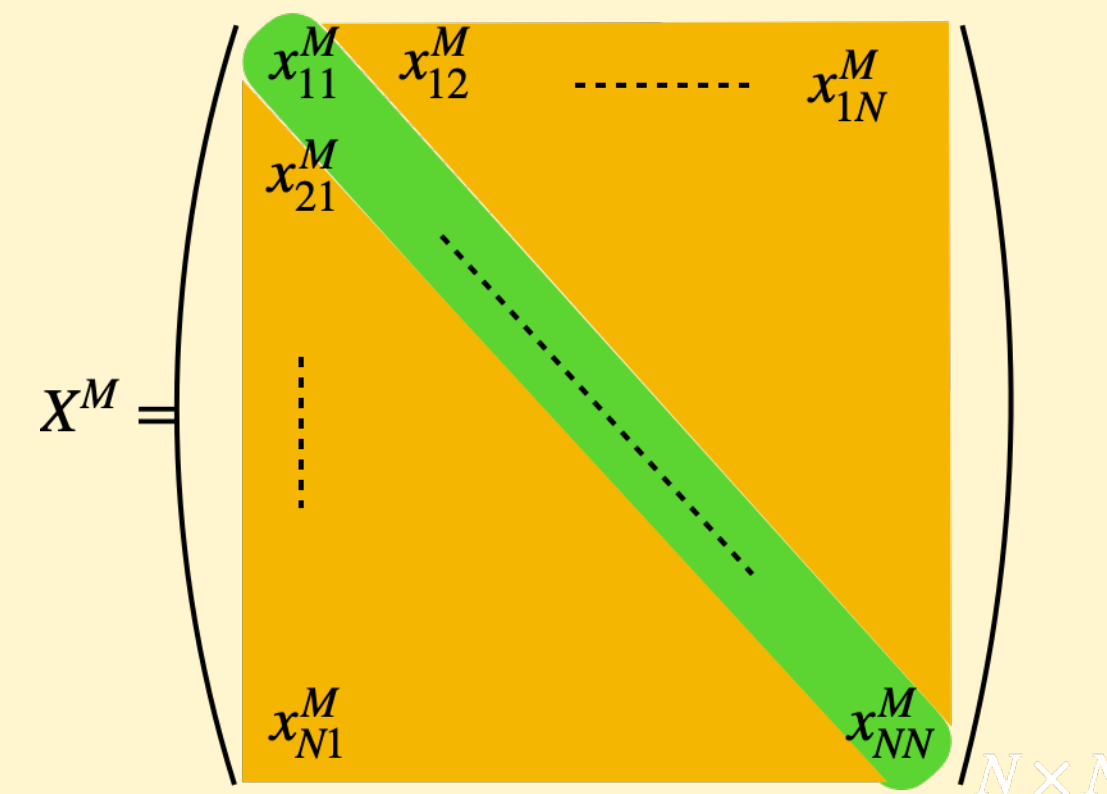


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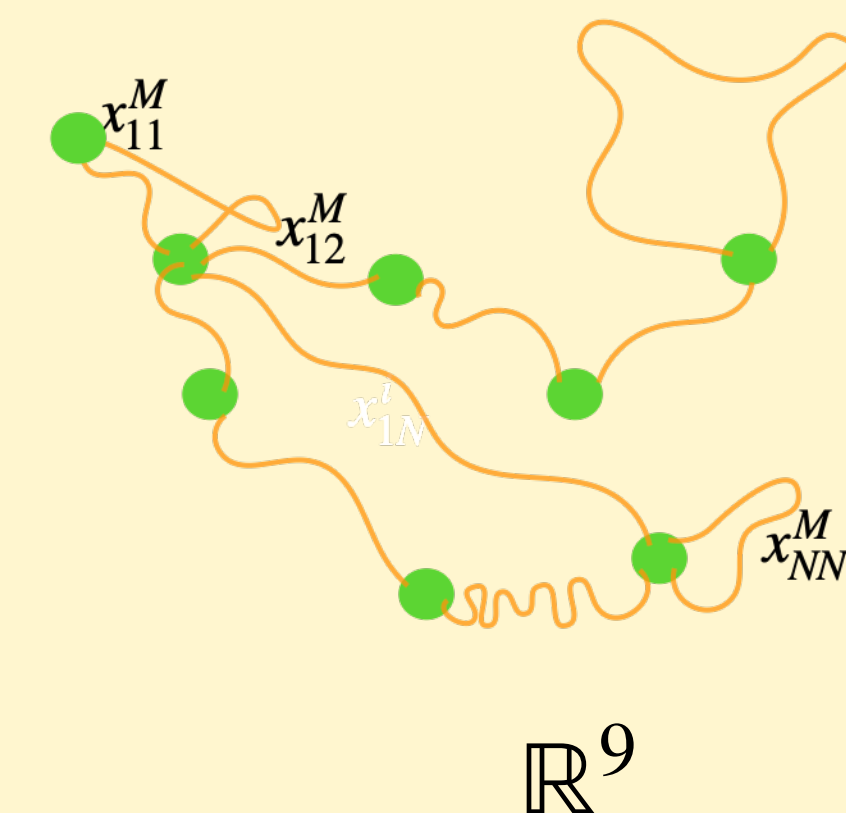
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large N limit

$$\begin{array}{|c|} \hline \lambda \sim N^0 \\ T \sim N^0 \\ \hline \end{array} \longleftrightarrow \begin{array}{|c|} \hline \lambda = 1 : \mathbf{fix} \\ \lambda^{-\frac{1}{3}} T \sim N^0 : \mathbf{fix} \\ \hline \end{array}$$



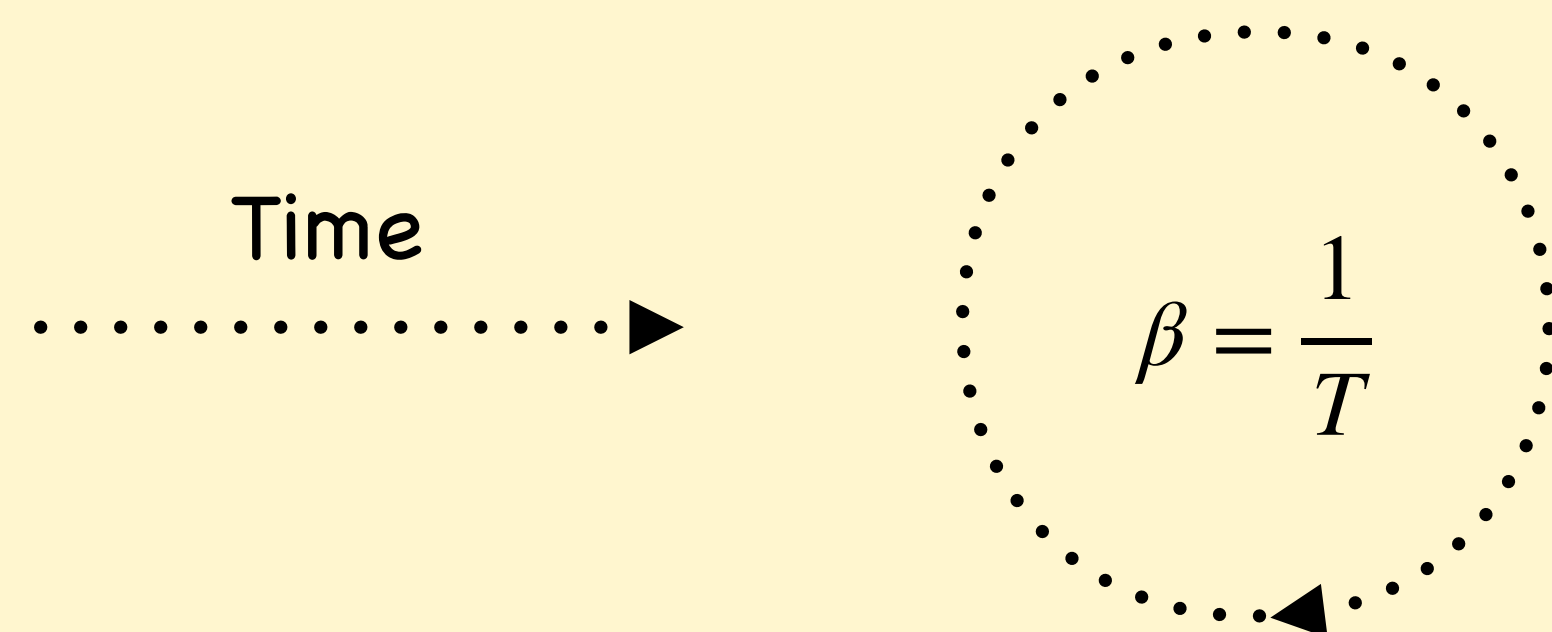
Witten 1995



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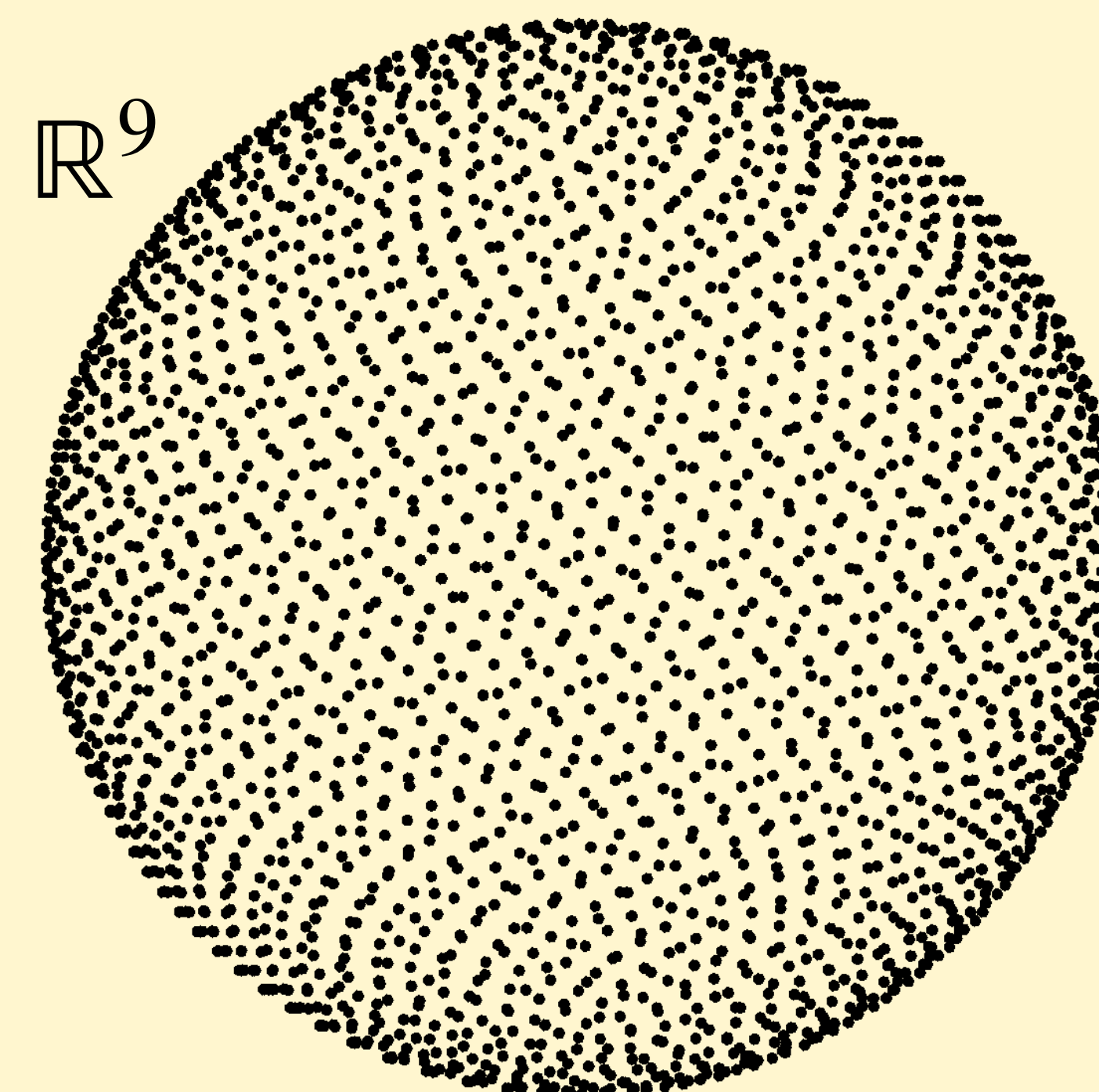


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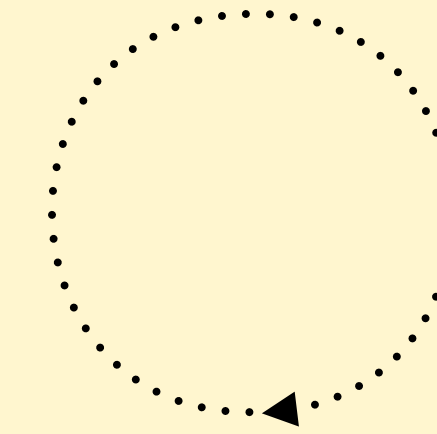


Confinement/deconfinement

- Polyakov loop

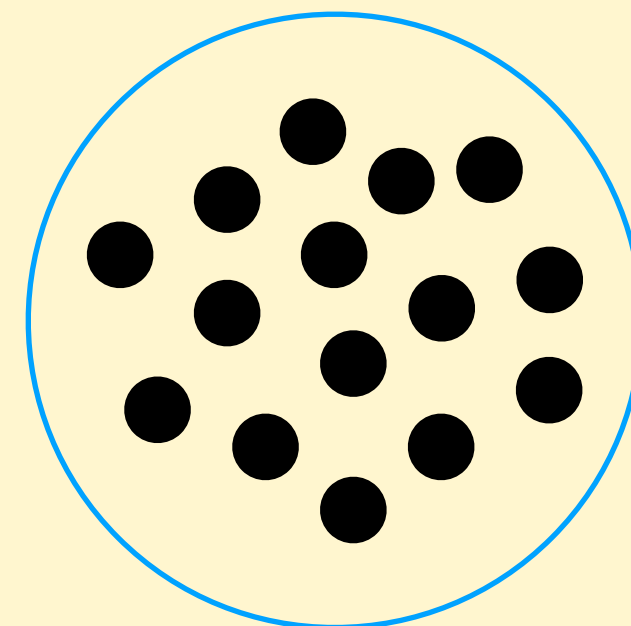
$$P = \frac{1}{N} \text{Tr} \left(\mathcal{P} \exp \left(i \int_0^\beta dt A_t \right) \right) = \frac{1}{N} \sum_{j=1}^N e^{i\theta_j}$$

On the lattice



- Restoration/breaking of $U(1)$ symmetry

- Intuition from $AdS_5 \times S^5$



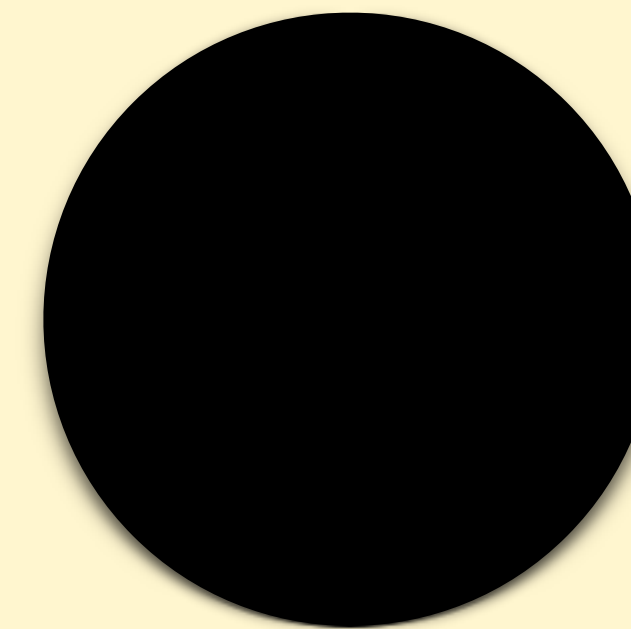
$$E, P = 0$$

Confinement
Graviton gas

Gravity side

MAGOO, Witten, Sundborg

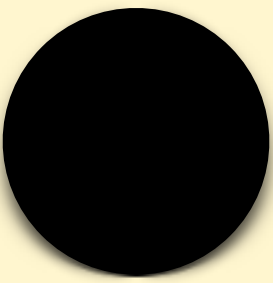
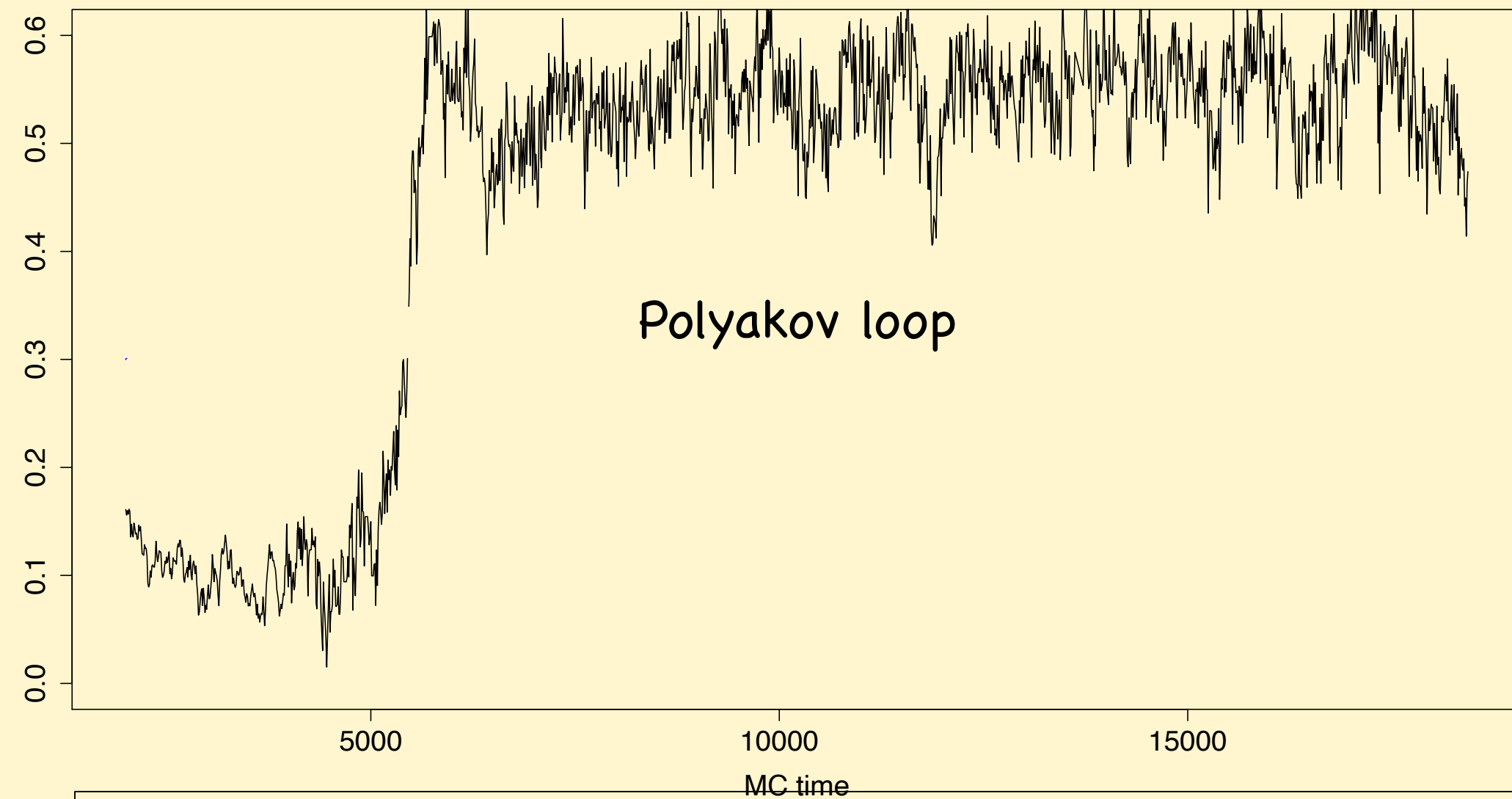
Aharony-Marsano-Minwalla-Papadodimas-Van
Raamsdonk, 2003



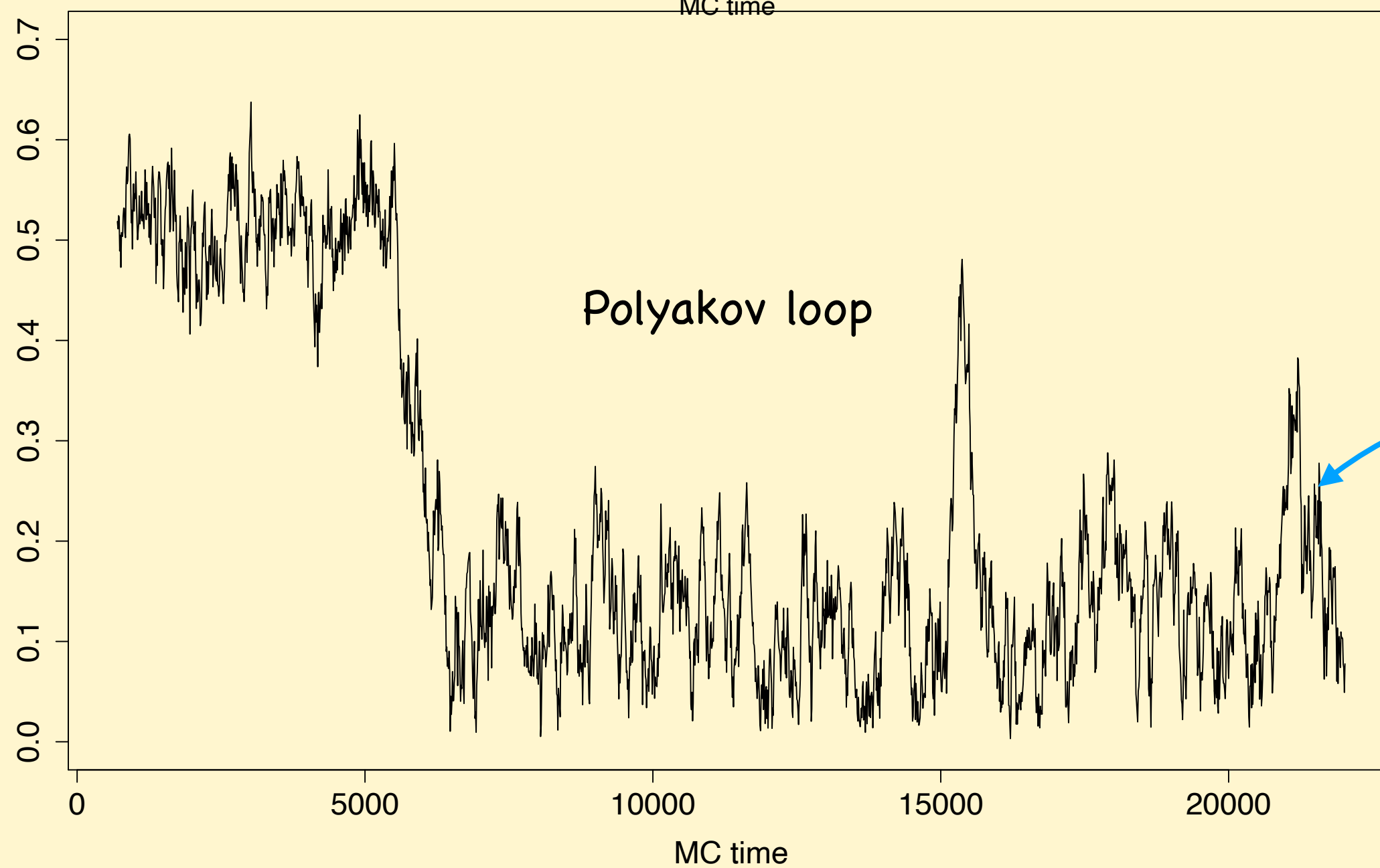
$$E \neq 0, P \gtrsim \frac{1}{2}$$

Deconfinement
Black holes

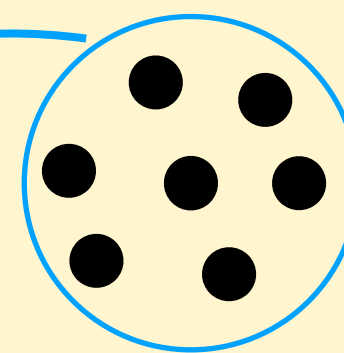
Confinement/deconfinement



deconfined

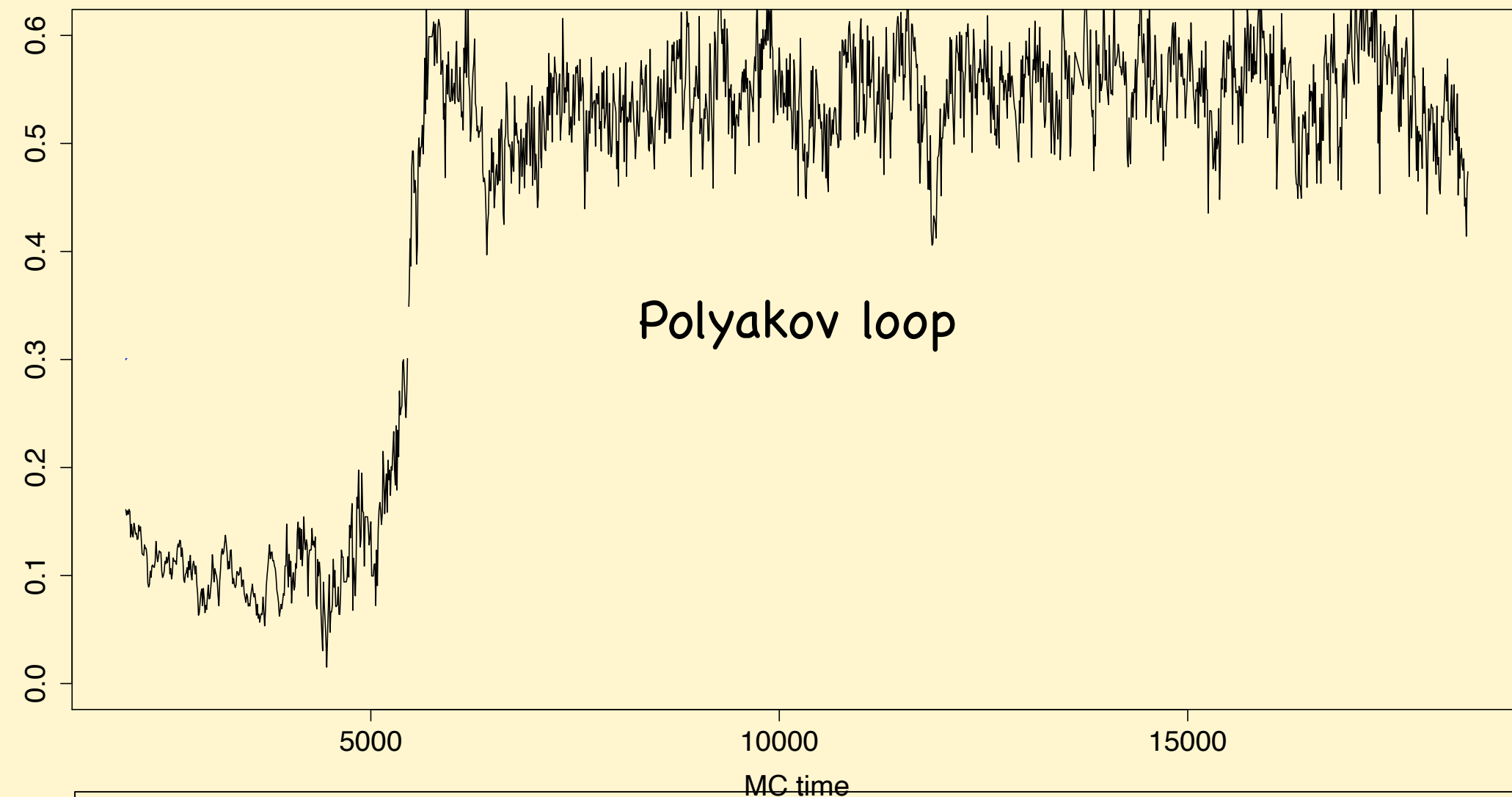
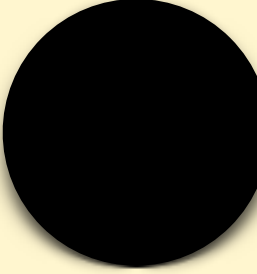
$$P \gtrsim \frac{1}{2}$$


Confined



$$P \approx 0$$

Confinement/deconfinement

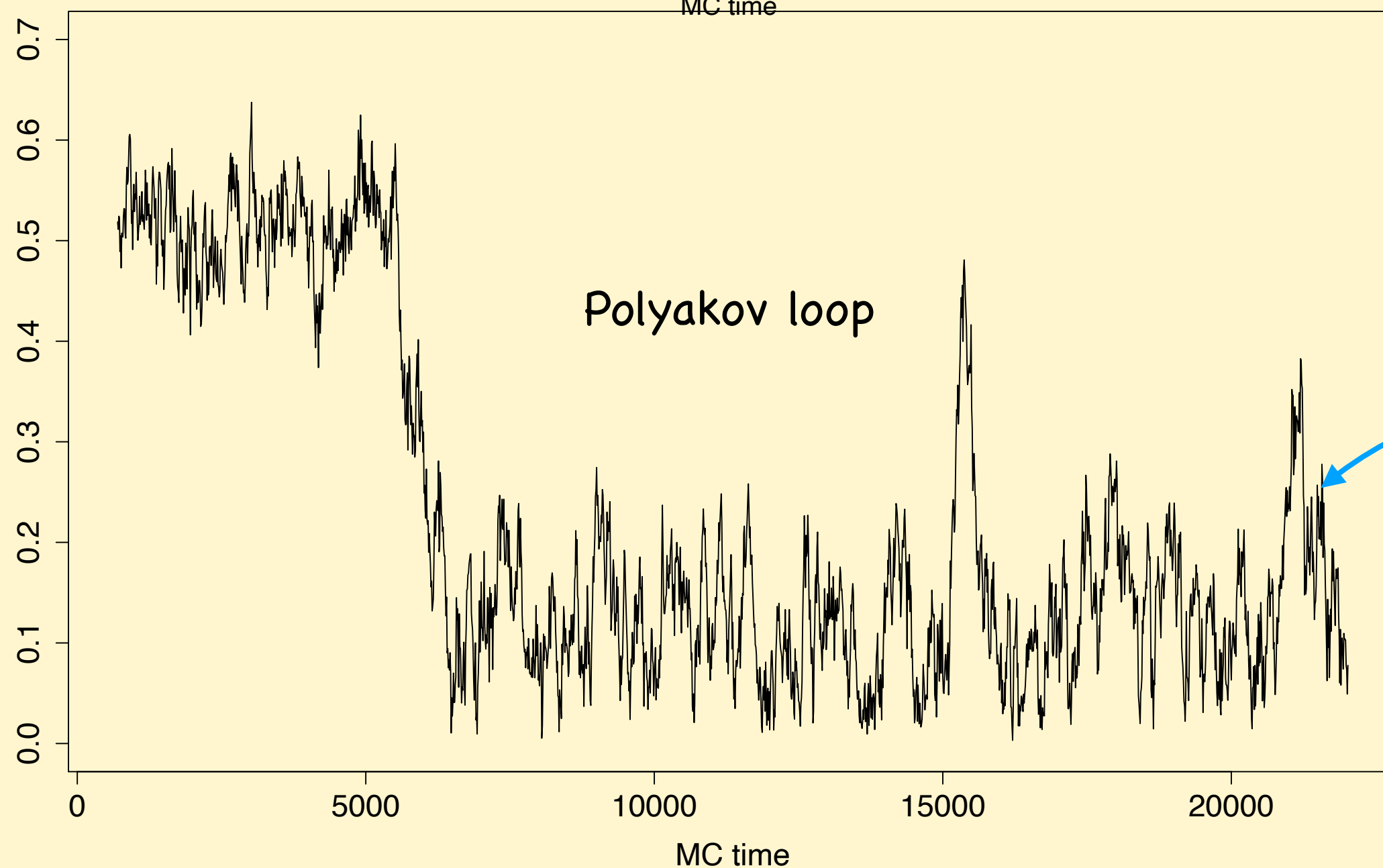



deconfined

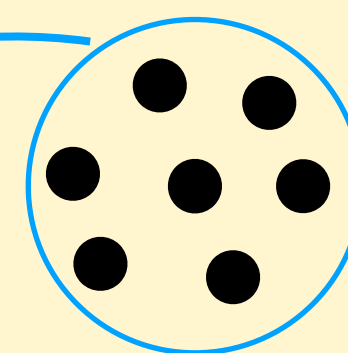
$$P \gtrsim \frac{1}{2}$$

The gravity theory predicts
always **deconfinement**

$$(E = 7.41N^2\lambda^{-3/5}T^{14/5})$$



Confined

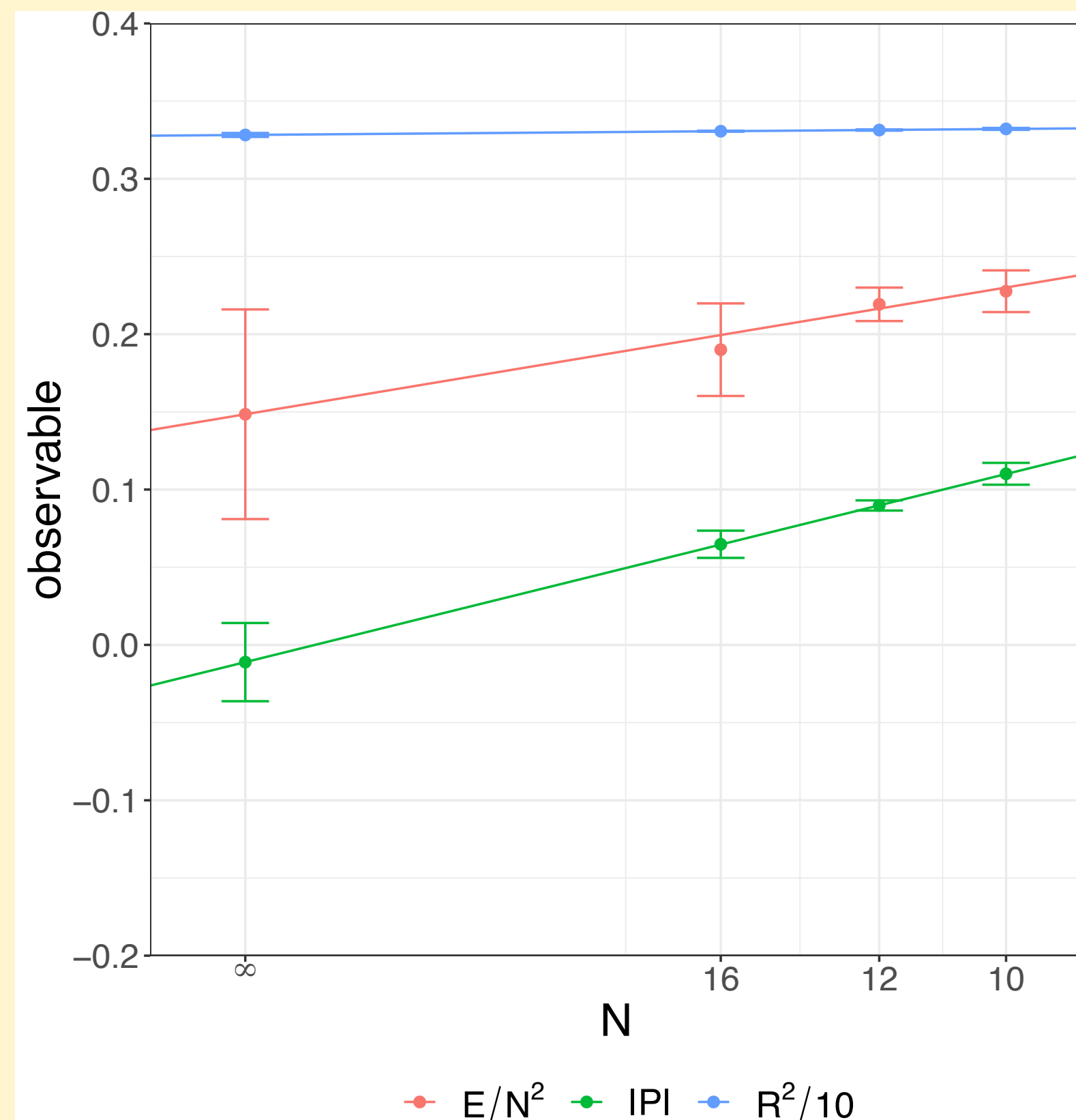


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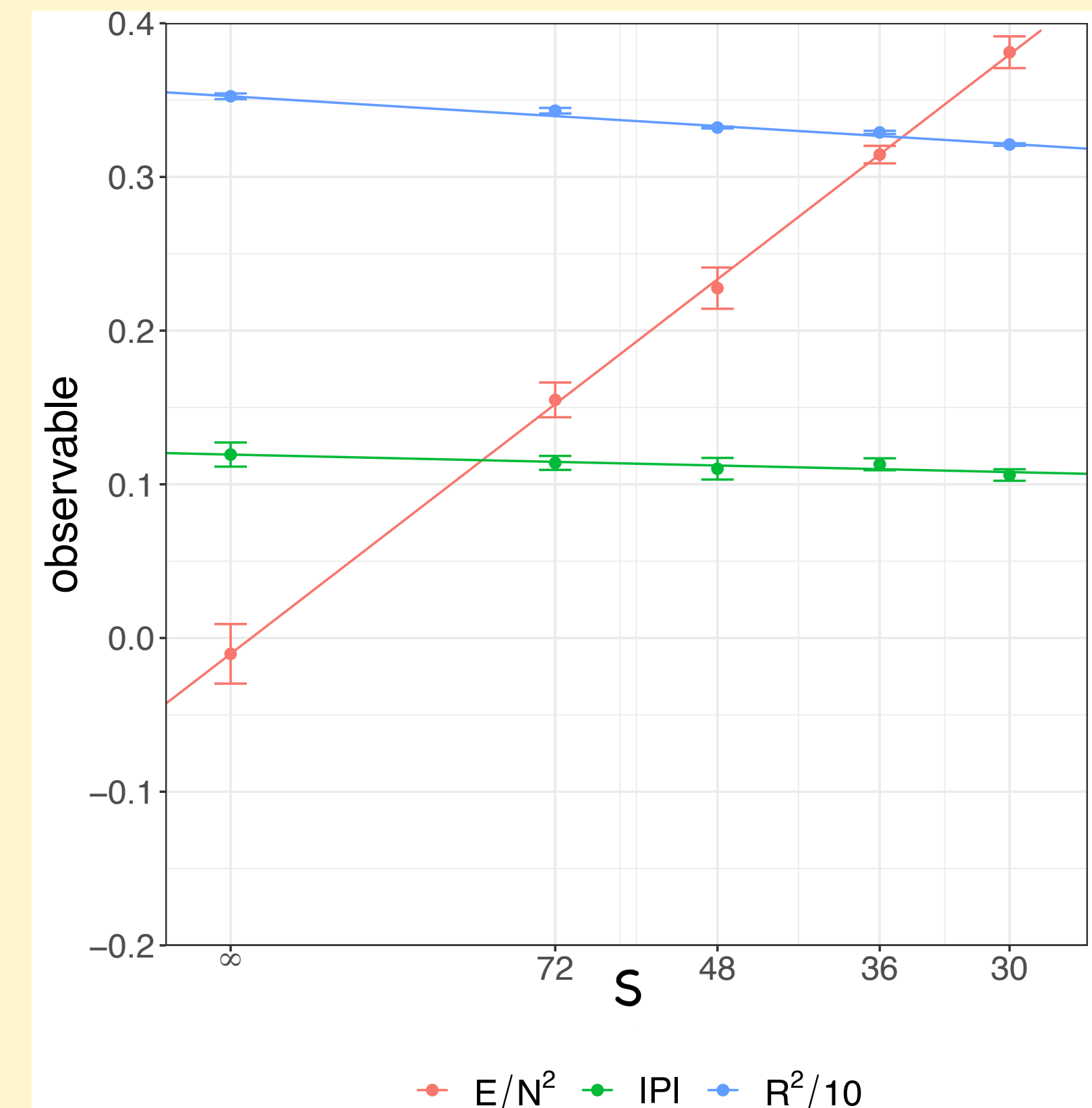
How to understand this?

Confined studies of D0-matrix model

Large N and S=48 @ T=0.2



Continuum and N=10 @ T=0.2



Combine both

Deconfined phase

$$\frac{E}{N^2} \simeq 7.41T^{\frac{14}{5}} \simeq 0.0818 \quad @ T=0.2$$

$$P \simeq 0.5 \quad @ T=0.2$$

Confined phase

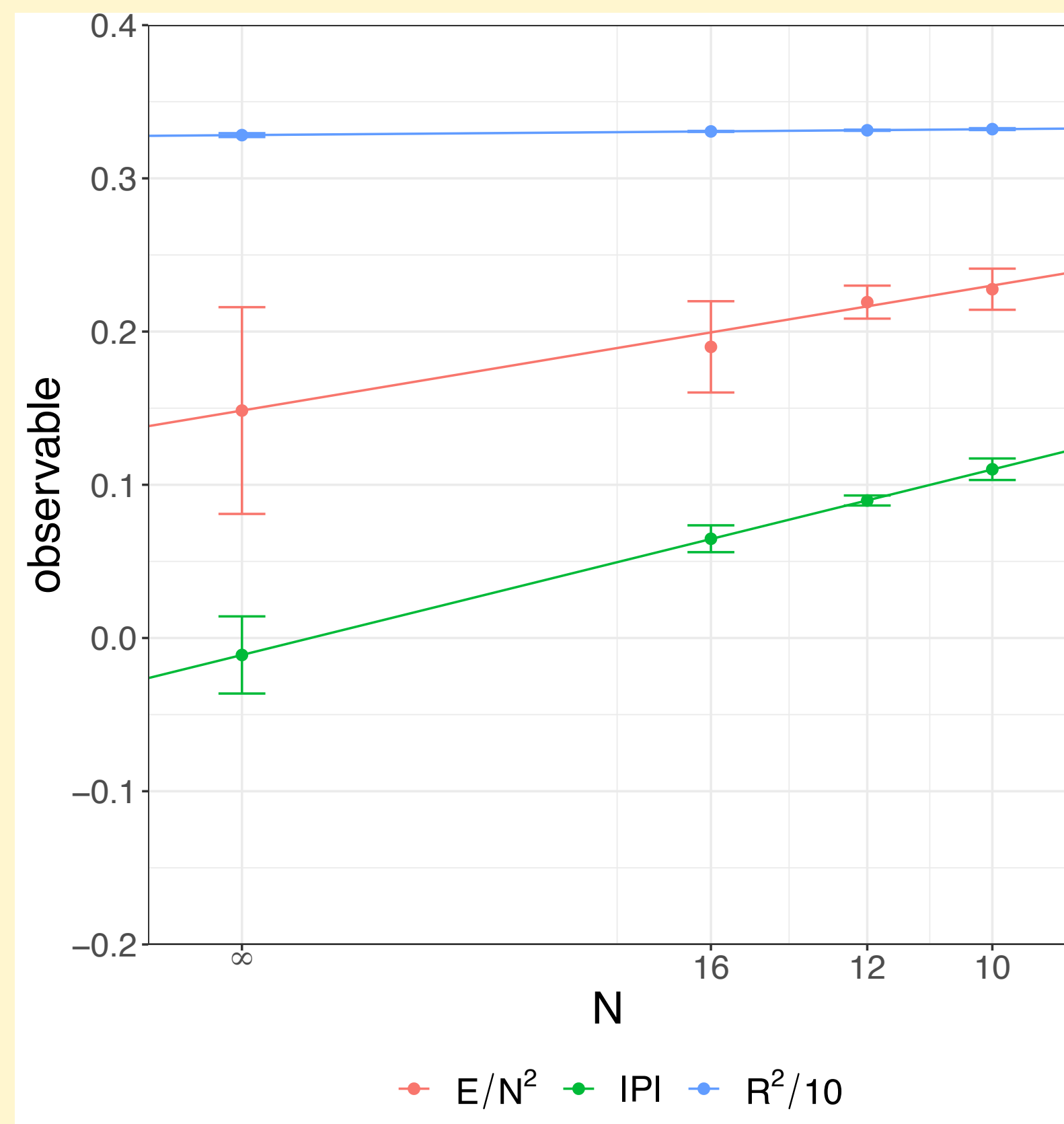
$$E \simeq 0$$

$$P \simeq 0$$

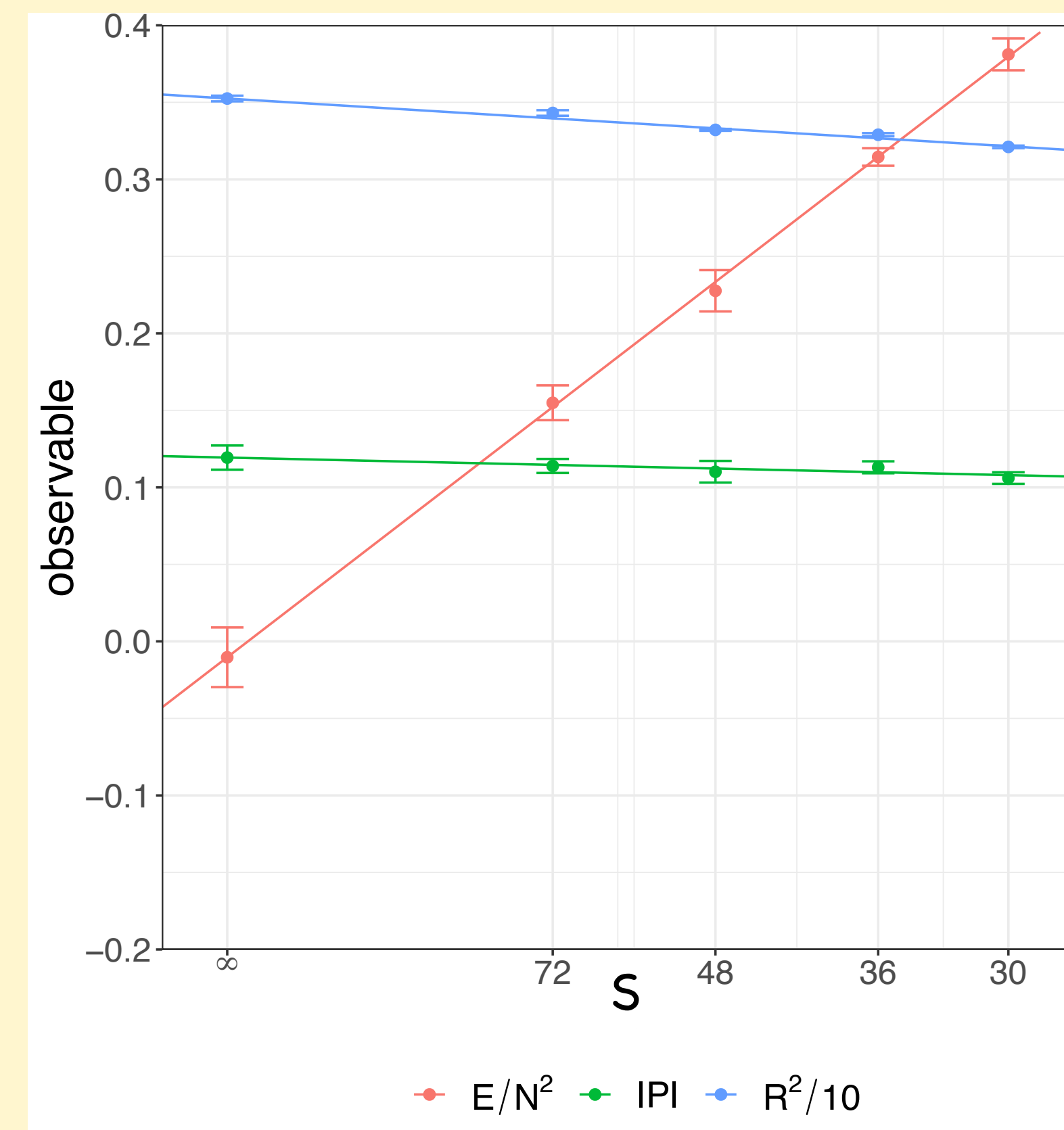
@ large N and continuum

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X

Deconfined phase

$$\frac{E}{N^2} \simeq 7.41T^{\frac{14}{5}} \simeq 0.0818 \quad @ T=0.2$$

$$P \simeq 0.5 \quad @ T=0.2$$

Confined phase

$$E \simeq 0$$

$$P \simeq 0$$

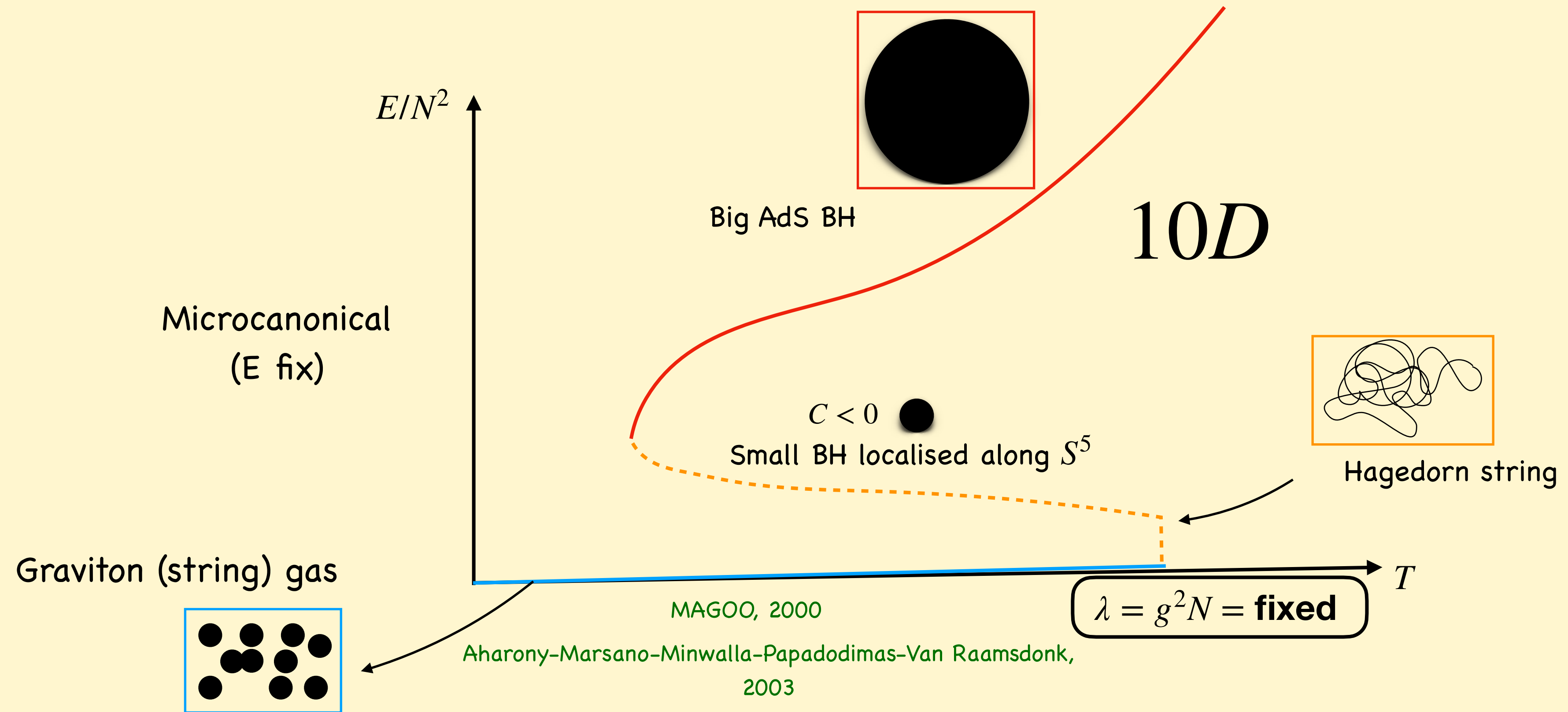
@ large N and continuum

Conventional holography

How to understand confinement?

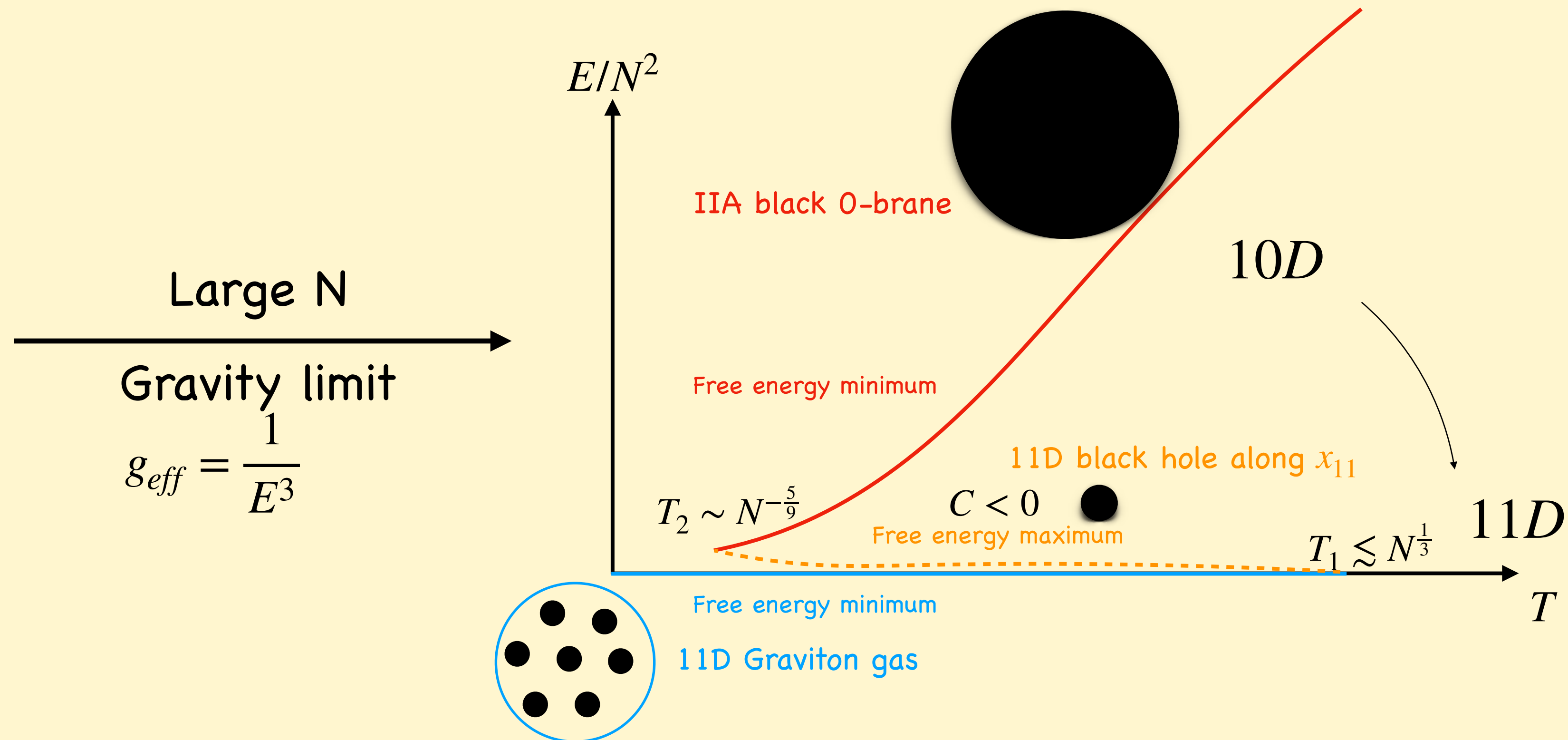
Motivation from

$$AdS_5 \times S^5$$

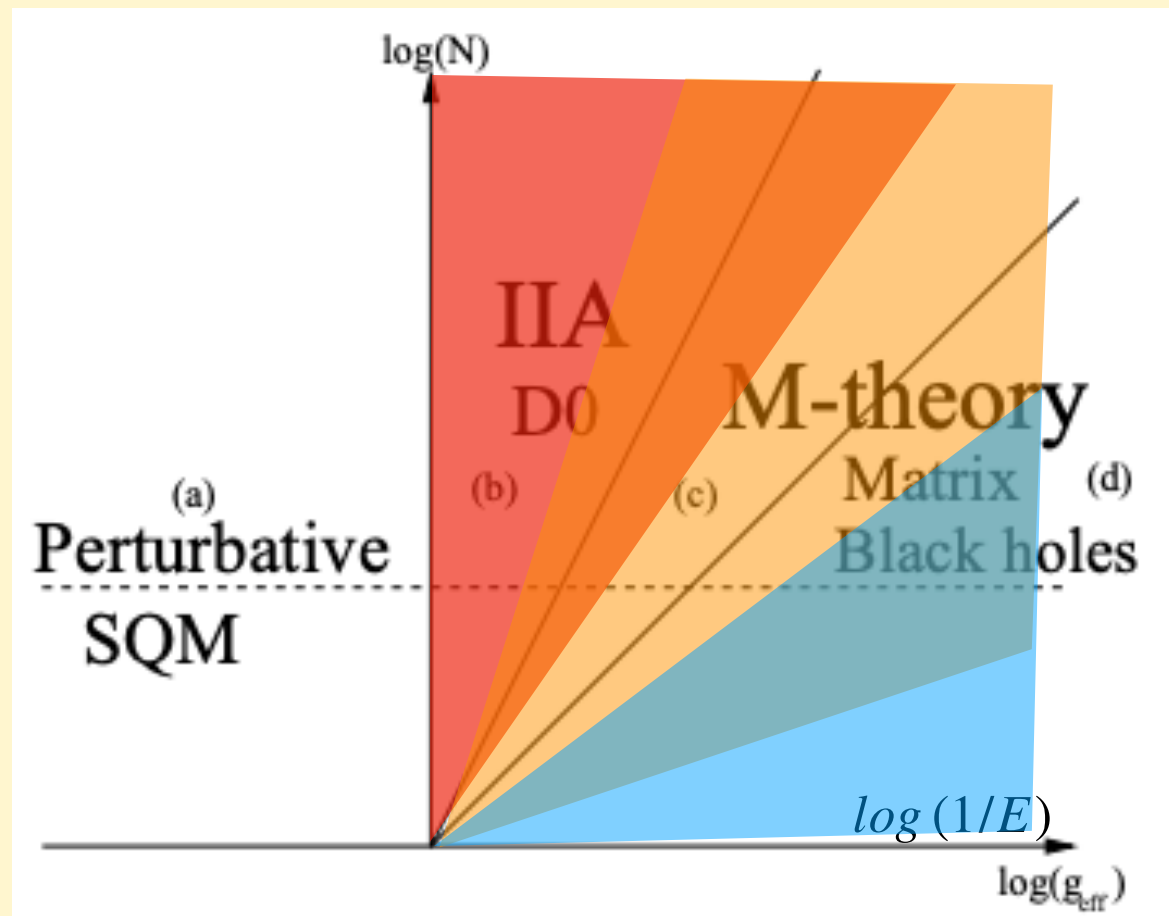


Confinement in the D0-matrix model

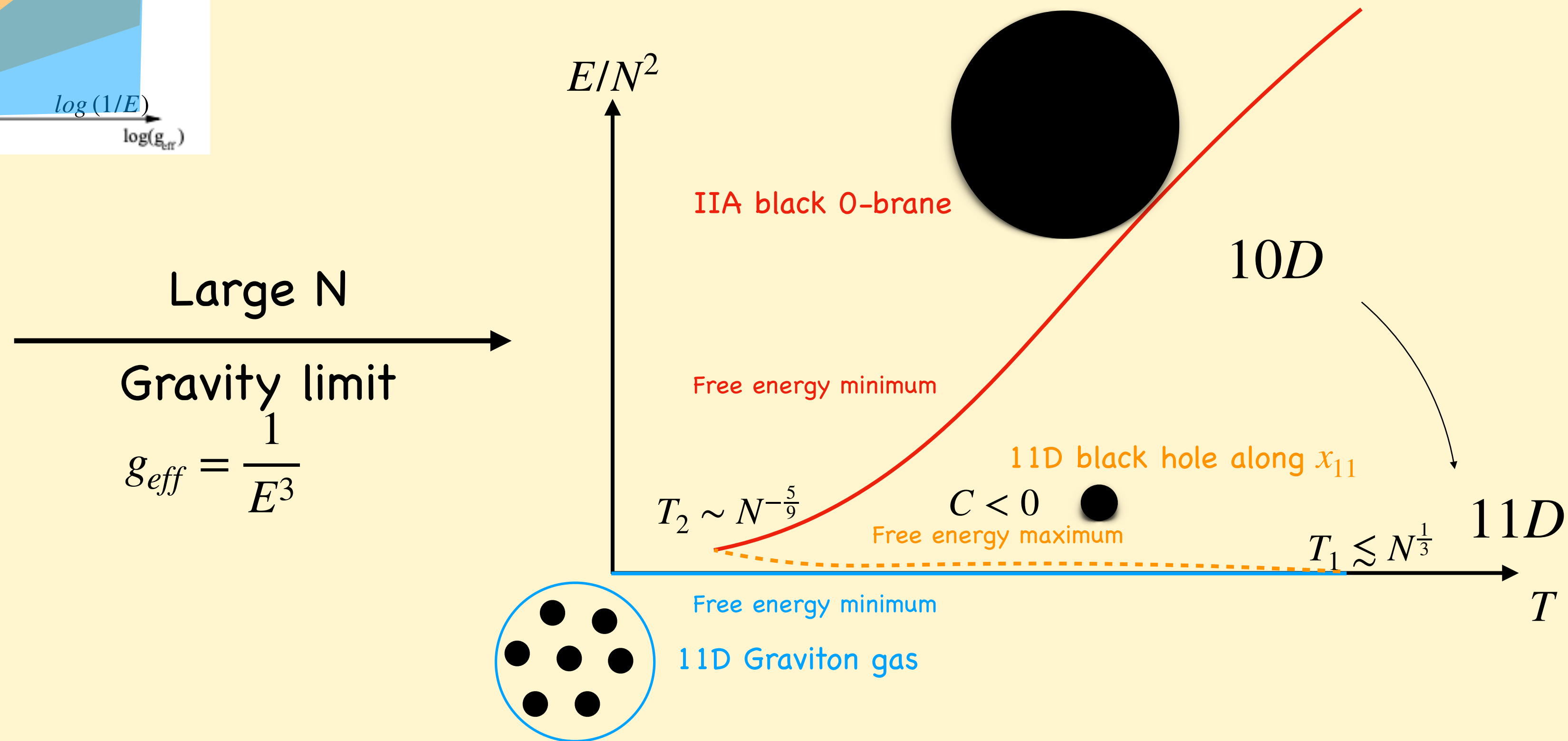
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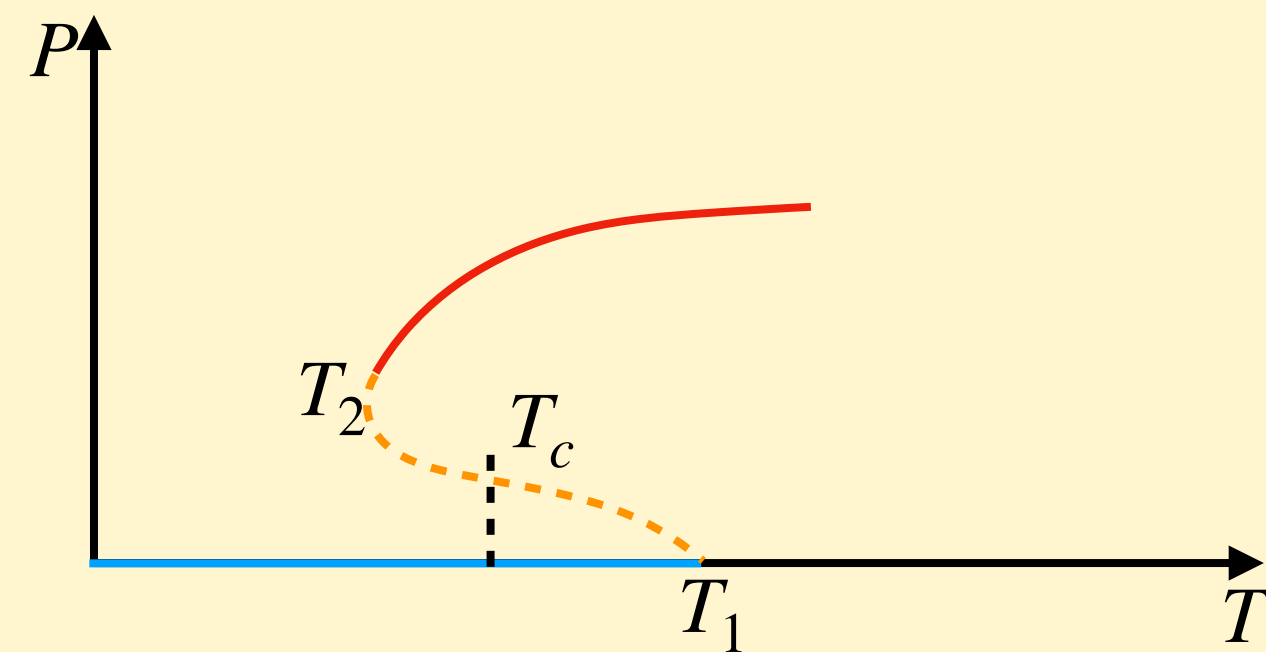
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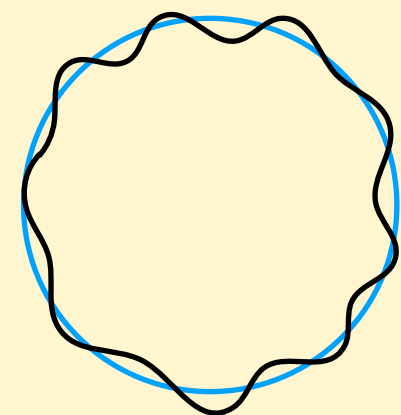


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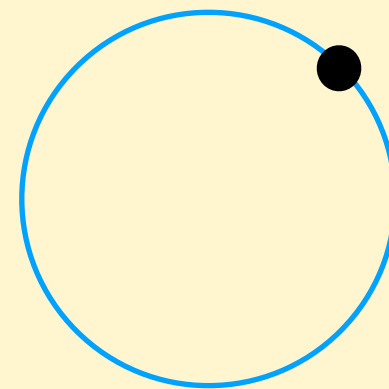


How to determine T_1 and T_2 ?

- T_2 corresponds to Gregory-Laflamme transition (or Gross-Witten-Wadia)



A black string wrapping S^1
 Collapses to a BH localised along S^1



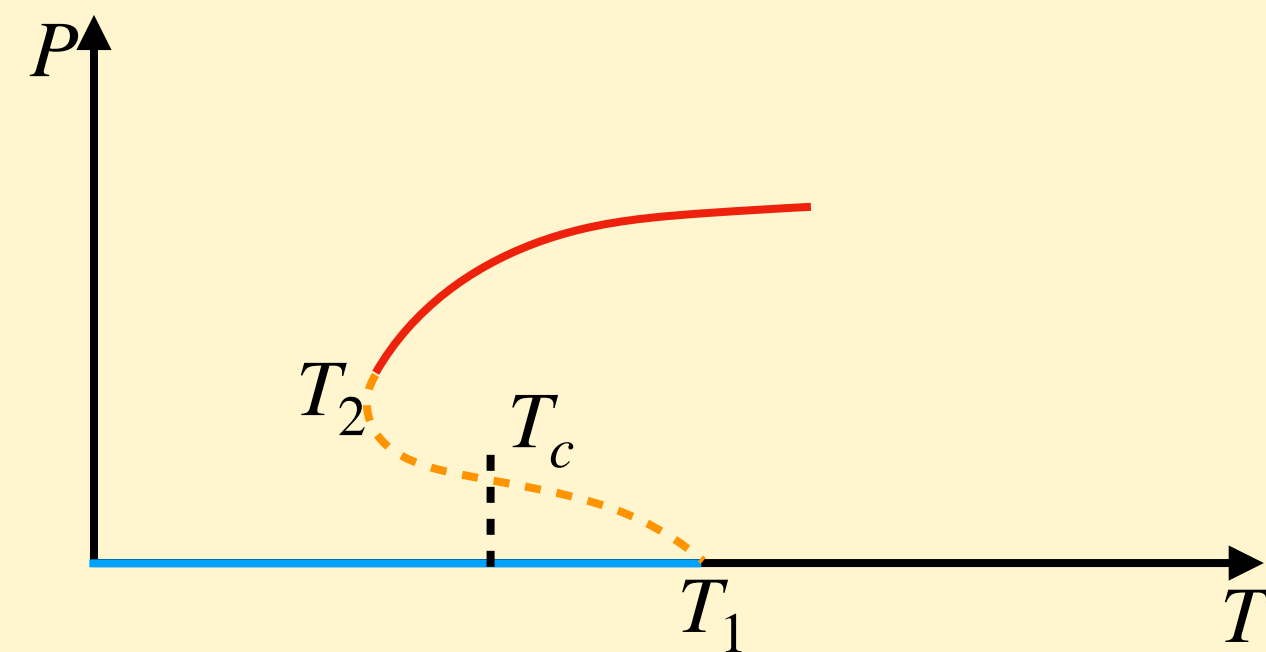
$$T_2 \sim N^{-5/9}$$

- T_1 corresponds to maximum/minimum **confinement/deconfinement** temperature

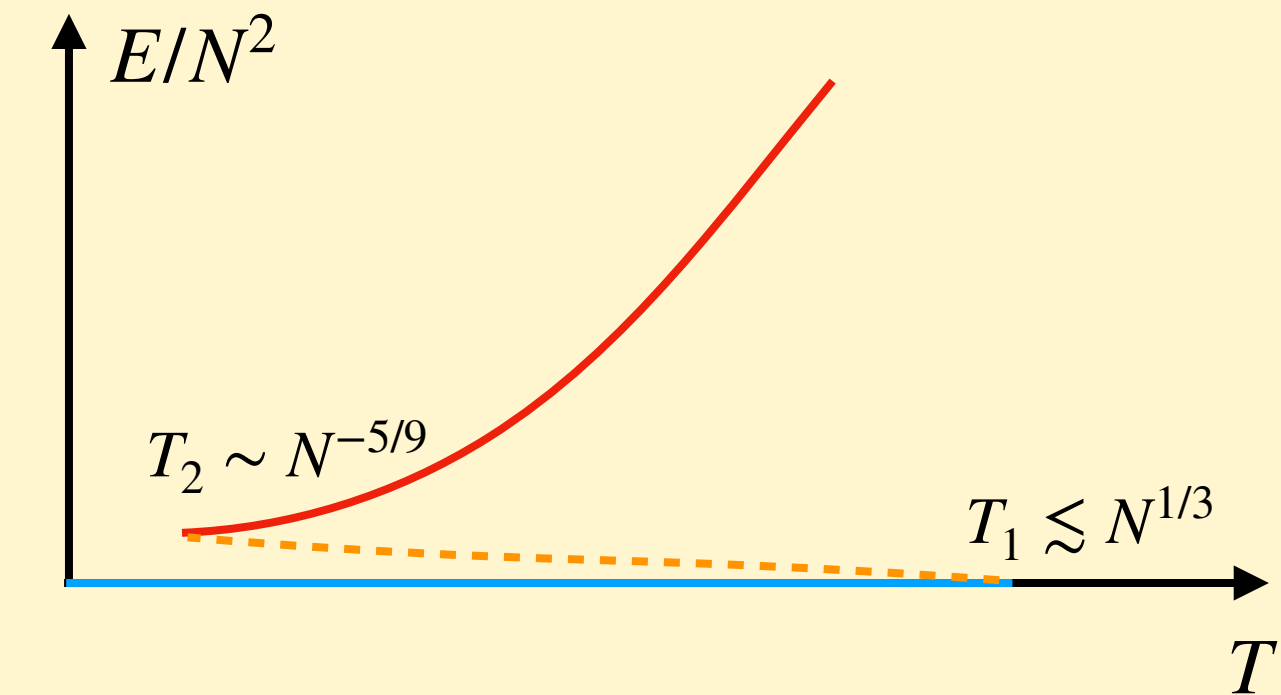
Schwarzschild BH in 11D with $M = M_{Pl}$ \longrightarrow $T_1 \lesssim N^{1/3}$

Gregory-Laflamme 1994, Gubser-Mitra 2001,
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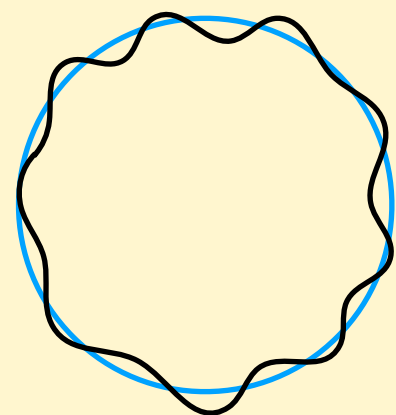
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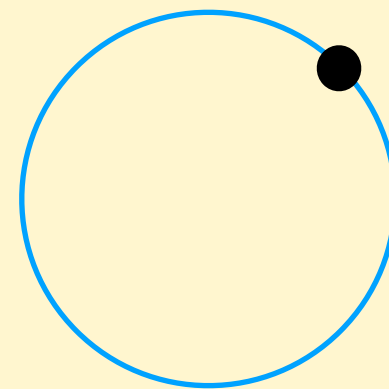
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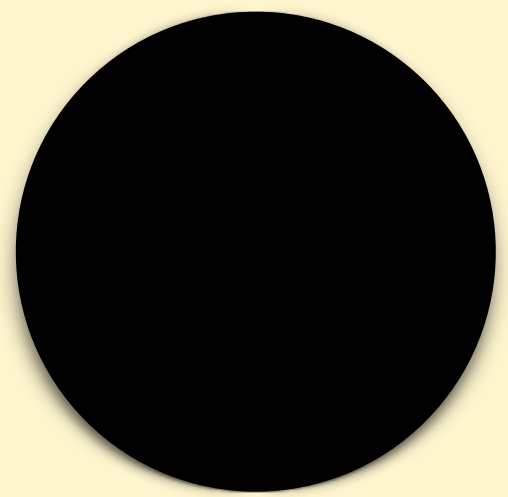
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Deconfined studies in the D0-matrix model

Tests of gauge/gravity duality

- One of the most precise tests of holography appeared in [arxiv/1606.04951](https://arxiv.org/abs/1606.04951)

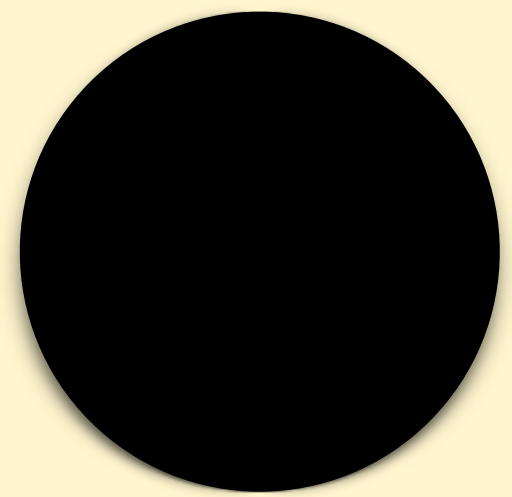


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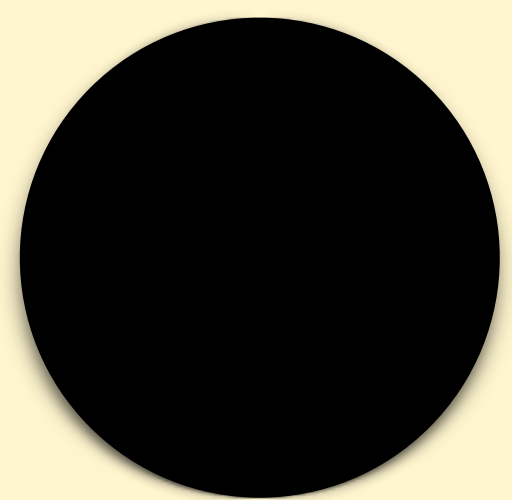
Reproduced by simulations of matrix quantum mechanics

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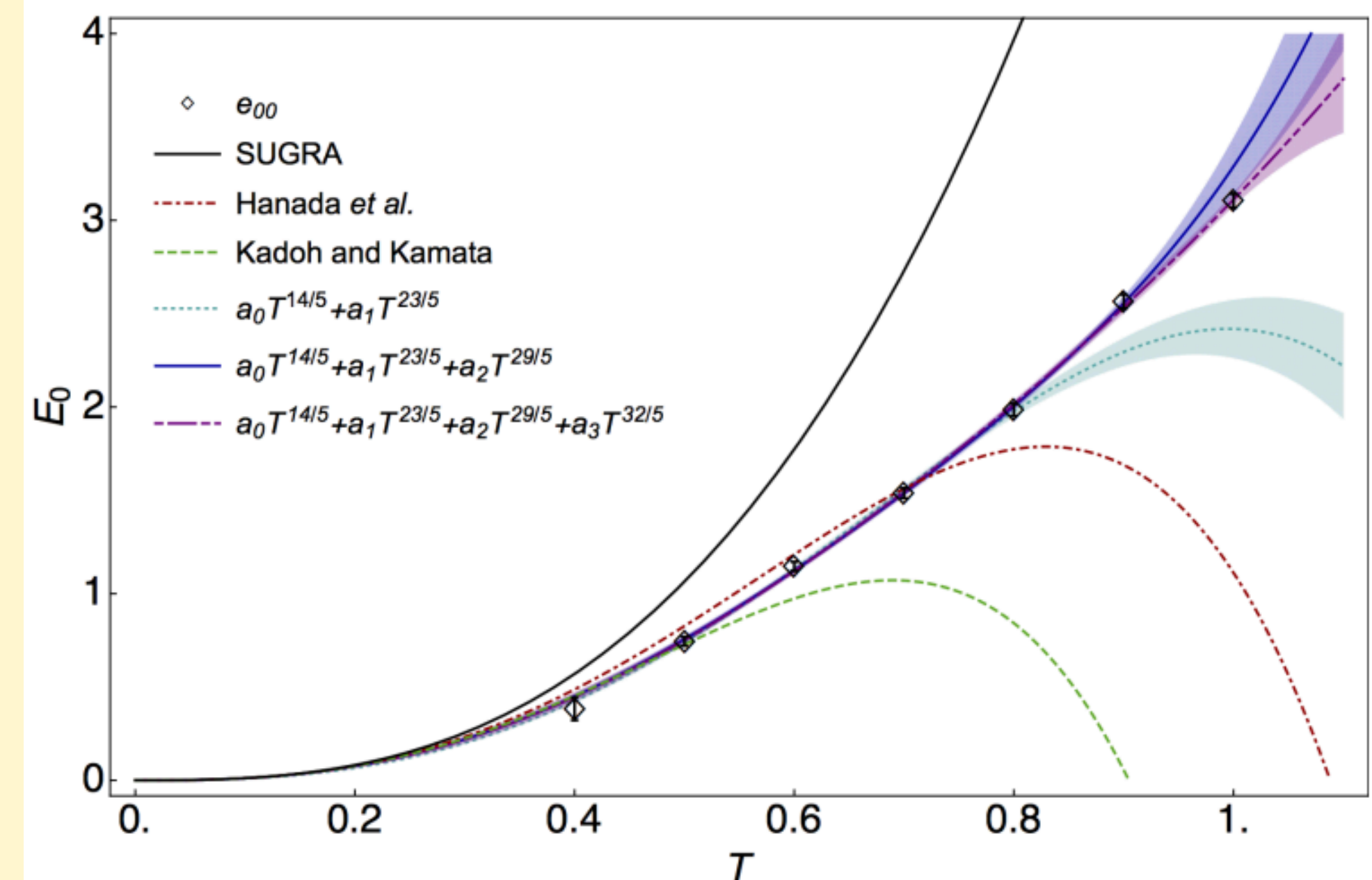
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$$\frac{E}{N^2} = \frac{\left(a_0 T^{\frac{14}{5}} + a_1 T^{\frac{23}{5}} + a_2 T^{\frac{29}{5}} + \dots \right)}{N^0} + \frac{\left(b_0 T^{\frac{2}{5}} + b_1 T^{\frac{11}{5}} + \dots \right)}{N^2} + \mathcal{O}(N^{-4}).$$

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Finite μ corrections for BMN supergravity

$$\frac{E}{N^2} \simeq a_0 T^{\frac{14}{5}} f(\mu)$$

Costa, Greenspan, Penedones, Santos

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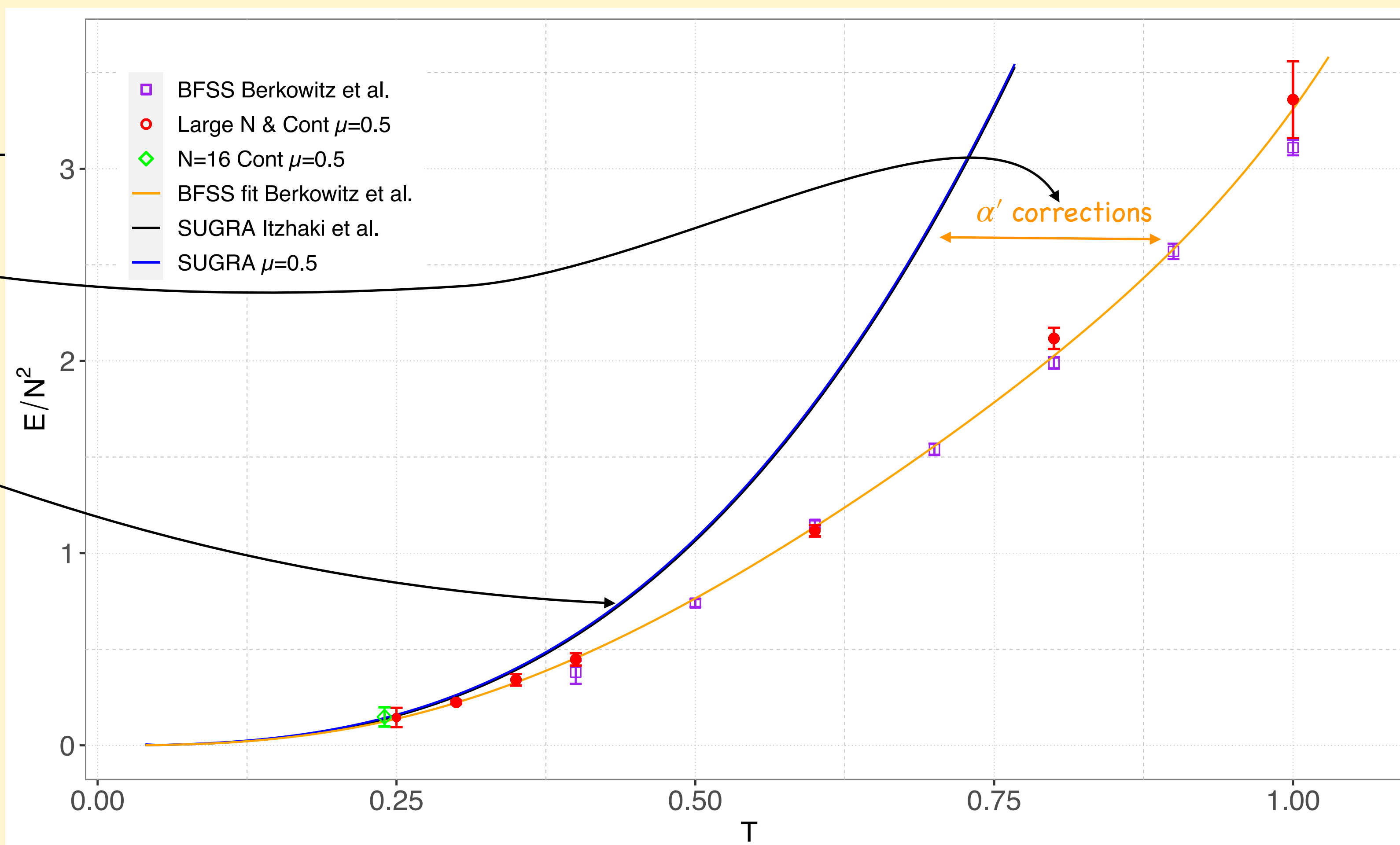
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Ongoing work



Is the singlet constraint important?

Maldacena-Milekhin conjecture

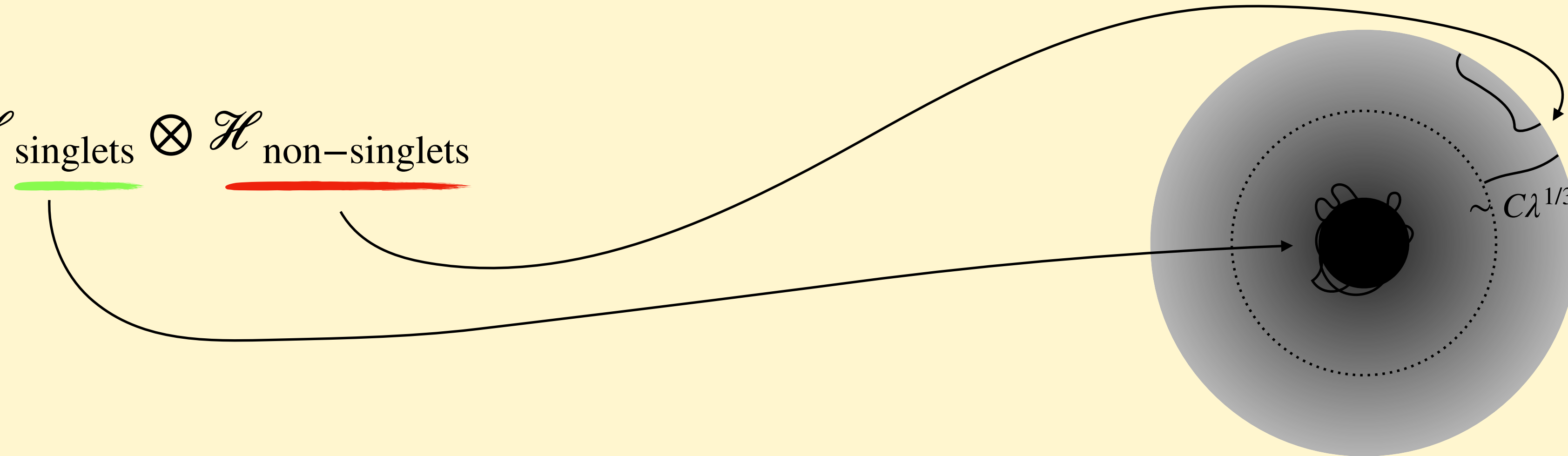
$$\begin{array}{ll}
 Z_{\text{gauged}} = \int [dX][d\psi][dA_t] e^{-S_{\text{matrix}}[X,\psi,A_t]} & \longrightarrow \text{Gauge singlet constraint: } \mathcal{G} := \frac{iN}{2\lambda} (2[\dot{X}_M, X_M] + [\bar{\psi}_\alpha, \psi_\alpha]) = 0 \\
 Z_{\text{ungauged}} = \int [dX][d\psi] e^{-S_{\text{matrix}}[X,\psi]} & \longrightarrow \text{No Gauge singlet constraint} \quad \text{Lattice} \longrightarrow \text{Gauge links} = 1
 \end{array}$$

Maldacena-Milekhin conjecture

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$$\mathcal{H}_{\text{total}} = \mathcal{H}_{\text{singlets}} \otimes \mathcal{H}_{\text{non-singlets}}$$



- Non-singlets do not contribute at low temperatures

$$\Delta Z = Z_{\text{gauged}} - Z_{\text{ungauged}} \simeq e^{-\frac{c\lambda^{1/3}}{T}}, \quad T \rightarrow 0$$

Maldacena–Milekhin conjecture

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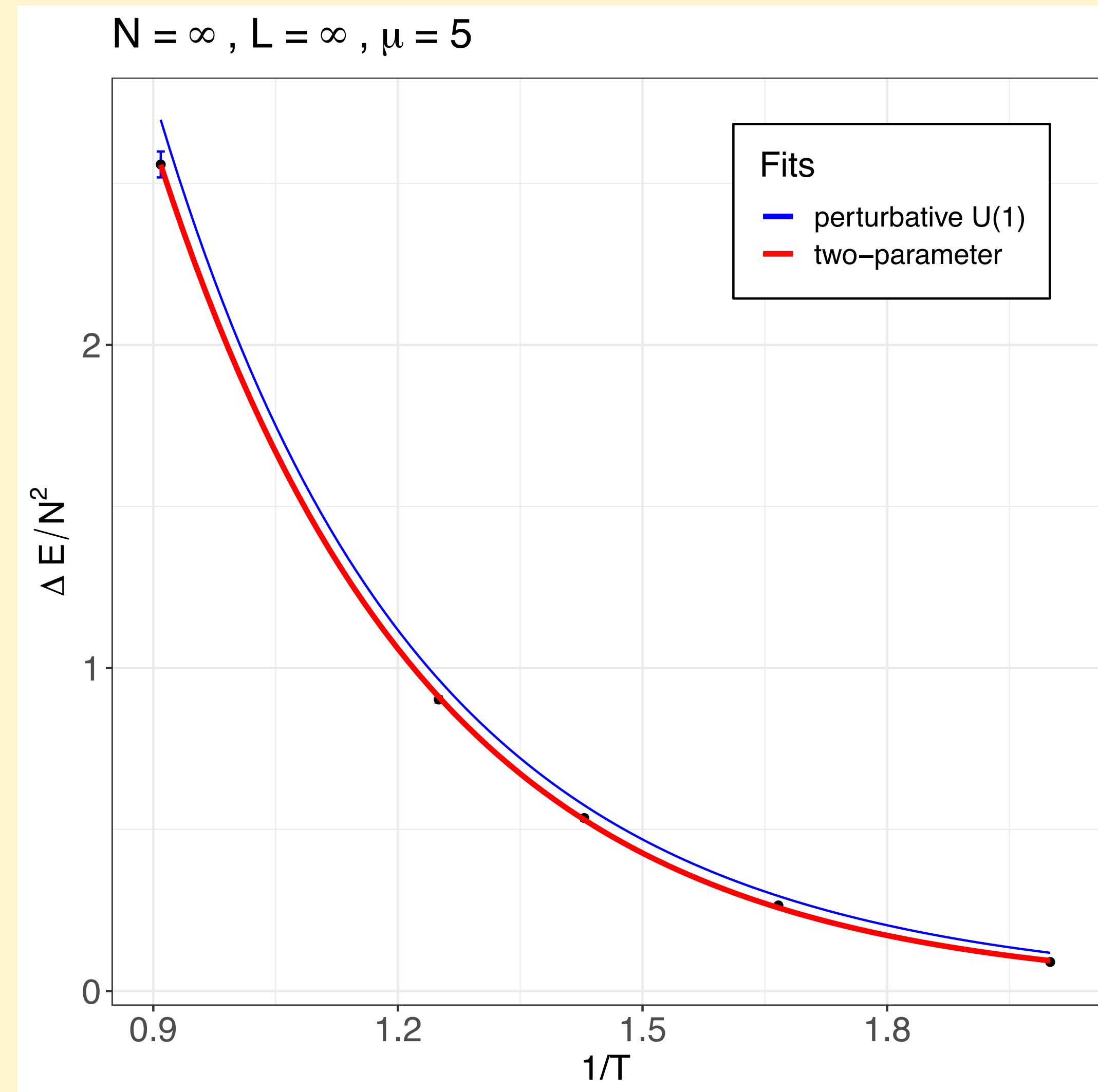
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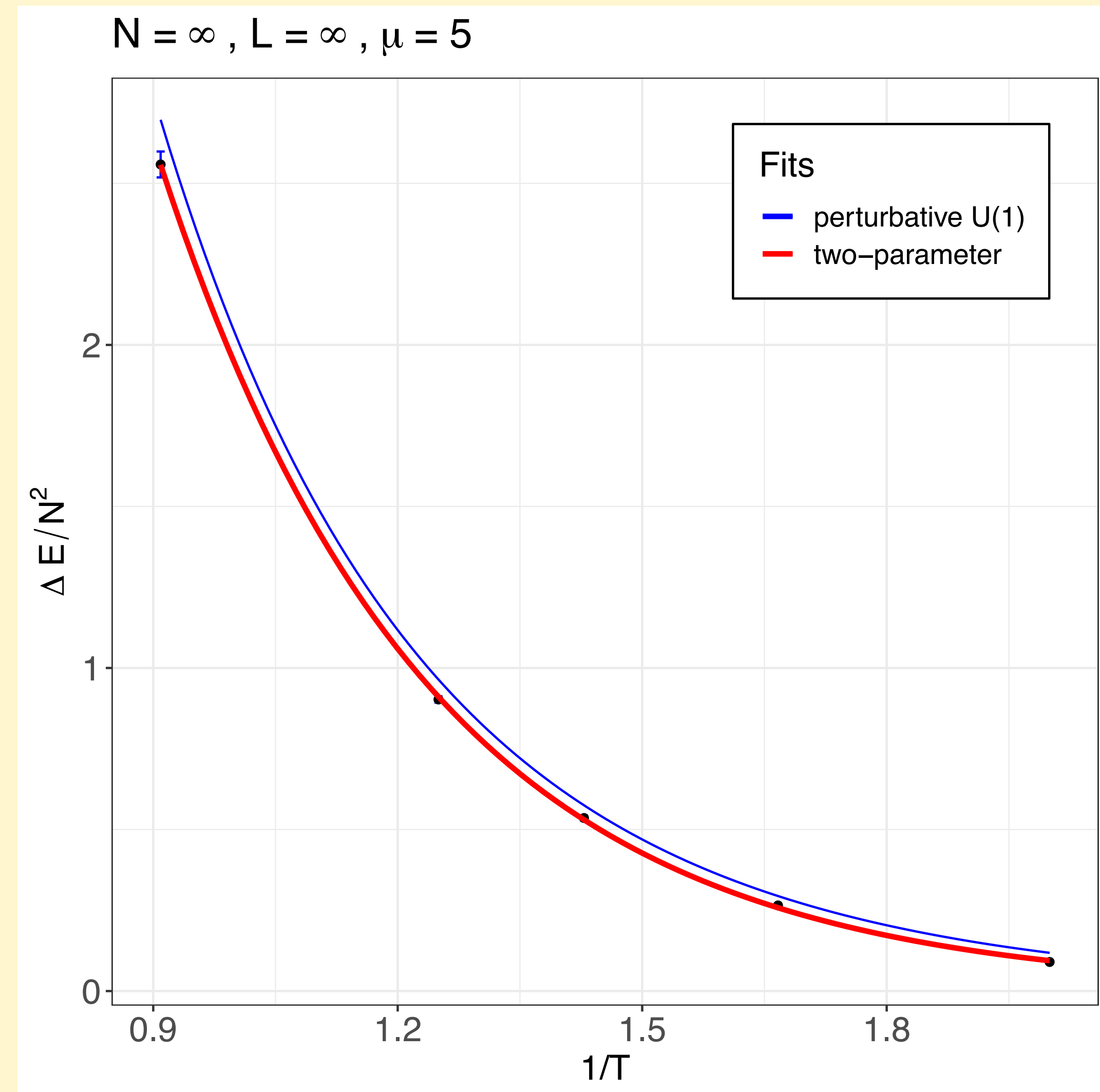
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$$\frac{E_{U(1)}}{N^2} = 6 \cdot \frac{\mu}{2} e^{-\frac{\mu}{2T}} + 8 \cdot \frac{3\mu}{4} e^{-\frac{3\mu}{4T}} + 3 \cdot \mu e^{-\frac{\mu}{T}} \quad , \quad \frac{\lambda}{T^3} \gg 1, \frac{\mu^3}{\lambda} \gg 1$$

$$\frac{\Delta E}{N^2} = E_{\text{ungauged}} - E_{\text{gauged}} = D \cdot e^{-\frac{c}{T}} \quad , \quad \frac{\lambda}{T^3} \gg 1, \frac{\mu^3}{\lambda} \ll 1$$



What do we learn?

- D0-matrix models interesting test examples for holography
- A stable **confined** phase has been observed for the first time
- Interesting possibility to probe contents of M-theory.
Study better the Schwarzschild BH. Membrane? Fivebrane?
- Low temperature precision test for holography (internal energies)
- Non-AdS/non-CFT, non-gauge/gravity, stringy corrections
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Thank you