

Scattering from production in 4d

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11-18th Sept. 2022*

PT, Zhiboedov JHEP 2021, and
PT, Zhiboedov, 2022, to appear

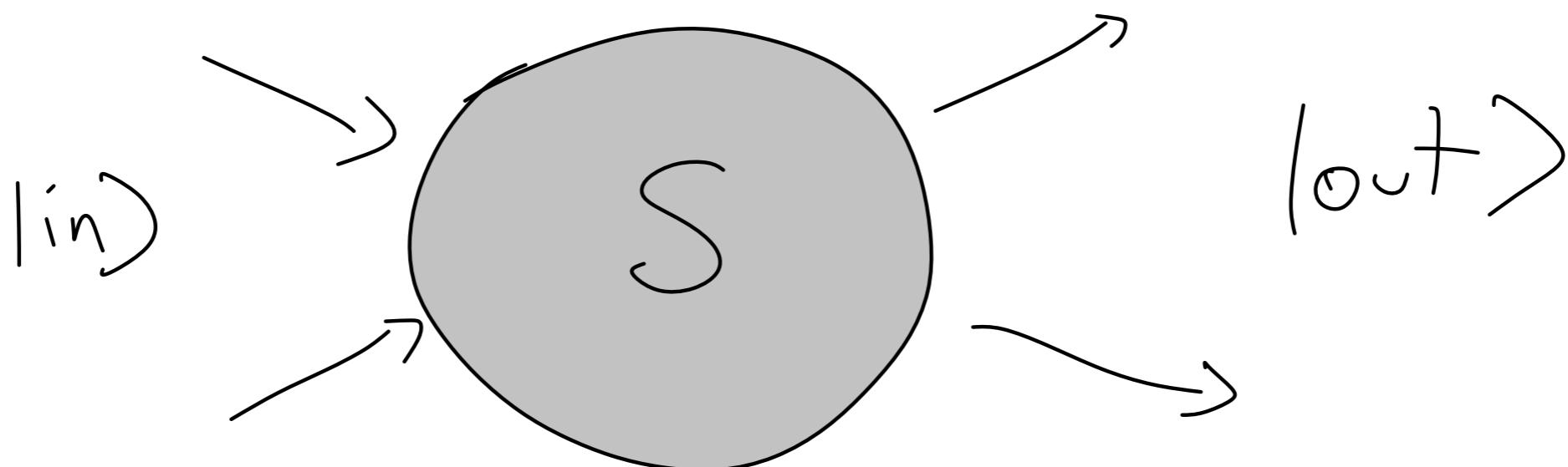


Outline

- Motivations and introduction
 - Non-perturbative unitarity
 - Scattering from production
- Results
 - numerical implementation
 - Aks physics (“scattering implies production”)

Introduction

- Find methods to compute S-matrices which satisfy analyticity, crossing, and non-pert. unitarity.

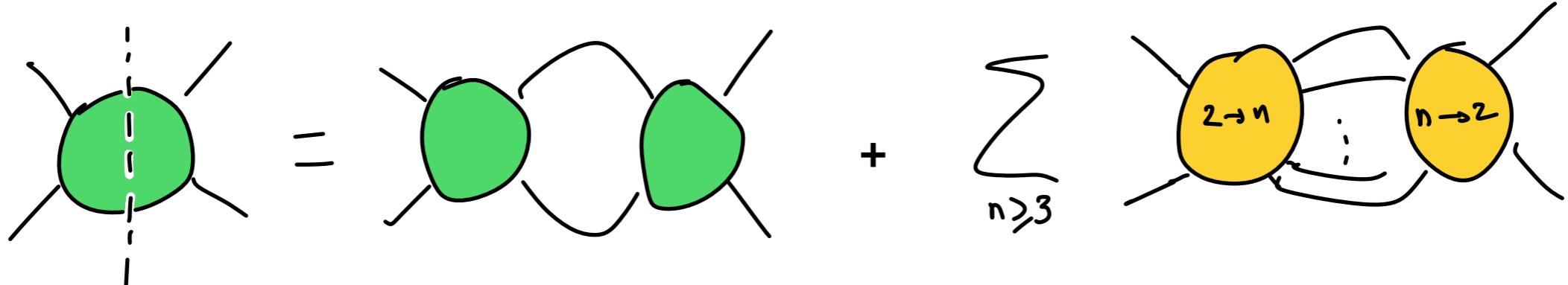


Introduction

- Perturbative unitarity : amplitudes' methods
- Non-pert. unitarity : given crossing, no generic method
- CFT numerical bootstrap has revived the hope that the S-matrix bootstrap of the 60's can be revisited today with modern computer's power.

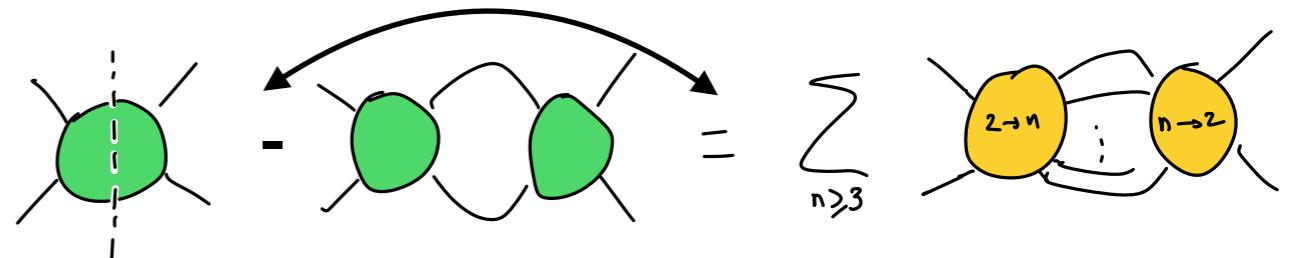
Introduction

- Construct full S-matrix (all n to m processes) is hard
- ↗ Compute 2 to 2 amplitudes $S=1+i T(s,t)$.
- Still non-trivial, because they contain info about all 2 to n processes via optical theorem



Introduction

- Scattering from production :



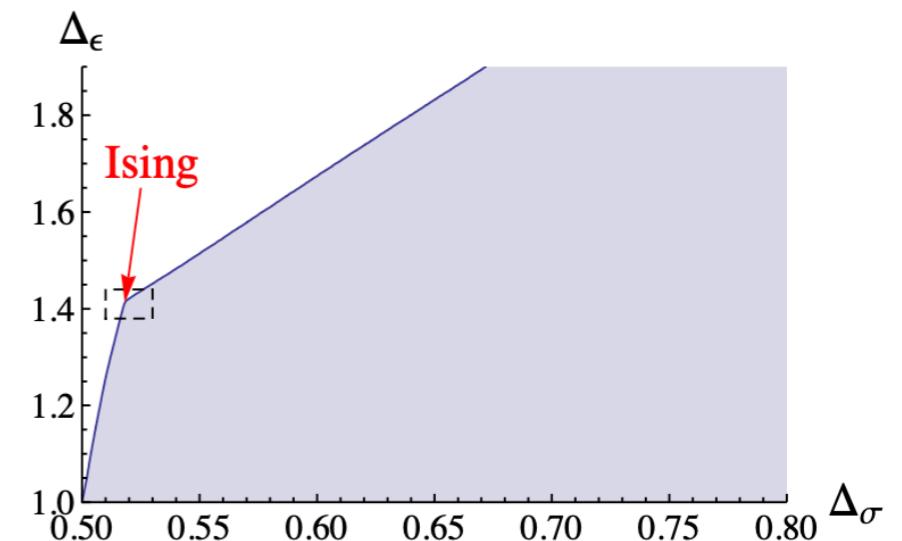
input

- bypasses complicated multipoint amplitudes

- scan all possible theories

- Change of paradigm compared to 60s: explore space of theories, rather than solve one theory

- Then, maybe, solve *extremal* theories



[arXiv:1203.6064] Phys.Rev. **D86** (2012) 025022
Solving the 3D Ising Model with the Conformal Bootstrap
S. El-Showk, M. F. Paulos, D. Poland, S. Rychkov, D. Simmons-Duffin, A. Vichi

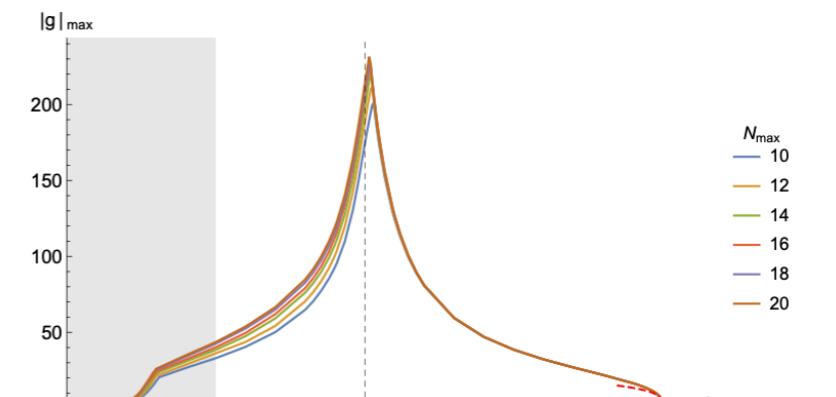
Introduction

- Many existing approaches, in particular [PPTvRV](#) apply strong unitarity constraints and derive bounds on various couplings.
- Problem: seems to find extremal theories which look purely elastic: tension with Aks theorem.
- Aks theorem ('64):

“Scattering implies production” (in $d>2$)

- Also: analytic structure (Landau curves) not built in and convergence to them seems impossible to achieve.
- Current status : no fully consistent 2-to-2 S-matrices could be constructed in $d>2$, even numerically, so far.

[M. F. Paulos, J. Penedones, J. Toledo, B. C. van Rees, P. Vieira, 2016, 2017](#)



gold standard

[H. Chen, A. L. Fitzpatrick, D. Karateev, 2022](#)

Main results

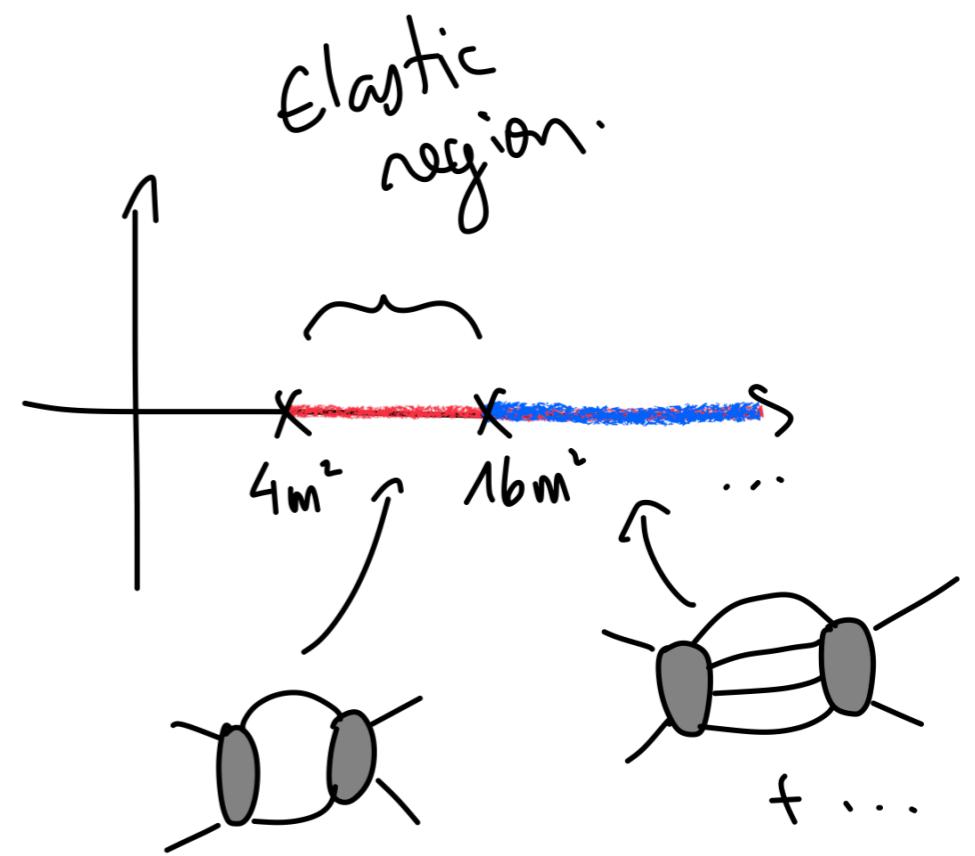
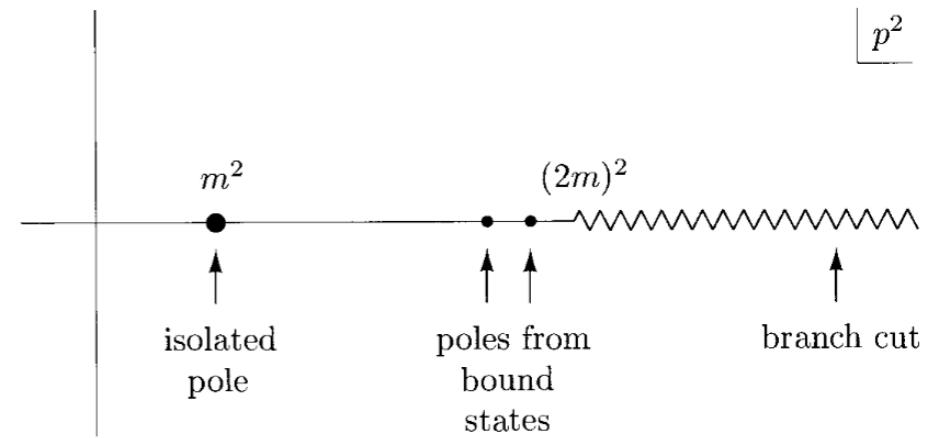
PT, Zhiboedov, 2022, to appear

- Numerical implementation of dispersive iterations which produces S-matrices that satisfy all known constraints.
- Aks / inelastic physics is correctly produced

Unitarity, analyticity

Set-up

- We consider the 2-to-2 scattering of lightest scalar states in a gapped QFT, with Z_2 symmetry (no cubic vertex)
- Goal: construct functions that satisfy the following S-matrix axioms: unitarity, crossing and Mandelstam analyticity
- In 4 dimensions, given crossing, one property is particularly difficult to enforce: *elastic unitarity*

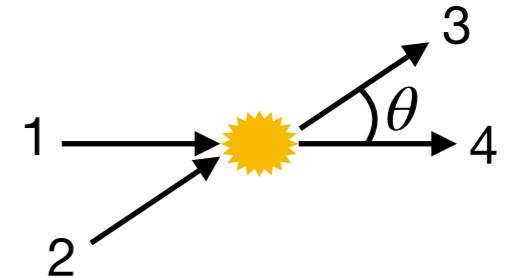


Partial wave unitarity :

$$S^\dagger S = 1$$

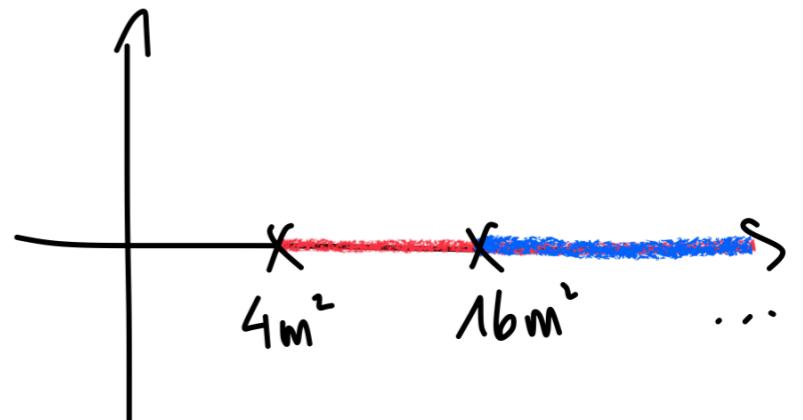
$$f_J(s) \sim \int_{-1}^1 P_J^{(d)}(\cos(\theta)) A(s, t) d\cos(\theta)$$

$$S_J(s) = 1 + \frac{(s-4)^{(d-3)/2}}{\sqrt{s}} f_J(s)$$



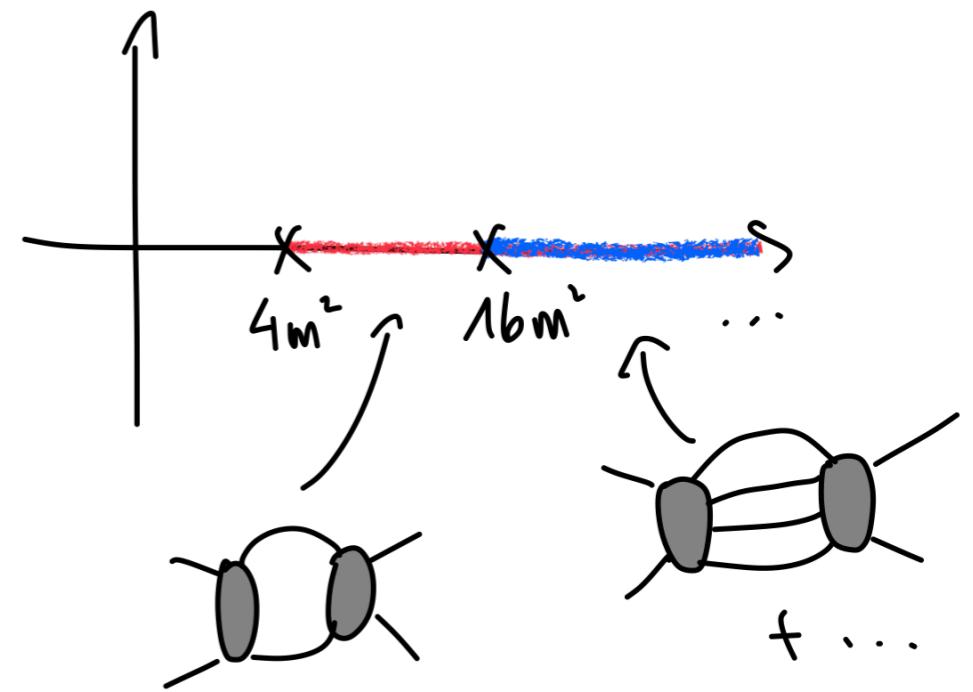
$$s, t, \cos(\theta) = 1 + 2t/s$$

- $|S_J(s)| = 1, s \in [4m^2; 16m^2]$
- $|S_J(s)| \leq 1, s \in [16m^2; +\infty]$
- *Straightforward to check*



Amplitude unitarity

- $S^\dagger S = 1$
- $S = 1 + iT \implies 2i \operatorname{Im} T = T^\dagger T$
- For 2 to 2:



$$2i \operatorname{Im} T_{2 \rightarrow 2} = \sum_{n=2}^{\infty} T_{2 \rightarrow n} T_{n \rightarrow 2}^* = |T_{2 \rightarrow 2}|^2 + \sum_{n \geq 3} |T_{2 \rightarrow n}|^2$$

Amplitude unitarity

for $4m^2 \leq s \leq 16m^2$,

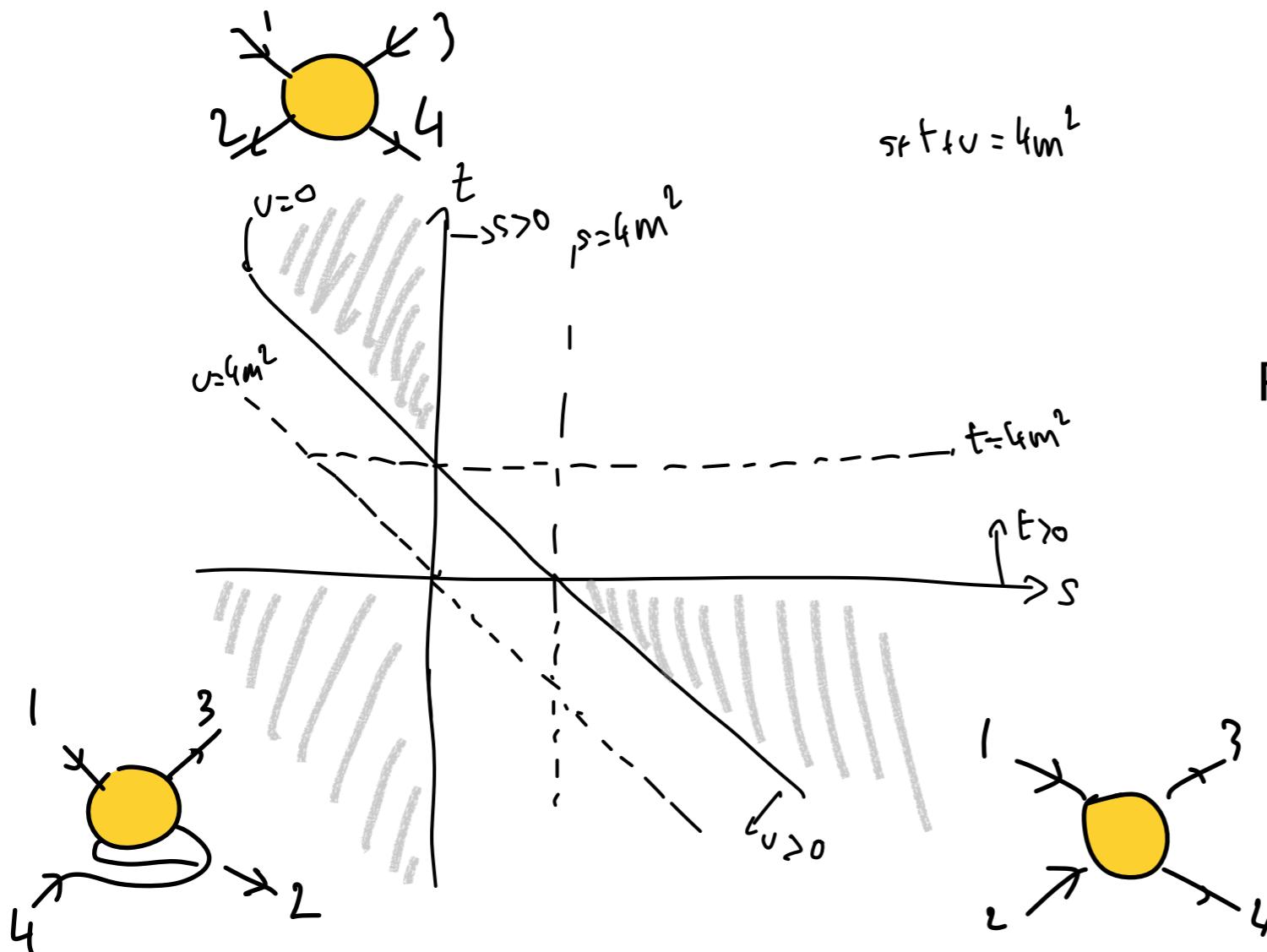
$$T_s(s, t) = \frac{1}{2} \int \frac{d^{d-1} \vec{q'}}{(2\pi)^{d-1} (2E_{\vec{q'}})} \int \frac{d^{d-1} \vec{q''}}{(2\pi)^{d-1} (2E_{\vec{q''}})} (2\pi)^d \delta^d(p_1 + p_2 - q' - q'') T^{(+)}(s, t') T^{(-)}(s, t'')$$

$z = \cos(\theta)$

$$T_s(s, t) = \frac{(s - 4m^2)^{\frac{d-3}{2}}}{8(4\pi)^{d-2} \sqrt{s}} \int_{-1}^1 dz' \int_{-1}^1 dz'' \mathcal{P}_d(z, z', z'') T^{(+)}(s, t(z')) T^{(-)}(s, t(z''))$$

\mathcal{P}_d Jacobian for change of variables

intermezzo : 2-2 4d kinematics



$$s = (p_1 + p_2)^2$$

$$t = (p_1 - p_3)^2$$

$$u = (p_1 - p_4)^2$$

Physical regime : $-1 \leq \cos(\theta) \leq 1$

$$\cos(\theta) = 1 + 2t/s$$

Amplitude unitarity

for $4m^2 \leq s \leq 16m^2$,

$$T_s(s, t) = \frac{1}{2} \int \frac{d^{d-1} \vec{q'}}{(2\pi)^{d-1} (2E_{\vec{q'}})} \int \frac{d^{d-1} \vec{q''}}{(2\pi)^{d-1} (2E_{\vec{q''}})} (2\pi)^d \delta^d(p_1 + p_2 - q' - q'') T^{(+)}(s, t') T^{(-)}(s, t'')$$

$z = \cos(\theta)$

$$T_s(s, t) = \frac{(s - 4m^2)^{\frac{d-3}{2}}}{8(4\pi)^{d-2} \sqrt{s}} \int_{-1}^1 dz' \int_{-1}^1 dz'' \mathcal{P}_d(z, z', z'') T^{(+)}(s, t(z')) T^{(-)}(s, t(z''))$$

\mathcal{P}_d Jacobian for change of variables

$$4m^2 - s < t < 0$$

Not crossing-friendly

- > discontinuity in s only in lhs
- > physical kinematics

double-disc

Take another disc !

$$T_s(s, t) = \frac{(s - 4m^2)^{\frac{d-3}{2}}}{8(4\pi)^{d-2}\sqrt{s}} \int_{-1}^1 dz' \int_{-1}^1 dz'' \mathcal{P}_d(z, z', z'') T^{(+)}(s, t(z')) T^{(-)}(s, t(z''))$$

double-disc

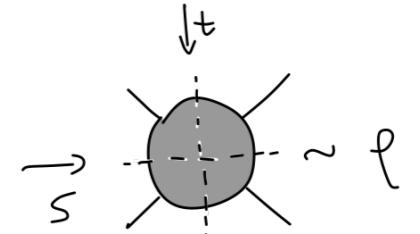
Take another disc !

$$\text{disc_t}(\, T_s(s, t) = \frac{(s - 4m^2)^{\frac{d-3}{2}}}{8(4\pi)^{d-2}\sqrt{s}} \int_{-1}^1 dz' \int_{-1}^1 dz'' \mathcal{P}_d(z, z', z'') T^{(+)}(s, t(z')) T^{(-)}(s, t(z'')) \,)$$

double-disc

Take another disc !

$$\text{disc_t}(\mathcal{T}_s(s, t) = \frac{(s - 4m^2)^{\frac{d-3}{2}}}{8(4\pi)^{d-2}\sqrt{s}} \int_{-1}^1 dz' \int_{-1}^1 dz'' \mathcal{P}_d(z, z', z'') T^{(+)}(s, t(z')) T^{(-)}(s, t(z'')))$$



Mandelstam equation

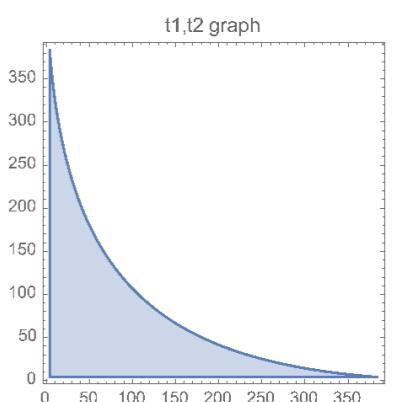
$$\rho(s, t) = \text{disc}_t \text{disc}_s T(s, t)$$

$$\rho(s, t) = \frac{(s - 4m^2)^{\frac{1}{2}}}{(4\pi)^2 \sqrt{s}} \int_{z_1}^{\infty} d\eta' \int_{z_1}^{\infty} d\eta'' T_t(s + i\epsilon, t(\eta')) T_t(s - i\epsilon, t(\eta'')) \mathcal{K}_d(s, z, \eta', \eta'')$$

$$\eta_{\pm} = \eta' \eta'' \pm \sqrt{\eta'^2 - 1} \sqrt{\eta''^2 - 1},$$

This is almost the equation we want to solve (lacks inelastic input)

But.. what do we do with ρ ?



Mandelstam representation

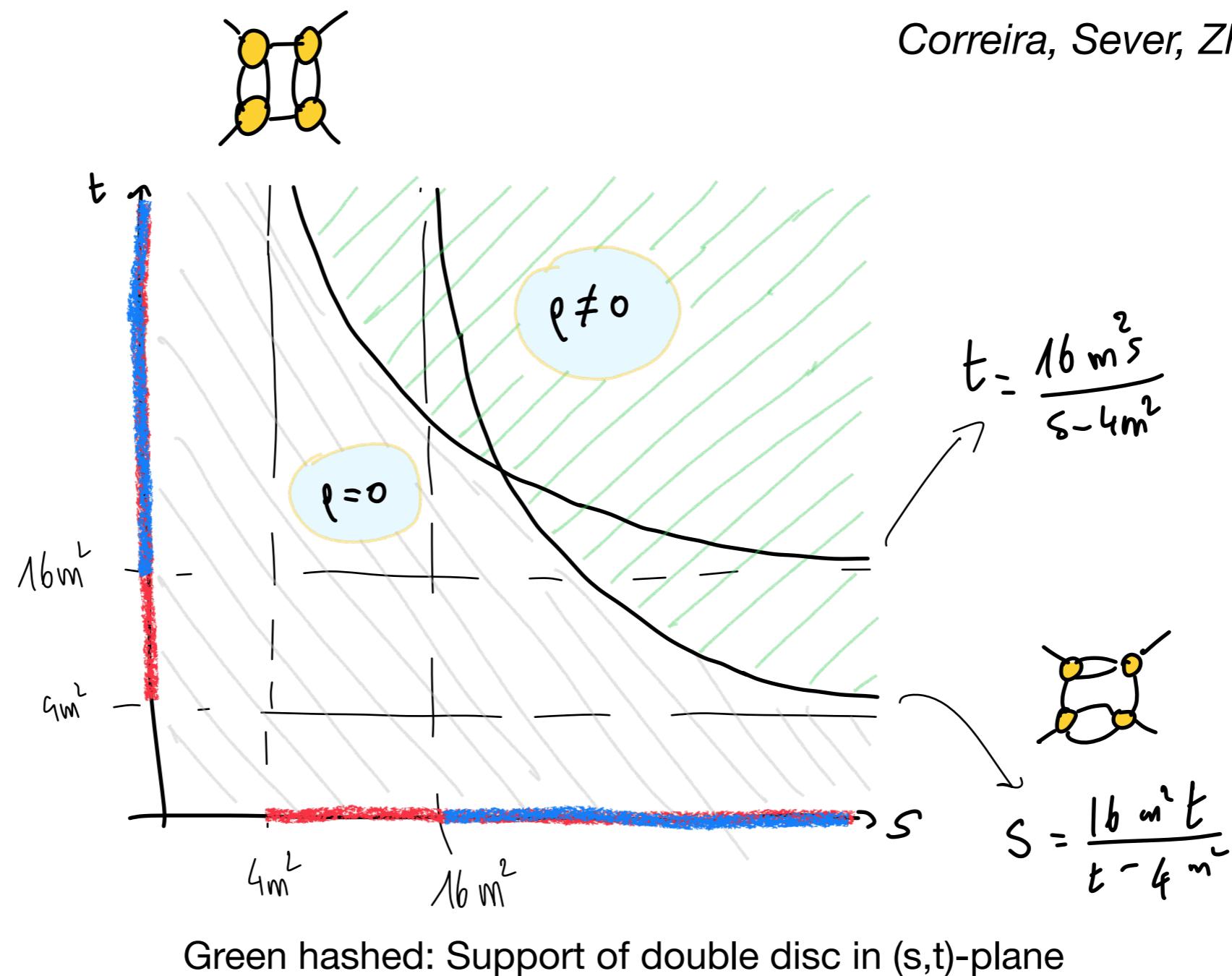
$$T(s, t, u) = B(s, t) + B(t, u) + B(u, s)$$

$$B(s, t) = \frac{1}{\pi^2} \iint_{4m^2}^{\infty} \frac{ds' dt' \rho(s', t')}{(s' - s)(t' - t)}$$

- Double-dispersive integral : in s and in t .
- Lightest Particle Maximal Analyticity
 - ➡ singularities are those given unitarity + single disc is polynomially bounded.

Elastic unitarity in 4d

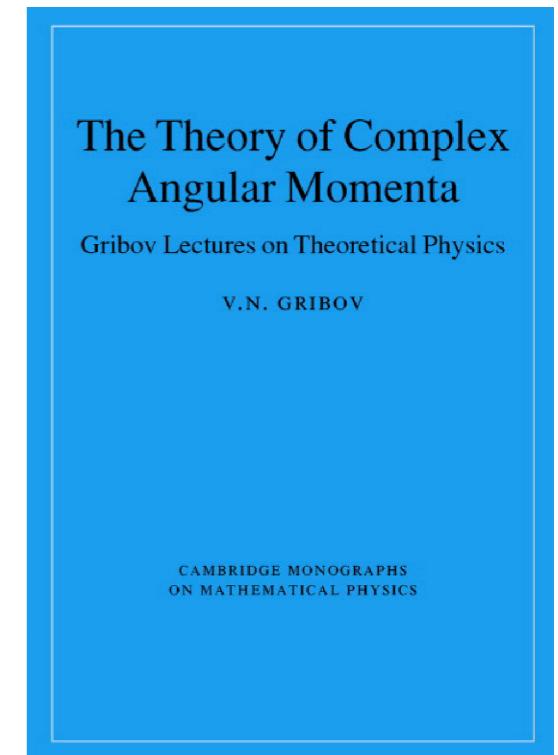
Correira, Sever, Zhiboedov '20



Elastic unitarity in 4d

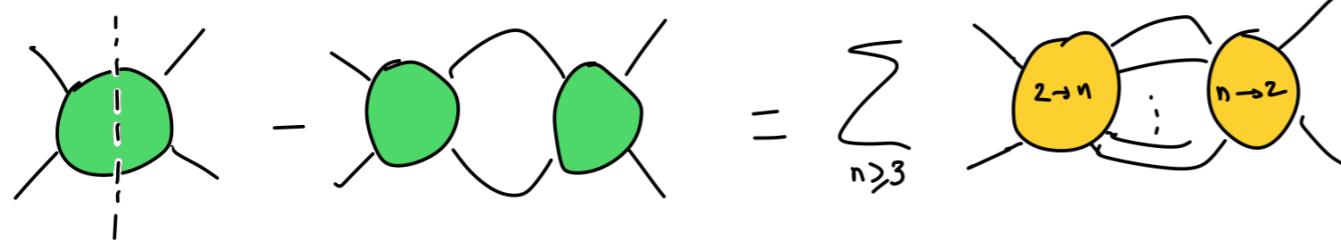
Correira, Sever, Zhiboedov '20

- Consequences of elastic unitarity + crossing are profound
 - Aks' theorem: scattering implies production in $d > 2$.
 - Gribov's theorem (disprove black disk diffraction model) $A_s(s, t) \neq sf(t)$ for $s \rightarrow \infty$ 60's
 - Bound on inelasticity *Correira, Sever, Zhiboedov '20*
- As it seems, only one scheme was proposed in the literature to construct amplitudes which satisfy elastic unitarity + crossing, by Atkinson; [1968-1970].

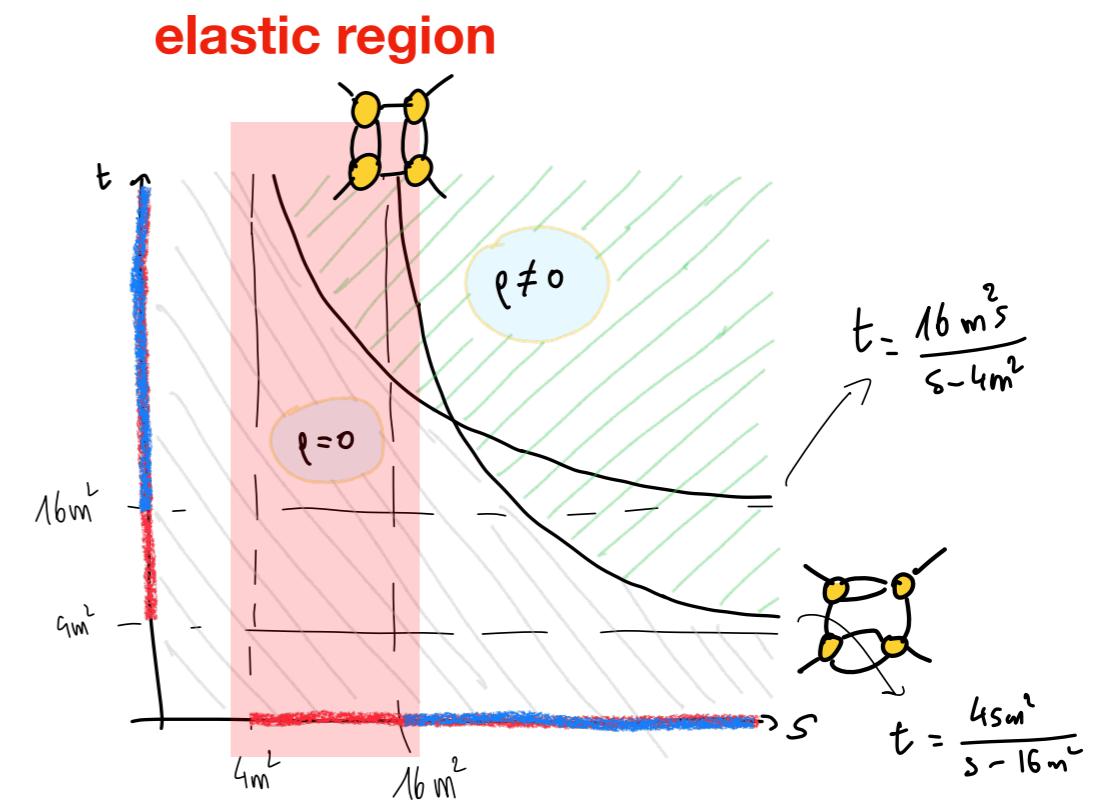


Elastic unitarity in 4d

Correira, Sever, Zhiboedov '20



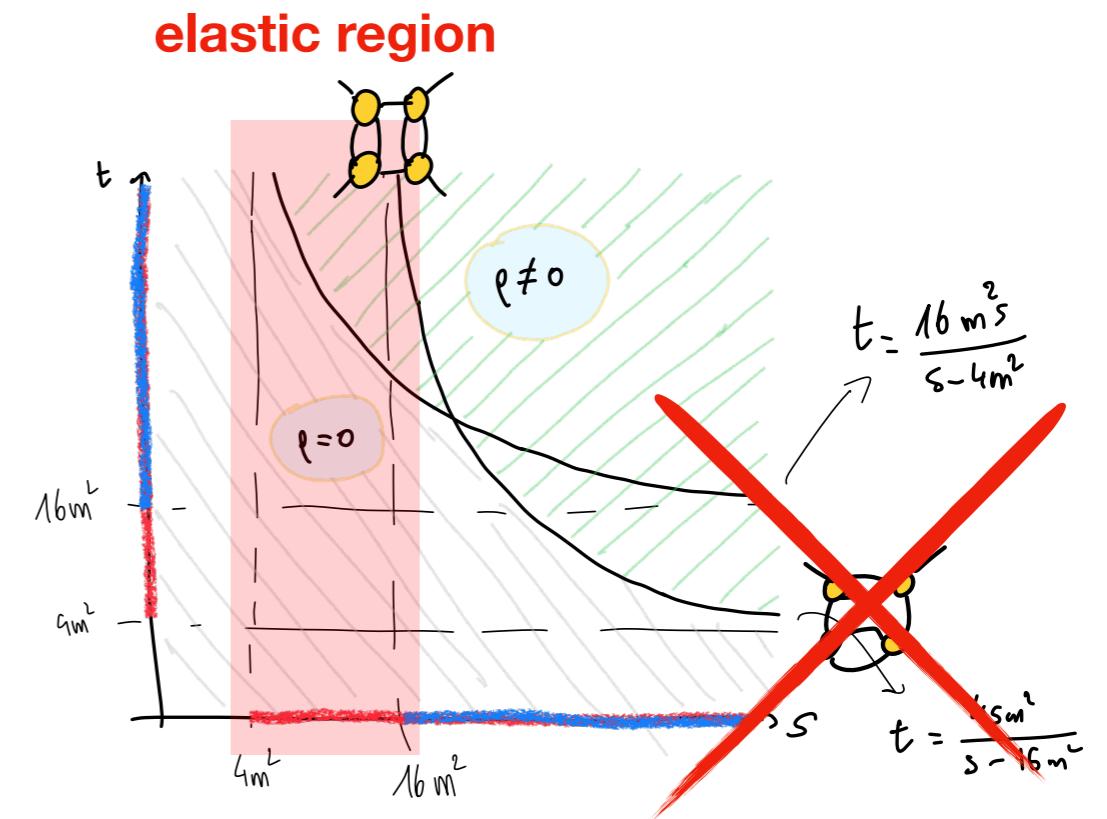
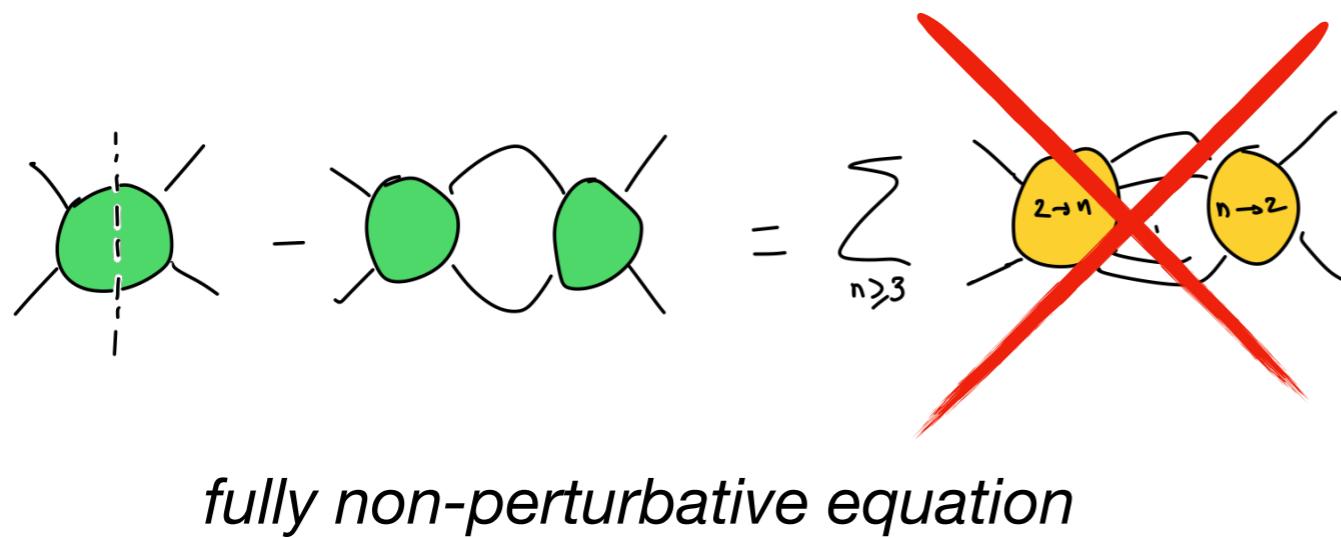
fully non-perturbative equation



Green hashed: Support of double disc in (s,t) -plane

Elastic unitarity in 4d

Correira, Sever, Zhiboedov '20

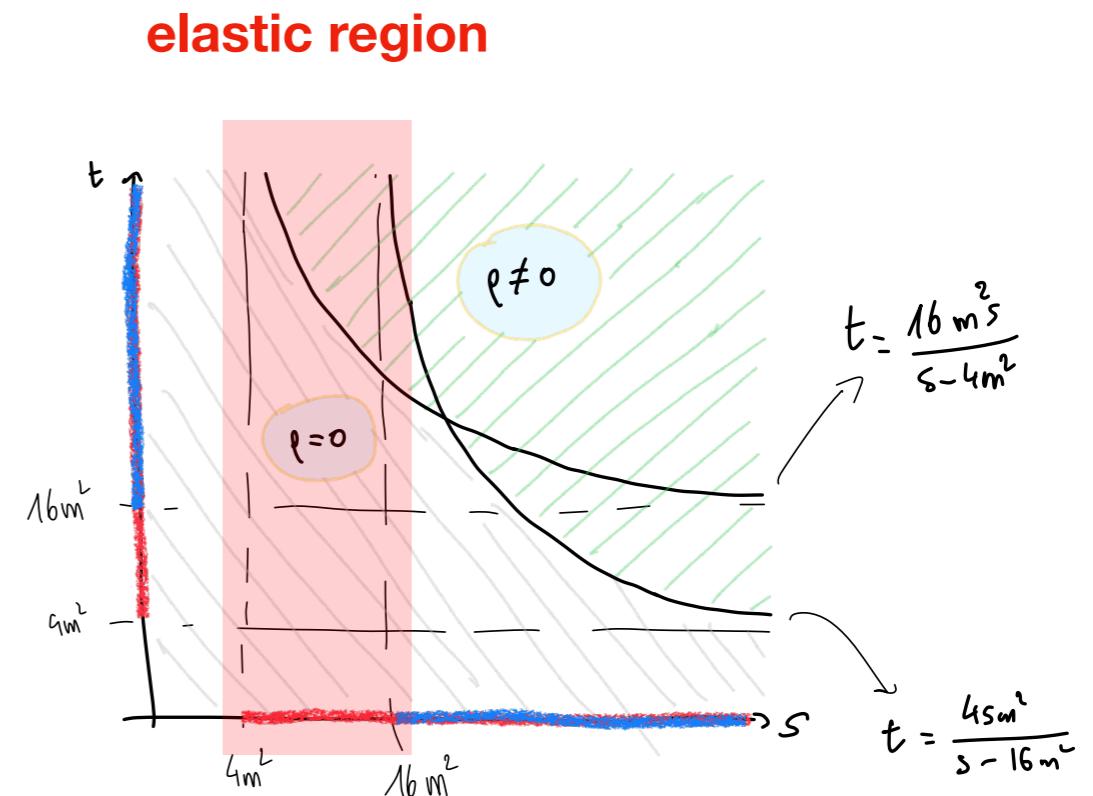


Elastic unitarity in 4d

Correira, Sever, Zhiboedov '20

$$\text{Diagram} - \text{Diagram} = 0$$

fully non-perturbative equation



Scattering from production and Atkinsons' theorems

Nucl.Phys. **B15** (1970) 331-331

A Proof of the Existence of Functions That Satisfy Exactly Both Crossing and Unitarity

D. Atkinson



Nucl.Phys. **B15** (1970) 331-331

A Proof of the Existence of Functions That Satisfy Exactly Both Crossing and Unitarity (ii) Charged Pions. No Subtractions

D. Atkinson

Nucl.Phys. **B13** (1969) 415-436

A Proof of the Existence of Functions That Satisfy Exactly Both Crossing and Unitarity (iii). Subtractions

D. Atkinson

Nucl.Phys. **B23** (1970) 397-412

A Proof of the Existence of Functions That Satisfy Exactly Both Crossing and Unitarity. Iv. Nearly Constant Asymptotic Cross-Sections

D. Atkinson

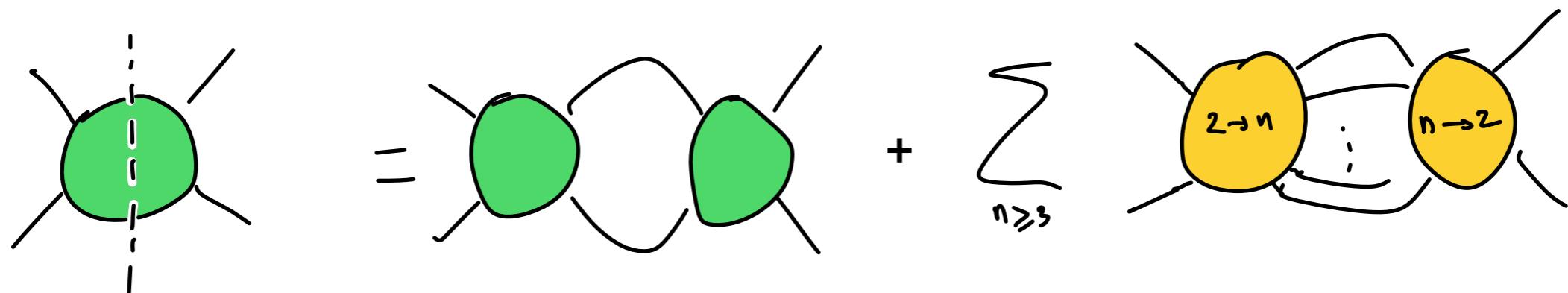
Lecture notes:

S Matrix Construction Project: Existence Theorems, Rigorous Bounds and Models

D. Atkinson

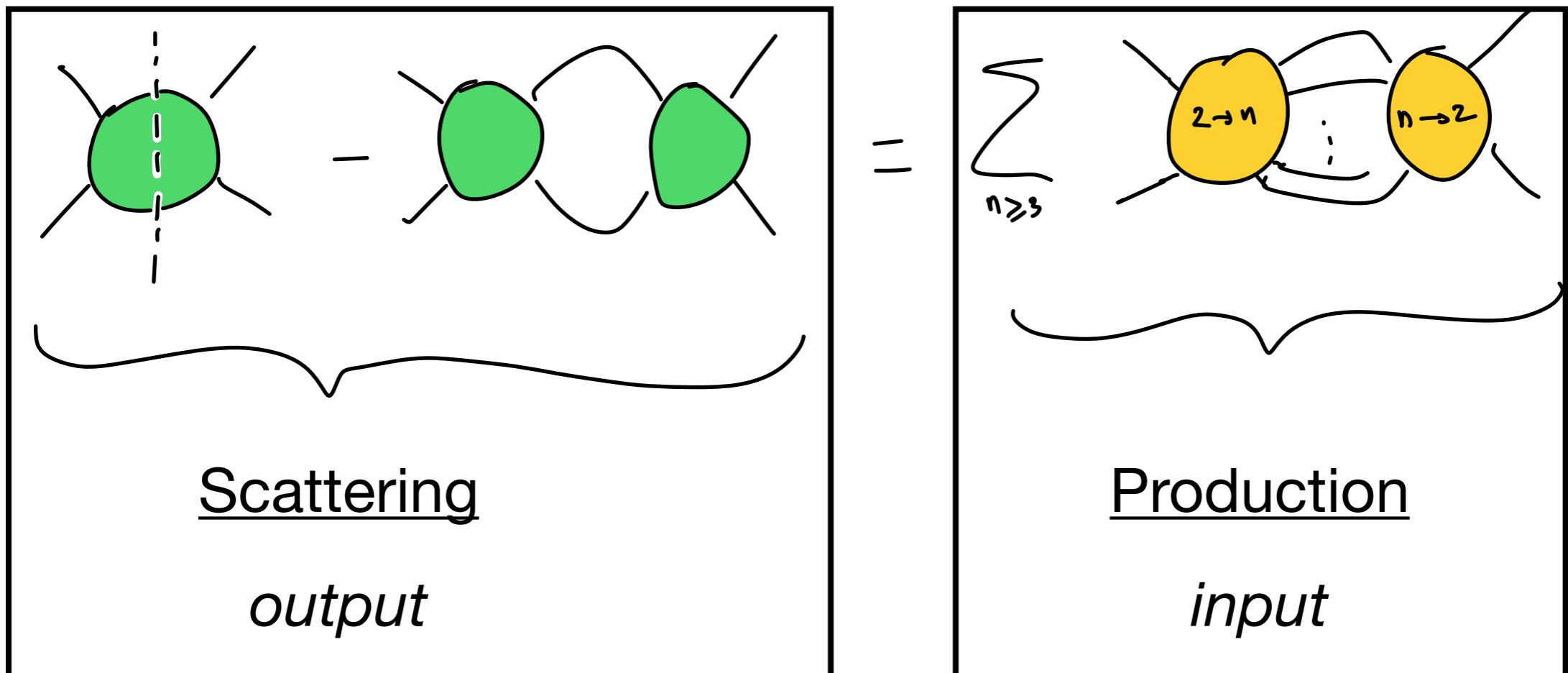
Scattering from production

Unitarity



Scattering from production

Unitarity



Atkison's machinery solves Scattering from Production.
How ?

Atkinson program

Look for fixed-point of the following map:

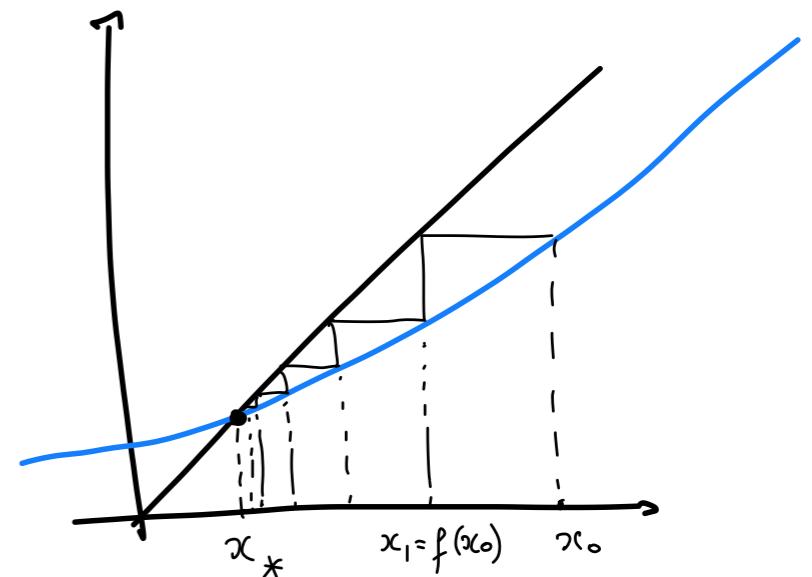
$$\Phi[\text{Diagram}] = \text{Diagram} + \sum_{n \geq 3} \text{Diagram}$$

At the fixed-point, $\Phi[\text{Diagram}] = \text{Diagram}$ satisfies unitarity

Atkison's solution is *iterative*

$$\text{Diagram}_{n+1} = \text{Diagram}_n + \sum_{n \geq 3} \text{Diagram}$$

$$x_{n+1} = f(x_n)$$



Atkinson's proof

- Start from the map $\Phi : L \mapsto L$ where L is a Banach space of Hölder continuous functions
- Hölder continuity :
 $\forall x, y \in [0; 1], |f(x) - f(y)| \leq k|x - y|^\alpha$
 for $0 < \alpha < 1$ and $k > 0$
- Let $B = \{f \in L, \|f\| \leq b\}$ an open ball for some $b > 0$
- If $\Phi[B] \subset B$, Leray-Schauder principle
 $\implies \exists$ fixed point of Φ
- If Φ is *contracting*, i.e.
 $\|\Phi[f_1 - f_2]\| \leq c\|f_1 - f_2\|$, then the solution is also unique in B .

Nucl.Phys. B15 (1970) 331-331
A Proof of the Existence of Functions That Satisfy Exactly Both Crossing and Unitarity
D. Atkinson

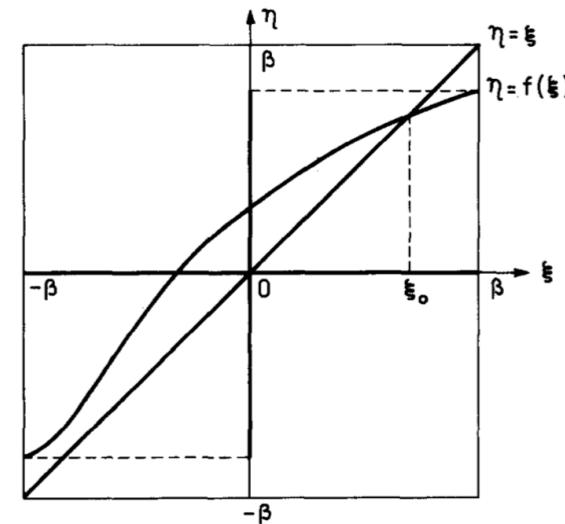
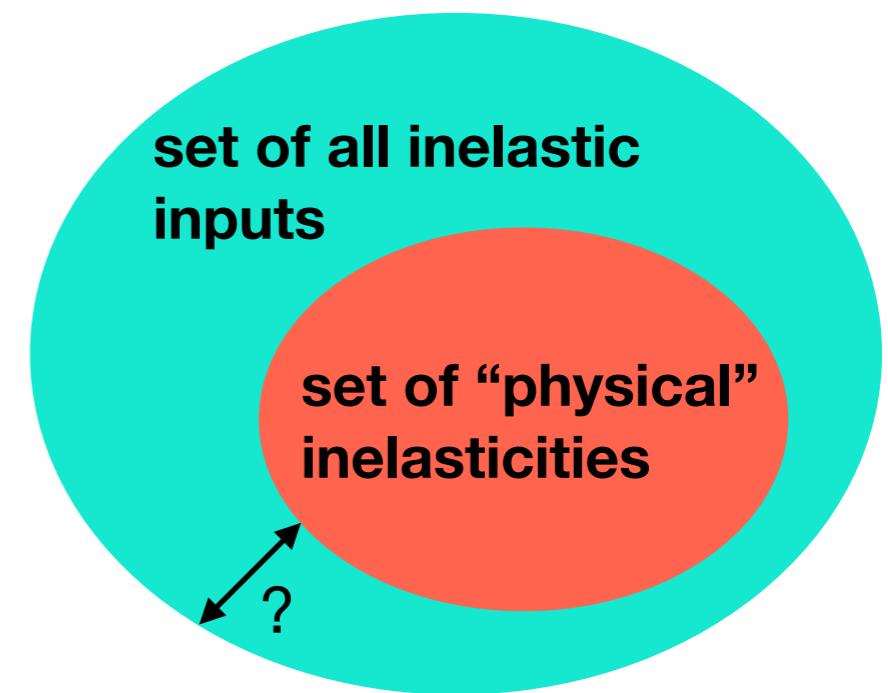


Fig. 1. Illustration of a fixed-point theorem. The image of the interval, $-\beta \leq \xi \leq \beta$, under the continuous, nonlinear mapping, f , is a subset of the same interval. Therefore the curve $\eta = f(\xi)$ intersects the line $\eta = \xi$ at least once, at a point ξ_0 , such that $\xi_0 = f(\xi_0)$.

Inelastic function

- In practice we don't "choose" all of the $T_{2 \rightarrow n}$ separately.
We choose a single function
 $v_{inel}(s, t) \sim \sum_{n \geq 3} |T_{2 \rightarrow n}|^2$
- The problem is *complete*: allowing any functions gives a set that contains all physical amplitudes
- Philosophy is geared towards bootstrap



Numerical implementation

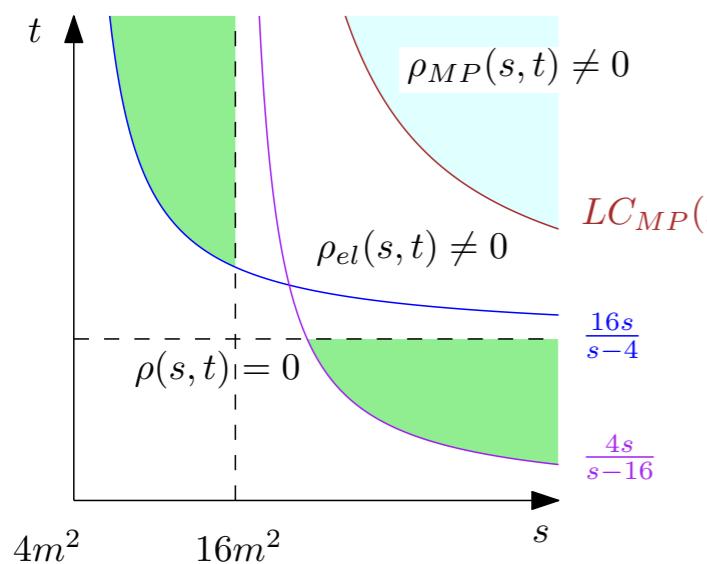
Numerics

- Laptop computations
- Mathematica
 - convenient for integrals and development
 - basic speed of `NIntegrate[]` close to default python `quad()`
 - Easy grid-computing environment `LaunchKernels[]`

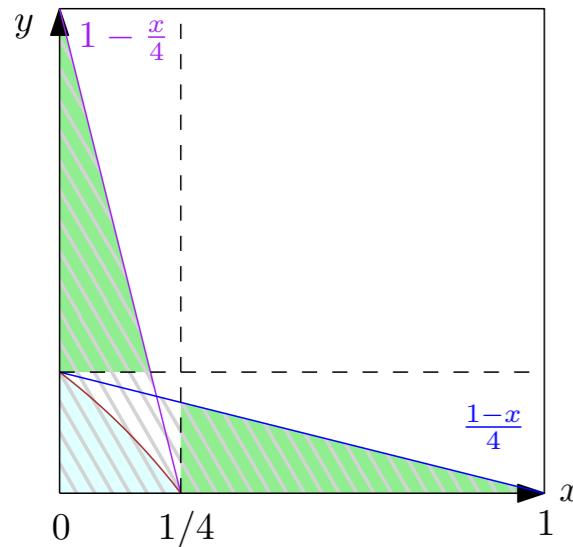
In[18]:= `Kernels[] // Length`

Out[18]= 42

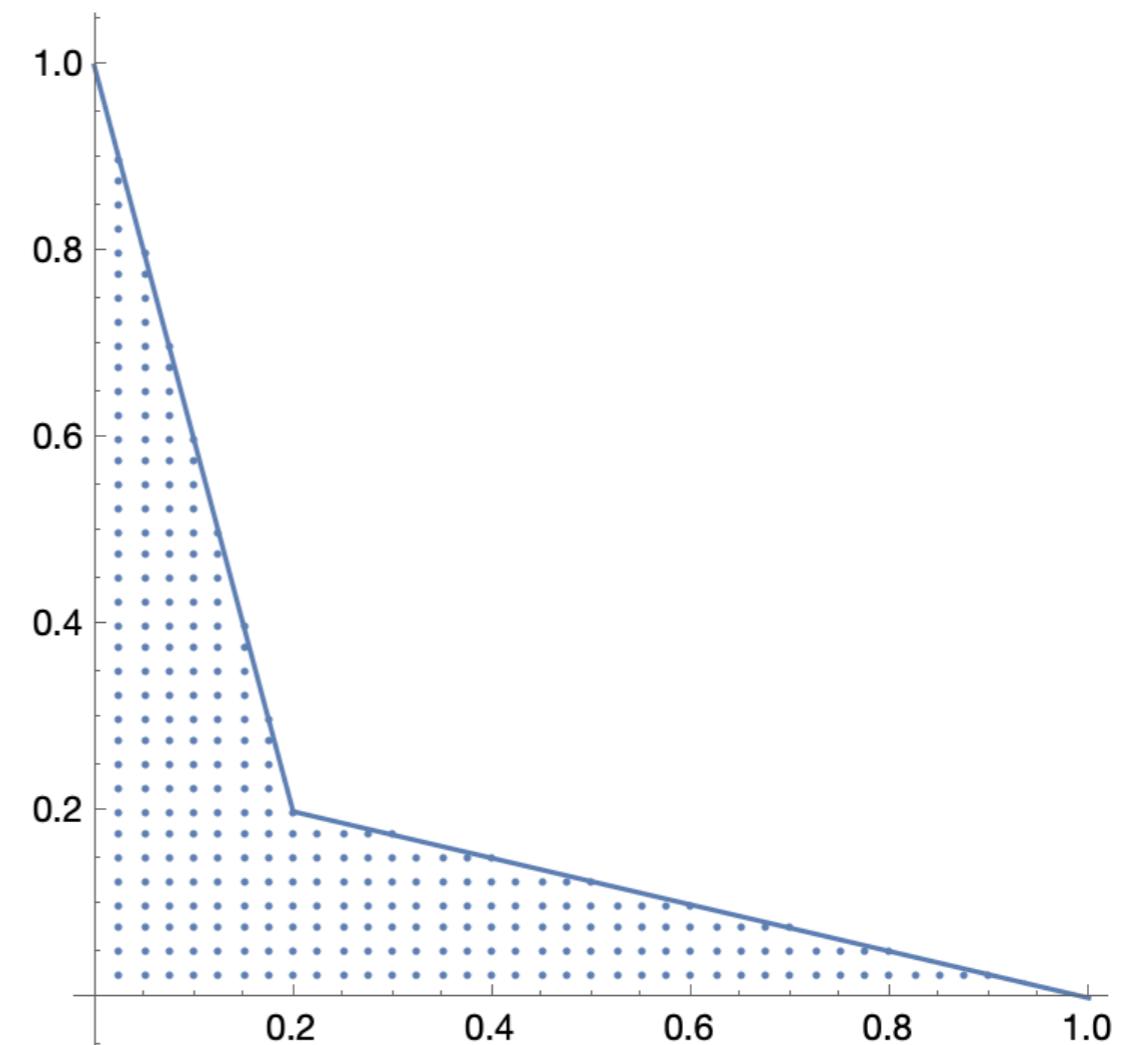
Numerical implementation



$$\downarrow \quad s, t \rightarrow x = 4m^2/s, y = 4m^2/t$$

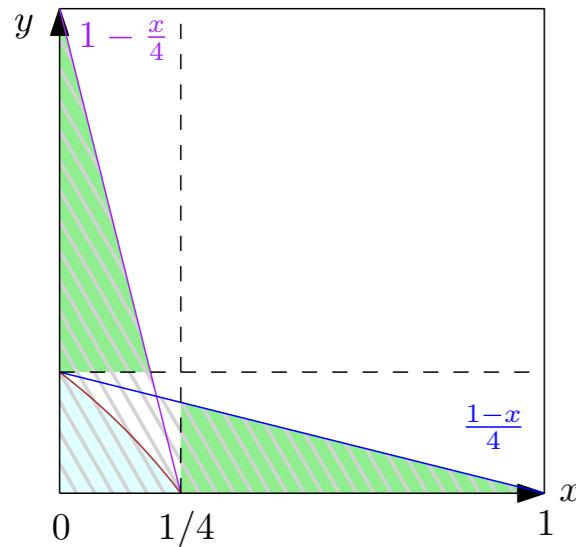
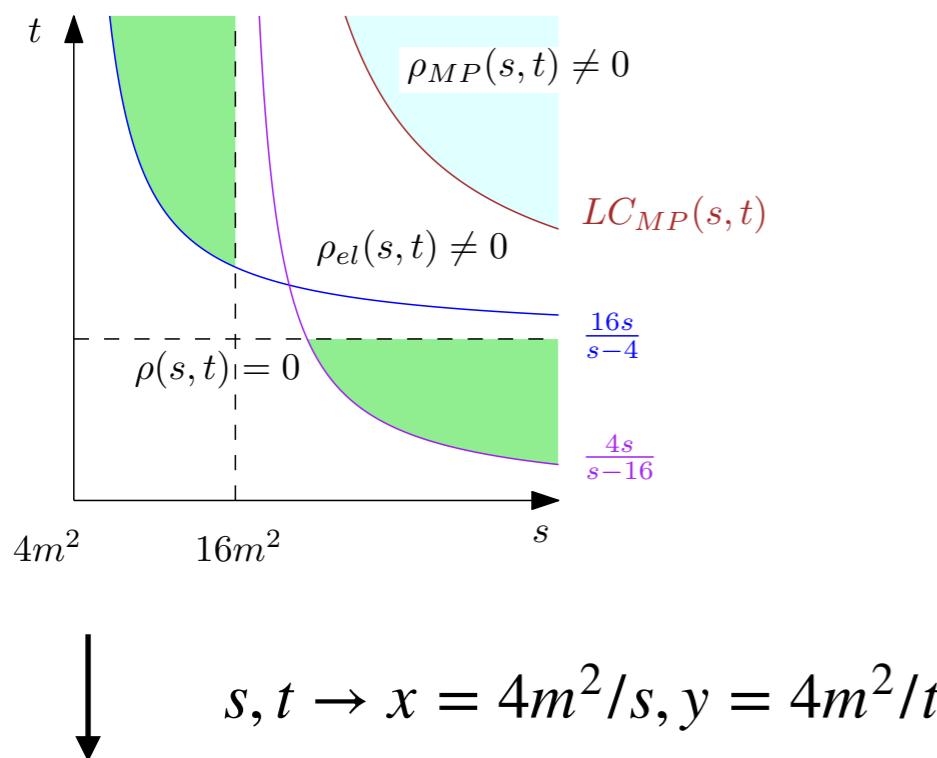


then:
discretize

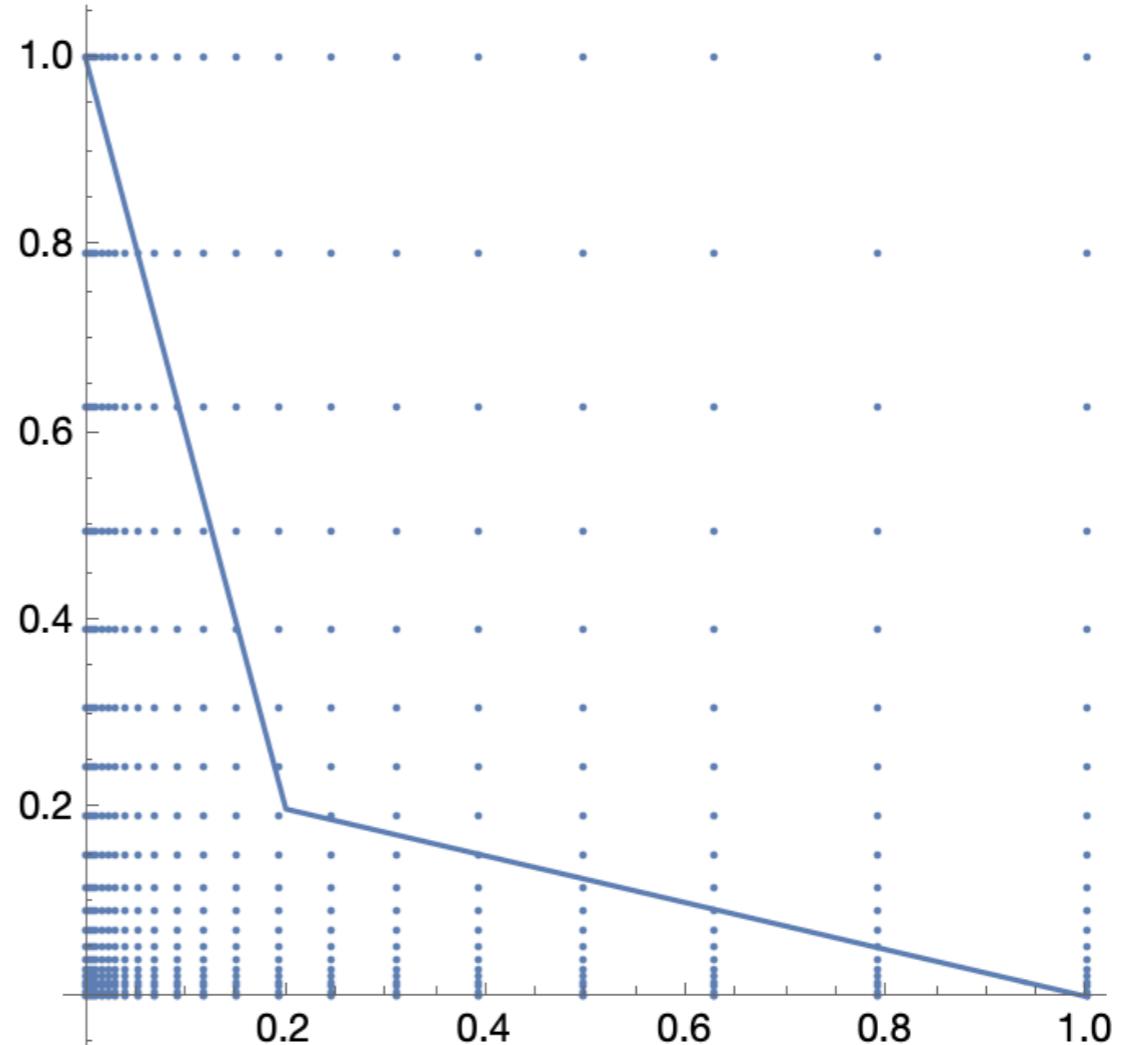


not good: poor resolution at large s, t

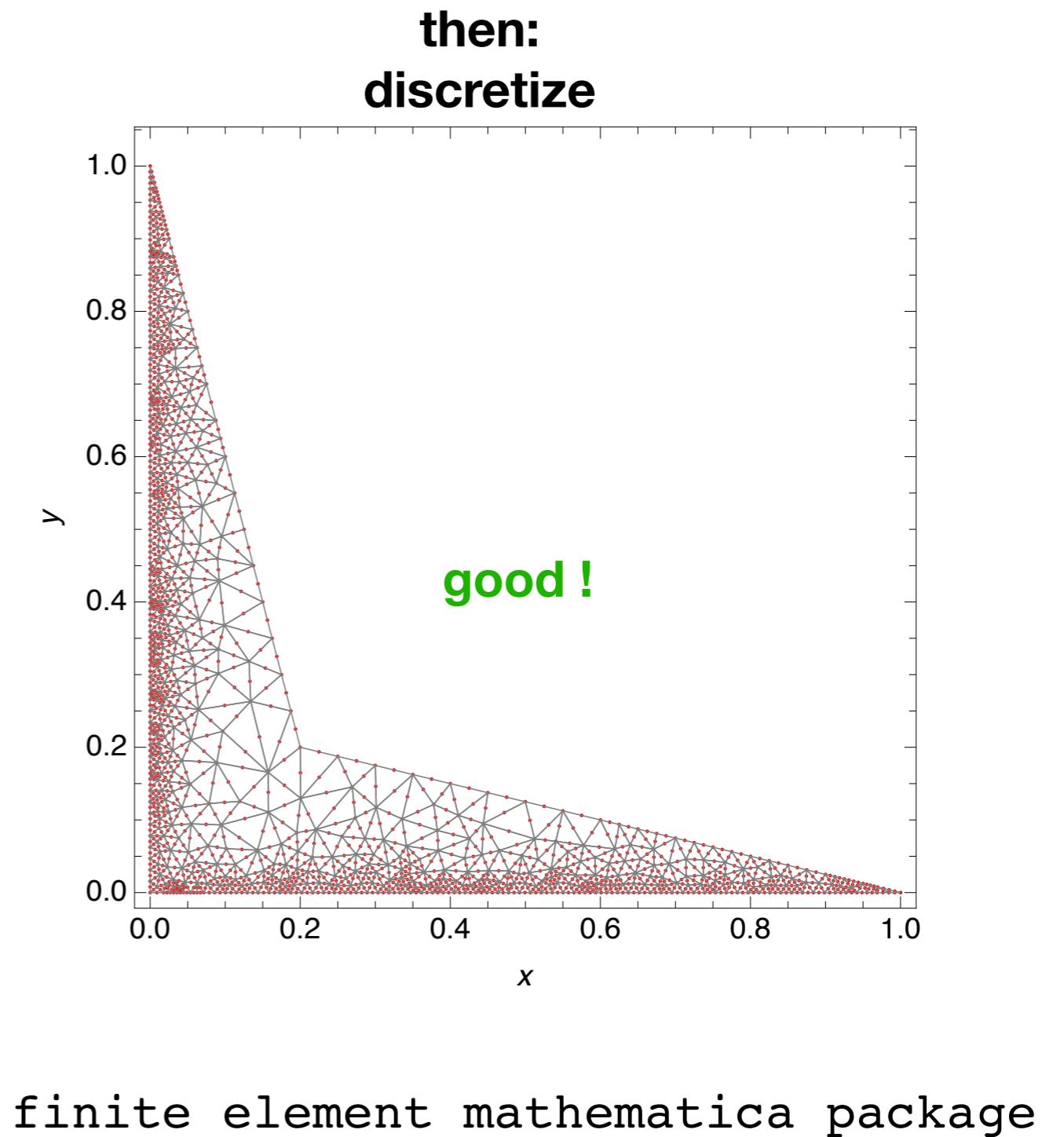
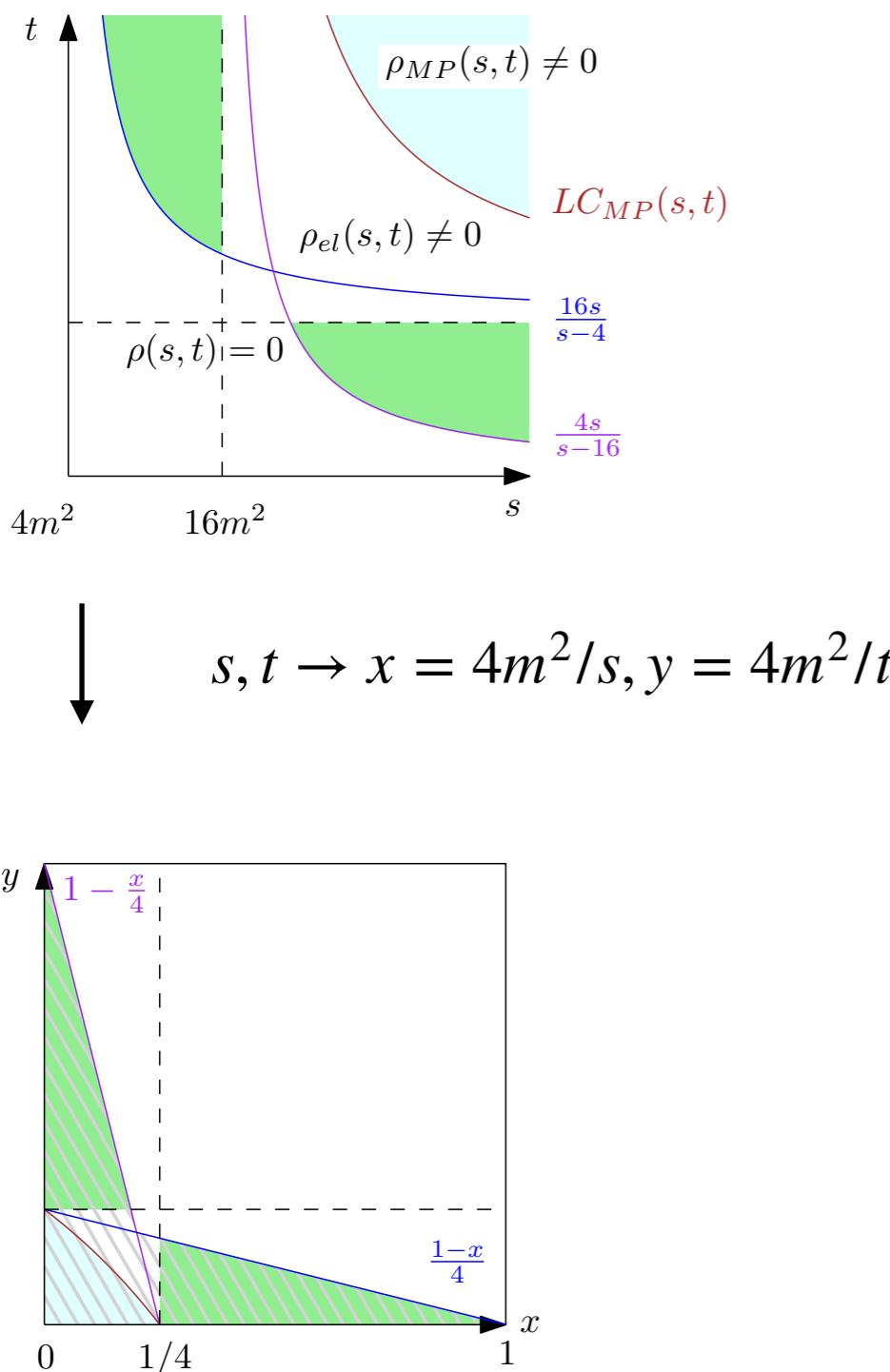
Numerical implementation



then:
discretize



Numerical implementation



Algorithm

Start from

$$\rho_0(s, t)$$

Calculate

$$D_0(s, t) = \frac{1}{\pi} \int_{4m^2}^{\infty} \rho_0(s', t) \left(\frac{1}{s' - s} + \frac{1}{s' - u} \right)$$

Calculate

$$\rho_{(1),el}(s, t) = \iint (D_0(s, t_1)(D_0(s, t_2) + \rho_0(s, t_1)\rho_0(s, t_2)) dt_1 dt_2$$

Define

$$\rho_{(1)}(s, t) = \rho_{(1),el}(s, t) + \rho_{(1),el}(t, s) + v_{inel}(s, t)$$

Iterate

Algorithm

Start from

$$\rho_{\textcolor{blue}{n}}(s, t)$$

Calculate

$$D_{\textcolor{blue}{n}}(s, t) = \frac{1}{\pi} \int_{4m^2}^{\infty} \rho_{\textcolor{blue}{n}}(s', t) \left(\frac{1}{s' - s} + \frac{1}{s' - u} \right)$$

Calculate

$$\rho_{(\textcolor{blue}{n+1}),el}(s, t) = \iint (D_{\textcolor{blue}{n}}(s, t_1)(D_{\textcolor{blue}{n}}(s, t_2) + \rho_{\textcolor{blue}{n}}(s, t_1)\rho_{\textcolor{blue}{n}}(s, t_2)) dt_1 dt_2$$

Define

$$\rho_{(\textcolor{blue}{n+1})}(s, t) = \rho_{(\textcolor{blue}{n+1}),el}(s, t) + \rho_{(\textcolor{blue}{n+1}),el}(t, s) + \textcolor{brown}{v}_{inel}(s, t)$$

Algorithm

Start from

$$\rho_{\textcolor{blue}{n}}(s, t)$$

Calculate

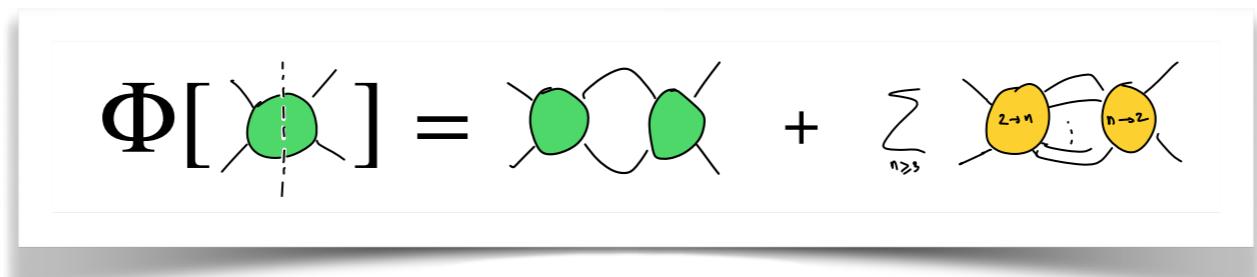
$$D_{\textcolor{blue}{n}}(s, t) = \frac{1}{\pi} \int_{4m^2}^{\infty} \rho_{\textcolor{blue}{n}}(s', t) \left(\frac{1}{s' - s} + \frac{1}{s' - u} \right)$$

Calculate

$$\rho_{(\textcolor{blue}{n+1}),el}(s, t) = \iint (D_{\textcolor{blue}{n}}(s, t_1)(D_{\textcolor{blue}{n}}(s, t_2) + \rho_{\textcolor{blue}{n}}(s, t_1)\rho_{\textcolor{blue}{n}}(s, t_2)) dt_1 dt_2$$

Define

$$\rho_{(\textcolor{blue}{n+1})}(s, t) = \rho_{(\textcolor{blue}{n+1}),el}(s, t) + \rho_{(\textcolor{blue}{n+1}),el}(t, s) + \textcolor{orange}{v}_{inel}(s, t)$$



Subtractions

$$T(s, t) = \lambda + B(s, t) + B(s, u) + B(t, u),$$

$$\begin{aligned} B(s, t) &= \frac{s - s_0}{2} \int_{4m^2}^{\infty} \frac{ds'}{\pi} \frac{\rho(s')}{(s' - s)(s' - s_0)} + \frac{t - t_0}{2} \int_{4m^2}^{\infty} \frac{dt'}{\pi} \frac{\rho(t')}{(t' - t)(t' - t_0)} \\ &\quad + (s - s_0)(t - t_0) \int_{4m^2}^{\infty} \frac{ds' dt'}{\pi^2} \frac{\rho(s', t')}{(s' - s)(t' - t)(s' - s_0)(t' - t_0)} \end{aligned}$$

$$s_0 = t_0 = u_0 = 4/3$$

Subtractions

$$T(s, t) = \lambda + B(s, t) + B(s, u) + B(t, u),$$

$$\begin{aligned} B(s, t) &= \frac{s - s_0}{2} \int_{4m^2}^{\infty} \frac{ds'}{\pi} \frac{\rho(s')}{(s' - s)(s' - s_0)} + \frac{t - t_0}{2} \int_{4m^2}^{\infty} \frac{dt'}{\pi} \frac{\rho(t')}{(t' - t)(t' - t_0)} \\ &\quad + (s - s_0)(t - t_0) \int_{4m^2}^{\infty} \frac{ds' dt'}{\pi^2} \frac{\rho(s', t')}{(s' - s)(t' - t)(s' - s_0)(t' - t_0)} \end{aligned}$$

$$s_0 = t_0 = u_0 = 4/3$$

Coupling λ to describe ϕ^4 -like theories

Subtractions

$$T(s, t) = \lambda + B(s, t) + B(s, u) + B(t, u),$$

$$\begin{aligned} B(s, t) &= \frac{s - s_0}{2} \int_{4m^2}^{\infty} \frac{ds'}{\pi} \frac{\rho(s')}{(s' - s)(s' - s_0)} + \frac{t - t_0}{2} \int_{4m^2}^{\infty} \frac{dt'}{\pi} \frac{\rho(t')}{(t' - t)(t' - t_0)} \\ &\quad + (s - s_0)(t - t_0) \int_{4m^2}^{\infty} \frac{ds' dt'}{\pi^2} \frac{\rho(s', t')}{(s' - s)(t' - t)(s' - s_0)(t' - t_0)} \end{aligned}$$

$$s_0 = t_0 = u_0 = 4/3$$

$\rho(s) \sim$ single disc \rightarrow gets its own inel input $\rho_{MP}(s)$

Have to solve unitarity separately for the S_0 partial wave

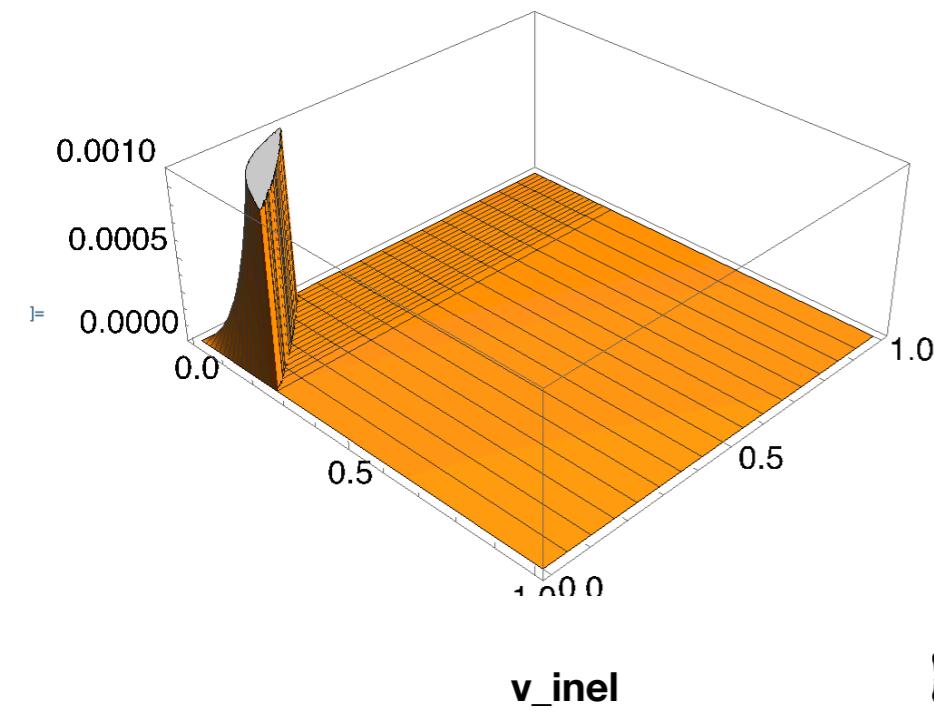
Subtractions

$$T(s, t) = \lambda + B(s, t) + B(s, u) + B(t, u),$$

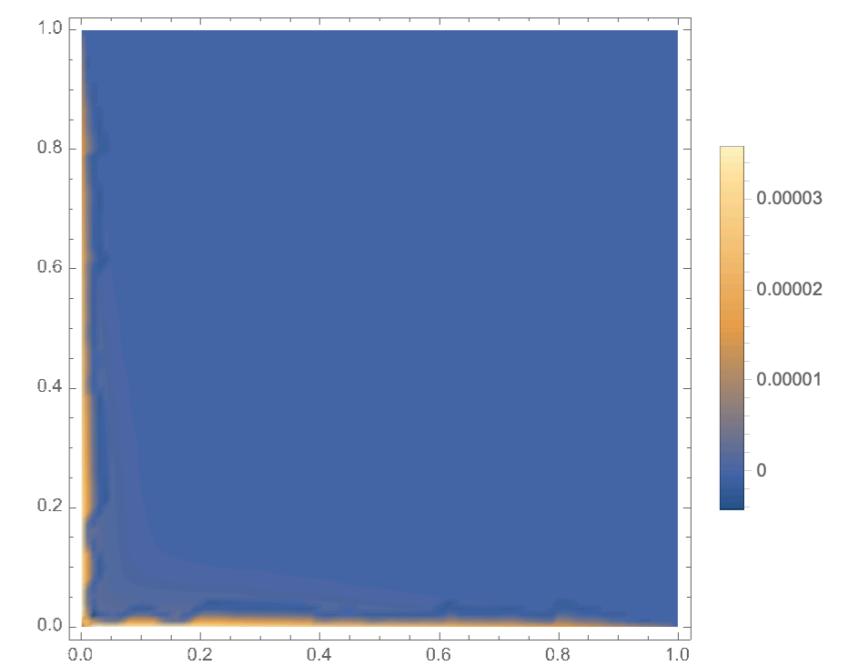
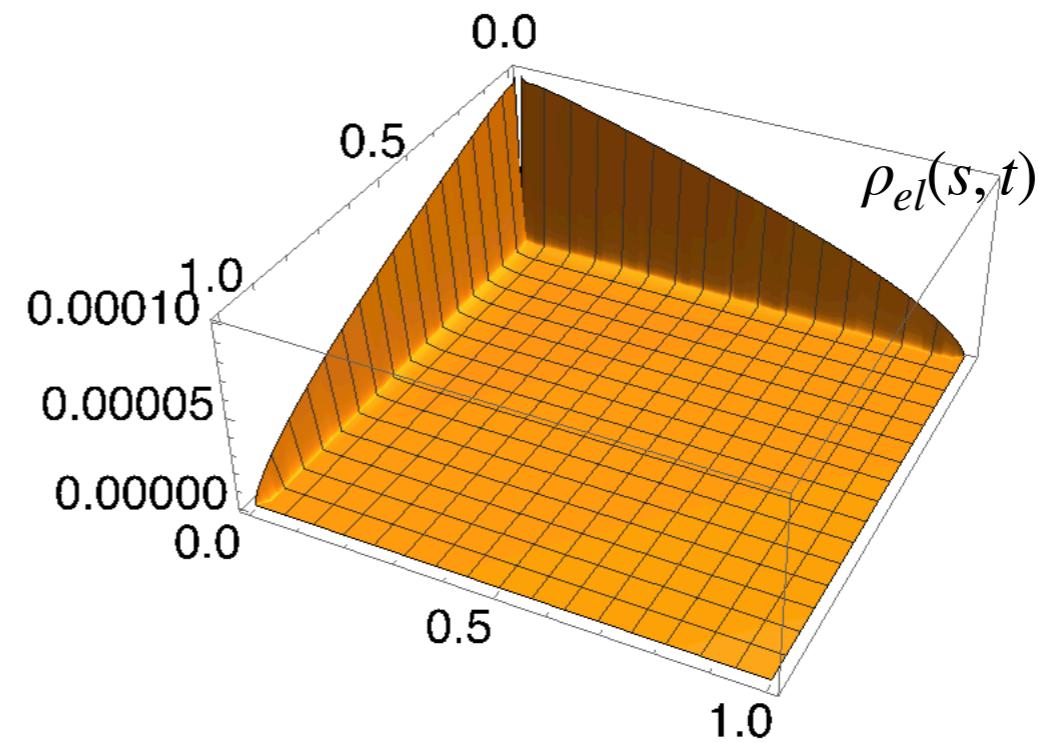
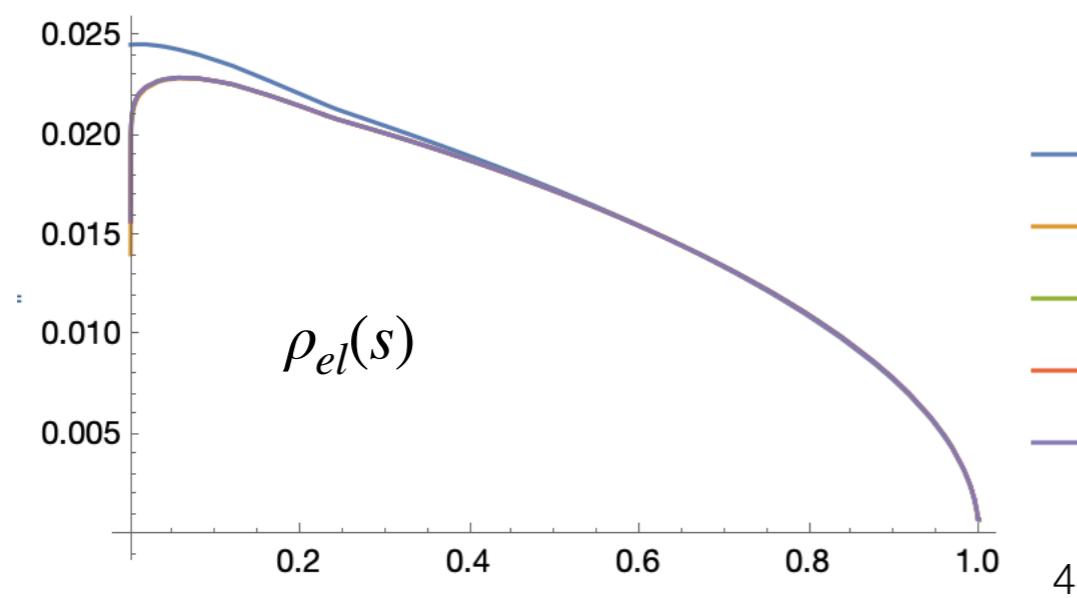
$$\begin{aligned} B(s, t) &= \frac{s - s_0}{2} \int_{4m^2}^{\infty} \frac{ds'}{\pi} \frac{\rho(s')}{(s' - s)(s' - s_0)} + \frac{t - t_0}{2} \int_{4m^2}^{\infty} \frac{dt'}{\pi} \frac{\rho(t')}{(t' - t)(t' - t_0)} \\ &\quad + (s - s_0)(t - t_0) \int_{4m^2}^{\infty} \frac{ds' dt'}{\pi^2} \frac{\rho(s', t')}{(s' - s)(t' - t)(s' - s_0)(t' - t_0)} \end{aligned}$$

$$s_0 = t_0 = u_0 = 4/3$$

Results

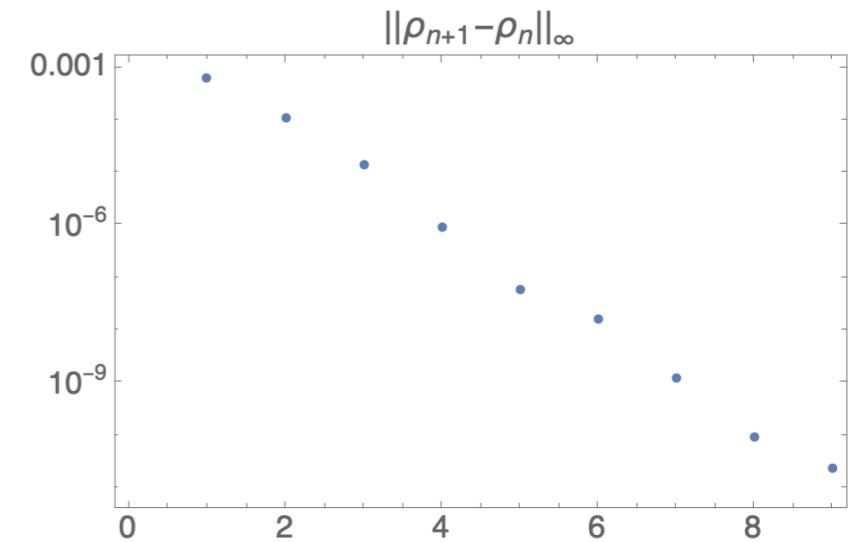


$$\lambda = \pi/2$$



Results

- Without subtractions, hard to converge to unitary partial waves.
- With subtraction, algorithm converges for $-\pi \leq \lambda \leq 5.5\pi$ and satisfies unitarity (far from PPTvRV).
- Amplitudes have correct Landau curves and analytic structure, by construction.
- How small can inelasticity be ? Set to zero, to see.
Produces a sort of ϕ^4 theory from dispersive iterations



$$\rho' = \begin{array}{c} \text{Diagram of } \rho' \text{ with dashed vertical line} \end{array} = \sum \begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array} + \sum \begin{array}{c} \text{Diagram 3} \\ \text{Diagram 4} \end{array} + \sum \begin{array}{c} \text{Diagram 5} \\ \text{Diagram 6} \end{array} + \dots + \begin{array}{c} \text{Diagram 7} \\ \text{Diagram 8} \end{array} + \sigma_{\text{inel}}$$

ρ' ρ^2

zero inel input

$$\rho' = \text{Diagram 1} + \text{Diagram 2} + \text{Diagram 3} + \text{Diagram 4} + \text{Diagram 5} + \text{Diagram 6} + \sigma_{\text{inel}}$$

The equation shows the decomposition of the operator ρ' into a sum of Feynman-like diagrams and an inelastic term. The diagrams involve four external legs labeled 1, 2, 3, and 4, and internal shaded circles representing interactions.

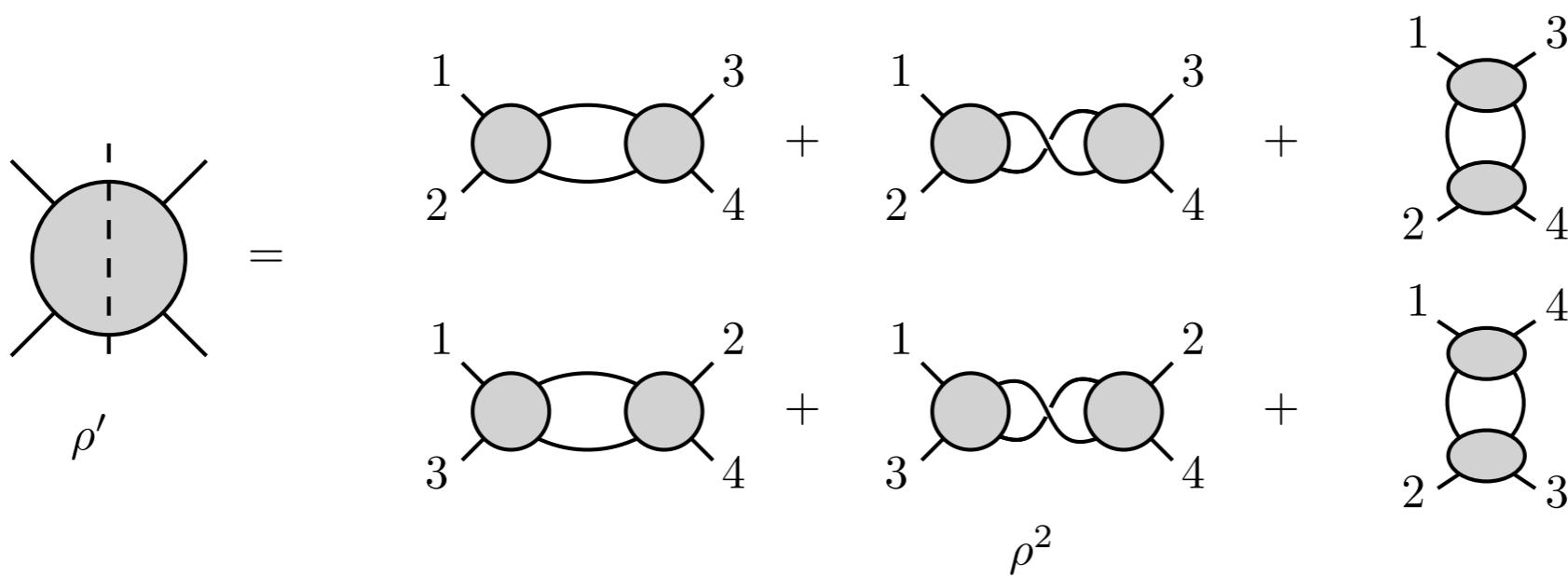
- Diagram 1:** Four external legs (1, 2, 3, 4) connected sequentially by two shaded circles.
- Diagram 2:** Four external legs (1, 2, 3, 4) connected sequentially by three shaded circles.
- Diagram 3:** Four external legs (1, 2, 3, 4) connected sequentially by two shaded circles, with leg 1 at the top and leg 3 at the bottom.
- Diagram 4:** Four external legs (1, 2, 3, 4) connected sequentially by three shaded circles, with leg 1 at the top and leg 3 at the bottom.
- Diagram 5:** Four external legs (1, 2, 3, 4) connected sequentially by two shaded circles, with leg 2 at the top and leg 4 at the bottom.
- Diagram 6:** Four external legs (1, 2, 3, 4) connected sequentially by three shaded circles, with leg 2 at the top and leg 4 at the bottom.
- σ_{inel} :** A black circle representing inelastic scattering.

zero inel input

$$\rho' = \sum_{\text{graphs}} + \sum_{\text{graphs}} + \sum_{\text{graphs}} + \dots$$

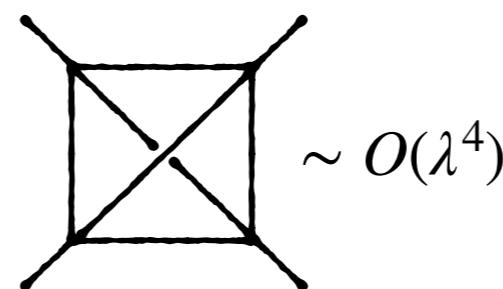
The equation shows the decomposition of the operator ρ' into a sum of Feynman-like diagrams. The first term is a single vertex with four external lines and a dashed internal line, labeled ρ' . The subsequent terms are sums of diagrams involving four vertices connected by horizontal lines. The vertices are shaded gray. The indices 1, 2, 3, 4 are assigned to the vertices and lines. The diagrams are categorized by the number of vertices and the arrangement of indices. The last term in the sequence is crossed out with a large red X.

zero inel input

$$\rho' = \sum_{\text{graphs}} + \sum_{\text{graphs}} + \sum_{\text{graphs}}$$


The diagram shows the expansion of the operator ρ' . It starts with a single vertex containing a dashed line, followed by an equals sign. Then it shows two terms separated by a plus sign. Each term consists of a vertex with four external lines, where each line is connected to a shaded circle. The top term has the lines labeled 1, 2, 3, 4 from left to right. The bottom term has the lines labeled 3, 1, 2, 4 from left to right. Below these two terms is another plus sign, followed by a third term consisting of two stacked vertices. The top vertex has lines 1, 2, 3, 4. The bottom vertex has lines 2, 1, 4, 3. Below this third term is a final plus sign.

will never produce graphs which have no 2-pt cuts, such as:



$\sim O(\lambda^4)$

zero inel input

$$\rho' = \text{Diagram 1} + \text{Diagram 2} + \text{Diagram 3} + \text{Diagram 4} + \text{Diagram 5} + \text{Diagram 6} + \sigma_{\text{inel}}$$

Diagram 1: A circular vertex with four external lines. The top-left line is labeled '1', the top-right '3', the bottom-left '2', and the bottom-right '4'. A dashed vertical line passes through the center of the circle.

Diagram 2: A circular vertex with four external lines labeled 1, 2, 3, 4 clockwise from top-left. Two horizontal edges connect the 1-2 and 3-4 pairs.

Diagram 3: Similar to Diagram 2, but the two horizontal edges cross each other.

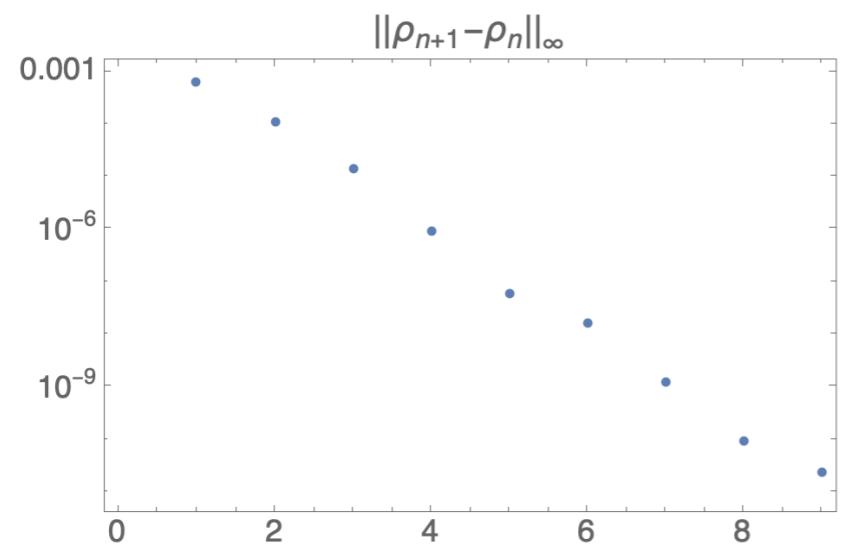
Diagram 4: Similar to Diagram 2, but the 1-2 and 3-4 edges are swapped.

Diagram 5: Similar to Diagram 3, but the 1-2 and 3-4 edges are swapped.

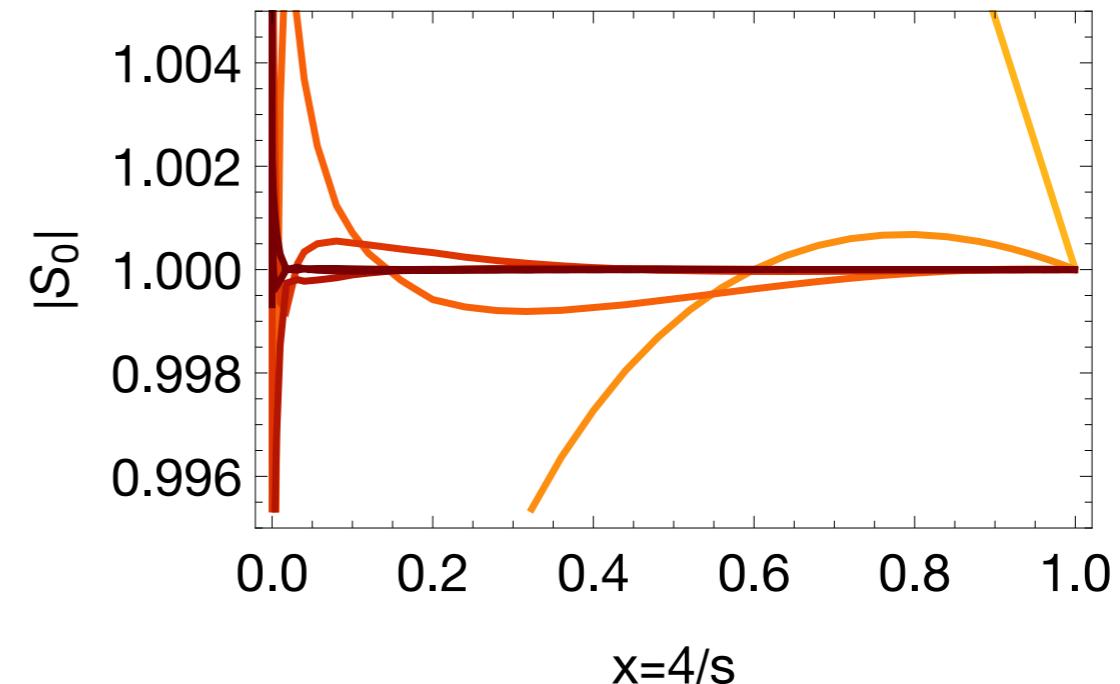
Diagram 6: A circular vertex with four external lines labeled 1, 2, 3, 4 clockwise from top-left. The 1-2 and 3-4 edges are swapped, and the 2-3 and 4-1 edges are swapped.

ρ^2

$$N_{n+1} = 6N_n^2 + 1 \quad N_n \sim (2.55)^L$$

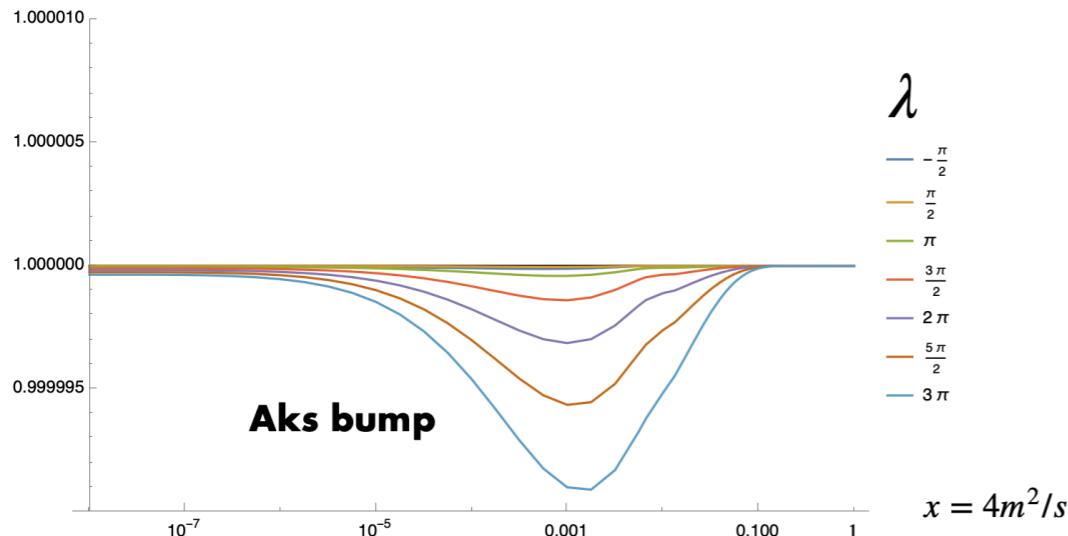


linear convergence

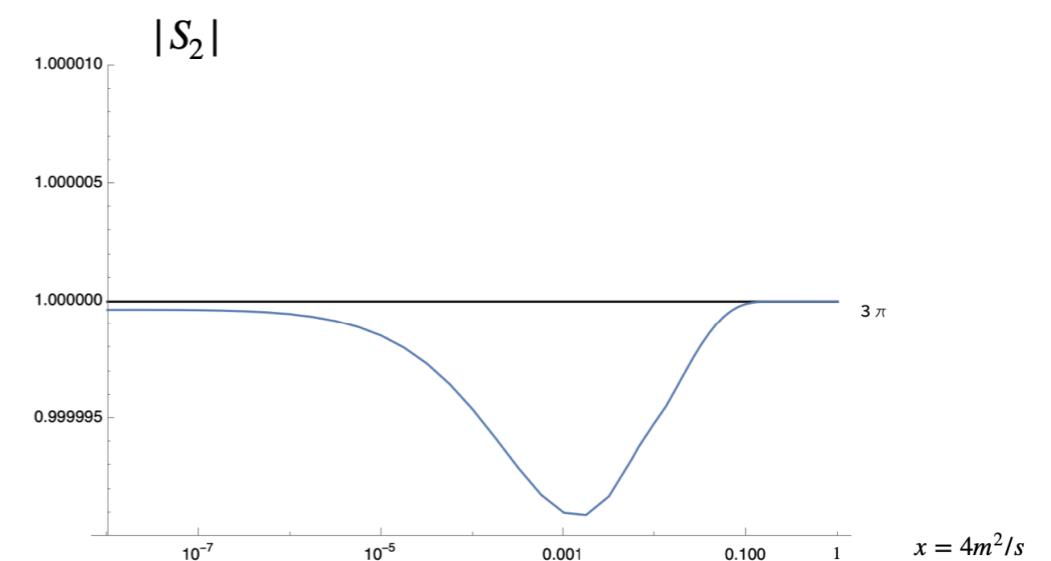


$S_0(s)$ converge to pure phases

color =
iteration
counter

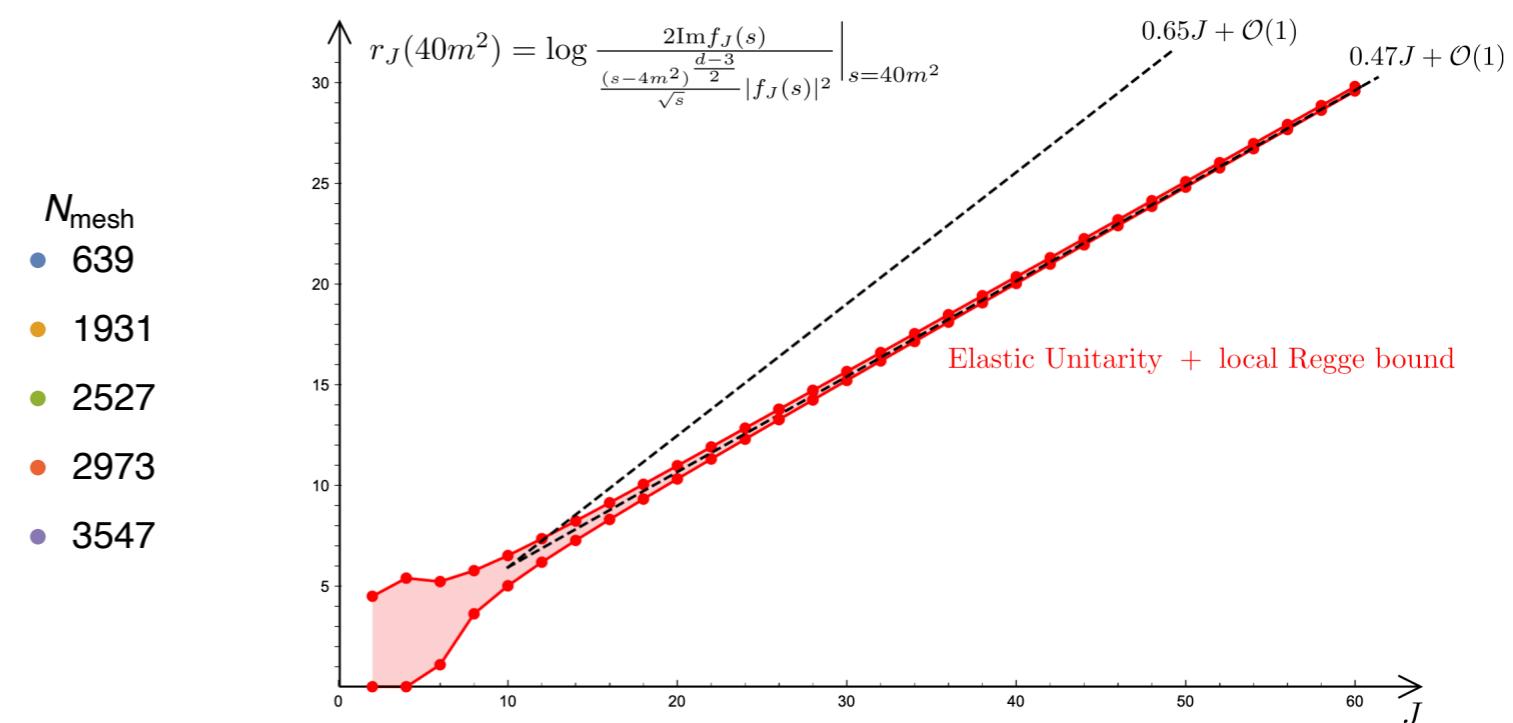
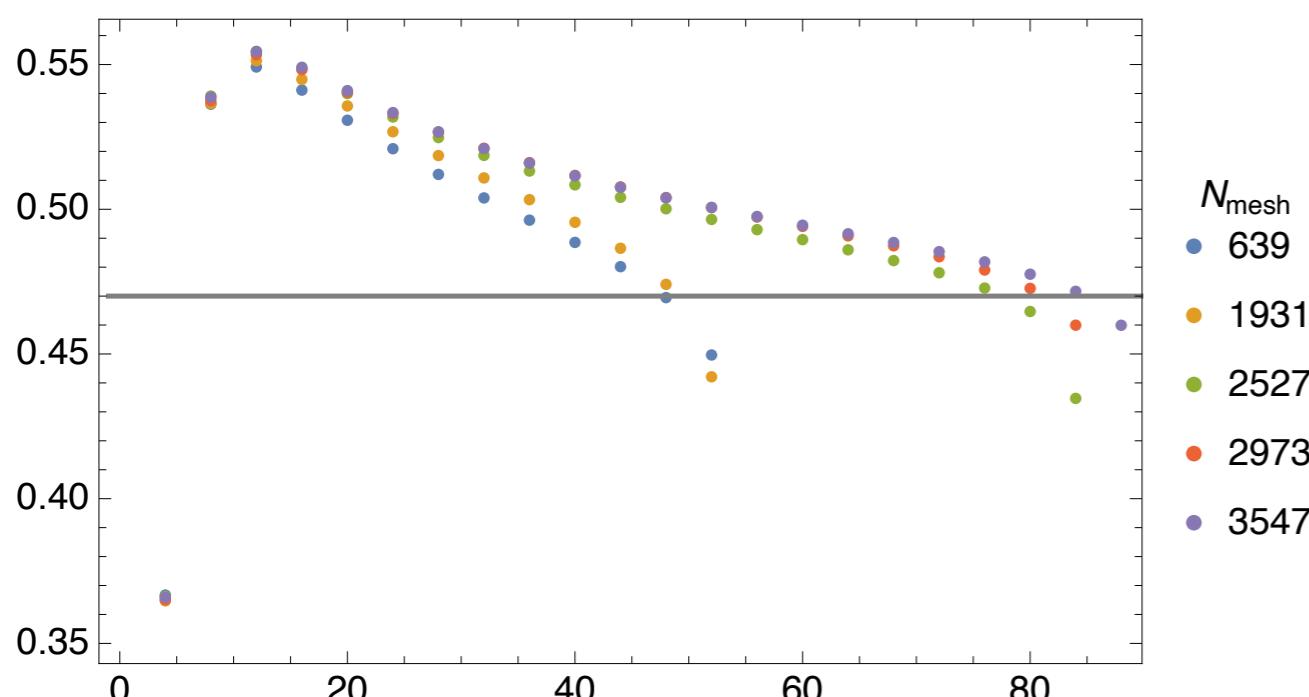


$S_2(s)$ have inelasticity



$S_2(s) < 1$ at infinity

Large J Dragt-Martin bootstrap seems compatible with results as grid size increase

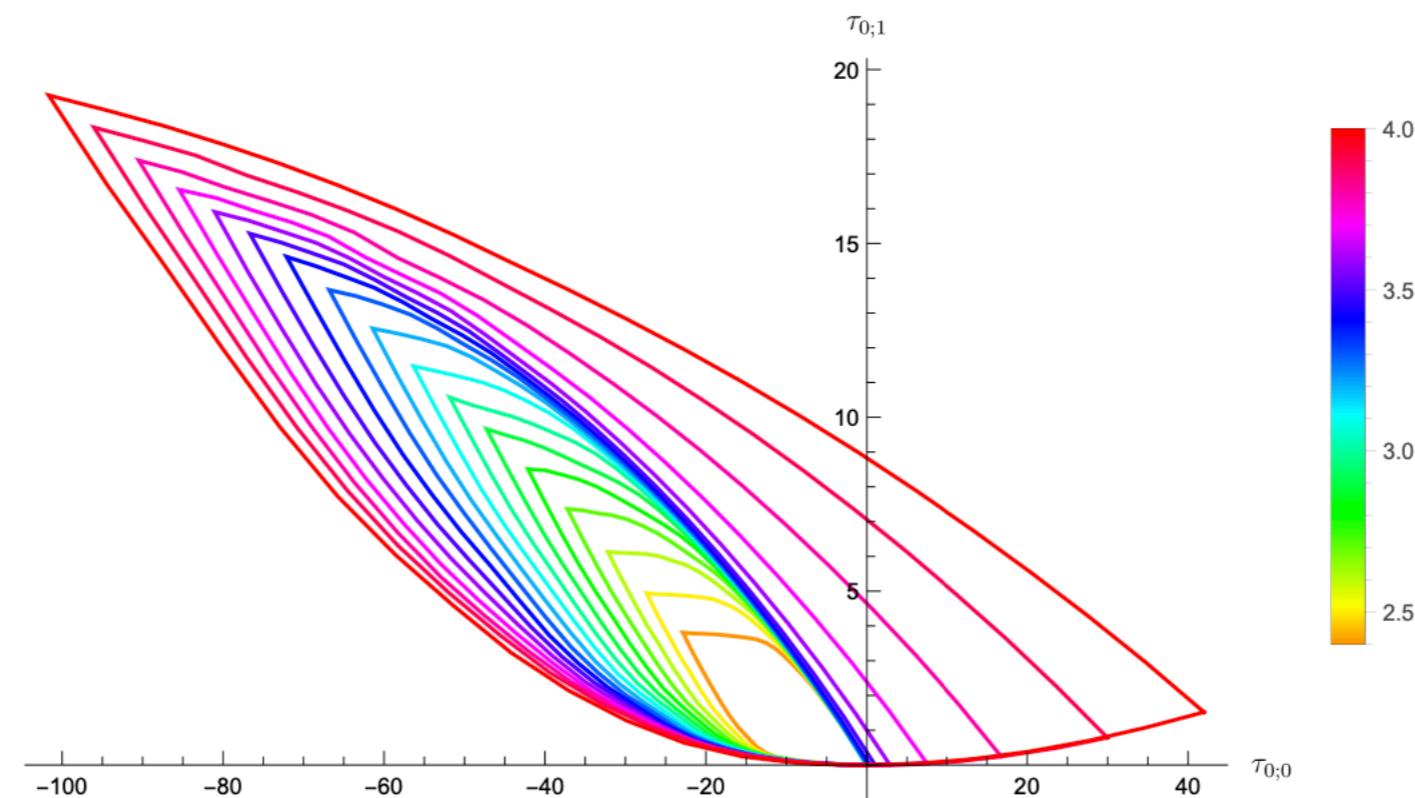


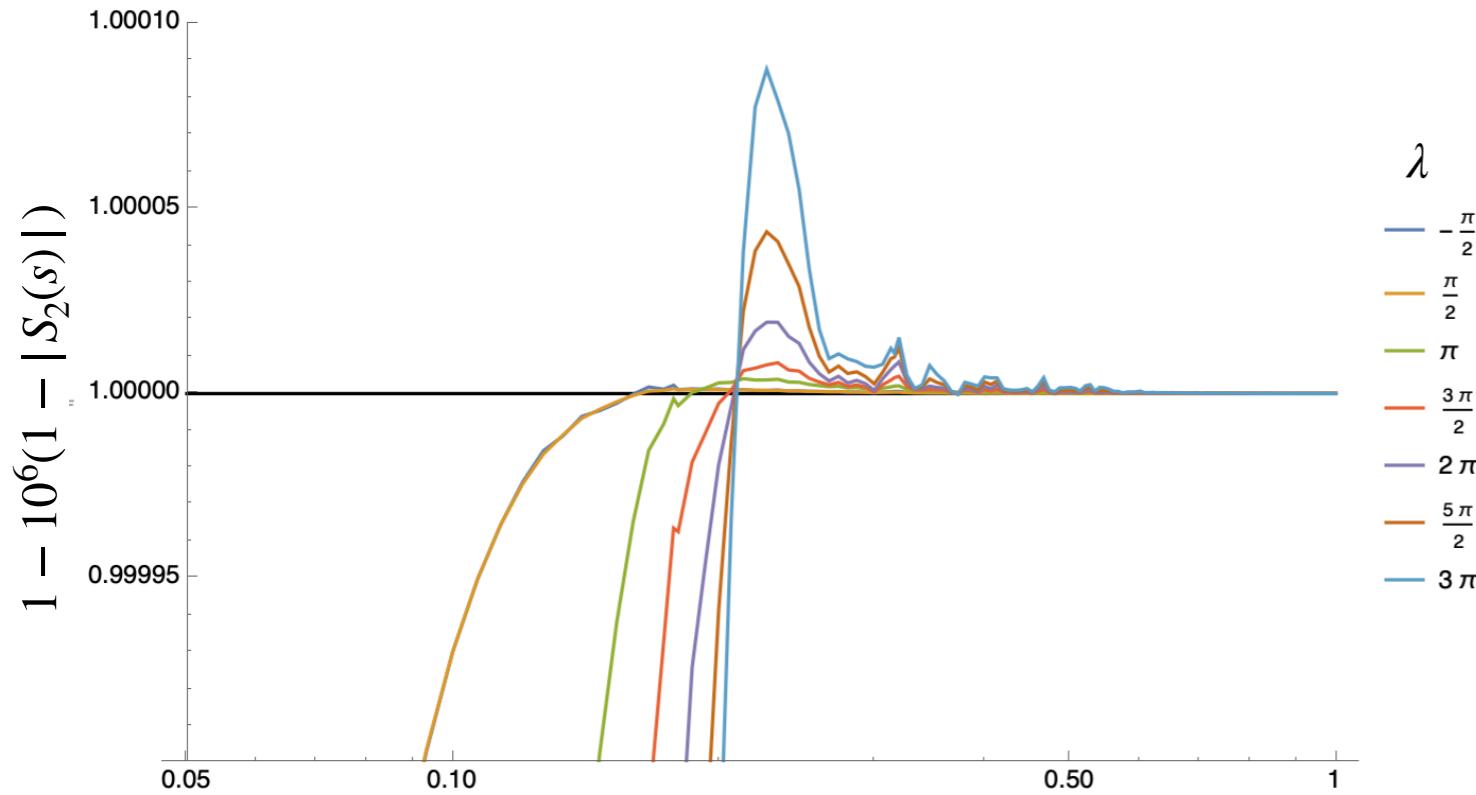
An Analytical Toolkit for the S-matrix Bootstrap
[M. Correia, A. Sever, A. Zhiboedov](#)

This “extremal” amplitude with zero inel input saturates the bounds from

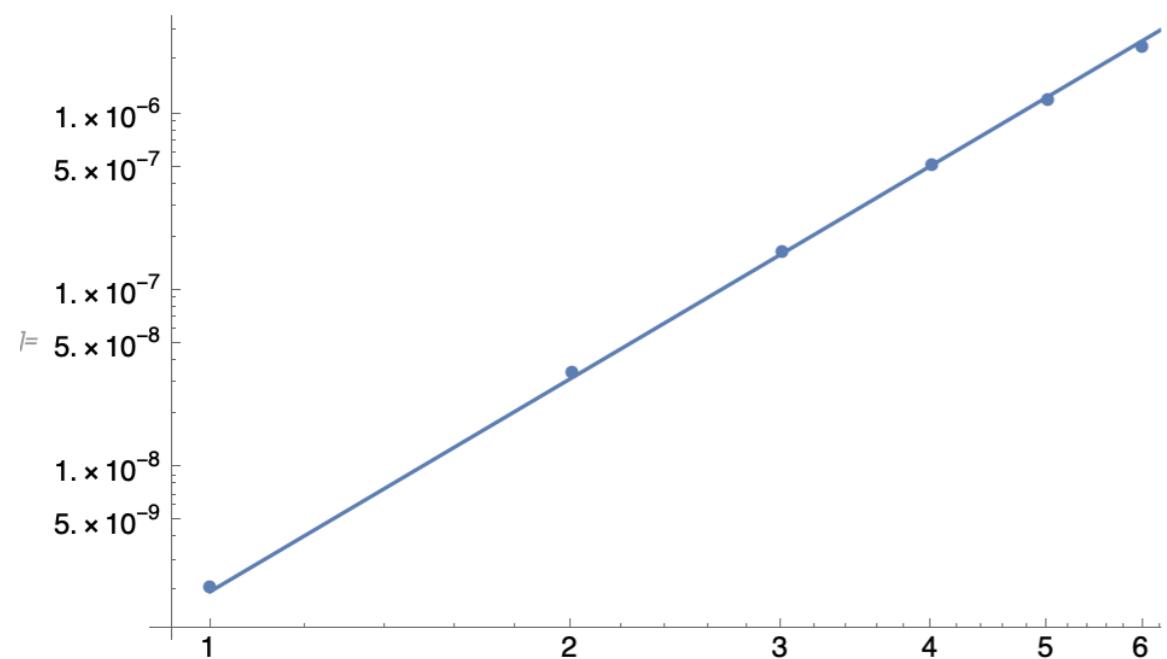
Nonperturbative Bounds on Scattering of Massive Scalar Particles in $d \geq 2$

[H. Chen, A. L. Fitzpatrick, D. Karateev](#)





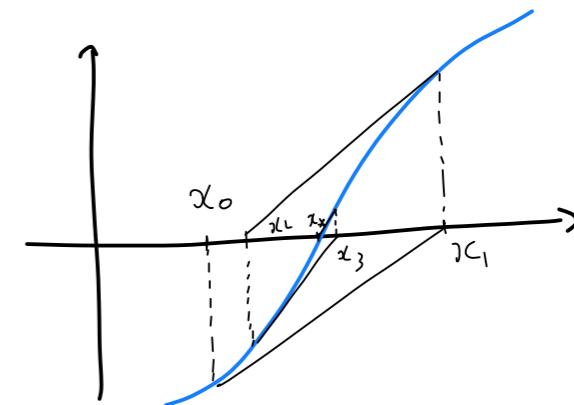
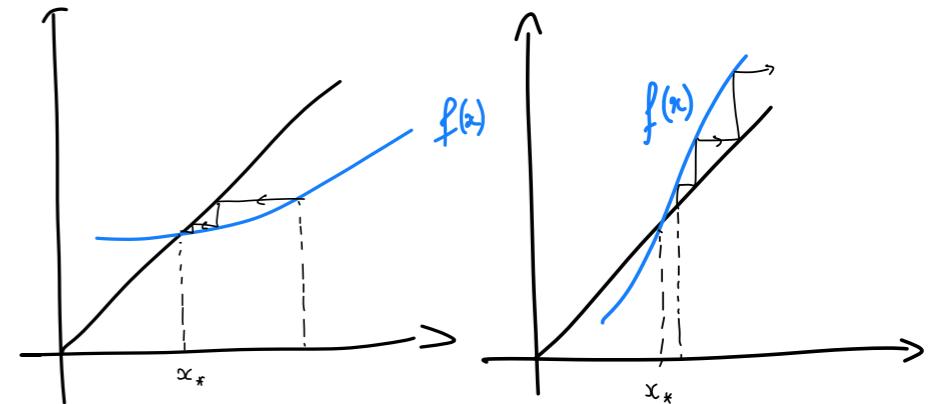
10^{10} violation of unitarity,
which is proportional to λ^4



- indicates that inel. input probably cannot be zero
- how small can it be ? -> stay tuned

Numerics : prospects

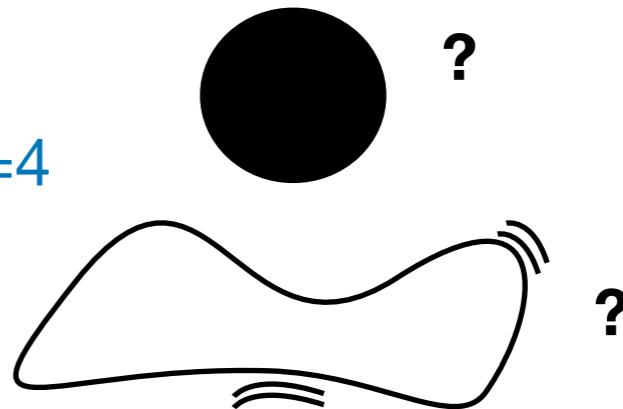
- Gradient descent / machine learning (Newton-Raphson)
- Make the double integral of Mandelstam's equation a matrix/tensorial multiplication.



$$\Psi[\rho] = \rho - \Phi[\rho]$$
$$\rho_{n+1} = \rho_n - \frac{1}{\Phi'(\rho_n)} \Phi(\rho_n)$$

Perspectives

- Reduce to tensor operation, to run on a GPU with gradient descent and release open-source code (python / julia)
- Other dimensions
- Produce amplitude that saturate Froissart bound
- Pion S-matrix
- Unitarise quantum gravity S-matrix in $d>4$, $d=4$
- Celestial formulation of
 - NP unitarity
 - scattering from production ?



Advertisement

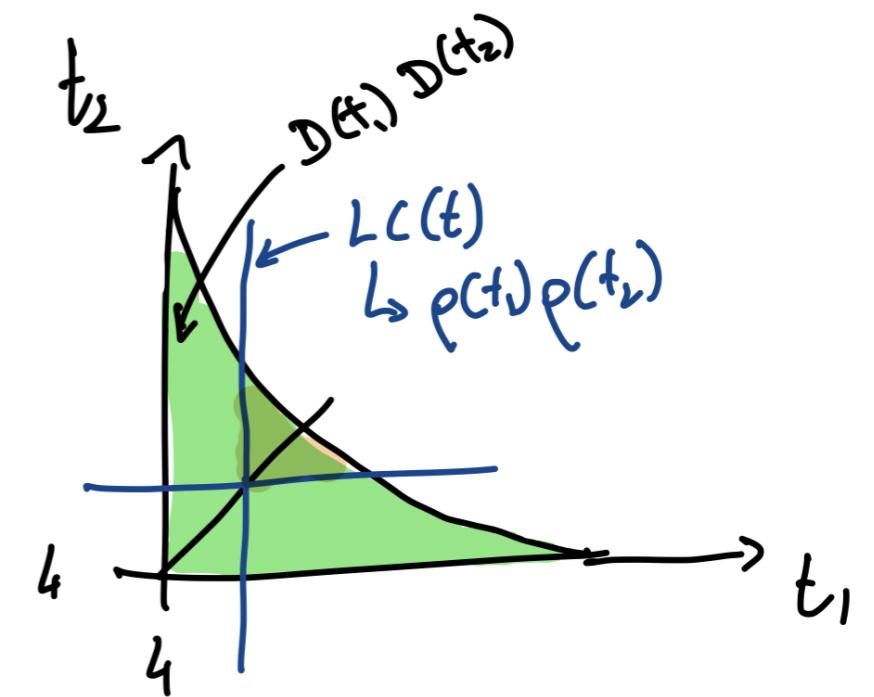
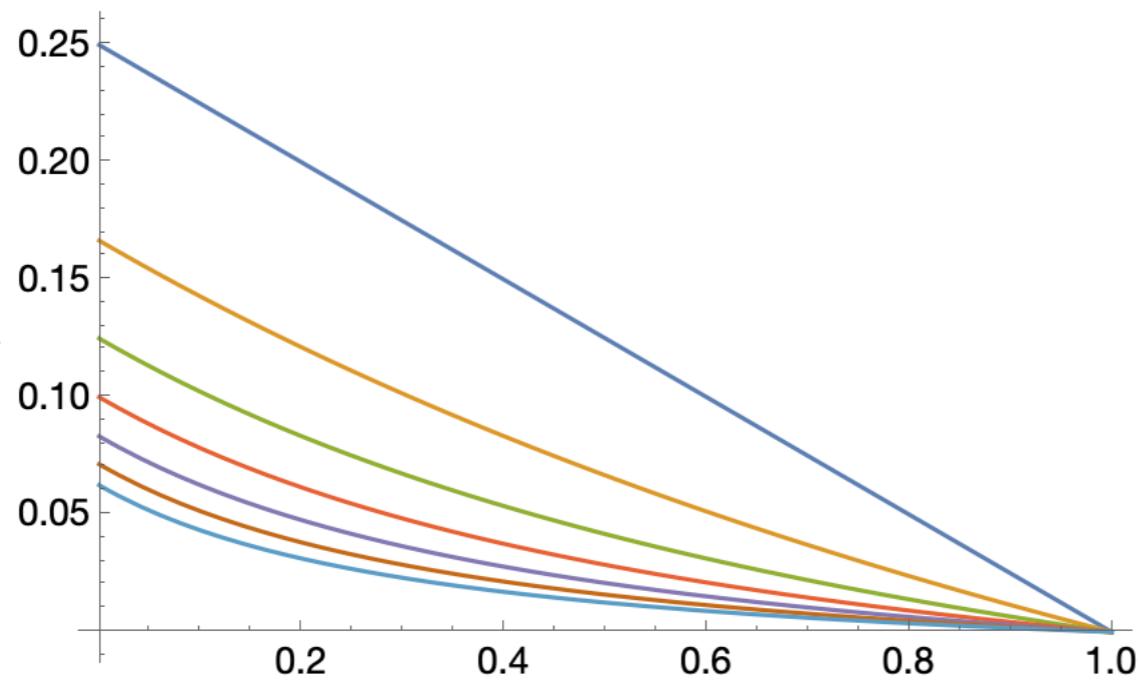
- Postdoc offer in LAPTh, starting fall 2023
- Group: PT, Felipe Figueroa, Christopher Eckner, Yihong Wang
- Also: D. Andriot, D. Chicherin, and BSM / cosmo group
- Well connected to Geneva



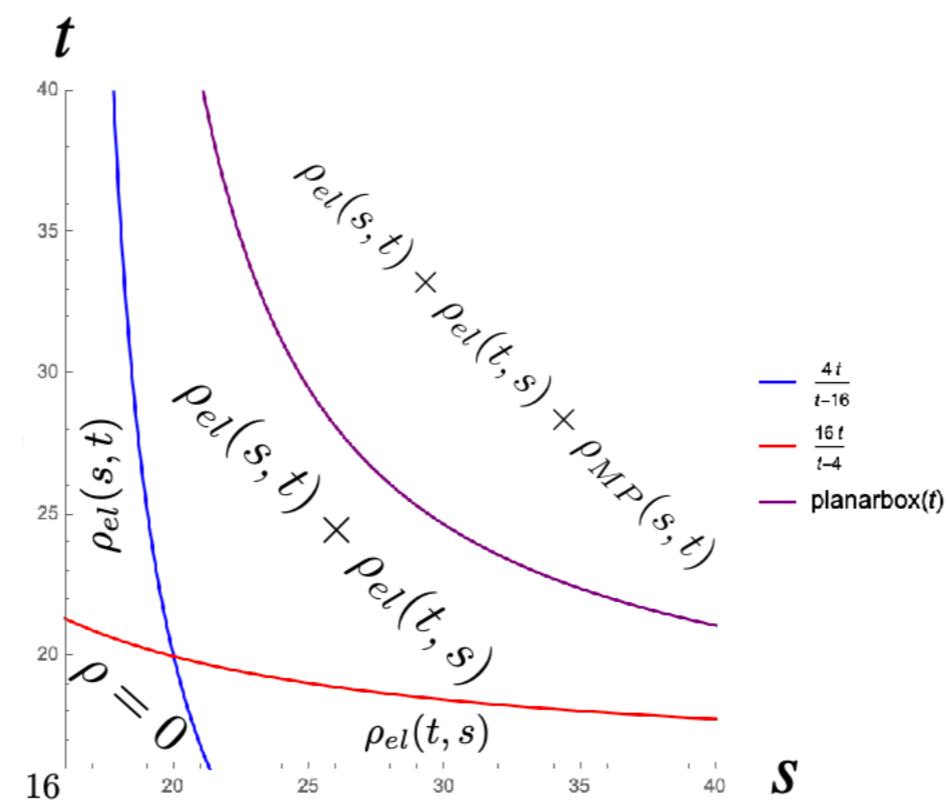
ευχαριστώ!

extras

Thin slit



Inelastic input



Unitarity violation at 10^{-10} : independence on grid

