On the old and new muon g-2 puzzles

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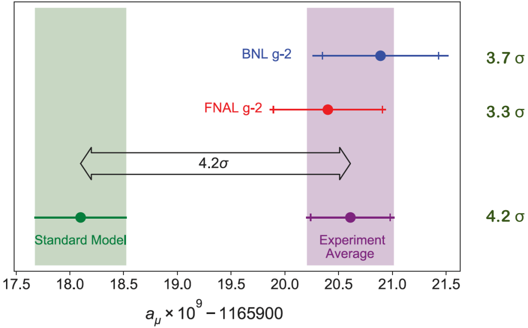
Workshop on the SM and Beyond
Corfu Summer Institute 2022 - 5/09/2022

Plan of the talk

1. **“Old muon g-2 puzzle” (pre BMW 2021)**
   - Possible new physics interpretations
   - Leptonic g-2, EDMs & LFV interrelationship
2. **“New muon g-2 puzzle” (post BMW 2021)**
   - Possible new physics interpretations
3. **Conclusions and future prospects**

Experimental status

- **April 7$^{\text{th}}$ 2021: Muon $g-2$ experiment at FNAL confirms BNL!**



$a_\mu^{EXP} = (116592089 \pm 63) \times 10^{-11}$ [0.54ppm] BNL E821

$a_\mu^{EXP} = (116592040 \pm 54) \times 10^{-11}$ [0.46ppm] FNAL E989 Run 1

$a_\mu^{EXP} = (116592061 \pm 41) \times 10^{-11}$ [0.35ppm] WA

- **FNAL aims at 16×10^{-11}. First 4 runs completed, 5th in progress.**
- **Muon $g-2$ proposal at J-PARC: Phase-1 with similar BNL precision.**

New Physics for the muon $g-2$: at which scale?

- **Δa_μ discrepancy at $\sim 4.2\,\sigma$ level:**

$$\Delta a_\mu = a_\mu^{\rm EXP} - a_\mu^{\rm SM} \equiv a_\mu^{\rm NP} = (2.51 \pm 0.59) \times 10^{-9}$$

$$\Delta a_\mu \equiv a_\mu^{\rm NP} \approx (a_\mu^{\rm SM})_{weak} \approx \frac{m_\mu^2}{16\pi^2 v^2} \approx 2 \times 10^{-9}$$

  - NP is at the weak scale ($\Lambda \approx v$) and weakly coupled to SM particles.*
  - NP is very light ($\Lambda \lesssim 1$ GeV) and feebly coupled to SM particles.
  - NP is very heavy ($\Lambda \gg v$) and strongly coupled to SM particles.

*Favoured by the *hierarchy problem* and by a WIMP DM candidate but disfavoured by the LEP and LHC bounds (supersymmetry being the most prominent example).

[For a through compilation of models, see Athron, Balazs, Jacob, Kotlarski, Stockinger, Stockinger-Kim, '21.]

$\Lambda \approx v$: SUSY and the muon $(g-2)$

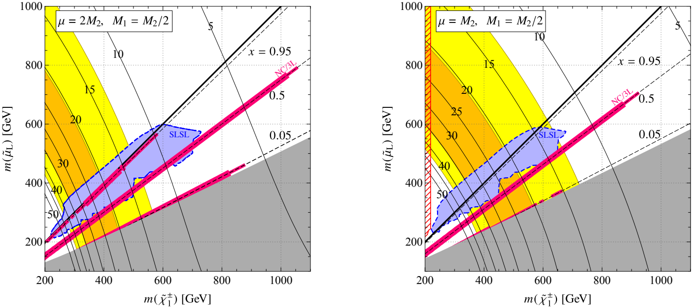
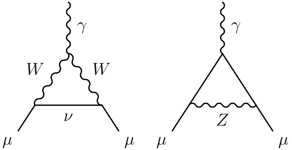
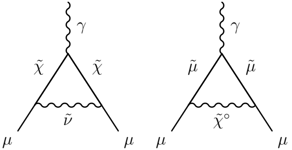


Figure: LHC Run 2 bounds on SUSY scenario for the muon $g-2$ anomaly for $\tan\beta = 40$. Orange (yellow) regions satisfy the muon $g-2$ anomaly at the 1σ (2σ) level [Endo et al., '20].





$$(a_\mu^{\rm SM})_{weak} \approx \frac{g^2 m_\mu^2}{32\pi^2 M_W^2} \approx 2\times 10^{-9}$$

$$a_\mu^{\rm SUSY} \approx \frac{g^2 m_\mu^2 \tan\beta}{32\pi^2 \tilde{m}^2} \underbrace{\approx 2\times 10^{-9}}_{\tilde{m}=500\,{\rm GeV}\ \&\ \tan\beta=40}$$

Axion-like Particle effective Lagrangian

$$\mathcal{L} = e^2 C_{\gamma\gamma} \frac{a}{\Lambda} F_{\mu\nu}\tilde{F}^{\mu\nu} + \frac{c_{\mu\mu}}{2}\frac{\partial^\nu a}{\Lambda}\bar{\mu}\gamma_\nu\gamma_5\mu$$

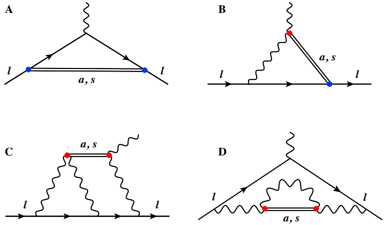


Figure: Contributions of a scalar 's' and a pseudoscalar 'a' ALP to the $(g-2)_\ell$.

[Marciano, Masiero, PP, Passera '16]

[Cornella, P.P., Sumensari '19]

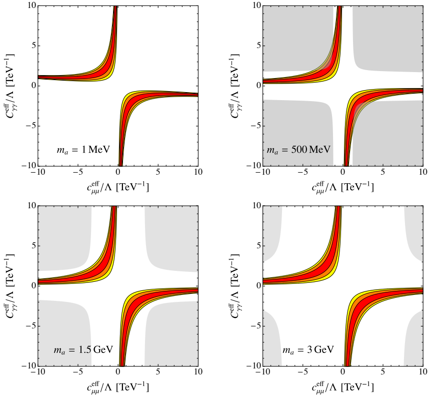
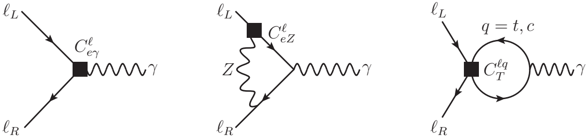


Figure: Δa_μ regions favoured at 68% (red), 95% (orange) and 99% (yellow) CL. Gray regions are excluded by the BaBar search $e^+e^- \to \mu^+\mu^- + \mu^+\mu^-$ [Bauer, Neubert, Thamm, '17]

$$\Delta a_\mu = \frac{m_\mu^2}{\Lambda^2}\left[\frac{12\alpha^3}{\pi} C_{\gamma\gamma}^2 \ln^2\frac{\Lambda^2}{m_\mu^2} - \frac{(c_{\mu\mu})^2}{16\pi^2} h_1\left(\frac{m_a^2}{m_\mu^2}\right) - \frac{2\alpha}{\pi} c_{\mu\mu} C_{\gamma\gamma} \ln\frac{\Lambda^2}{m_\mu^2}\right]$$

- **SMEFT Lagrangian relevant for Δa_ℓ**

$$\mathcal{L} = \sum_{V=B,W} \frac{C^\ell_{eV}}{\Lambda^2} \left(\bar{\ell}_L \sigma^{\mu\nu} e_R\right) H V_{\mu\nu} + \sum_{q=c,t} \frac{C_T^{\ell q}}{\Lambda^2} (\bar{\ell}_L \sigma_{\mu\nu} e_R)(\overline{Q}_L \sigma^{\mu\nu} q_R) + h.c.$$



$$\Delta a_\ell \simeq \frac{4 m_\ell^2}{e\Lambda^2} \frac{v}{m_\ell} \left(C^\ell_{e\gamma} - \frac{3\alpha}{2\pi} \frac{c_W^2 - s_W^2}{s_W c_W} C^\ell_{eZ} \log \frac{\Lambda}{m_Z} \right) - \sum_{q=c,t} \frac{4 m_\ell^2}{\pi^2} \frac{m_q}{m_\ell} \frac{C_T^{\ell q}}{\Lambda^2} \log \frac{\Lambda}{m_q}$$

$$\frac{|\Delta a_\mu|}{3\times 10^{-9}} \approx \left(\frac{250\,\mathrm{TeV}}{\Lambda}\right)^2 |C^\mu_{e\gamma}| \qquad \frac{|\Delta a_\mu|}{3\times 10^{-9}} \approx \left(\frac{50\,\mathrm{TeV}}{\Lambda}\right)^2 |C^\mu_{eZ}|$$

$$\frac{|\Delta a_\mu|}{3\times 10^{-9}} \approx \left(\frac{100\,\mathrm{TeV}}{\Lambda}\right)^2 |C_T^{\mu t}| \qquad \frac{|\Delta a_\mu|}{3\times 10^{-9}} \approx \left(\frac{10\,\mathrm{TeV}}{\Lambda}\right)^2 |C_T^{\mu c}|$$

- **Strongly coupled NP:** $C^\mu_{e\gamma}, C_T^{\mu t} \sim g^2_{\rm NP}/16\pi^2 \lesssim 1$ implying $\Lambda \lesssim$ *few* x 100 TeV, beyond the direct production reach of any foreseen collider.

[Buttazzo and P.P., '20]

- **SMEFT Lagrangian relevant for Δa_ℓ**

$$\mathcal{L} = \sum_{V=B,W} \frac{C_{eV}^{\ell}}{\Lambda^2} (\bar{\ell}_L \sigma^{\mu\nu} e_R) H V_{\mu\nu} + \sum_{q=c,t} \frac{C_T^{\ell q}}{\Lambda^2} (\bar{\ell}_L \sigma_{\mu\nu} e_R)(\overline{Q}_L \sigma^{\mu\nu} q_R) + h.c.$$

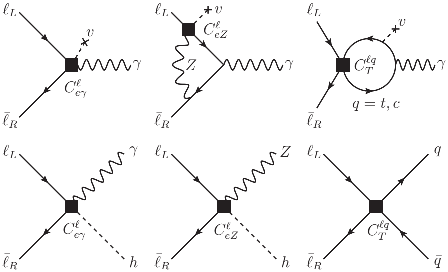


Figure: Connection between the Feynman diagrams for leptonic g-2 (upper row) and high-energy scattering processes (lower row) within the SMEFT: $\boldsymbol{H = v + h/\sqrt{2}}$

$$\boldsymbol{\Delta a_\mu \sim \frac{m_\mu v}{\Lambda^2} C_{eV,T}} \quad \Longleftrightarrow \quad \boldsymbol{\sigma_{\mu\mu\to f} \sim \frac{s}{\Lambda^4} |C_{eV,T}|^2} \qquad \boldsymbol{(f = e\gamma, eZ, q\bar{q})}$$

- **At high energy $\sigma_{\mu\mu\to f}$ can compete with Δa_μ to test the very same NP!**

The muon g-2 at a Muon Collider [Buttazzo and P.P., '20]

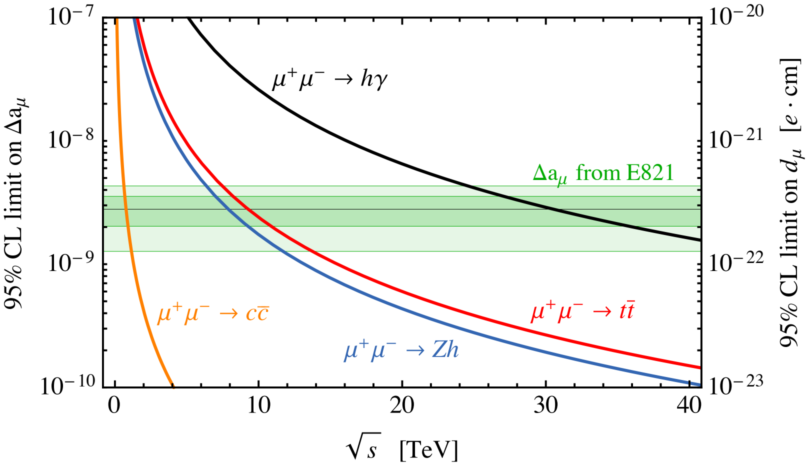


Figure: 95% C.L. reach on Δa_μ, as well as on the muon EDM d_μ, as a function of $\sqrt{s}$ from various processes for the reference integrated luminosity $\mathcal{L} = \left(\sqrt{s}/10\,\mathrm{TeV}\right)^2 \times 10\,\mathrm{ab}^{-1}$.

- **NP effects are encoded in the effective Lagrangian**

$$\mathcal{L} = e\frac{m_\ell}{2}\left(\bar{\ell}_R \sigma_{\mu\nu} A_{\ell\ell'} \ell'_L + \bar{\ell}'_L \sigma_{\mu\nu} A^\star_{\ell\ell'} \ell_R\right) F^{\mu\nu} \qquad \ell, \ell' = e, \mu, \tau\,,$$

  - **Branching ratios of $\ell \to \ell'\gamma$**

$$\frac{\mathrm{BR}(\ell \to \ell'\gamma)}{\mathrm{BR}(\ell \to \ell'\nu_\ell\bar{\nu}_{\ell'})} = \frac{48\pi^3\alpha}{G_F^2}\left(|A_{\ell\ell'}|^2 + |A_{\ell'\ell}|^2\right).$$

  - **Δa_ℓ and leptonic EDMs**

$$\Delta a_\ell = 2m_\ell^2\,\mathrm{Re}(A_{\ell\ell}), \qquad \frac{d_\ell}{e} = m_\ell\,\mathrm{Im}(A_{\ell\ell})\,.$$

  - **“Naive scaling”:** a broad class of NP theories contributes to Δa_ℓ and d_ℓ as

$$\frac{\Delta a_\ell}{\Delta a_{\ell'}} = \frac{m_\ell^2}{m_{\ell'}^2}, \qquad \frac{d_\ell}{d_{\ell'}} = \frac{m_\ell}{m_{\ell'}}\,.$$

Model-independent predictions

- $\mathrm{BR}(\ell_i \to \ell_j\gamma)$ **vs.** $(g-2)_\mu$

$$\mathrm{BR}(\mu \to e\gamma) \approx 3\times 10^{-13}\left(\frac{\Delta a_\mu}{3\times 10^{-9}}\right)^2\left(\frac{\theta_{e\mu}}{10^{-5}}\right)^2$$

$$\mathrm{BR}(\tau \to \mu\gamma) \approx 4\times 10^{-8}\left(\frac{\Delta a_\mu}{3\times 10^{-9}}\right)^2\left(\frac{\theta_{\mu\tau}}{10^{-2}}\right)^2$$

- **EDMs vs.** $(g-2)_\mu$

$$d_e \simeq \left(\frac{\Delta a_\mu}{3\times 10^{-9}}\right) 10^{-29}\left(\frac{\phi_e^{CPV}}{10^{-5}}\right)\ e\,\mathrm{cm}\,,$$

$$d_\mu \simeq \left(\frac{\Delta a_\mu}{3\times 10^{-9}}\right) 2\times 10^{-22}\ \phi_\mu^{CPV}\ e\,\mathrm{cm}\,,$$

- **Main messages:**
  - $\Delta a_\mu \approx (3\pm 1)\times 10^{-9}$ **requires a nearly flavor and CP conserving NP**
  - **Large effects in the muon EDM** $d_\mu \sim 10^{-22}\ e\,\mathrm{cm}$ **are still allowed!**

[Giudice, P.P., & Passera, '12, Isidori et al., '22]

Breakdown of SM contributions

- a_μ from WP20 (w/o BMWc lattice result)

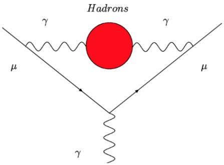
[Colangelo EPS-HEP2021 proceeding]

Contribution	Value $\times 10^{11}$	References
Experiment (E821)	116 592 089(63)	Ref. [3]
Experiment (FNAL)	116 592 040(54)	Ref. [1]
Experiment (World-Average)	116 592 061(41)	
HVP LO (e^+e^-)	6931(40)	Refs. [6–11]
HVP NLO (e^+e^-)	−98.3(7)	Ref. [11]
HVP NNLO (e^+e^-)	12.4(1)	Ref. [12]
HVP LO (lattice, *udsc*)	7116(184)	Refs. [13–21]
HLbL (phenomenology)	92(19)	Refs. [22–34]
HLbL NLO (phenomenology)	2(1)	Ref. [35]
HLbL (lattice, *uds*)	79(35)	Ref. [36]
HLbL (phenomenology + lattice)	90(17)	
QED	116 584 718.931(104)	Refs. [37, 38]
Electroweak	153.6(1.0)	Refs. [39, 40]
HVP (e^+e^-, LO + NLO + NNLO)	6845(40)	
HLbL (phenomenology + lattice + NLO)	92(18)	
Total SM Value	116 591 810(43)	
Difference: $\Delta a_\mu := a_\mu^{\text{exp}} - a_\mu^{\text{SM}}$	251(59)	

→ HVP LO is the bottle-neck of the SM prediction

HLO contribution from $e^+e^- \to hadrons$

- dominated by $e^+e^- \to \pi^+\pi^-$ channel (70% of the full hadronic)



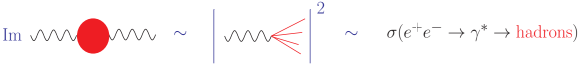
$$(a_\mu^{\rm HVP})_{e^+e^-} = \frac{\alpha}{\pi^2}\int_{m_{\pi^0}^2}^{\infty} \frac{ds}{s} K(s)\, {\rm Im}\,\Pi_{\rm had}(s) = \frac{1}{4\pi^3}\int_{m_{\pi^0}^2}^{\infty} ds\, K(s)\sigma_{\rm had}(s)$$

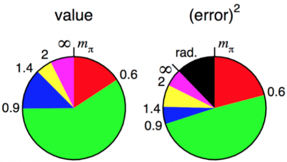
dispersion relations

optical theorem

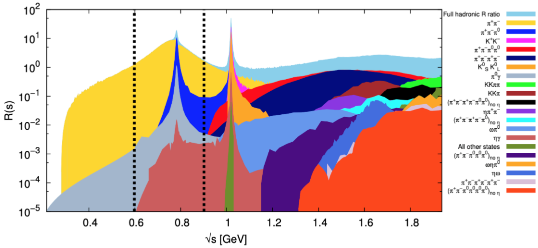
kernel function

$K(s) \approx m_\mu^2/3s$ for $\sqrt{s} \gg m_\mu$





Keshavarzi, Nomura, Teubner 2018



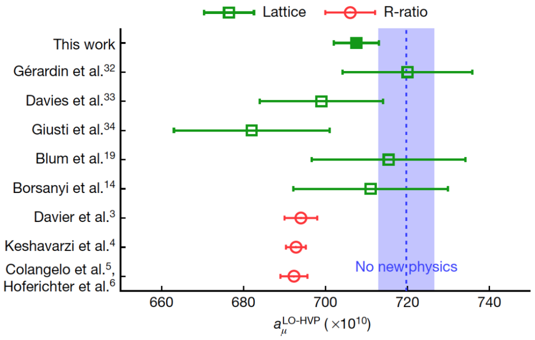
$$a_{\mu,e^+e^-}^{\rm HLO} = 6931(40) \times 10^{-11} (0.6\%)$$ [WP20]

HLO contribution from lattice QCD

- **Great progress also in lattice QCD, where spacetime is modeled as a discrete grid of points. The BMW collaboration reached a 0.8% precision!**

$$a_\mu^{HLO} = 7075(23)_{stat}(50)_{syst}\ [55]_{tot} \times 10^{-11}$$

- **2–2.5σ tension with the "data-driven" evaluations.**

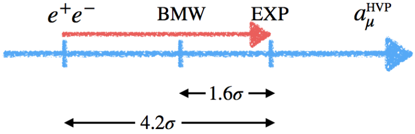


Borsanyi et al (BMWc), Nature 2021

$(a_\mu^{\rm HVP})_{\rm EXP} = a_\mu^{\rm EXP} - a_\mu^{\rm SM,\,rest}$

$(a_\mu^{\rm HVP})_{e^+e^-}^{\rm WP20} = 6931(40) \times 10^{-11}$

$(a_\mu^{\rm HVP})_{\rm BMW} = 7075(55) \times 10^{-11}$



“new puzzle”: if BMW is correct, the “old” g-2 discrepancy (4.2σ) would be basically gone

→ *however, this brings in a new tension with e^+e^- data (2.2σ)*

Here, NP in $\sigma_{\rm had}(e^+e^- \to \text{hadrons})$ such that [LDL, Masiero, Paradisi, Passera 2112.08312]

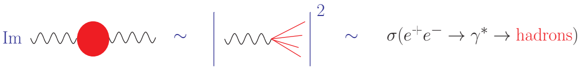
1. $(a_\mu^{\rm HVP})_{e^+e^-}^{\rm WP20} \approx (a_\mu^{\rm HVP})_{\rm EXP}$
2. the approximate agreement between BMW and EXP is not spoiled
3. w/o a direct contribution $a_\mu^{\rm NP}$ (i.e. NP not in muons)

Consequences of the BMW result

- **Can Δa_μ be due to missing contributions in $\sigma(e^+e^- \to had)$?**
  - An upward shift of $\sigma(s)$ also induces an increase of $\Delta\alpha^{(5)}_{\rm had}(M_Z)$ defined by:

$$\alpha(M_Z) = \frac{\alpha}{1 - \Delta\alpha(M_Z) - \Delta\alpha^{(5)}_{\rm had}(M_Z) - \Delta\alpha_{\rm top}(M_Z)}$$

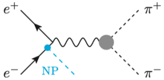
$$a_\mu^{\rm HLO} \simeq \frac{m_\mu^2}{12\pi^3}\int_{4m_\pi^2}^{\infty} ds\, \frac{\sigma(s)}{s}\,, \qquad \Delta\alpha^{(5)}_{\rm had} = \frac{M_Z^2}{4\pi\alpha^2}\int_{4m_\pi^2}^{\infty} ds\, \frac{\sigma(s)}{M_Z^2 - s}$$



- **A change in $\sigma(e^+e^- \to had)$ is strongly disfavoured by:**
  - **EW-fit for $\sqrt{s} \gtrsim$ 1 GeV** [Marciano, Passera, Sirlin, '08, Keshavarzi, Marciano, Passera, Sirlin, '20, Crivellin, Hoferichter, Manzari, Montull, '20]. A shift of $\sigma(e^+e^- \to had)$ to accomodate the Δa_μ anomaly would necessarely require new physics to show up in the EW-fit!
- **A check of the BMW results by other lattice QCD (LQCD) coll. is worth.**
- **LQCD coll. should provide $\Delta\alpha^{\rm LQCD}_{\rm had}$ to be compared with $\Delta\alpha^{e^+e^-}_{\rm had}$.**

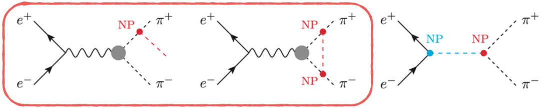
Light New Physics in $\sigma_{\rm had}$

- Light new physics inducing a sub-GeV modification of $\sigma_{\rm had}$ is the only possibility



1. NP coupled only to electrons → severe bounds

[See however
Darmé, Grilli di Cortona, Nardi 2112.09139
NP in Bhabha scattering ? → backup slides]

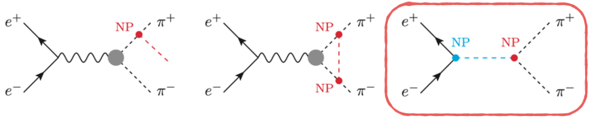


2. NP coupled only to hadrons

FSR effects due to NP should be included into $\sigma_{\rm had}(s)$, not easy to be accounted for…
(depend on exp. cuts and mass of NP)

→ *however, we know that in the QED case*

$$(a_\mu^{\rm HVP})^{\rm FSR}_{e^+e^-} \approx 50 \times 10^{-11} \quad \longleftrightarrow \quad |(a_\mu^{\rm HVP})_{\rm BMW} - (a_\mu^{\rm HVP})^{\rm WP20}_{e^+e^-}| \approx 150 \times 10^{-11}$$

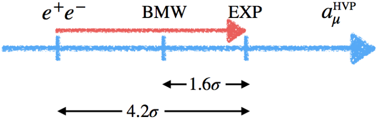


3. NP coupled both to hadrons and electrons

$$(a_\mu^{\rm HVP})_{e^+e^-} = \frac{\alpha}{\pi^2}\int_{m_{\pi^0}^2}^{\infty} \frac{{\rm d}s}{s} K(s)\, {\rm Im}\,\Pi_{\rm had}(s) = \frac{1}{4\pi^3}\int_{m_{\pi^0}^2}^{\infty} {\rm d}s\, K(s)\sigma_{\rm had}(s)$$

$$\sigma_{\rm had} = \sigma_{\rm had}^{\rm SM} + \Delta\sigma_{\rm had}^{\rm NP}$$

$$\sigma_{\rm had} - \Delta\sigma_{\rm had}^{\rm NP}$$



should be "subtracted" by NP,
since NP does not contribute to HVP at the LO
(but it contributes at the LO to the x-section)

→ *a positive sift on* $(a_\mu^{\rm HVP})_{e^+e^-}$ *requires* $\Delta\sigma_{\rm had}^{\rm NP} < 0$ *(negative interference)*

A new light Z' vector boson

- Requirements:

1. $\sigma_{\rm had}$ modified at $\sqrt{s} \lesssim 1\,\mathrm{GeV}$ (mostly $\pi^+\pi^-$ channel) → sub-GeV mediator; → couples to u, d quarks (and electrons)
2. A sizeable negative interference with the SM

sizeable → tree-level mediator

negative interference → couples via a vector current

→ *a light spin-1 mediator with vector couplings to first generation SM fermions*

$$\mathcal{L}_{Z'} \supset (g_V^e\, \bar{e}\gamma^\mu e + g_V^q\, \bar{q}\gamma^\mu q) Z'_\mu \qquad q = u, d \qquad m_{Z'} \lesssim 1\ \mathrm{GeV}$$

- It can be shown that (neglecting iso-spin breaking corrections due to NP)

$$\frac{\sigma_{\pi\pi}^{\rm SM+NP}}{\sigma_{\pi\pi}^{\rm SM}} = \left| 1 + \frac{g_V^e (g_V^u - g_V^d)}{e^2} \frac{s}{s - m_{Z'}^2 + i m_{Z'} \Gamma_{Z'}} \right|^2$$

A new light Z' vector boson

1. Semi-leptonic processes

$e^+e^- \to q\bar{q}$ *has been measured with per-cent accuracy at LEP-II*

$$\frac{\sigma_{qq}^{\rm SM+NP}}{\sigma_{qq}^{\rm SM}} \approx 1 + 2\frac{g_V^e g_V^q}{e^2 Q_q} \longrightarrow |g_V^e g_V^q| \lesssim 4.6 \cdot 10^{-4} |Q_q| \qquad (\epsilon \lesssim 3.3 \cdot 10^{-3})$$

2. Leptonic processes

- for $m_{Z'} \lesssim 0.3$ GeV ($Z' \to e^+e^-$ is the main decay mode)

$e^+e^- \to \gamma Z'$ *@ BaBar* $\longrightarrow$ $g_V^e \lesssim 2 \cdot 10^{-4}$

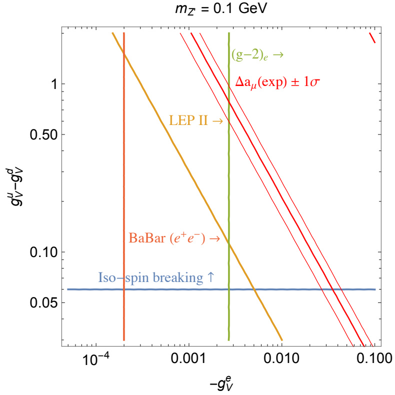
3. Iso-spin breaking observables

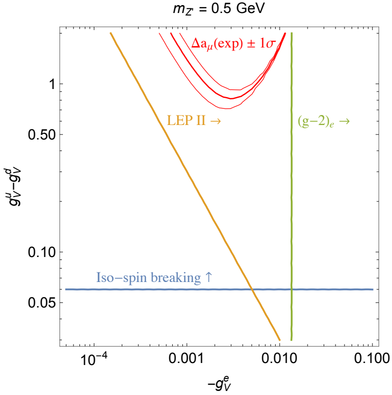
charged vs. neutral pion mass^2 difference $\Delta m^2 = m_{\pi^+}^2 - m_{\pi^0}^2$

$$(\Delta m^2)_{Z'} \sim \frac{(g_V^u - g_V^d)^2}{(4\pi)^2} \Lambda_\chi^2 \qquad (\Lambda_\chi \approx 1\ {\rm GeV})$$

$\longrightarrow$ $|g_V^u - g_V^d| \lesssim 0.06$ [Rescaling lattice QCD calculation of Frezzotti et al 2112.01066]

A new light Z' vector boson





At least two independent bounds prevent to solve the "new muon g-2 puzzle"!

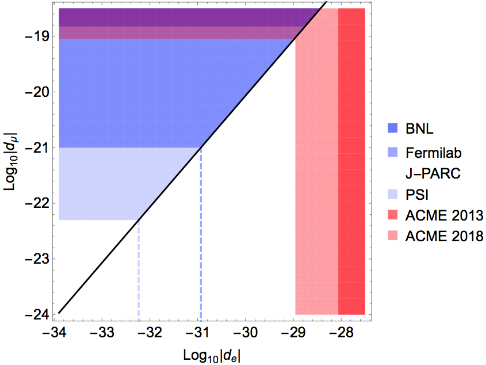
Outlook

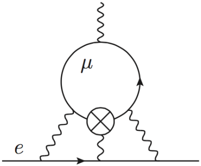
- The muon $g-2$ represents the most longstanding hint of New Physics now, thanks to the E989 experiment at FNAL, growing to 4.2σ.
- LQCD results by the BMWc weaken the muon $g-2$ discrepancy to 1.6σ but they are in tension with the EW-fit and $e^+e^- \to hadrons$ experimental data:
  - The MUonE experiment can provide an independent measure of $\Delta\alpha_{\rm had}$ which is not contaminated by new physics effects [Masiero, P.P. & Passera, '20].
- Both heavy New Physics ($v \lesssim \Lambda \lesssim 100\,\mathrm{TeV}$) and ligh New Physics ($\Lambda \lesssim 1\mathrm{GeV}$) scenarios have the potential to account for the muon $g-2$ anomaly.
  - Different scenarios can be disentangled by dedicated searches at running or future experiments such as Belle II and a high-energy Muon Collider .
- If the muon $g-2$ anomaly will survive, we expect relevant enhancements in leptonic EDMs (especially in the muon EDM) and LFV physics.
- Testing New Physics effects in the electron $g-2$ at the 10^{-13} is not too far! This will bring a_e to play a pivotal role in probing New Physics in the leptonic sector.

Message: an exciting Physics program is in progress at the Intensity Frontier!

Backup slides

Experimental status of the muon EDM





$$d_\mu \lesssim 10^{-21}\, e\,\mathrm{cm} \left(\frac{d_e}{10^{-31}\, e\,\mathrm{cm}}\right)$$

[Crivellin, Hoferichter & Schmidt-Wellenburg, ’18]

$$d_\mu \simeq \left(\frac{\Delta a_\mu}{3\times 10^{-9}}\right) 2\times 10^{-22}\, \phi_\mu^{CPV}\, e\,\mathrm{cm}\,,$$

[Giudice, PP & Passera, ’12]

Testing new physics with the electron $g-2$

- **Longstanding muon $g-2$ anomaly**

$$\Delta a_\mu = a_\mu^{\rm EXP} - a_\mu^{\rm SM} \equiv a_\mu^{\rm NP} = (2.51 \pm 0.59) \times 10^{-9}$$

$$\Delta a_\mu \equiv a_\mu^{\rm NP} \approx (a_\mu^{\rm SM})_{weak} \approx \frac{m_\mu^2}{16\pi^2 v^2} \approx 2 \times 10^{-9}$$

- **Testing the muon $g-2$ anomaly through the electron $g-2$**

$$\frac{\Delta a_e}{\Delta a_\mu} = \frac{m_e^2}{m_\mu^2} \qquad \Longleftrightarrow \qquad \Delta a_e = \left(\frac{\Delta a_\mu}{3 \times 10^{-9}}\right) 0.7 \times 10^{-13}$$

  - a_e has never played a role in testing NP effects. From $a_e^{\rm SM}(\alpha) = a_e^{\rm EXP}$, we extract α which was is the most precise value of α up to 2018!
  - The situation has now changed thanks to th. and exp. progresses.
  - α can be extracted from atomic physics and a_e used to perform NP tests!

[Giudice, P.P, & Passera, '12]

- **Status of Δa_e as of 2012**

$$\Delta a_e = a_e^{\rm EXP} - a_e^{\rm SM} = -9.2\,(8.1)\times 10^{-13},$$

$$\delta a_e \times 10^{13}:\quad (0.6)_{\rm QED4},\ (0.4)_{\rm QED5},\ (0.2)_{\rm HAD},\ (7.6)_{\delta\alpha},\ (2.8)_{\delta a_e^{\rm EXP}}.$$

  - The errors from QED4 and QED5 will be reduced soon to 0.1×10^{-13} [Kinoshita]
  - We expect a reduction of $\delta a_e^{\rm EXP}$ to a part in 10^{-13} (or better). [Gabrielse]
  - Work is also in progress for a significant reduction of $\delta\alpha$. [Nez]

- **Status of Δa_e as of 2018: 2.4σ discrepancy** [Parker et al., Science, '18]

$$\Delta a_e = a_e^{\rm EXP} - a_e^{\rm SM}(\alpha_{\rm Berkeley}) = -8.8\,(3.6)\times 10^{-13}$$

$$\delta a_e \times 10^{13}:\quad (0.1)_{\rm QED5},\ (0.1)_{\rm HAD},\ (2.3)_{\delta\alpha},\ (2.8)_{\delta a_e^{\rm EXP}}.$$

- **Status of Δa_e as of 2020: 1.6σ discrepancy** [Morel et al., Nature, '20]

$$\Delta a_e = a_e^{\rm EXP} - a_e^{\rm SM}(\alpha_{\rm LKB2020}) = 4.8\,(3.0)\times 10^{-13}$$

$$\delta a_e \times 10^{13}:\quad (0.1)_{\rm QED5},\ (0.1)_{\rm HAD},\ (0.9)_{\delta\alpha},\ (2.8)_{\delta a_e^{\rm EXP}}.$$

- **$\Delta a_e \lesssim 10^{-13}$ is not too far! This will bring a_e to play a pivotal role in probing new physics in the leptonic sector.** [Giudice, P.P, & Passera, '12]

Pion matrix element

- Vector form factor (VFF) defined via

$$\langle \pi^\pm(p')|J^\mu_{\rm em}(0)|\pi^\pm(p)\rangle = \pm(p'+p)^\mu F^V_\pi(q^2)$$

$$J^\mu_{\rm em} = \tfrac{2}{3}\bar{u}\gamma^\mu u - \tfrac{1}{3}\bar{d}\gamma^\mu d$$

$$q = p' - p$$

Using iso-spin and C invariance

$$\langle \pi^\pm|J^\mu_{\rm em}|\pi^\pm\rangle = \langle \pi^\pm|\bar{u}\gamma^\mu u|\pi^\pm\rangle = -\langle \pi^\pm|\bar{d}\gamma^\mu d|\pi^\pm\rangle$$

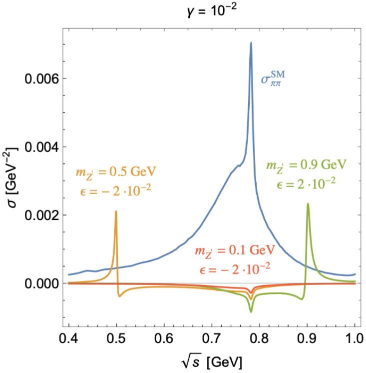
we can cast the matrix element of the Z' quark current in terms of the VFF

$$\langle \pi^\pm(p')|J^\mu_{Z'}(0)|\pi^\pm(p)\rangle = \pm(p'+p)^\mu F^V_\pi(q^2)(g^u_V - g^d_V)$$

$$J^\mu_{Z'} = g^u_V\bar{u}\gamma^\mu u + g^d_V\bar{d}\gamma^\mu d$$

$$\frac{\sigma^{\rm SM+NP}_{\pi\pi}}{\sigma^{\rm SM}_{\pi\pi}} = \left|1 + \frac{g^e_V(g^u_V - g^d_V)}{e^2}\frac{s}{s - m^2_{Z'} + i m_{Z'}\Gamma_{Z'}}\right|^2$$

- Typical benchmarks solving the g-2 discrepancy



$$\Delta a_\mu = \frac{1}{4\pi^3}\int_{s_{\rm exp}}^{\infty} {\rm d}s\, K(s)(-\Delta\sigma_{\rm had}^{\rm NP}(s))$$

$\sqrt{s_{\rm exp}} \approx 0.3$ GeV
for $\pi^+\pi^-$ channel

$$\Delta\sigma_{\rm had}^{\rm NP}(s) \approx \sigma_{\pi\pi}^{\rm SM}(s) \times \frac{2\epsilon s(s-m_{Z'}^2)+\epsilon^2 s^2}{(s-m_{Z'}^2)^2+m_{Z'}^4\gamma^2}$$

$$\epsilon \equiv g_V^e(g_V^u - g_V^d)/e^2$$

$$\gamma \equiv \Gamma_{Z'}/m_{Z'}$$

- What if the measurement of the KLOE luminosity is affected by NP ?

[Darmé, Grilli di Cortona, Nardi 2112.09139]

$$\mathcal{L}^{\rm SM}_{e^+e^-} = \frac{N_{\rm Bha}}{\sigma^{\rm SM}_{\rm eff}} \quad \longrightarrow \quad \mathcal{L}_{e^+e^-} = \mathcal{L}^{\rm SM}_{e^+e^-}\frac{\sigma^{\rm SM}_{\rm eff}}{\sigma_{\rm eff}}$$

$$\sigma_{\rm eff} = \sigma^{\rm SM}_{\rm eff}(1+\delta_R)$$

$$\sigma_{\rm had} \propto N_{\rm had}/\mathcal{L}_{e^+e^-} \quad \longrightarrow \quad \sigma_{\rm had} \to \sigma_{\rm had}(1+\delta_R)$$

$$a_\mu^{\rm LO,HVP} \to a_\mu^{\rm LO,HVP}(1+\delta_R)$$

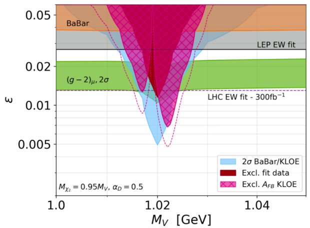


Figure 3. Parameter range compatible at 2σ with the experimental measurement of Δa_μ (green region) resulting from a redetermination of the KLOE luminosity, for $\alpha_D = 0.5, m_{\chi_2} = 0.95 m_V$ and $m_{\chi_1} = 25$ MeV. In the blue region the KLOE and BaBar results for $\sigma_{\rm had}$ are brought into agreement at 2σ. The red region corresponds to a shift of the KLOE measurement in tension with BaBar (and with the other experiments) at more than 2σ.