

Noisy SUSY

Stam Nicolis

CNRS–Institut Denis–Poisson(UMR7013)
Université de Tours, Université d'Orléans
Parc Grandmont, 37200 Tours, France

Workshop on the Standard Model and Beyond
Corfu 2022

6 September 2022

*Based on arXiv:1302.2361, arXiv:1405.0820, arXiv:1712.07045,
arXiv:1912.12925, arXiv:2107.03194 and forthcoming.*

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40th anniversaries

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- ▶ 1982–2022: The 40th anniversary of the Corfu workshops: Many happy returns!
- ▶ 1982–2022: The 40th anniversary of the Parisi-Sourlas paper that implied that SUSY is an *inevitable* property of any physical system, in equilibrium with a bath of fluctuations: There's, still, a lot to understand (in particular how to evade obstructions for $D > 2$)!

Noisy SUSY

Since its invention/discovery, SUSY seems to be considered an *optional* feature of natural phenomena;

is there any way in which it might be understood as an *inevitable* feature of natural phenomena?

Forty years ago G. Parisi and N. Surlas, in “Supersymmetric field theories and stochastic differential equations”, *Nucl. Phys.* **B206** (1982) 321

made the case that supersymmetry is an inevitable property of a physical system in equilibrium with a bath of fluctuations.

A key role is played by a quantity introduced, some years previously, by H. Nicolai—within the context of supersymmetric theories—and known, since, as “the Nicolai map”.

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The story of a physical system and its fluctuations

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The description of the properties of a physical system relies on *two* distinct, but equally important, groups: The dynamical degrees of freedom, that describe the “classical” dynamics and their *superpartners*, that can resolve the fluctuations, with which they are in equilibrium.

These are (some of) their stories. . .

The story starts with the noise. . .

The noise fields

They are defined by their partition function. For white noise, this means

$$Z = \int [\mathcal{D}\eta_I(x)] e^{-\int d^D x \frac{1}{2} \frac{\eta_I(x)\eta_J(x)\delta^{IJ}}{\sigma^2}} \equiv 1$$

by definition of the measure.

This expression is equivalent to

$$\begin{aligned}\langle \eta_I(x) \rangle &= 0 \\ \langle \eta_I(x)\eta_J(x') \rangle &= \sigma^2 \delta_{IJ} \delta(x - x')\end{aligned}$$

and the other correlation functions are given by Wick's theorem.

We may choose units such that $\sigma = 1$.

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The physical meaning of the noise

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- ▶ $\sigma^2 = \hbar$: The bath describes quantum fluctuations.
- ▶ $\sigma^2 = k_B T$: The bath describes thermal fluctuations.
- ▶ $\sigma^2 =$ strength of the disorder: (Annealed) Disordered systems.

A non-trivial issue concerns combining different baths.

From noise to dynamical fields

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Now we must provide a map between the noise fields and putative dynamical fields.

The dynamics

Let us consider N scalar fields, ϕ_I ,

$$\eta_I(x) \equiv \sigma_{IJ}^\mu \partial_\mu \phi_J + \frac{\partial W}{\partial \phi_I}$$

where $\mu = 1, 2, \dots, D$ and $I, J = 1, 2, \dots, N$.

This was introduced by H. Nicolai in 1980, for Wess–Zumino models, and is known, since, as the “Nicolai map”.

For the moment, D and N can take any integer values and the σ_{IJ}^μ are, just, required to be real.

If we interpret this relation—as did Parisi and Sourlas—as an injunction to realize a change of variables in the partition function for the noise, however, we notice something quite interesting. . .

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Changing variables

If we perform the change of variables in the partition function, we find

$$Z = 1 = \int [\mathcal{D}\phi_I] \left| \det \frac{\delta\eta_I}{\delta\phi_J} \right| e^{-S[\phi_I]}$$

and we notice that, absent anomalies, the value of the integral does not change. Therefore the absolute value of the determinant describes *all* of the fluctuations of the action of the scalars, $S[\phi_I]$.

Now we can write

$$\left| \det \frac{\delta\eta_I}{\delta\phi_J} \right| = e^{-i\theta_{\det}} \det \frac{\delta\eta_I}{\delta\phi_J} = e^{-i\theta_{\det}} \int [\mathcal{D}\psi_I][\mathcal{D}\chi_I] e^{\int d^D x \psi_I \left\{ \sigma_{IJ}^\mu \partial_\mu + \frac{\partial^2 W}{\partial\phi_I \partial\phi_J} \right\} \chi_J}$$

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From the point of view of the (super)partners

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$$Z = 1 = \int [\mathcal{D}\phi_I][\mathcal{D}\psi_I][\mathcal{D}\chi_I] e^{-i\theta_{\text{det}}} \times \\ e^{-S[\phi_I] + \int d^D x \psi_I \left\{ \sigma_{IJ}^\mu \partial_\mu + \frac{\partial^2 W}{\partial \phi_I \partial \phi_J} \right\} \chi_J}$$

This expression can be understood in two, equivalent, ways:

- ▶ The fluctuations of the scalars are described by the action of the anticommuting fields—along with the phase of the determinant!
- ▶ The fluctuations of the anticommuting fields, in interaction with the scalars, are described by the phase of the determinant, along with the action of the scalars.

From the point of view of the (super)partners

Said differently:

The anticommuting fields resolve the bath of fluctuations, with which the scalars are in equilibrium, as do the scalars for the anticommuting fields, when they are part of a supermultiplet.

It is in this way that the no-go theorem pertaining, in particular, to Bell's inequalities can be evaded; this was, in fact, noted by P. G. O. Freund, already, in 1981 in the paper "Fermionic hidden variables and EPR correlations", *Phys. Rev.* **D24** (1981) 1526.

Curiously, this idea wasn't followed up—nor was the relation to the work of Parisi and Sourlas, after it appeared, investigated further...

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Worldvolume and target space fermions

If we expand the action for the scalars, we will find that there are two cases of particular interest:

- ▶ The σ_{IJ}^μ commute. Then the anticommuting fields are not target space fermions, they're worldvolume fermions; this is relevant for particle models.
- ▶ The σ_{IJ}^μ generate a Clifford algebra,

$$\{\sigma^\mu, \sigma^\nu\} = 2\delta^{\mu\nu}$$

Then the anticommuting fields are target space fermions. This is the case relevant for particle physics. N.B. We are working in Euclidian signature; this means that the σ^μ realize a Majorana representation iff $D \equiv 2 \pmod{8}$.

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Doubling

If $D \not\equiv 2 \pmod{8}$, (e.g. $D = 3$ or $D = 4$ spacetime dimensions) then the σ_{IJ}^μ have imaginary entries, therefore, in the map,

$$\eta_I = \sigma_{IJ}^\mu \partial_\mu \phi_J + \frac{\partial W}{\partial \phi_I}$$

the RHS is complex, so the LHS must be, too. Therefore, we must introduce the complex conjugate:

$$\eta_I^\dagger = \sigma_{JI}^\mu \partial_\mu \phi_J^\dagger + \left(\frac{\partial W}{\partial \phi_I} \right)^\dagger$$

(since the σ^μ are Hermitian) and the partition function for the noise fields is, now,

$$Z = 1 = \int [\mathcal{D}\eta_I][\mathcal{D}\eta_I^\dagger] e^{-\int d^D x \frac{\eta_I(x)\eta_J(x)^\dagger \delta^{IJ}}{\sigma^2}}$$

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Testing the idea in practice

The question is, whether the fluctuations of the scalars can reproduce the absolute value of the determinant, i.e. the Jacobian between the scalars and the noise fields. This can be answered by computing the correlation functions of the noise fields, $\eta_I[\phi]$, (resp. $\eta_I[\phi, \phi^\dagger]$, $\eta_I[\phi, \phi^\dagger]^\dagger$), sampled with the action of the scalars, i.e. with the measure

$$[\mathcal{D}\phi_I] e^{-S[\phi_I]}$$

(resp. for the generalization, when $D \not\equiv 2 \pmod{8}$) and checking that

$$\langle \eta_I(\phi(x)) \rangle \stackrel{?}{=} 0$$
$$\langle (\eta_I(x) - \langle \eta_I(x) \rangle) (\eta_J(x') - \langle \eta_J(x') \rangle) \rangle \stackrel{?}{=} \text{const} \delta_{IJ} \delta(x - x')$$

and the higher order correlators of the η_I should be given by Wick's theorem.

Any significant deviation is the signal for the appearance of anomalies.

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Tests

- ▶ For probability distributions, these identities *do* have anomalies, that can be understood (cf. arXiv:1302.2361[hep-th]). In addition, while the Jacobian does do the job expected of it, it can't be generated by the fluctuations, since the identities aren't satisfied.
- ▶ For a non-relativistic particle, these identities do *not* show anomalies—the identities are satisfied to numerical precision and up to lattice artifacts (cf. arXiv:1405.0820[hep-th]).
- ▶ For two dimensional scalar field theories, these identities do not show anomalies, either (cf. arXiv:1712.07045[hep-th]).
Work for the cases $D = 3$ and $D = 4$ is ongoing—the simulations take considerably more time. . .

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How about gauge theories?

Gauge theories, with compact gauge group, can be described by scalar fields, taking values on the group manifold. The “natural” noise distribution isn’t a Gaussian, with ultra-local 2-point function, but uniform over the group manifold. This has been studied on the lattice, through the so-called “trivializing maps”, introduced by Lüscher. These are, indeed, the avatars of the Nicolai map for the group manifolds.

However their construction is, still, work in progress.

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Abelian gauge theories

For abelian gauge fields it's possible to take a shortcut (in Lorenz–Feynman gauge):

$$\begin{aligned}\eta_I &= \sigma_{IJ}^\mu \partial_\mu \phi_J \\ \xi_I &= \sigma_{IJ}^\mu \nabla_\mu \varphi_J + \frac{\partial W}{\partial \varphi_I} \\ \xi_I^\dagger &= \sigma_{JI}^\mu [\nabla_\mu \varphi_J]^\dagger + \left(\frac{\partial W}{\partial \varphi_I} \right)^\dagger \\ \nabla_\mu &\equiv \partial_\mu - iqA_\mu \equiv \partial_\mu - iq\phi_\mu\end{aligned}$$

where

$$\phi_\mu \equiv \phi_I \equiv A_\mu$$

and q is the charge of the matter fields under the gauge field. Here φ_I are the scalar superpartners of the fermions of the hypermultiplet(s).

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The partition function for (S)QED

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$$Z = \int \underbrace{[\mathcal{D}\eta_I][\mathcal{D}\xi_I][\mathcal{D}\xi_I^\dagger]}_{[\mathcal{D}h_I]} e^{-\int d^D x \left\{ \frac{1}{2} \eta_I \eta_J \delta^{IJ} + \xi_I \xi_J^\dagger \delta^{IJ} \right\}} = 1 =$$
$$\int \underbrace{[\mathcal{D}\phi_I][\mathcal{D}\varphi_I][\mathcal{D}\varphi_I^\dagger]}_{[\mathcal{D}\Phi_I]} \left| \det \frac{\delta h_I}{\delta \Phi_J} \right| e^{-S[\phi_I, \varphi_I, \varphi_I^\dagger]}$$

The fermions are “hidden” in the determinant and “emerge” upon introducing it in the exponent.

For $D = 4$, we must double the degrees of freedom correspondingly.

Conclusions and Outlook

Any field theory (and that includes particle models, in the path integral formalism), whose fields take all possible real values, can be understood as providing a mapping between white noise fields and commuting fields; the anticommuting fields “emerge” from the Jacobian. The relation between the commuting and anticommuting fields is that they are superpartners. This is extended supersymmetry.

The superpartners may be thought of as “BSM” particles; but, in fact, they are part of the SM, since they resolve the quantum fluctuations of the fields of the SM!

That’s the essence of the proposal of Parisi and Sourlas; and the way to understand it, in practice, is by computing the identities that should be satisfied by the Nicolai map.

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Beware of “false prophets”, however!

The position, $x^\mu(\tau)$ and the spin, $\psi^\mu(\tau)$, of the spinning relativistic particle are related by target space SUSY;

however they *don't* resolve the fluctuations of the other!

The fluctuations of the position are *different* anticommuting fields, say $\chi^\mu(\tau)$, and the fluctuations of the spin define a *different* commuting fields, $\phi^\mu(\tau)$.

Of course they are all related—and how is an interesting exercise to solve. . .

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Consequences for the Standard Model:

- ▶ One scalar field is a semi-classical property, relevant within perturbation theory; in a relativistic field theory, it's not possible to describe, fully, the fluctuations of just one scalar field; there are, inevitably, more—in $D = 4$ the least number is 8, which leads to, at least, two “Higgs-like” scalars (the other scalars becoming, for example, the longitudinal polarization states of gauge bosons).
- ▶ Flavor (non-)universality can be straightforwardly accommodated, since the fermion determinant doesn't, inevitably, “factorize” over the flavors. How it does is of interest to spell out.
- ▶ Chiral fermions can be described using the domain–wall construction, that leads to “partial” SUSY breaking to $\mathcal{N} = 1$ on the brane.

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Another issue of practical significance is that, insofar as the absolute value of the determinant—that describes the contribution of the fermions—is generated by the fluctuations, this means that it is possible, in principle, to express fermionic correlators in terms of the correlators of their bosonic superpartners, sampled using the bosonic action, which is much easier to do, than the fermionic action. This remains to be spelled out for practical applications.

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There's a "natural" way to understand the relevance of SUSY for any field theory and the SM, in particular. There's, still, considerable work to be done to understand how this approach can be realized for non-abelian gauge theories and how this can lead to search strategies in real experiments. However SUSY isn't an optional property of Nature (or of the SM) but an inevitable part of it. It's necessary to learn how to see it. How it can be realized can be quite unexpected (recall that the quarks cancel the gauge anomalies of the leptons and vice versa!)

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*All theories are supersymmetric.
Some theories are born supersymmetric;
some become supersymmetric;
some have supersymmetry thrust upon them...*