Gravitational positivity bounds on dark photons

Sota Sato

Kobe University

Toshifumi Noumi, Junsei Tokuda, S.S.
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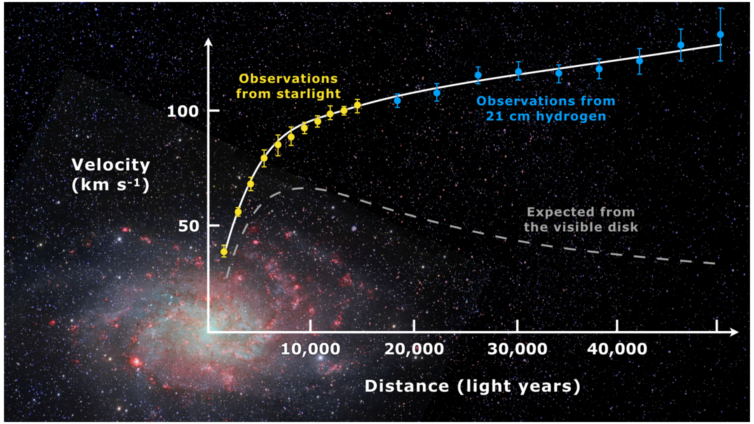
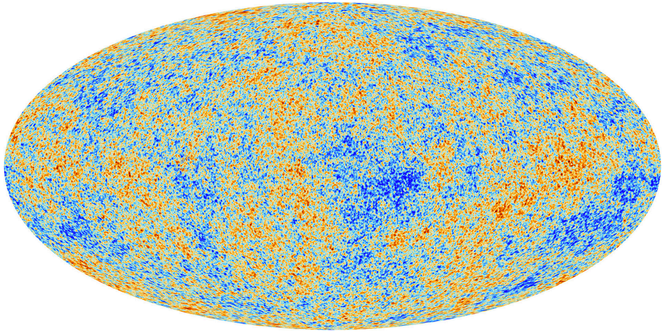
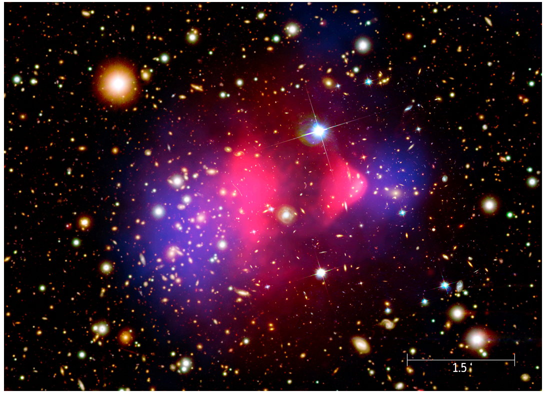
Introduction

- Dark matter exists





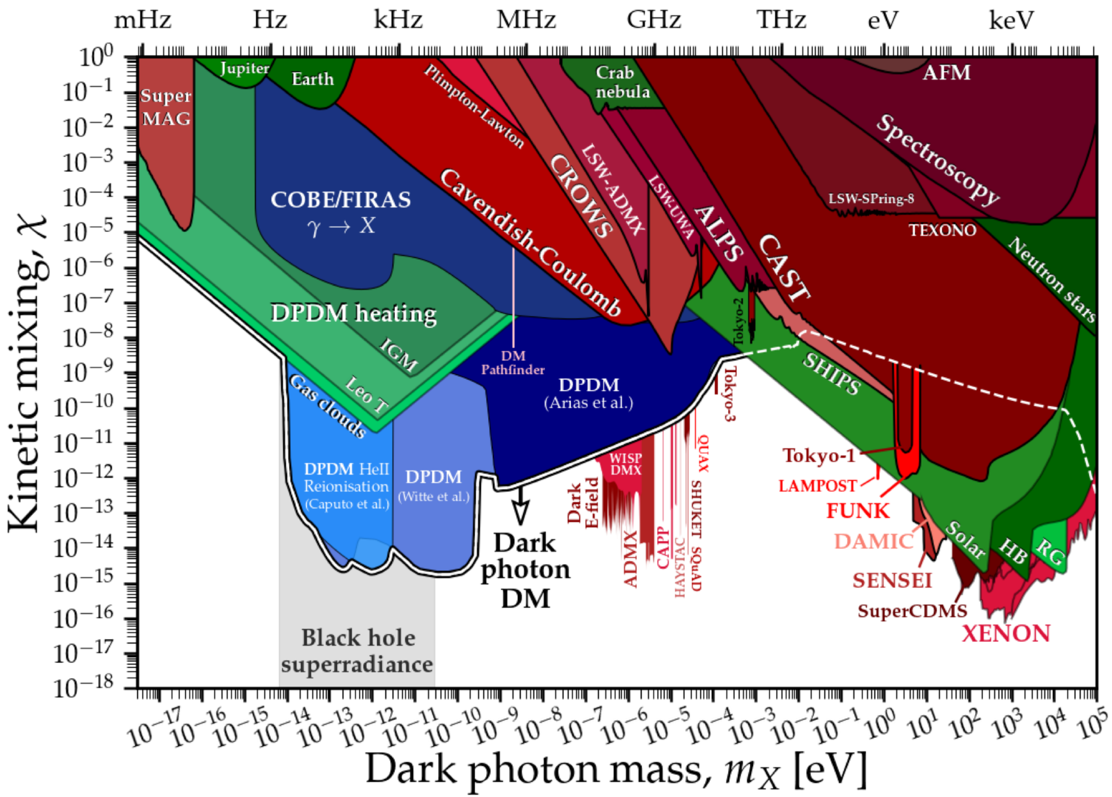


- But we don’t know its nature

Introduction

- Parameter space of dark matter is large

the strength of the interaction with SM



Introduction

- In this work, we constrain parameter space of dark matter models theoretically: dark photon model as an illustrative example for dark sector physics

- Positivity bounds: Consistency conditions on low energy EFTs

  → Constrain parameter spaces of EFTs

- Positivity bounds in the presence of gravity (gravitational positivity bounds)
  - ➢Condition for field theories to be compatible with quantum gravity
  - ➢Strong bound on parameter space of dark photon model

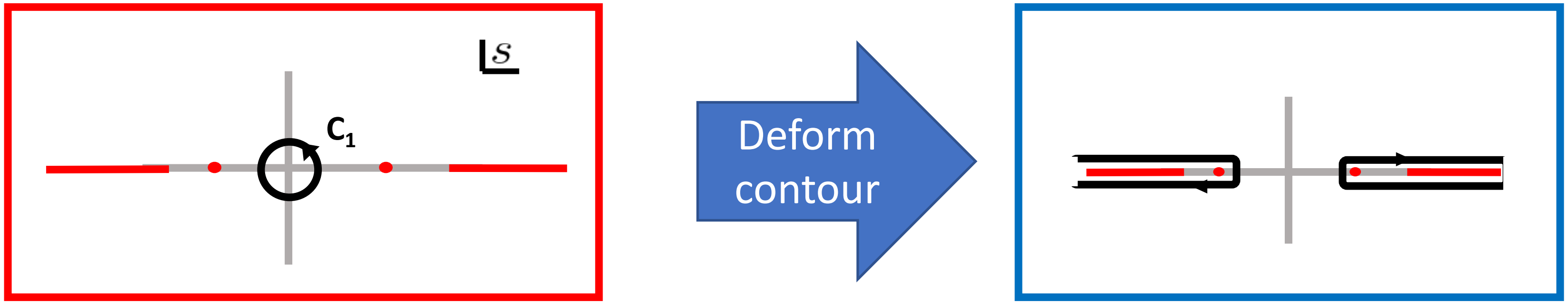
Outline

- Positivity bound without gravity
- Positivity bound with gravity(gravitational positivity bound)
  - ➢Technical difficulty with gravity & additional assumptions
  - ➢Implication: gravity should be weak!
- Application to dark photon models

Positivity bound

Positivity bound

- Low energy expansion of amplitude: $\mathcal{M}(s,0) = c_0 + c_1 s + c_2 s^2 + \cdots$
- Unitarity → $\mathrm{Im}\mathcal{M}(s,0) > 0$ (Optical theorem)
- C2 is equal to integration of $\mathrm{Im}\mathcal{M}(s,0)$ → $c_2 > 0$



positivity bound Adams+ '06

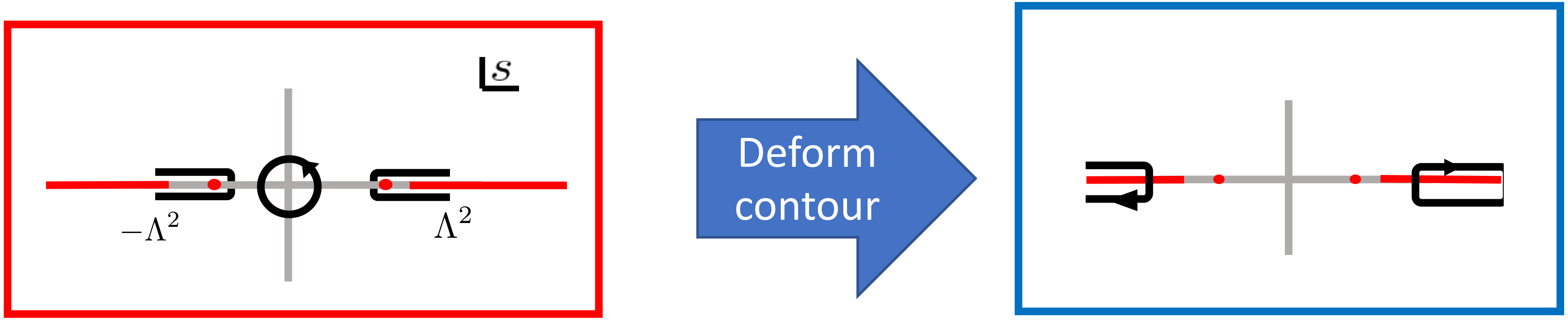
$$c_2 = \frac{1}{2\pi i}\oint_{C_1} ds' \frac{\mathcal{M}(s',0)}{s'^3} = \frac{2}{\pi}\int ds' \frac{\mathrm{Im}\,\mathcal{M}(s',0)}{s'^3} > 0$$

Improved positivity bound

- If EFT is valid below Λ, integral of $\mathrm{Im}\mathcal{M}(s,0)$ is calculable up to Λ^2

$$c_2 = \frac{2}{\pi}\int^{\Lambda^2} ds' \frac{\mathrm{Im}\,\mathcal{M}\,(s',0)}{s'^3} + \frac{2}{\pi}\int_{\Lambda^2}^{\infty} ds' \frac{\mathrm{Im}\,\mathcal{M}\,(s',0)}{s'^3}$$

Calculable



Improved positivity bound Bellazini '16, de Rham+ '17

$$B^{(2)}(\Lambda) := c_2 - \frac{2}{\pi}\int^{\Lambda^2} ds' \frac{\mathrm{Im}\,\mathcal{M}\,(s',0)}{s'^3} = \frac{2}{\pi}\int_{\Lambda^2}^{\infty} ds' \frac{\mathrm{Im}\,\mathcal{M}\,(s',0)}{s'^3} > 0$$

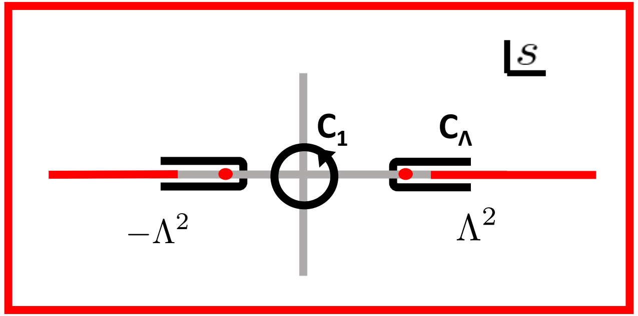
Gravitational positivity bound

Technical problem with gravity

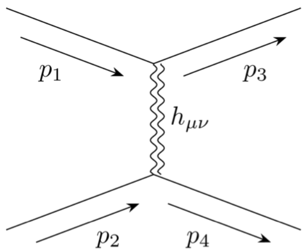
- Positivity bound w/ gravity: non-trivial consistency condition with quantum gravity (relation with swampland program)
- Technical problem due to massless spin-2 particle i.e. graviton:

<u>Divergence in the forward limit</u>

$$\frac{1}{2\pi i}\oint_{\mathcal{C}_1+\mathcal{C}_\Lambda} ds' \frac{\mathcal{M}(s',0)}{s'^3} = \lim_{t\to 0}\left(-\frac{1}{M_{\rm Pl}^2 t} + B^{(2)}(\Lambda)\right) = \frac{2}{\pi}\int_{\Lambda^2}^{\infty} ds' \frac{\mathrm{Im}\,\mathcal{M}(s',0)}{s'^3}$$



t-channel graviton exchange → diverge in forward limit



$$\sim -\frac{s^2}{M_{\rm Pl}^2 t} \xrightarrow[t\to 0]{} \infty$$

Gravitational positivity bound

- Additional assumptions to remove the divergence in the forward limit

Assumption(1) $\mathrm{Im}\,\mathcal{M}(s,t) \sim f(t)\left(\frac{\alpha' s}{4}\right)^{2+j(t)} \quad \text{for } s > M_*^2$

Regge behavior at the high energy
Cancel out the divergent term

Assumption(2) $\left|\frac{f'}{f}\right|, \left|\frac{j''}{j'}\right|, |j'| \ll \frac{1}{\Lambda^2}$

- Positivity bound holds approximately:

Gravitational positivity bound

Tokuda, Aoki, Hirano '20

$$B^{(2)}(\Lambda) := c_2 - \frac{2}{\pi}\int^{\Lambda^2} ds' \frac{\mathrm{Im}\,\mathcal{M}(s',0)}{s'^3} \gtrsim 0$$

Gravitational positivity bound

- Additional assumptions to remove the divergence in the forward limit

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Regge behavior at the high energy
Cancel out the divergent term

$$B^{(2)}(\Lambda) > \underline{\frac{1}{M_{\mathrm{Pl}}^2}\left[\frac{f'}{f} + j' \ln\left(\frac{\alpha' M_*^2}{4}\right) - \frac{j''}{j'}\right]}$$

The remaining term

Gravitational positivity bound

Tokuda, Aoki, Hirano '20

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Gravitational positivity bound

- Additional assumptions to remove the divergence in the forward limit

Assumption(1) $\mathrm{Im}\,\mathcal{M}(s,t) \sim f(t)\left(\frac{\alpha' s}{4}\right)^{2+j(t)} \quad \text{for } s > M_*^2$

Regge behavior at the high energy
Cancel out the divergent term

Assumption(2) $\left|\frac{f'}{f}\right|, \left|\frac{j''}{j'}\right|, |j'| \ll \frac{1}{\Lambda^2}$

The remaining term is small

- Positivity bound holds approximately:

Gravitational positivity bound

Tokuda, Aoki, Hirano '20

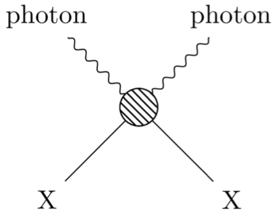
$$B^{(2)}(\Lambda) := c_2 - \frac{2}{\pi}\int^{\Lambda^2} ds' \frac{\mathrm{Im}\,\mathcal{M}(s',0)}{s'^3} \gtrsim 0$$

Structure of $B^{(2)}(\Lambda)$

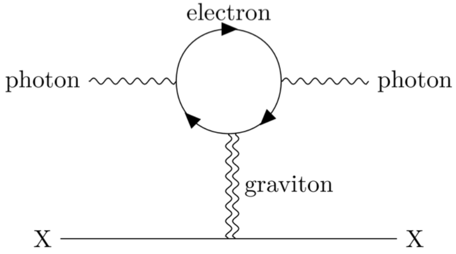
- Focus on scattering of photon and some particle X in gravitational theory

$$B^{(2)}(\Lambda) = B^{(2)}_{\text{non-grav}}(\Lambda) - \left| B^{(2)}_{\text{grav}}(\Lambda) \right|$$

non-gravitational part, positive



graviton-exchange part, negative



$$\sim \mathcal{O}\left(\frac{1}{M_{\text{Pl}}^2 m_e^2}\right)$$

Implication of gravitational positivity bound

- Non-gravitational interaction is bounded below by gravitational interaction

→ gravity should be weak!

$$B^{(2)}_{\text{non-grav}}(\Lambda) > \left| B^{(2)}_{\text{grav}}(\Lambda) \right|$$

- Relation with weak gravity conjecture   Tolley+ ’20 (See also Cheung+ ‘14, Hamada+ ’18)

- Lower bounds on interactions in dark sector models → useful for phenomenology?

Application to dark photon model

Dark photon models

- Additional $U(1)$ gauge field A'
- Massive if there is SSB
- Interaction with SM particles is the same as photon, and "kinetic mixing"ε represents its strength

$$\mathcal{L} \supset -\frac{1}{4}F'^{\mu\nu}F'_{\mu\nu} + \frac{1}{2}m_{A'}^2 A'^{\mu}A'_{\mu} + \epsilon e A'^{\mu} J_{\mu}^{EM}$$

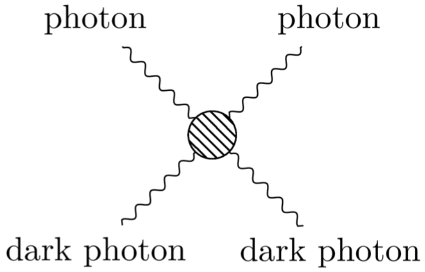
- We consider simple model: SM + dark photon + gravity

$$S = S_{\rm SM} + S_{\rm dark\ photon} + \frac{M_{\rm Pl}^2}{2}\int \sqrt{-g}R$$

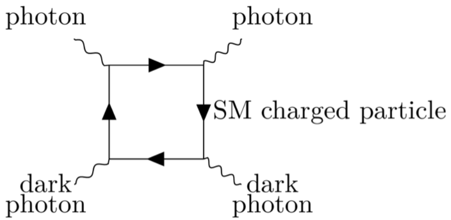
$$g_{\mu\nu} = \eta_{\mu\nu} + \frac{1}{M_{\rm Pl}}h_{\mu\nu}$$

Application of positivity

- Calculate photon-dark photon scattering @ 1-loop (*neglect QCD sector)
- Dark photon has transverse mode & longitudinal mode
- Calculate $B^{(2)}(\Lambda) = B^{(2)}_{\text{non-grav}}(\Lambda) - \left| B^{(2)}_{\text{grav}}(\Lambda) \right|$

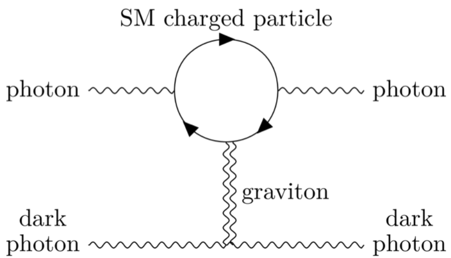


non-gravitational part



depends on charge, mass and spin of charged particle
→ W boson loops dominate

gravitational part



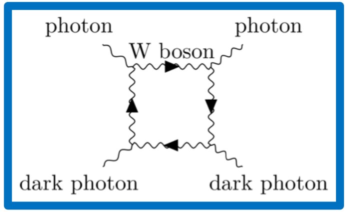
depends on charge & mass of charged particle
→ electron loops dominate

Results: transverse mode

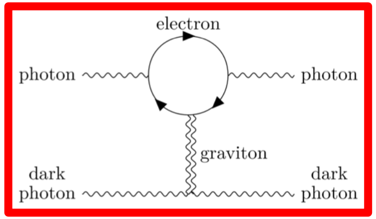
- Calculate Photon-Dark photon scattering in SM+Dark photon + Gravity

$$B^{(2)}(\Lambda) = \frac{32\alpha^2\epsilon^2}{m_W^2\Lambda^2} - \frac{11\alpha}{180\pi m_e^2 M_{P1}^2} > 0$$

α:fine structure constant
m_W:mass of W boson
m_e:mass of electron
M_{pl}:Planck mass
Λ: EFT cut-off



non-grav part: w boson



grav part: electron

"non-gravitational part>gravitational part"

$$\Rightarrow \quad \epsilon > \sqrt{\frac{11}{5760\pi\alpha}\frac{m_W\Lambda}{m_e M_{\rm pl}}} = 3.72\times 10^{-12}\left(\frac{\Lambda}{1\rm TeV}\right)$$

Λ: EFT cut-off

Results: longitudinal mode

- Calculate Photon-Dark photon scattering in SM+Dark photon + Gravity

$$B^{(2)}(\Lambda) = \frac{8\alpha^2\epsilon^2 m_{A'}^2}{\Lambda^2 m_W^4} - \frac{11\alpha}{180\pi m_e^2 M_{\rm Pl}^2} > 0$$

α:fine structure constant
m_W:mass of W boson
m_e:mass of electron
M_{pl}:Planck mass
Λ: EFT cut-off

$$\# \times (\text{transverse}) \times \frac{m_{A'}^2}{m_W^2}$$

non-grav part: suppression for light dark photon

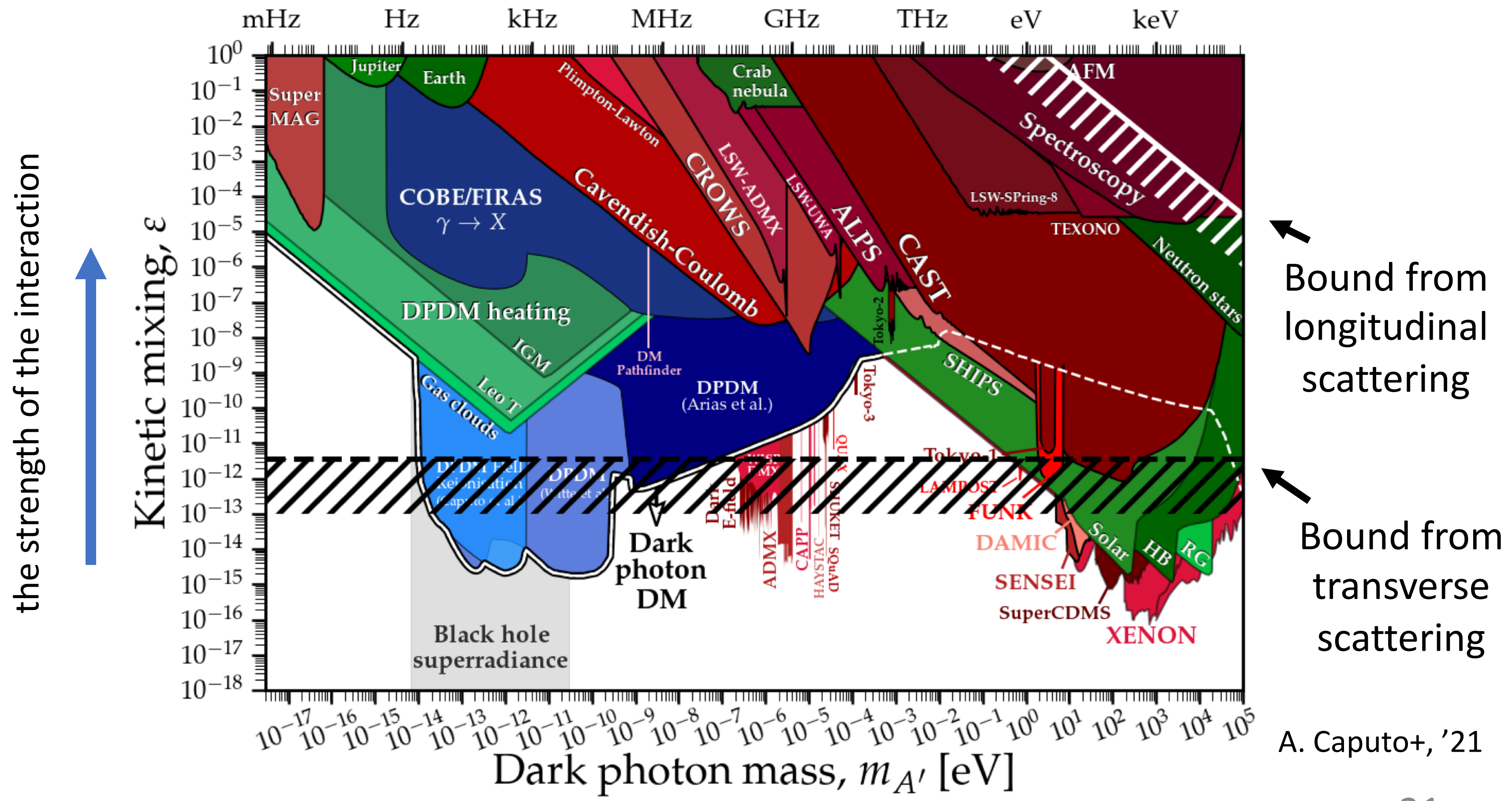
grav part: same as transverse case

“non-gravitational part>gravitational part”

$$\Rightarrow \quad \epsilon > \sqrt{\frac{11}{1440\pi\alpha}\frac{m_W^2\Lambda}{m_{A'}m_e M_{\rm Pl}}} = 3.0\times10^{-3}\times\left(\frac{\Lambda}{1\rm TeV}\right)\left(\frac{1\rm keV}{m_{A'}}\right)$$

Λ: EFT cut-off

Results



A. Caputo+, ’21

Summary

- Gravitational positivity bound: non-trivial consistency condition on the theory with gravity

- Application to dark photon models

  → Strong lower bound on the interaction with dark sector and standard model particles

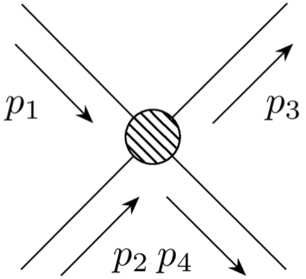
- The bound from the scattering of longitudinal mode is stringent

backup slides

Positivity bound

- Non-trivial consistency condition on low energy EFT   Adams+ '06
- Consider 2 to 2 scattering in some EFT

$$\mathcal{M}(s,t) =$$



$s = -(p_1 + p_2)^2 \sim (\text{CM energy})^2$

$t = -(p_1 - p_3)^2 \sim$ scattering angle

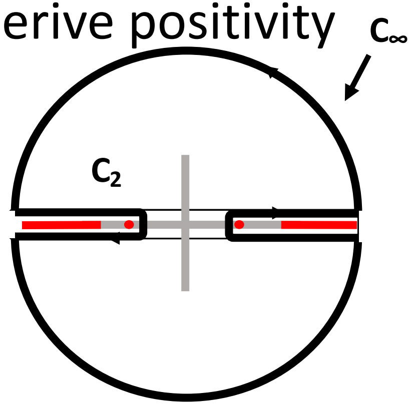
$t = 0:$ forward scattering

- Low energy expansion of amplitude: $\mathcal{M}(s,0) = c_0 + c_1 s + c_2 s^2 + \cdots$
- Positivity bound: If UV completion of EFT is "standard" theory (Unitary, Lorentz invariant , Analytic, Local), $\boxed{c_2 > 0}$

e.g. $-\frac{1}{2}\partial^\mu\phi\partial_\mu\phi + \lambda(\partial^\mu\phi\partial_\mu\phi)^2 + \cdots$ ⇒ $\lambda > 0$

s^2 bound

- s^2 bound: $\lim_{s\to\infty} |\mathcal{M}(s,0)| < s^2$ should be satisfied to derive positivity



$$c_2 = \frac{1}{2\pi i}\int_{C_1} ds' \frac{\mathcal{M}(s',0)}{s'^3} = \frac{1}{2\pi i}\left(\int_{C_2} + \int_{C_\infty}\right)\frac{\mathcal{M}(s,0)}{s'^3} ds'$$

- s^2 bound is guaranteed by Froissart bound for gapped theory

Froissart, '61

Azimov, '11

- For theory with gravity, see Caron-Huot+, '21, Zhiboedov+, '22

Technical problem with gravity

$$c_2 = \lim_{t\to 0}\left(\frac{2}{\pi}\int ds' \frac{\mathrm{Disc}\,\mathcal{M}(s',t)}{s'^3} + \frac{1}{M_{\mathrm{Pl}}^2 t}\right)$$

- Assume Regge behavior in the high-energy limit (Realized in string theory)

$$\mathrm{Disc}\,\mathcal{M}(s,t) \sim f(t)\left(\frac{\alpha' s}{4}\right)^{2+j(t)} = f(t)\left(\frac{\alpha' s}{4}\right)^{2+j' t+\cdots} \quad \text{for } s > M_*^2$$

>0 $\qquad$ "$\infty - \infty$"

$$c_2 = \lim_{t\to 0}\left(\frac{2}{\pi}\int^{M_*^2} ds' \frac{\mathrm{Disc}\,\mathcal{M}(s',t)}{s'^3} + \frac{2}{\pi}\int_{M_*^2}^{\infty} ds' \frac{\mathrm{Disc}\,\mathcal{M}(s',t)}{s'^3} + \frac{1}{M_{\mathrm{Pl}}^2 t}\right)$$

$$> \frac{1}{M_{\mathrm{Pl}}^2}\left[\frac{f'}{f} + j' \ln\left(\frac{\alpha' M_*^2}{4}\right) - \frac{j''}{j'}\right] = \pm\mathcal{O}\left(\frac{1}{M_{\mathrm{Pl}}^2 M^2}\right)$$

the remaining part, $\mathcal{O}\left(\frac{1}{M^2}\right)$

Tokuda, Aoki, Hirano '20

- The implication depends on the M

Our assumption

$$\sim \mathcal{O}\left(\frac{1}{M_{\rm Pl}^2 m_e^2}\right)$$

$$B^{(2)}_{\text{non-grav}}(\Lambda) > \left|B^{(2)}_{\text{grav}}(\Lambda)\right| \pm \mathcal{O}\left(\frac{1}{M_{\rm Pl}^2 M^2}\right)$$

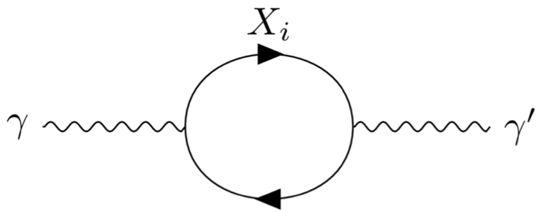
- the sign of third term and the scale M is determined by the high-energy behavior of the amplitude
- different possible implication
  1. $M \sim m_e$

     Non-trivial high energy behavior of the scattering amplitude
  2. $M \gg m_e$

     $$B^{(2)}_{\text{non-grav}}(\Lambda) > \left|B^{(2)}_{\text{grav}}(\Lambda)\right|$$
- In our work, we assume 2: justifying this assumption is future work

Kinetic mixing

- Kinetic mixing is induced by new particles X_i which have both charges of $U(1)_{EM}$ and $U(1)_{A'}$



$$\epsilon = -\frac{1}{16\pi^2}\sum_i g_i g_i' \ln \frac{M_i^2}{\mu^2}$$

B. Holdem, '86

Decoupling of the longitudinal mode

- A polarization vector of the longitudinal mode is proportional to the momentum in the $m_{A'} \to 0$ limit

$$\epsilon_\mu = (k, 0, 0, \sqrt{k^2 + m^2})/m = k_\mu + \mathcal{O}\left(\frac{m_{A'}}{k}\right)$$

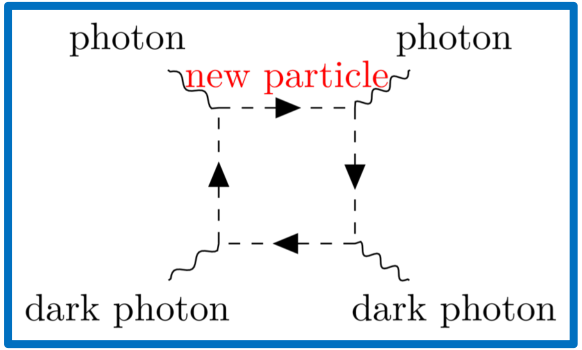
- $k_\mu \mathcal{M}^\mu = 0$ by the Ward identity → $\mathcal{M} \propto m_{A'}$

$$\mathcal{M} = \epsilon_\mu \mathcal{M}^\mu = \left(k_\mu + \mathcal{O}\left(\frac{m_{A'}}{k}\right)\right) \mathcal{M}^\mu \to \mathcal{O}\left(\frac{m_{A'}}{k}\right)$$

Adding new massive particle

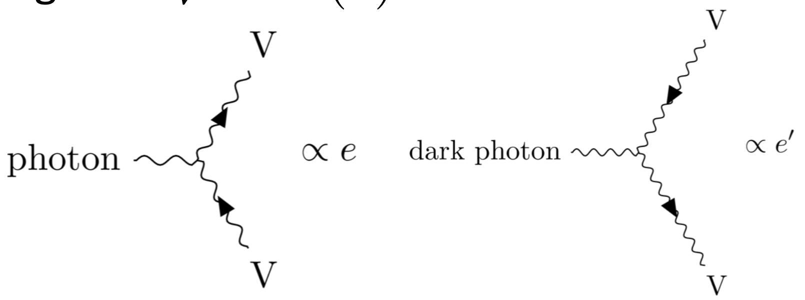
- Is non-zero direct coupling with SM necessary?

- Instead of the direct coupling with SM, new massive particle might mediate scattering of photon and dark photon
→ Dark photon with zero kinetic mixing?



Adding new massive particle

- Introduce new vector particle V with following properties
  1. $\mathcal{O}(1)$ coupling to both photon and dark photon
  2. massive enough: $m_V > \mathcal{O}(1)\text{TeV}$



$e, e' = \mathcal{O}(1)$

$$S = S_{\text{SM}} + S_{\text{dark photon}} + \frac{M_{\text{Pl}}^2}{2}\int\sqrt{-g}R + S_{\text{V}}$$

- We consider the scenario with new massive particles and obtain upper bound on the cut-off Λ

Results for new massive particle scenario

- Transverse mode

$$\Lambda < 24\sqrt{\frac{5}{11}\frac{m_e}{m_V}}e' M_{\rm Pl} = \underline{2.0\times 10^{13}\mathrm{GeV}} \times e' \left(\frac{1\mathrm{TeV}}{m_V}\right)$$

- Longitudinal mode

$$\Lambda < 12\sqrt{\frac{5}{11}\frac{m_e m_{A'}}{m_V^2}}e' M_{\rm Pl} = \underline{10\mathrm{TeV}} \times e' \left(\frac{m_{A'}}{1\mathrm{keV}}\right)\left(\frac{1\mathrm{TeV}}{m_V}\right)^2$$

- Implication
  1. Massless dark photon: cut-off is high enough
  2. Massive dark photon: cut-off scale is still too small

  → Light but massive dark photon is incompatible in this scenario too
  (Reece '18: Similar conclusion in the context of swampland)

Outlook

- Justification of the assumption that the "remaining term" of $B^{(2)}_{\text{non-grav}}(\Lambda) > \left|B^{(2)}_{\text{grav}}(\Lambda)\right| \pm \underline{\mathcal{O}\left(\frac{1}{M_{\text{Pl}}^2 M^2}\right)}$ is small
- Why gravitational part has negative sign?
- Incorporate QCD effect
- More on dark photon model
  - ➢New massive particle might be necessary for the existence of dark photon
    → What is parameter space of this particle?
  - ➢Is the suppression factor of longitudinal mode $\frac{m_{A'}^2}{m_W^2}$ unavoidable?
- Application to other models(B-L gauge field, axion, …)