UV/IR mixing, misaligned SUSY and the running of couplings in string theory

Steve Abel *(Durham IPPP), Corfu* 09/22

Mainly based on work with Keith Dienes and Luca Nutricati, arXiv:2XXX.YYYYY and arXiv:2106.04622, and related to …

- **w/ Dienes+Mavroudi *Phys.Rev. D* 91, (2015) 126014, arXiv:1502.03087**
- **SAA JHEP 1611 (2016) 085, arXiv:1609.01311**
- **Aaronson, SAA, Mavroudi, *Phys. Rev. D* 95, (2016) 106001, arXiv:1612.05742**
- **w/ Stewart, *Phys.Rev.D* 96 (2017) 10, 106013 arXiv:1701.06629**
- **w/ Dienes+Mavroudi *Phys.Rev.D* 97 (2018) 12, 126017 arXiv: 1712.06894**

Motivation: suppose I declare that I never want to use an EFT but want to live within string theory — what happens?

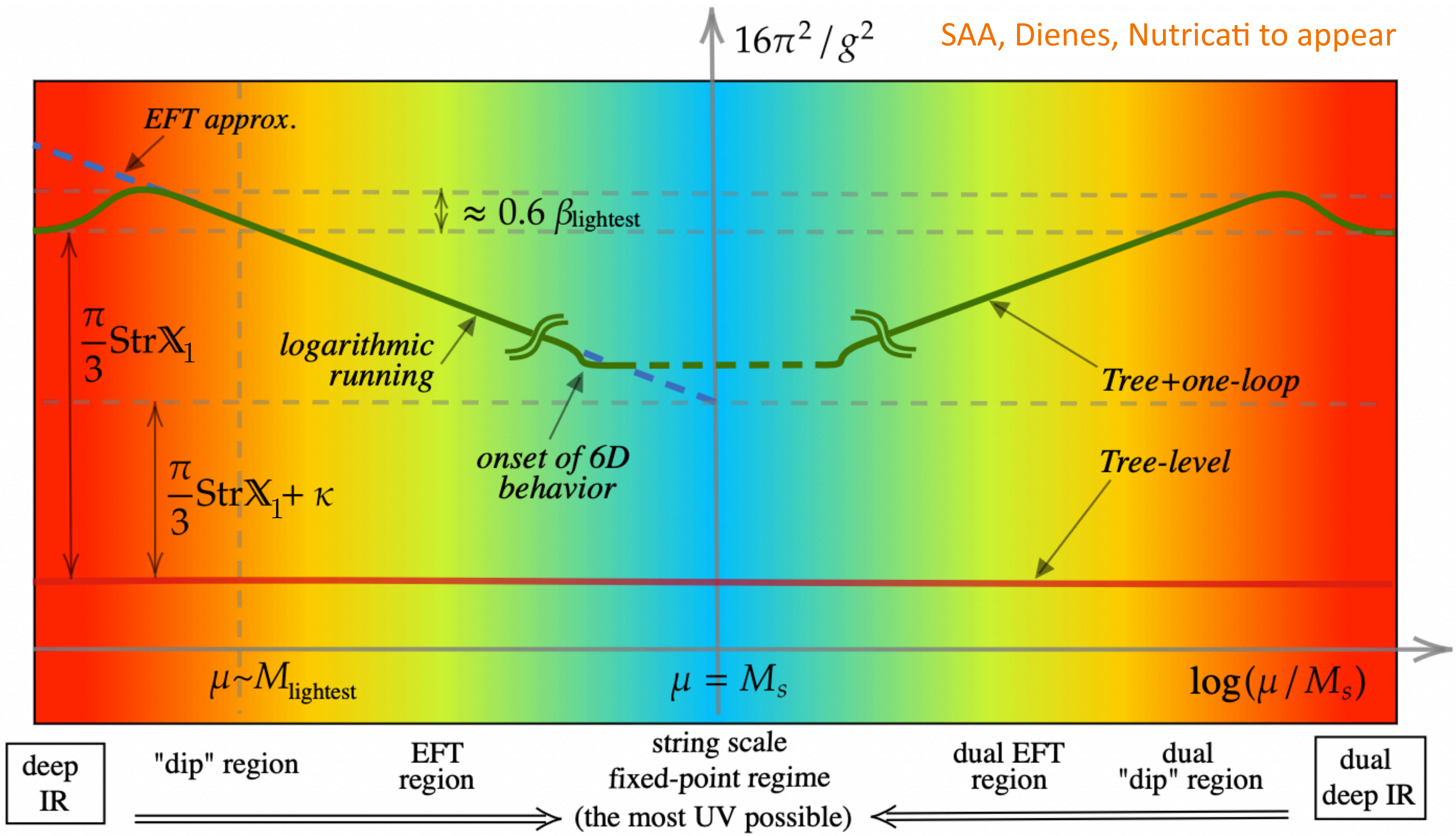
- Modular invariance in closed strings: a modular transformation mixes the entire spectrum of states up.
- As UV/IR mixed as a theory could be, yet the usual assumption is the opposite — "String theory just provides a UV-completion to a Wilsonian EFT".

- *How can these both be true?!!*
- *Need to understand how EFT and RG running emerge in string theory*
- *Expect no ambiguity — e.g. there can be no RG "scheme" in a UV-complete theory - is there a "correct" scheme we should be using?*
- *It will have profound implications for naturalness.*

Motivation: suppose I declare that I never want to use an EFT but want to live within string theory — what happens?

Mainly focus on the well trodden path of 1-loop gauge coupling corrections in this talk (but at the end we will see that the discussion applies to the entire theory including the Higgs SAA, Dienes `21 **)**

SAA, Dienes, Nutricati to appear



This talk:

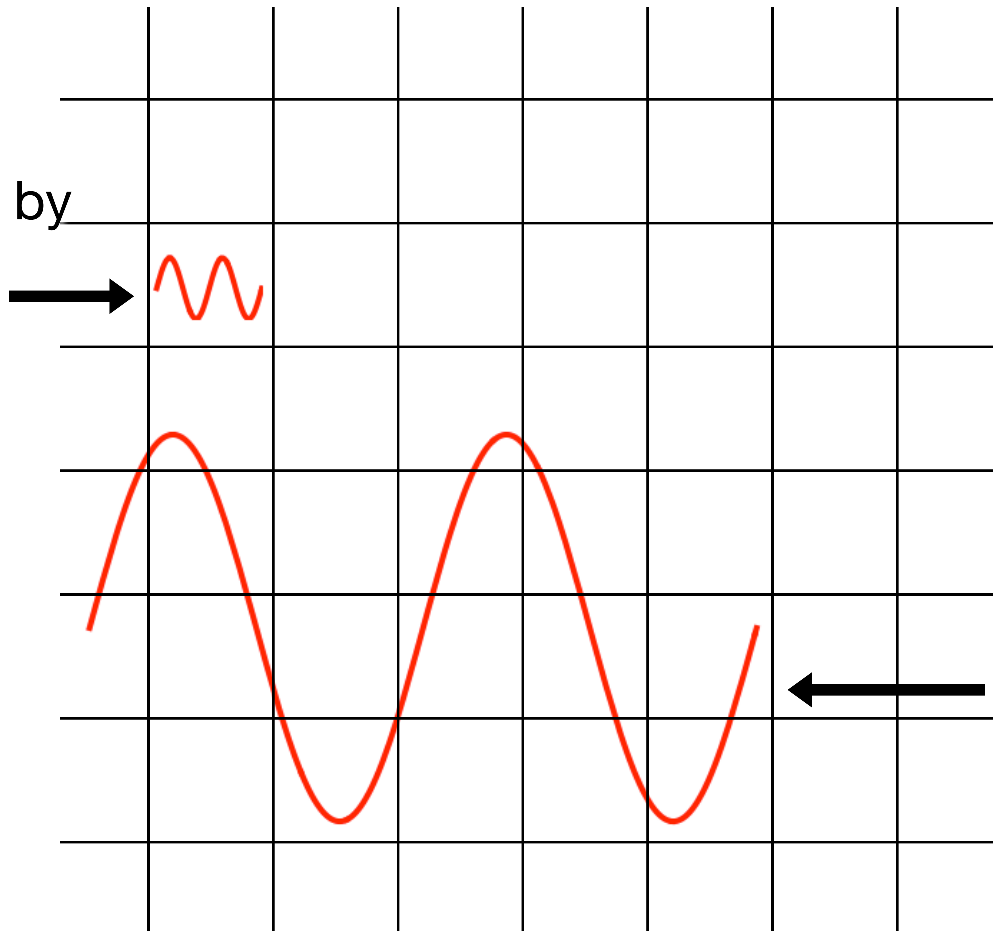
Mainly focus on the well trodden path of 1-loop gauge coupling corrections in this talk (but at the end we will see that the discussion applies to the entire theory including the Higgs)

- Functional RG (FRG) approach to renormalisation in a particle theory
- Towards the same approach in strings: the unregulated case
- The UV complete (stringy) version of RG: the regulated case
- Higgs potential and naturalness
- Conclusions: a string naturalness condition

1. Background: RG in particle theory

Let’s begin by considering a version of Functional RG. According to Kadanoff and Wilson, running of couplings emerges because we have to integrate out all modes smaller than our lattice:

Too small to be described by fluctuations on the lattice

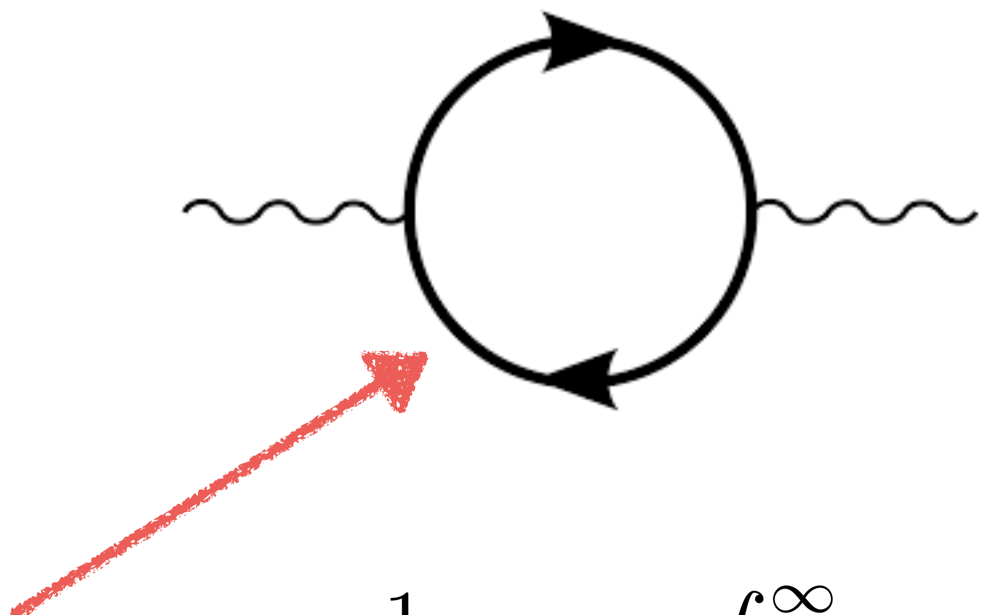


Larger modes can be described by the lattice so belong in the EFT *at this scale*

In practice we do this by including only the small modes in the one-loop integral: e.g. consider …

Vacuum polarization diagram (in Schwinger worldline formalism):

Feynman; Abbot;
Affleck, Alvarez, Manton;
Bern, Kosower;
Strassler;
Schmidt, Schubert



$$\Delta(p^2) = \frac{1}{p^2 + m^2} = \int_0^\infty dt\ e^{-t(p^2+m^2)}$$

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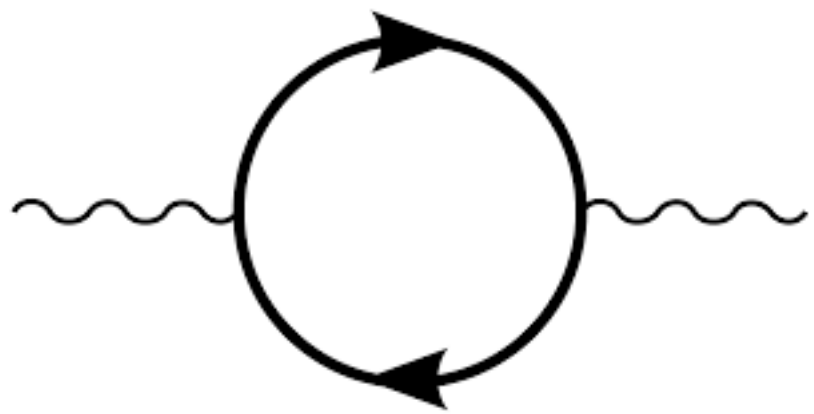

The unregulated case: we see there is a UV divergence for everyone and an IR one for massless states:

Indeed substitute $t = t_1 + t_2$ and assume external momenta near to on-shell

$x = t_1/(t_1 + t_2)$

$$\Pi^{\mu\nu} \approx \sum_i \frac{b_i g^2}{16\pi^2} \delta^{ab} (p_1^\mu p_2^\nu - p_1 . p_2 \eta^{\mu\nu}) \int_0^1 dx \int_0^\infty \frac{dt}{t} e^{t\left(p_1 \cdot p_2 x(1-x) - M_i^2\right)}$$

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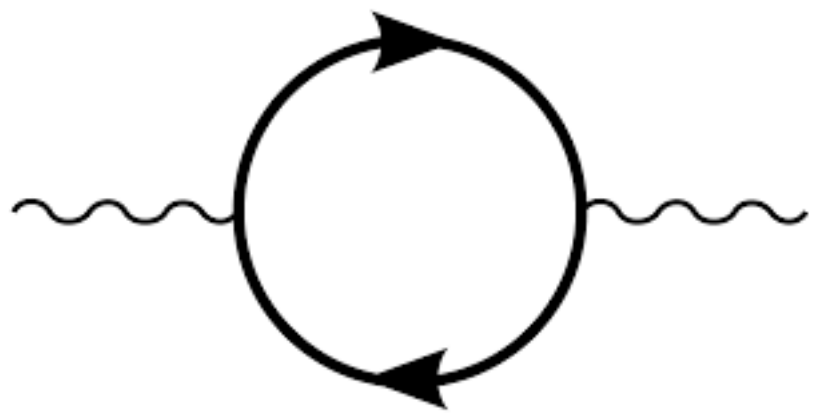

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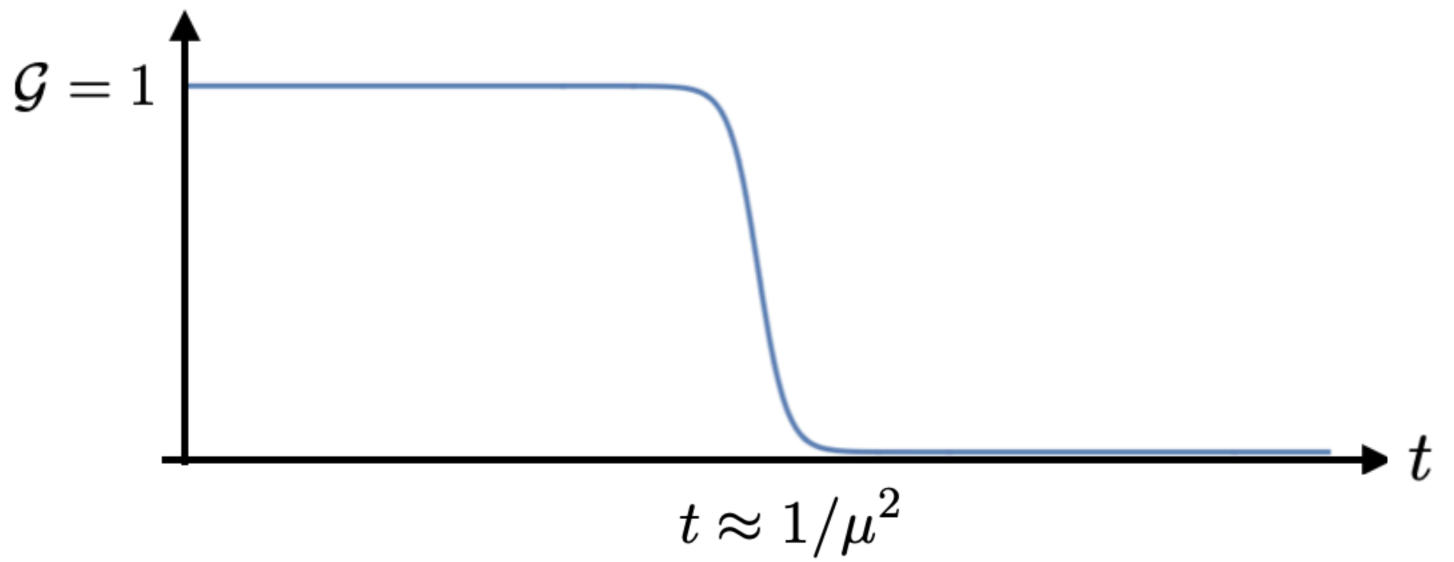
$$b_i = 2(-1)^{F_i} \left[1/12 - s_i^2\right] Q_G^2$$

one loop beta function coefficient: $$= \left(\frac{1}{6}, \frac{1}{3}, -\frac{11}{6}\right) Q_G^2$$

According to Wilson we should truncate the large size loops: add an *IR* regulator

$$\Pi^{\mu\nu} \approx \sum_i b_i \frac{g^2}{16\pi^2} \delta^{ab} (p_1^\mu p_2^\nu - p_1 . p_2 \eta^{\mu\nu}) \int_0^\infty \frac{dt}{t} e^{-tM_i^2} \; \mathcal{G}(\mu, t)$$

where $\mathcal{G}(\mu, t)$ looks like …



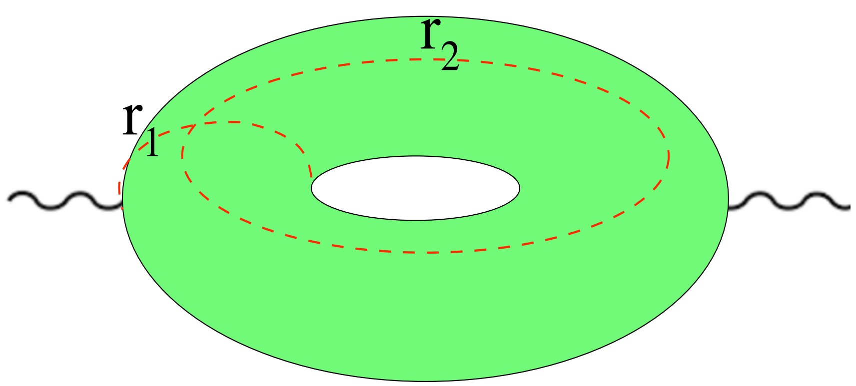
The result is a logarithmic dependence on the energy scale for any states lighter than it: of course the UV is still divergent so we traditionally decide to ignore all the UV crap by doing this …

$$\frac{16\pi^2}{g^2(\mu^2)} - \frac{16\pi^2}{g^2(M_s^2)} = -b_0 \log \frac{\mu^2}{M_s^2} + \sum_{M_i > \mu} b_i \int_{M_s^{-2}}^{\mu^{-2}} \frac{dt}{t} e^{-M_i^2 t} \mathcal{G}$$

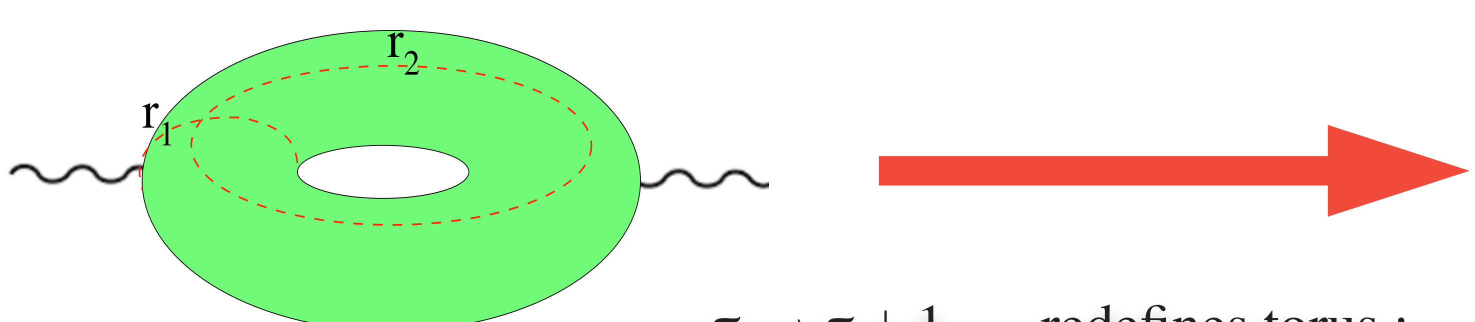
$$\equiv -b_0 \log \frac{\mu^2}{M_s^2} + \Delta_G$$

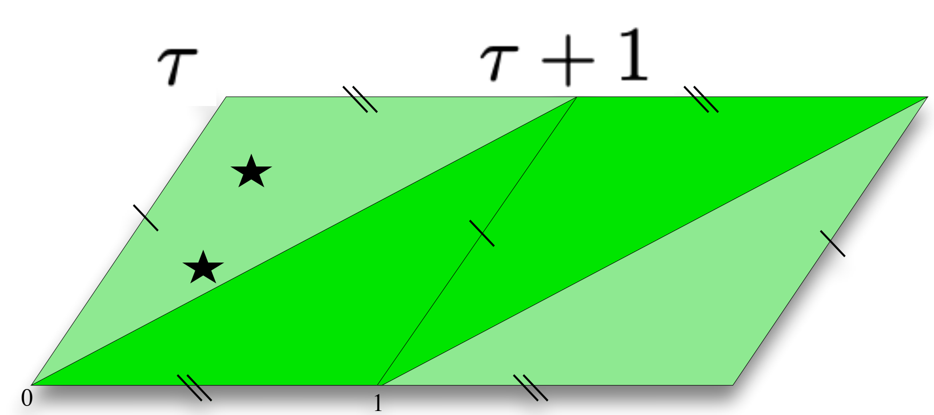
2. Towards the same FRG approach in closed string theory:
The unregulated case

The equivalent of the Schwinger parameter are the two parameters describing torus radii:



Thanks to conformal symmetry can be mapped to parallelogram in complex plane, with single parameter τ , but theory invariant under ***modular transformations***:





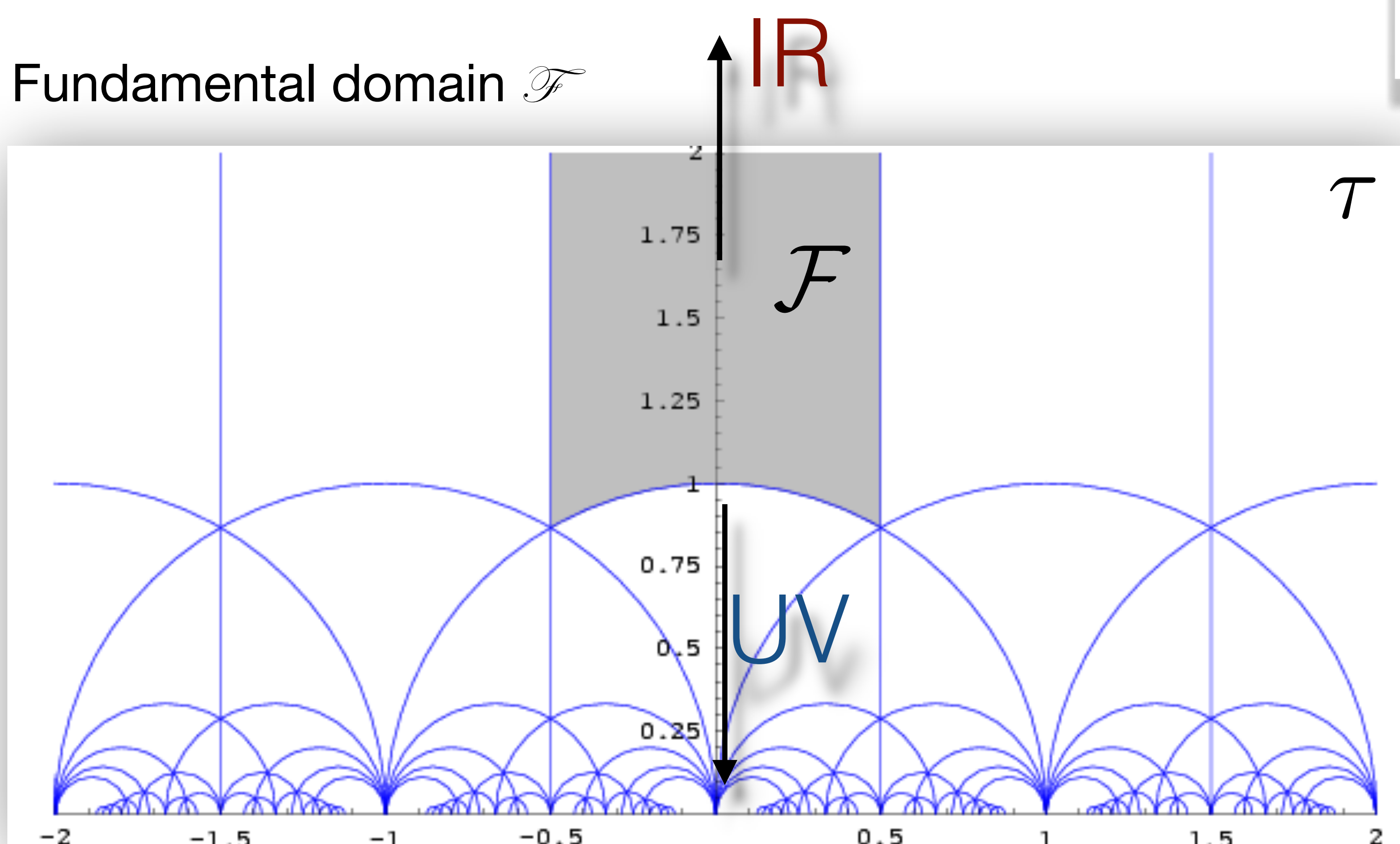
$\tau \to \tau + 1$ redefines torus :

$\tau \to -1/\tau$ swops σ_1 and σ_2 and just reorients torus

So then we have to integrate over all inequivalent tori, i.e. over τ, and we will find that the imaginary part of τ plays the role of the Schwinger parameter (overall torus volume):

$$\tau = \tau_1 + i\tau_2$$

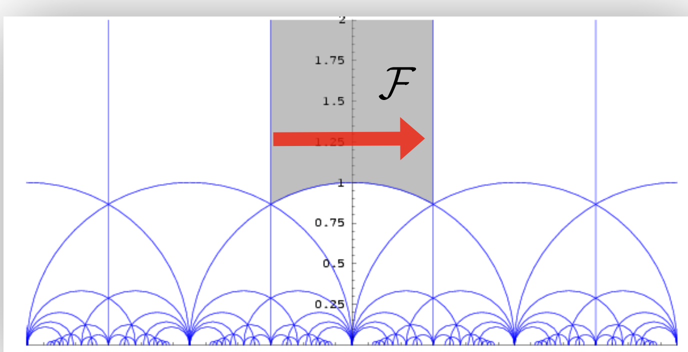
Fundamental domain $\mathscr{F}$



The identification of Schwinger parameter is clear at large imaginary tau: the one-loop integral must be of the general form :

$$\Delta_G = \langle \mathbb{X} \rangle = \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^2} \tau_2^{-1} \sum_{m,n} \mathbb{X}\, a_{mn} q^m \overline{q}^n \qquad \text{where} \qquad q = e^{2\pi i \tau}$$

for some operator $\mathbb{X}$ that encapsulates the vertex operators. But at large tau2, the tau1 integral just projects onto the physical states of the low energy particle theory …



$$\langle \mathbb{X} \rangle \approx \int_{\sim 1}^{\infty} \frac{d\tau_2}{\tau_2^2} \tau_2^{-1} \sum_n \mathbb{X}_{nn}\, a_{nn} e^{-\pi\alpha' M_n^2 \tau_2}$$

Counts physical (level matched) states weighted by statistics at each level

So we can infer the operator required $\mathbb{X}$ by modular completing the particle Schwinger integral:

$$\Delta_G = -2 \left\langle \tau_2^2 \left(S^2 - \frac{1}{12}\overline{E}_2 \right) \left(Q_G^2 - \frac{\xi}{4\pi\tau_2} \right) \right\rangle$$

SAA, Dienes and Nutracati, to appear

Before we regulate it, let's discuss the UV/IR mixing: First note that according to Wilson these *unregulated* integrals simply give terms in the *effective action*: i.e. the deep IR.

There is an astonishing identity for these due to Rankin, Selberg (1939-40) and Zagier (1981) … let

$$g(\tau_2) = \tau_2^{-1} \sum_n a_{nn} \mathbb{X}_{nn} e^{-\pi\alpha' M_n^2 \tau_2}$$

Rankin, (1939), Selberg (1940), Zagier (1981)
Angelantonj, Cardella, Elitzur, and Rabinovici
Angelantonj, Florakis, and Pioline

i.e. precisely the integrand that appears for the entire tower of just the physical states. Then …

$$\langle \mathbb{X} \rangle = \lim_{\tau_2 \to 0} \frac{\pi}{3} g(\tau_2)$$

When *X=1* for example this matches the known relation for the cosmological constant:

$$\Lambda = -\frac{\mathcal{M}^4}{2} \langle 1 \rangle = \frac{1}{24} \mathcal{M}^2 \mathrm{Str} M^2$$

Dienes, 1994

1) This is an IR fixed point for the cosmological constant (that corresponds to the UV limit of g).

2) Note that in other words before we even think about RG we know where the theory will end up!!

The incredible fact that this infinite supertrace is finite can then be put down to the fact that the "particle partition function" … behaves as follows in the UV (i.e. as $\tau_2 \to 0$):

$$g(\tau_2) \sim \tau_2^{-1} \mathrm{Str}\,(e^{-\tau_2 M^2}) \longrightarrow c_1$$

Dienes showed in 1994 that the above property is due to Misaligned SUSY, which ensures that *Str(1)=0* even though there is no level by level cancellation and the nett (Boson-Fermion) numbers of states in each level are completely crazy!

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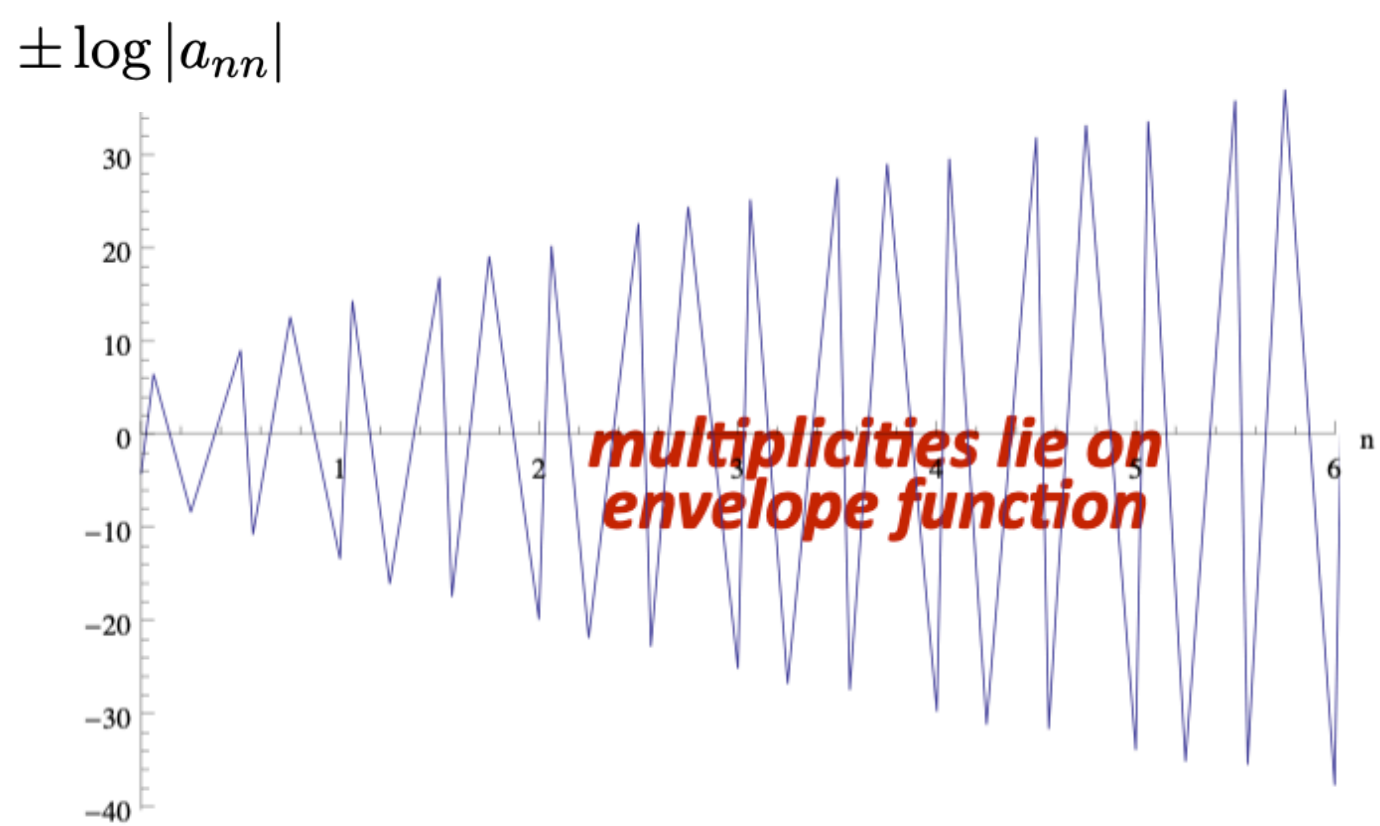


Dienes, Misaligned SUSY, 1994

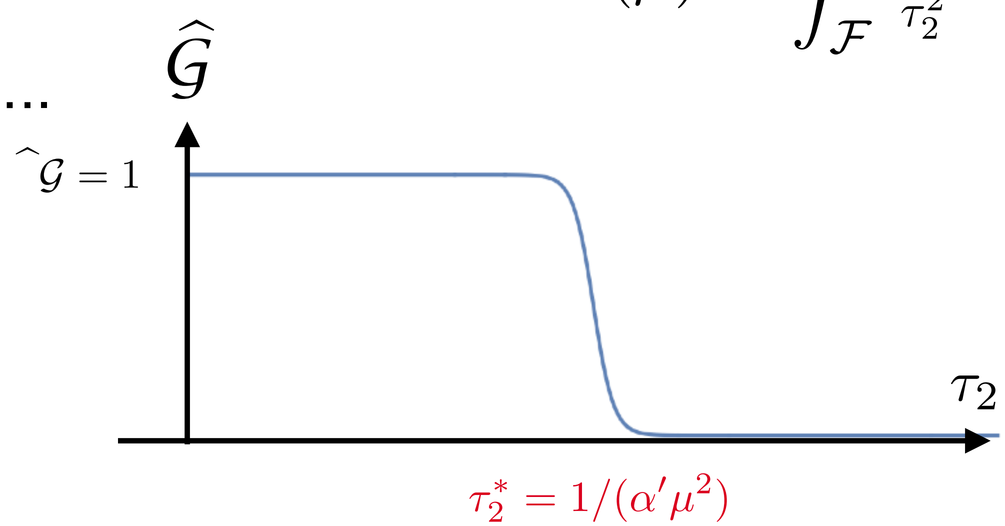
Dienes, Moshe, Myers 1995

3. The UV complete (stringy) version of RG: The regulated case

Again need a “Wilsonian” regulator, to introduce an IR cut-off around EVERY cusp $\widehat{\mathcal{G}}$:

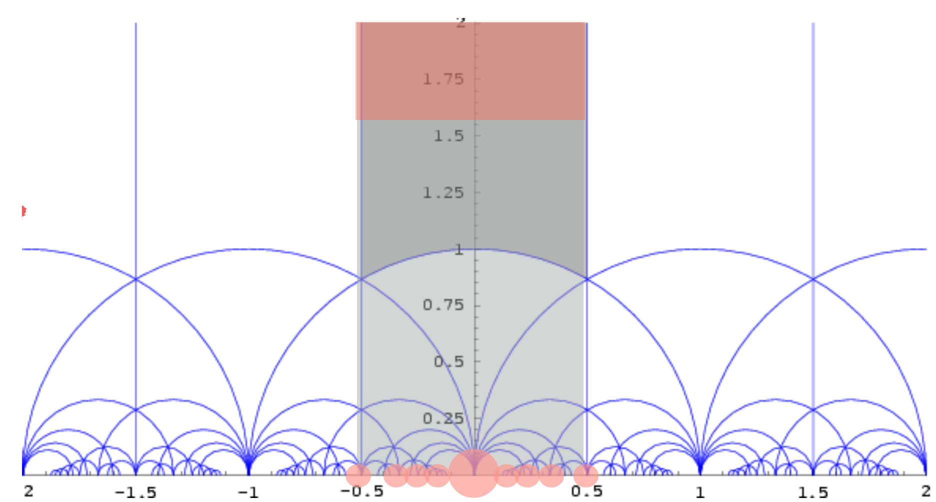
$$\widehat{I}(\mu) = \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^2}\, \widehat{\mathcal{G}}(\mu,\tau,\overline{\tau})\, F(\tau,\overline{\tau})$$

- a) Is itself a modular function
- b) Should look roughly like this ….



- c) As our goal is to write everything as supertraces which ultimately means an integral over the critical strip …This only makes sense if actually all the cusps are crushed equally. In other words: all the cusps are equivalent IR cusps, implying…

$$\tau_2^* \equiv 1/\tau_2^* \implies \widehat{\mathcal{G}}(\mu,\tau,\overline{\tau}) = \widehat{\mathcal{G}}(M_s^2/\mu,\tau,\overline{\tau})$$



We modify a (geometrically derived) cut-off function of Costas et al

(Kiritsis, Kounnas, Petropoulos, Rizos)

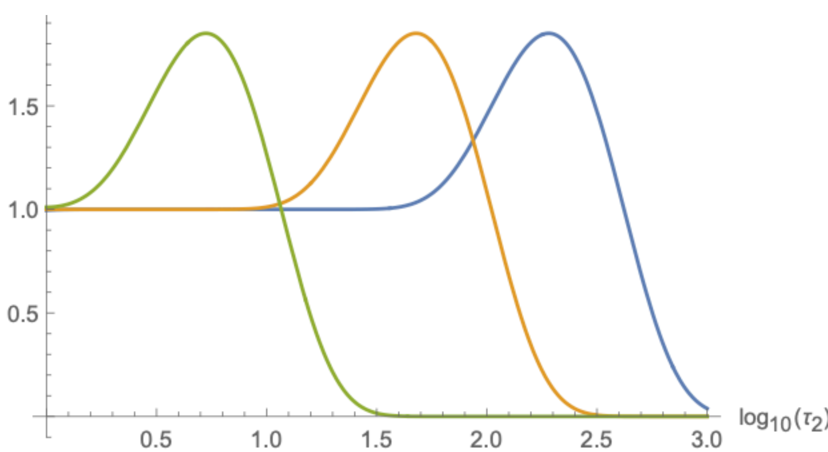
- Take the circle partition function with radius defined by parameter $a \equiv \sqrt{\alpha'}/R$:

$$Z_{\text{circ}}(a,\tau) = \sqrt{\tau_2} \sum_{m,n\in\mathbb{Z}} \overline{q}^{(ma-n/a)^2/4}\, q^{(ma+n/a)^2/4}$$

- Then a suitable cut-off function that obeys all the required properties is …

$$\widehat{\mathcal{G}}(a,\tau) = \frac{2a^2}{1+2a^2}\frac{\partial}{\partial a}\left(Z_{\text{circ}}(2a,\tau) - Z_{\text{circ}}(a,\tau)\right)$$

SAA, Dienes, \`21



$$\mu^2(a) = \frac{2a^2}{\alpha'} \implies \tau_2^* = 1/2a^2$$

Results for gauge coupling:

The correction we want is $\widehat{\Delta}_G = \langle (\tau_2 \mathbb{X}_1 + \tau_2^2 \mathbb{X}_2) \widehat{\mathcal{G}}_\rho(\tau, a) \rangle$

where
$$\mathbb{X}_1 \equiv \frac{\xi}{2\pi}\left(S^2 - \frac{\overline{E}_2}{12}\right)$$
$$\mathbb{X}_2 \equiv -2\left(S^2 - \frac{\overline{E}_2}{12}\right) Q_G^2 .$$

Evaluate $P(a) = \langle (\tau_2 \mathbb{X}_1 + \tau_2^2 \mathbb{X}_2)\, Z_{\text{circ}}(a,\tau) \rangle$ by unfolding against $Z_{\text{circ}}(a,\tau)$ gives

$$\begin{aligned} P(a) \;=\; & \frac{\pi}{3a} \text{Str} \mathbb{X}_1 + \underset{M=0}{\text{Str}}\ \mathbb{X}_2 \left[-\frac{2}{a} \log\, a \right] \\ & + \underset{M\geq 0}{\text{Str}}\ \mathbb{X}_1 \left[\frac{2}{\pi} \sum_{r=1}^{\infty} \left(\frac{M}{r\mathcal{M}} \right) K_1 \left(\frac{rM}{a\mathcal{M}} \right) \right] \\ & + \underset{M>0}{\text{Str}}\ \mathbb{X}_2 \left[\frac{4}{a} \sum_{r=1}^{\infty} K_0 \left(\frac{rM}{a\mathcal{M}} \right) \right] \end{aligned}$$

SAA, Dienes `21

… and then take the required derivatives in *a* …

1) EFT behaviour emerges at the required scale …

Contribution from absolutely massless states

$$\widehat{\Delta}_G \approx \quad constant$$
$$+ \operatorname*{Str}_{M=0} 4\left(S^2 - \frac{1}{12}\right) Q_G^2 \left[\log\left(\frac{\mu}{2\sqrt{2}eM_s}\right)\right]$$
$$+ \operatorname*{Str}_{0<M\lesssim\mu} 4\left(S^2 - \frac{1}{12}\right) Q_G^2 \left[\log\left(\frac{1}{\sqrt{2}}e^{-(\gamma+1)}\frac{\mu}{M}\right)\right]$$

SAA, Dienes, Nutricati to appear

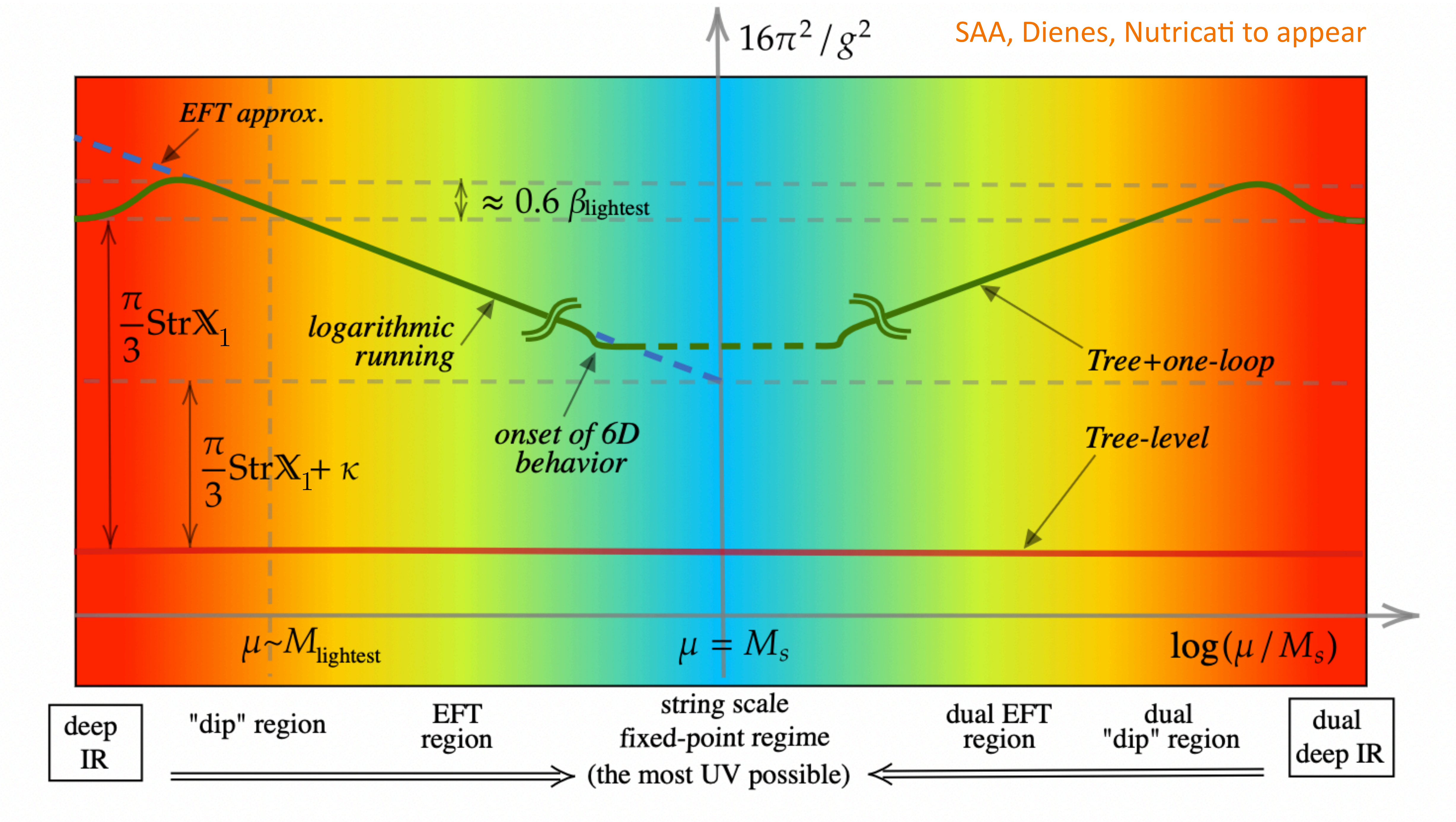
Contribution from every massive state running up from its mass with the log from pile-up of Bessels (Paris)

The corrections makes the theory run in the EFT. If there are no massless states charged under the gauge symmetry it has an IR fixed point

$$\frac{16\pi^2}{g^2(\mu)} - \frac{16\pi^2}{g^2_{\text{tree}}} = \beta_{\frac{16\pi^2}{g^2}}(0)\log\left(\frac{\mu}{M_s}\right) + \tilde{\Delta}$$

$$\tilde{\Delta} = \frac{\pi}{3}\operatorname{Str}\mathbb{X}_1 + \kappa$$

hence we find the following picture for gauge coupling:



2) Comparison with famous case of large volume 2d thresholds …

Dixon, Louis, Kaplunovsky
c.f. Angelantonj, Florakis, Tsulaia

- Moduli dependence of string threshold corrections for 2D compactifications
- DKL and traditionally simply match to the EFT, so lose mod.inv and all UV completeness, and not able to get running nor emergence of the EFT …
- Nevertheless the moduli dependence should be in the coupling corrections … we find …

it indeed emerges in the deep-IR value of the running …

$$\widehat{\Delta}_G \overset{a\to 0}{\approx} -\log(c\,T_2U_2|\eta(T)\eta(U)|^4) - 2\log\left(\frac{\mu}{M_s}\right)$$

and also in the deep dual-IR running …

$$\widehat{\Delta}_G \overset{a\to\infty}{\approx} -\log\left(c\,T_2U_2|\eta(T)\eta(U)|^4\right) + 2\log\left(\frac{\mu}{M_s}\right)$$

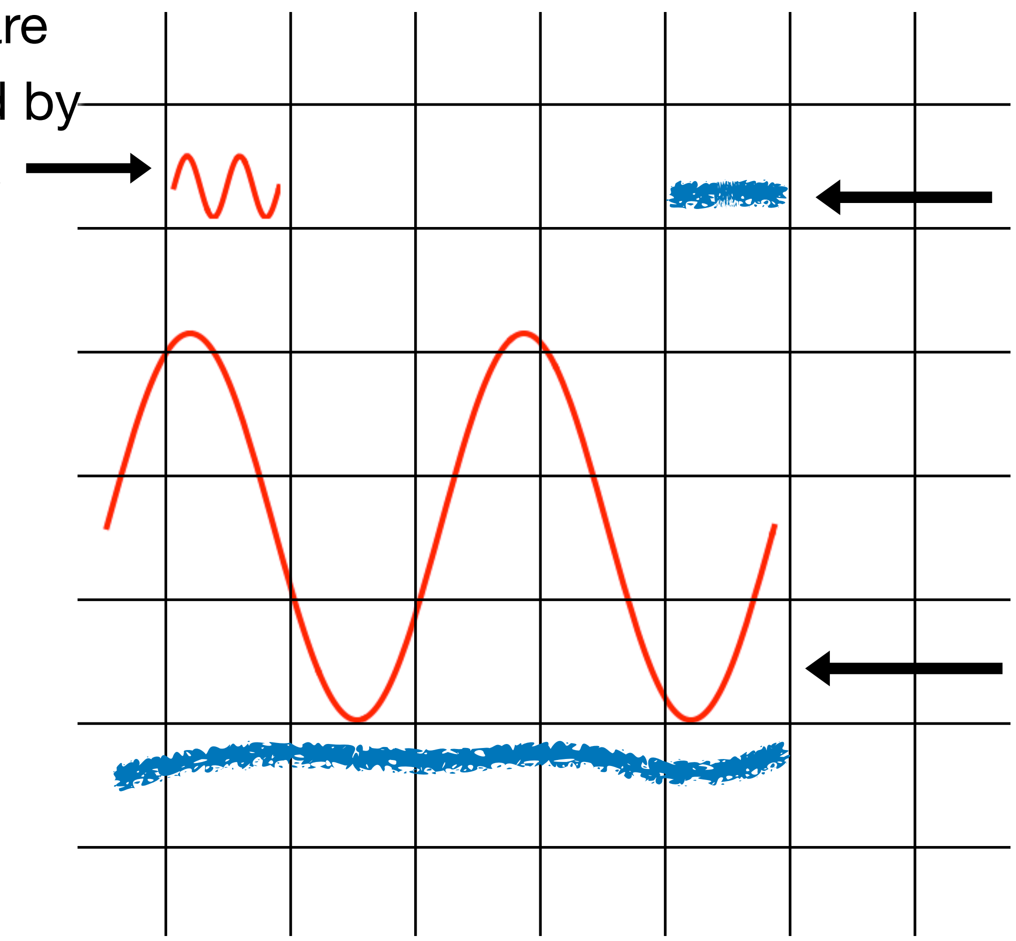
… but it never really disappears at any scale … (no power law running)

$$\widehat{\Delta}_G \overset{a\sim 1}{\approx} \tfrac{\pi}{3}T_2$$

The meaning of the turn-over?

Momentum modes that are too small to be described by fluctuations on the lattice

$\mu_{IR} \gtrsim 1/R$



Extended “winding” modes too small to be described by fluctuations on the lattice

$\mu_{UV} \lesssim RM_s^2$

Larger modes can be described by the lattice so belong in the EFT at this scale

$$\mu_{UV} = M_s^2/\mu_{IR}$$

4. The Higgs potential and naturalness

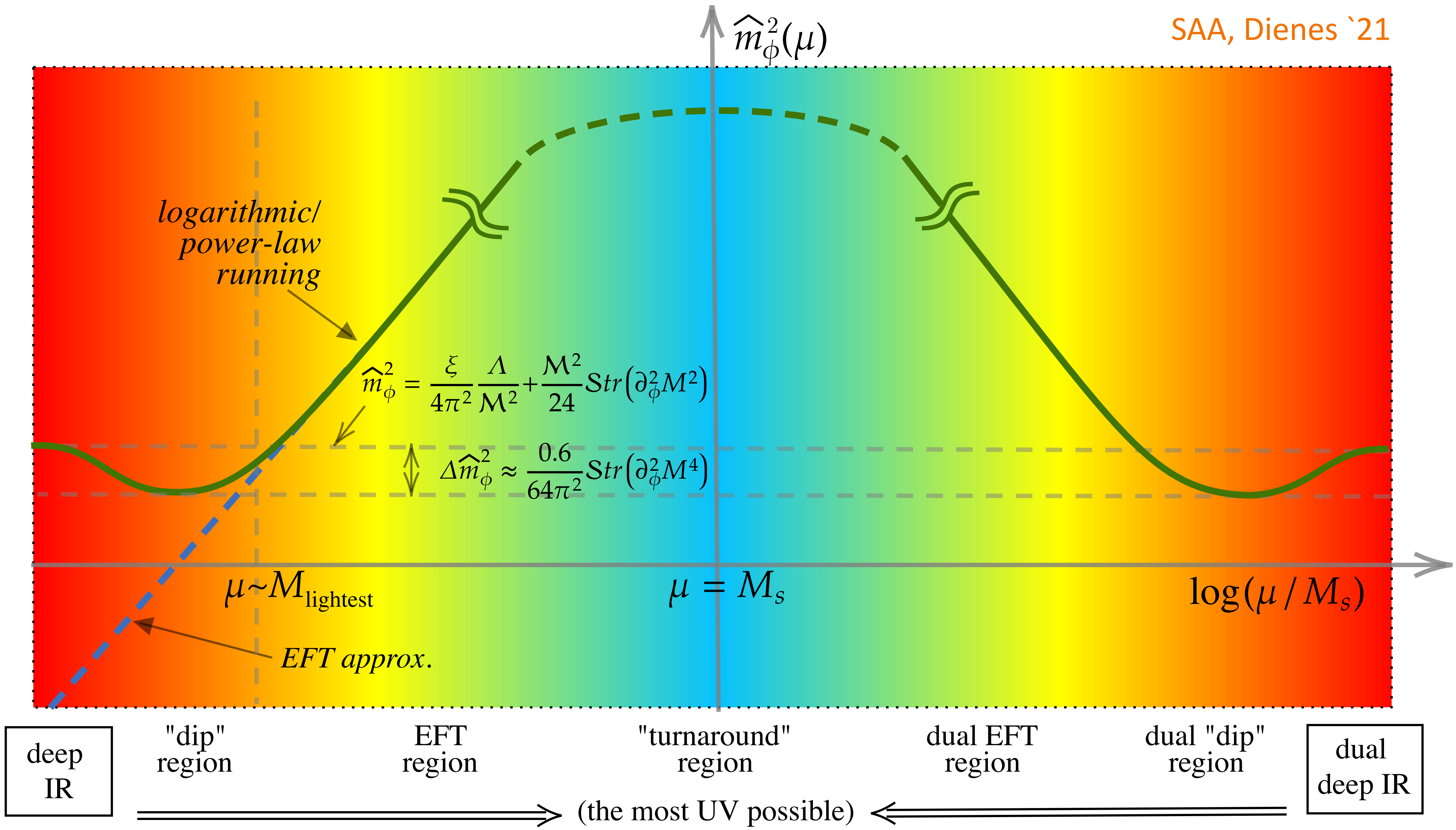
The cosmological constant is similar infinite sum of Bessel functions, but it has the following magical behaviour with emergent Coleman-Weinberg potentials …

SAA, Dienes `21

$$\widehat{\Lambda}(\mu,\phi) \,=\, \frac{1}{24}\mathcal{M}^2\,\mathrm{Str}\,M^2 - c'\underset{M\gtrsim\mu}{\mathrm{Str}}\,M^2\mu^2 \quad - \underset{0\leq M\lesssim\mu}{\mathrm{Str}}\left[\frac{M^4}{64\pi^2}\log\left(c\frac{M^2}{\mu^2}\right)+c''\mu^4\right]$$

$c = 2e^{2\gamma+1/2}$, $c' = 1/(96\pi^2)$, and $c'' = 7c'/10$

This is a fully UV complete effective potential which holds for any modular invariant theory.
Below the mass of all states (that couple to the Higgs) they do not contribute to the running.
At some intermediate energy scale the result is a sum over all states *as **if they had all logarithmically run up from their mass***.



$$\lim_{\mu\to 0} \hat{m}_\phi^2(\mu) = \frac{\xi}{4\pi^2}\frac{\Lambda}{\mathcal{M}^2} + \frac{1}{24}\mathcal{M}^2 \text{Str}\partial_\phi^2 M^2$$

5. Conclusions

- We have developed formalism for RG and extracting EFT within fully UV complete theory
- A modular invariant regulator provides a natural “Wilsonian energy cut-off” and a definition of RG scale. Gives meaning where the EFT fails, and retains the predictivity of the UV complete theory.
- Results rendered as infinite sums of particle Bessel function contributions
- Operators such as the gauge couplings and Higgs mass can be thought of as “running” to its predetermined IR value (fixed point): this is actually both a UV and IR asymptote as it should be.
- Recovers famous old results but importantly there is only log running (even for the Higgs mass).
- Relevant for many old and new pheno ideas: e.g. a stringy naturalness (Veltman) condition:

$$\mathrm{Str}\,\partial_\phi^2 M^2 \lesssim \frac{24}{\mathcal{M}^2} M_W^2$$